PARALLEL SESSION ON THEORY OF STRONG INTERACTIONS AT HIGH ENERGIES

THE FUNDAMENTAL LENGTH AS A KEY TO PHYSICS AT VERY HIGH ENERGIES

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Many years ago strong interactions were described by the Yukawa potential. As higher velocities become essential relativistic description of the interactions was necessary. As a result instead of the Yukawa potential one had to use the Feynman propagator:

$$\frac{I}{m^2 - p^2}\ . \qquad (I)$$

It is important to remark that in (I) a new fundamental constant - the velocity of light c appears.*

In this point a question arises. What will happen if we increase further the energy? Will the propagator modify once more at very high energies and will a new fundamental constant appear in the theory? If it is so this should be crucial in the analysis and understanding of the physics at these energies.

Let us try to answer the question. The propagator (I) can be written as the Fourier transform of the vacuum expectation value of the time-ordered product of two free field operators:

$$F\{\langle 0|T\ \phi(x_1)\ \phi(x_2)|0\rangle\}. \qquad (2)$$

Under the usual locality condition this expression is Lorentz invariant because the sign of the relative time is invariant for timelike intervals:

$$\varepsilon = \frac{t}{|t|} = \text{invar.},\ t=t_I-t_2,\ \text{when}\ x^2=(x_I-x_2)^2>0. \quad (3)$$

Let us notice, however, that in the time-ordered product quantities related to higher than Lorentz symmetries are used. Namely the interval x^2 is Casimir operator of the Poincaré group of the fourdimensional momentum space

$$p' = \Lambda p + k \qquad (4)$$

and $\varepsilon=\frac{t}{|t|}$ is additional invariant of its unitary representations when $x^2 > 0$.

Therefore one can say that the usual properties of $x^2, \frac{t}{|t|}$ and finally of the propagator(2) itself, are closely connected with the Euclidean character of momentum space or more briefly with the Euclidean character of the parallel shifts in this space:

$$\frac{I}{m^2 - p^2 - i\varepsilon} \longleftrightarrow x^2, \frac{t}{|t|} \longleftrightarrow p'=p+k. \quad (5)$$

But as the mathematicians teach, there exist noneuclidean geometries with different definitions of the parallel shifts:

$$p' = p \oplus k \qquad (6)$$

(Recall the story of the fifth postulate of Euclid).

Because the transformation (6) is nonlinear it is obvious from dimensional reasoning that a new universal constant ℓ with dimension of length appears. We shall call it fundamental length.

If $p,k \ll \frac{1}{\ell}$, then $p \oplus k \longrightarrow p + k$.

A new question arises: Could the noneuclidean shifts $p \oplus k$ produce a generalization of x^2, invariant $\frac{t}{|t|}$, the corresponding T-product and at last a new propagator substituting the old one at very high energies $|p| \gtrsim \frac{1}{\ell}$?

The answer is positive and is illustrated in the following Table:

Euclidean	Shifts	Noneuclidean
$p'=p+k$		$p'=p \oplus k$
	Lorentz group $p'=\Lambda p$ (common to both)	
$x^2=\begin{cases} >0 \\ =0\ \text{cone} \\ >0 \end{cases}$	Casimir Operator	$x^2=\begin{cases} L(L+3) \\ \text{no cone} \\ -\frac{9}{4} - \Lambda^2 \end{cases}$
time t is continuous	Time	time n is discrete
$\frac{t}{\lvert t\rvert}$ = invar.		$\frac{n}{\lvert n\rvert}$ = invar.
T_t- product	Time-ordered Product	T_n- product
Propagator: $\frac{I}{m^2 - p^2}$		Propagator: $\frac{\ell^2}{2(\sqrt{I-\ell^2p^2}-\sqrt{I-\ell^2m^2})}$

Later on we shall put also $l^2=I$.

Let us briefly enumerate the properties of the new propagator:

I) The mass of elementary particles is bounded from above i.e.

$$m^2 \leq I \qquad (7)$$

2) At $p^2, m^2 \ll I$

$$\frac{I}{2(\sqrt{1-p^2}-\sqrt{1-m^2})} \longrightarrow \frac{I}{m^2 - p^2}\ .$$

3) In the timelike region $p^2 \leq I$, which means that the one-photon annihilation cross section vanishes at $p^2 \gtrsim I$ (fig.I)

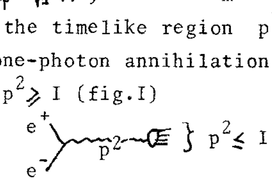
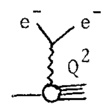


Fig.I

4) In the spacelike region if $|Q^2| \gg I$ the propagator is proportional to $\frac{I}{2|Q|}$ and the cross section decreases with Q^2, slower than usual.



The propagator is $\sim \frac{I}{2|Q|}$

Fig.2

5) The fermion propagator has the form:

$$\frac{I}{\not p - (I-\sqrt{I-p^2})\gamma^5 - \mu} \qquad (8)$$

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*/ We already put in (I) $\hbar$=c=I.

The γ^5 suggests that parity violation effects should be observed at very high energies and their magnitude should be of the order of $p^2\ell^2$.

Therefore, it is very tempting and natural to identify our fundamental length with Fermi constant

$$\ell \approx \ell_F = \sqrt{\frac{G}{\hbar c}} \simeq 6.10^{-17} \text{ cm.} \qquad (9)$$

The introduction of fundamental length ℓ in the theory changes many of its basic relations. For example, series of qualitative predictions could be made on the basis of the new relation between the momentum transfer and interaction distances:

$$r \sim \frac{I}{\chi_p}, \qquad (I0)$$

where

$$ch\chi_p = I + \frac{\vec{p}^2}{2}, \qquad (II)$$

$\vec{p}^2$ being the momentum transfer. If $|\vec{p}| << I$, $r \sim \frac{I}{|\vec{p}|}$. But if $|\vec{p}| >> I$, then $r \sim \frac{I}{\ln|\vec{p}|}$. From the last formula one sees that at very high momentum transfers the role of momentum transfer is played by its logarithm

$$\sqrt{t} \to \ln \sqrt{-t}. \qquad (I2)$$

In other words one may expect that all the exponent $\sqrt{-t}$ dependences of the standard theory should change in this domain to power dependences. For example:

$$\frac{d\tau}{dt} \sim ae^{-n\sqrt{-t}} \longrightarrow \frac{a}{(\sqrt{-t})^n}. \qquad (I3)$$

The exact calculation confirms this qualitative speculation/I/.

The relation (I0) is of principal importance, namely distances $r \lesssim \ell$ can not be practically reached (the $\ln(|\vec{p}|)$ in the dominator!). Therefore if $l \sim l_F$/see eq.(9)/ the gravitational effects occurring at $r << l_F$ are irrelevant to particle physics.

In this short talk we sketched some of the implications of quantum field theory with fundamental length. The detailed analysis of the theory can be found in refs./I-5/

Some features of this new quantum field theory are close to the original ideas of M.A.Markov (I940,I958)/6/ concerning the possible formulation of nonlocal field theories. From the other hand, it is close to the more recent field theories on a space-time lattice/7/.

In conclusion we would like to stress that reasoning from purely group-theoretical point of view we introduced in quantum field theory a new universal constant and drastically modified the interaction at very high energies(small distances). This can be done in Lorentz and translation invariant way (ref./4/). Therefore if a fundamental length exists we shall have a completely new physics at very high energies. Reversing the argument - a possible key for understanding the very high energy phenomena is the fundamental length.

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RECENT PROGRESS IN REGGEON FIELD THEORY

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I cannot describe all of the recent work in Reggeon Field Theory (RFT) in the time allotted to me, so I will just discuss a few Topics which I have found particularly interesting. I apologize to those whose work I will have time to cover.

What Gribov did when he invented RFT[1] was to teach us how to systematically evaluate Regge cut contributions to high energy scattering amplitudes. The cuts play a crucial role in processes in which the Pomeron can be exchanged because for small momentum transfers all of the multi-Pomeron branch points are close to each other and to the Pomeron pole. It is essential to take them all into account in order to have a realistic description of diffraction scattering at very high energies.

Two years ago at the Landau conference we heard about the work of Migdal, Polyakov and Ter-Martirosyan [2] and of Abarbanel and Bronzan [3], who showed how renormalization group techniques can be used to sum the multi-Pomeron cut contributions when the Pomeron intercept is exactly equal to one. They obtained a set of scaling laws for high energy scattering amplitudes. For example, they found that the elastic scattering amplitude has the asymptotic form

$$M(s,t) \underset{s\to\infty}{\to} i s\, g_A(t) g_B(t)\, c_1 (\ln s)^{\eta} F\left[c_2 t (\ln s)^{z}\right] \quad (1)$$

Here η and z are critical indices which are independent of the underlying parameters in the theory such as the triple Pomeron coupling constant. Similarly F is a universal scaling function. The only dependence on the underlying parameters is in the constants C_1 and C_2, and in the couplings of the Pomeron to the external particles g_A and g_B.

The renormalization group analysis which leads to eq. (I) is valid provided the theory has an infra-red stable fixed point. In the original work on the renormalization group[2,3] the theory was studied in $D = 4-\varepsilon$ transverse dimensions, rather than in the physical number $D = 2$. It was shown that for D near 4, i.e., for small ε, the theory does have an infra-red stable fixed point. The critical indices and the scaling function can be expand in a power series in ε and one finds that [2-5]

$$\eta = \frac{\varepsilon}{12}\left[1 + .45\varepsilon + o(\varepsilon^2)\right] \quad (2)$$

$$z = 1 + \frac{\varepsilon}{24}\left[1 + .39\varepsilon + o(\varepsilon^2)\right].$$

For $D = \varepsilon = 2$ the second order terms in the ε - expansion are of the same magnitude as the first order ones. So, although it is generally believed that the qualitative predictions of the ε - expansion are correct, one does not know what to make of the quantitative predictions.

The situation is even worse if one tries to use the ε - expansion to determine whether the theory has a fixed point in the physical number of dimensions. One can study this problem in a theory with only a bare triple Pomeron coupling constant. Then the theory will have an infra-red stable fixed point provided the Gell-Mann Low β function has a zero with a positive slope. If one computes β to first order in ε and then sets $\varepsilon = 2$, one finds that such a zero exists, but it disappears when the ε^2 term is taken into account [5]. Similarly, if one works directly in two dimensions, there is a zero in the one loop approximation to β but not in the two loop one.

The situation can be clarified by studying the theory for $D = 0$, where it is possible to compute β exactly [6]. One finds that β does have an infrared stable zero. However, if one makes the loop expansion for β one finds the same pattern as in two dimensions. There is a zero if one stops after an odd number of loops, but not if one stops after an even number. The difficulty is that the perturbation series diverges as one might expect in a non-linear field theory. The perturbation series is an asymptotic expansion for small values of the coupling constant, but it is meaningless in the vicinity of the zero. One can use information from perturbation theory to calculate β accurately in the vicinity of the zero by making use of Pade and Pade-Borel approximants.

The same techniques appear to be applicable in two dimensions. Recently Harrington has calculated β through three loops (seventh order in perturbation theory) for $D = 2$. The (1,1), (1,2) and (2,1) Pade-Borel approximants obtained from his calculation [6] are shown in Fig.1. The convergence is surprisingly rapid. I believe that this calculation constitutes impressive evidence for the validity of the renormalization group approach to RFT. One can also use the information from the three loop calculation to determine the critical indices. One finds [6]

$$\eta \simeq .22, \quad z \simeq 1.13. \tag{3}$$

If one believes that scaling laws such as eq. (1) hold, then the next problem is to determine the energy domain in which they are applicable. For a theory with only a bare triple Pomeron coupling constant the energy scale is set by the parameter $\lambda = (r_0^2/\alpha_0') \ln s$, where r_0 is the bare triple Pomeron coupling constant and α_0' is the bare slope parameter. For $\lambda \ll 1$ one can make a perturbation expansion in r_0 and for $\lambda \gg 1$ one must sum the perturbation series and obtain the scaling laws. It is difficult to make a precise estimate of the energy at which multi-Pomeron effects will start to become important. Because of the small size of r_0, it would appear that one should use perturbation theory at present accelerator energies.

If we are not presently in the asymptotic domain, then we cannot be certain that the intercept of the physical or renormalized Pomeron, $\alpha(0)$ is exactly equal to one. I believe that it is important to consider all possible predictions of the theory. $\alpha(0)=1$ only when the bare intercept, α_0, takes on a certain critical value, α_{0c}. For $\alpha_0<\alpha_{0c}$, $\alpha(0)<1$. The data will certainly not allow $\alpha(0)$ to be very much below one, which means that the cutts will still be near the pole for small momentum transfers. They must still be taken into account at all but the highest energies. Under these circumstances it is again possible to use renormalization group to sum the cut contributions. One finds, for example, that the elastic scattering amplitude satisfies a generalized scaling law [8]

$$\begin{aligned} M(s,t) \underset{\substack{s\to\infty \\ \alpha_0 \lesssim \alpha_{0c}}}{\longrightarrow} & \; i s\, g_A(t)\, g_B(t)\, C_1 (\alpha_{0c}-\alpha_0)^{\frac{1+\eta}{1-K}} \\ & \times F'\left[C_2 \ln s\, (\alpha_{0c}-\alpha_0)^{\frac{1}{1-K}},\; C_3\, t\, (\alpha_{0c}-\alpha_0)^{\frac{-z}{1-K}}\right], \end{aligned} \tag{4}$$

η and z are the same indices encountered in eq . (I). K is a new critical index and F' a generalized scaling function. In the limit $\alpha_0 \to \alpha_{0c}$ eq. (4) goes over smoothly into eq. (I). There are now two dimensionless parameters which set the energy scale: λ and $\mu = [1-\alpha(0)] \ln s$. Again for $\lambda<1$ one needed take into account only the first few terms in the perturbation series. For $\lambda>1$ and $\mu<1$ all the cut contributions must be taken into account and eq. (4) is applicable. In this intermediate energy region it will be difficult to distinguish the subcritical Pomeron from the critical one described by eq. (I). For $\mu>1$ eq. (4) still holds, but the pole dominates the cuts and the total cross section decreases like $s^{\alpha(0)-1}$.

Up to this point there is a close parallel between RFT and the problem of second order phase transitions in statistical mechanics. The bare intercept plays a role similar to that of the temperature in statistical mechanics. The analogy can be made more complete by studying correlations among particle produced in the central region /9/ . For $\alpha_0 = \alpha_{0c}$ which corresponds to $T = T_c$ in statistical mechanics, one finds that there are long range correlations in rapidity while for $\alpha_0 < \alpha_{0c}$ $(T > T_c)$ there are only short range correlations. The correlation length is $[1-\alpha(0)]^{-1}$. However, for $\mu < 1$ the correlation length is longer than the available rapidity and the results for the subcritical Pomeron are similar to those for the critical one.

Let me now turn to the case of $\alpha_0 > \alpha_{0c}$. It is clear from eq. (4) that the scattering amplitude has a singularity at $\alpha_0 = \alpha_{0c}$. We must ask whether it is possible to continue past this singularity and still obtain a sensible theory. The Green's functions in statistical mechanics also have singularities at $T = T_c$. They arise because the ground state of the system is degenerated for $T < T_c$. One can remove the singularity by introducing an external field which breaks the degeneracy and then smoothly continue past the critical point. It is at this point that the analogy between RFT and statistical mechanics breaks down. Recent work by Amati, Giafaloni, Marshesini, Le Bellac and Parisi /10,11/ has shown that the nature of the phase transition in RFT is completely diffirent from the one found in the Ising model. Amati will discuss this work in detail. Let me just state that their final result corresponds to the scattering from a grey disc with a radius which grows like $\ln s$ Amati et al. consider a theory in which there is only a bare triple Pomeron coupling. Universality arguments strongly suggest that this is sufficient for studying the high energy, small momentum transfer behaviour when α_0 is at or near its critical value. On the other hand in session AI Ter-Martirosyan discussed the problem of $\alpha_0 > \alpha_{0c}$ in a theory with all possible multi-Pomeron couplings /12/ . He finds a black disc with a radius which also grows like $\ln s$. The connection between this work and that of Amati et al. is unclear to me at the present time.

Let me conclude by summarizing the present status of the theory of the Pomeron in RFT. For $\alpha_0 \leq \alpha_{0c}$ the theory is in very good shape. It appears to satisfy both s and t- channel unitarity and to avoid all of the decreases which plagued the simple pole model of the Pomeron. For $\alpha_0 > \alpha_{0c}$ the situation is less clear, partly because the work is more recent. The challenging problem is whether the expanding disc solutions satisfy the multi-particle t-channel unitarity /13/ .

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THE POMERON AS A NONRELATIVISTIC GOLDSTONE PARTICLE

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Usually to fit the experimental data the vacuum t-channel is characterized in the compex j-plane by the two poles different disposed at $t=0$: $P(\omega=j-1=0)$ and $P'(\omega=-\Delta<0)$. Only the contribution of P (and its cuts) is important for high energies S while the P'-contribution decreases with energy as $s^{-\Delta}$. If the Pomeron P is located exactly at $\omega=0$ for $t=0$ it becomes an analogue of the nonrelativistic massless excitation. The spectrum of two states in this case is similar to that one coming from the spontaneous symmetry breaking of the simplest nonrelativistic theory with two fields ($O(2)$ symmetrical theory). This situation suggests to suppose that the P and P' spectrum is also originated by the symmetry breaking for the system of vacuum reggeons. The Pomeron appears then as the Goldstone particle.

We can obtain all possible forms of the Lagrangian for such a system /1/ . The procedure can be described as follows.

For the Lagrangian of this reggeon system before the breaking we can use a very general O(2)-invariant form with arbitrary number of reggeon operator $\psi_i(\xi,\vec{\rho})$, $\psi_i^+(\xi,\vec{\rho})$ and their derivatives ($i=1$ for P', $i=2$ for P, $\xi=\ln\frac{s}{s_0}$, $\vec{\rho}$ is an impact vector). Nonhermitian character of reggeon interactions unambiguously chooses the operator with a finite vacuum expectation value

$$\psi_i^+ - \psi_i \to \psi_i^+ - \psi_i + \varphi\,\delta_{i1}. \qquad (1)$$

After the excursion of φ the Lagrangian is required to have the nonrelativistic form without any terms allowing annihilation and creation of reggeons in vacuum ψ_i, ψ_i^+, ψ_i^{+2} ... This requirement provides the system of equations selecting possible forms for the Lagrangian. Their main properties are

a) they contain only such interactions which leave P massless and stable at $t=0$: all vertices of the P- decay are proportional to the P momentum $\vec{K}$ $(\vec{K} \equiv \vec{K}_\perp)$;

b) these forms contain the set of conditions on the different coupling constants (see /2/) which serve to cancel the infrared divergences existing in connection with the massless character of P .

The simplest Lagrangian obtained by this procedure is

$$L = -\frac{1}{2}\left(\psi_i^+ \frac{\partial \psi_i}{\partial \xi} - \psi_i \frac{\partial \psi_i^+}{\partial \xi}\right) - \alpha'_P \nabla \psi_i^+ \nabla \psi_i - \Delta \psi_1^+ \psi_1 + \sqrt{\lambda\Delta}\left(\psi_1^+ \psi_i^2 - \psi_1 \psi_i^{+2}\right) + \lambda \psi_i^2 \psi_k^2 . \quad (2)$$

More complicated models enable to describe P and P' with different slopes α'_P and $\alpha'_{P'}$ and so on.

The use of P' in the asymptotic theory (simultaneously with P) can give, of course, only the model description of some qualitative features of the scheme. But it is important that P' provides the theory with the natural energy scale which shows where the real asymptotic begins to work:

$$\Delta \xi > 1. \quad (3)$$

As P' is located on the Pomeron cut it is really an unstable state decaying into 2P,3P ... Its properties can be represented by the P' propagator. The approximate equation for it in the simplest case (2) is of the form $(\lambda > 0)$

$$G_{P'}^{(-1)} = \omega + \alpha'_P K^2 + \frac{\Delta}{1 - \gamma(\omega, K^2)} ;$$

$$\gamma(\omega, K^2) = \frac{\lambda}{4\pi} \ln \frac{2\alpha'_P L}{\omega + \frac{1}{2}\alpha'_P K^2} . \quad (4)$$

The contribution of (4) to σ_{tot} contains not only the positive term decreasing with energy as $\xi^{-\tilde{\Delta}}$ x) ($\tilde{\Delta}$ can be different from Δ) but slowly decreasing $\left(\sim \frac{1}{\xi}\right)$ negative contribution from the cut. This cut contribution to σ_{tot} is proportional to the same experimentally large coefficient $g^2_{P'}$ as the term $\xi^{-\tilde{\Delta}}$. Moreover the P-loops which form the cuts in the P'- propagator can be considered as coming not immediately from the hadron vertices but after several steps in rapidity (P'- line). There are some experimental evidences that in this case the cut-off momentum L for the reggeon loop is large $(\alpha'_P L \gtrsim 1)$ (all references are in /2/). Thus the P - contribution contains the negative terms of the type

$$- \frac{g^2_{P'}}{\Delta} \frac{\lambda}{\xi + 1/\alpha'_P L} \quad (5)$$

which is large ($g^2_{P'}$ is large) and strongly depends on ξ $(\alpha'_P L \gtrsim 1)$. This effect could explain, in principle, in more complicated version of the model (2) (even for the constant contribution of the Pomeron) large and fast growth of σ_{tot} at modern energies.

The most interesting effect appears in P'- propagator (4) at $t = -K^2 < 0$. The pole in $G_{P'}$ comes here out of the cut. It is located near the cut at the point

$$x_0 = \omega_0 + \frac{1}{2}\alpha'_P K^2 \approx 2\alpha'_P L \exp\left\{-\frac{\eta}{K^2} - \frac{1}{\lambda}\right\}; \quad (6)$$

$$\eta = \frac{2\Delta}{\lambda \alpha'_P} .$$

Only in the Goldstone P theory we can choose the parameters so that the pole appears in $G_{P'}$ only for $K^2 \neq 0$ and never comes to the region $\omega > 0$.

The contribution of this pole to the scattering amplitude is

$$\sim -\lambda \frac{g^2_{P'}(t)}{\Delta}\left(\frac{\eta}{K^2}\right) \exp\left\{-\frac{\eta}{K^2} - \frac{1}{2}\alpha'_P K^2 \xi + x_0 \xi\right\} \quad (7)$$

This contribution has the maximum and the attractive possibility is that the pole (6) plays an important role (together with P and the cut in (4)) in forming the minimum

x) In the simple model (2) these terms are very different from the experimental picture of P.

and the second maximum in hadron elastic cross-sections (pp-elastic scattering). The dependence $\exp\{-h/\kappa^2\}$ helps this idea very much. Reasonable choice of model parameters easily allows one to reconcile the qualitative behaviour near the maximum-minimum positions. Of course, the strict quantitative description for such large t ($t \sim$ 1.5-3 GeV^2) strongly depends on the model but the existence of the "Pomeron bound state" is very probable property of the considered scheme.

Papers /1/ and /2/ contain all necessary references.

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THEORY OF THE POMERON WITH INTERCEPT LARGER THAN 1

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What I will tell you is contained in two recent CERN preprints Ref. Th 2152 and 2185. The physicists involved are M.Ciafaloni, M.Le Bellac, G.Marchesini, G.Parisi and myself.

I will not discuss the motivation for this research and, neither, the Gribov lagrangian that we adopt for the self interacting pomeron. This is given by

$$\mathcal{L} = \bar{\psi}\,\partial_y \psi + \alpha' \vec{\nabla}\bar{\psi}\cdot\vec{\nabla}\psi + \mu\bar{\psi}\psi - i\lambda(\bar{\psi}^2\psi + \bar{\psi}\psi^2), \quad (1)$$

where $\mu = \alpha(0) - 1$ and $\psi(y, \vec{b})$ is defined in a space with an (imaginary) time y and a two dimensional space $\vec{b}$. For $\mu < 0$ the perturbative series is meaningful and gives rise to the Gribov calculus. The pomeron propagator is given by

$$D_0(y, \vec{b}) \propto \exp\left(\mu y - \frac{\vec{b}^2}{4\alpha' y}\right). \quad (2)$$

For $\mu > 0$ the perturbative series loses its meaning in the sence that every order dominates over the preceding. This is however not true outside of the disc defined by:

$$b = \sqrt{4\mu\alpha'}\, y \,. \quad (3)$$

Indeed if $b > \sqrt{4\mu\alpha'}\, y$, we see from (2) that D_0 does not explode and one can check that higher iterations are even smaller. In other words, for all μ there is a region in $\vec{b}$ space in which-at fixed y- the perturbative expansion of $\mathcal{L}$ as given by (1) is meaningful.

But if we want to understand asymptotic properties of the theory $(y \to \infty)$ we will be forced to find nonperturbative solution of it if $\mu > 0$.

We will use a hamiltonian method. The time evolution of the theory is given by e^{-yH} and, therefore, the asymptotic properties will be controlled by the lower lying eigenstates. To determine them together with their eigenvalues, will be our task.

Let us first remark, that due to the i in front of the trilinear term in (1), H will not be hermitean. However it exists a unitary operator L that transforms H into H^{+} i.e.

$$L H L^{-1} = H^{+}, \quad L \Psi L^{-1} = -\bar{\Psi}^{+}. \tag{4}$$

We will introduce a lattice in $\vec{b}$ space (inter-lattice distance a). Then

$$H = \frac{i\lambda}{a}\sum_{j} \bar{\Psi}_j \left(\bar{\Psi}_j + \Psi_i - \frac{\mu}{i\lambda}\right)\Psi_j - \frac{\alpha'}{a^2}\sum_{(j,\ell)} \bar{\Psi}_j \Psi_\ell , \tag{5}$$

where (j,ℓ) means near neighbours. Our procedure is to first solve the single site dynamics and, then, to introduce the intersite coupling. The single site dynamics was solved in earlier papers (V.Alessandrini, D.Amati and R.Jengo. Nucl.Phys. B108,425 (1976); R.Jengo. Nucl.Phys. B108, 447 (1976); J.Bronsan, J.Shapiro and R.Sugar. Santa Barbara preprint (1976)). The outcoming spectrum has the structure of Fig.1.

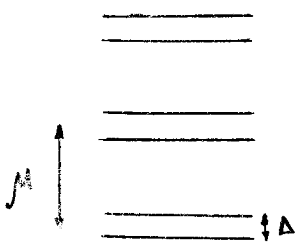

Fig.1

where

$$\Delta \propto e^{-\frac{\mu^2}{4\lambda^2}} \tag{6}$$

If we call χ_0 and χ_1 the eigenfunctions of the two lowest eigenstates we have that the matrix elements of Ψ and $\bar{\Psi}$ in this basis are given by

$$\langle \bar{\Psi} \rangle = \frac{\mu}{i\lambda}\begin{pmatrix} 0 & -1 \\ 0 & 1 \end{pmatrix}, \quad \langle \Psi \rangle = \frac{\mu}{i\lambda}\begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}. \tag{7}$$

The lowest lying state is the single site vacuum i.e. x)

$$\Psi_i \chi_0^i = \bar{\chi}^i \bar{\Psi}_i = 0 .$$

We will now introduce the interlattice interaction by keeping at each site only the two lowest lying levels. This approximation is expected to correctly represent the lowest lying levels of the whole system and, in particular, the phase transition. Introducing the representations (7) for the fields we can write the hamiltonian with a spin formalism as:

$$H = \Delta \sum_i \left(1 - \sigma_3^i\right) + \sum_{(i,j)} J_{ij}\, \frac{1 + \vec{\sigma}^i \vec{n}_Q}{2} \cdot \frac{1 + \vec{\sigma}^j \vec{n}_P}{2} \tag{8}$$

where

$$n_{P,Q} = \left(\pm 1, -i, -1\right), \quad J = \sum_{(i,j)} J_{ij} = \alpha' \frac{\mu^2}{\lambda^2} . \tag{9}$$

The spectrum of H looks as fig.2

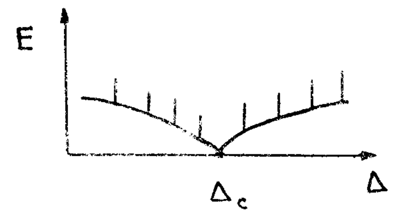

Fig.2

For $\Delta > \Delta_c$ we have a single vacuum $\Phi_0 = \prod_i \chi_0$ and a continuum with a gap that goes to zero as $\Delta \to \Delta_c$. For $\Delta < \Delta_c$ we find two degenerate ground states. One is still Φ_0 , the other Ψ_0 is a negative metric one which is a product of local states only if $\Delta = 0$.

x) The definition of $\bar{\chi}_0$ is $\chi_0^{+} L$ where L is the metric operator that appears in eq. (4).

Ψ_0 and Φ_0 are connected by field operator, i.e.

$$(\bar{\Psi}_0 \, \bar{\psi} \, \Phi) = (\Phi_0 \, \psi \, \Psi_0) = \frac{\mu}{i\lambda}\, \delta , \tag{10}$$

where δ is a number that plays the role of the order parameter. The S-matrix can be easily calculated. If the external particles are considered as pomeron sources, located at y_1 and y_2 respectively ($Y = y_2 - y_1$ is the total rapidity) and with structures given by $f(\vec{\ell})$, $g(\vec{B}-\vec{\ell})$ respectively ($\vec{B}$- total impact parameter), then

$$S = \left(\bar{\Phi}_0 \, e^{-i\alpha \Sigma f_j \psi_j} \; e^{-yH} \; e^{-i\alpha \Sigma g_i \bar{\psi}_i} \; \Phi_0\right) = \tag{11}$$

$$\underset{\substack{y\to\infty \\ B \text{ fixed}}}{=} 1 - \left(1 - \exp\left(-\frac{\delta\mu}{\lambda}\int f(\vec{\ell})\,d\vec{\ell}\,\right)\right) \times \left(1 - \exp\left(-\frac{\delta\mu}{\lambda}\int g(\vec{B}-\vec{\ell})\,d\vec{\ell}\right)\right) + o\left(e^{-\omega t}\right)$$

where ω is the collective excitation frequency (i.e., the continuum of fig.2). Eq. (11) shows an asymptotic scattering amplitude described in terms of a $\vec{B}$ independent positive and factorized opacity. The structure of the spectrum (fig.2) is similar to that of the Ising model - related to a φ^4- theory - given by eq. (8) but with $n_{\underset{a}{\rho}} = (\pm 1, 0, 0)$. However the structure of the two lower lying states appearing for $\Delta < \Delta_c$ is very different. Indeed, in the Ising case none of them is the original vacuum; in both of them the fields would have non vanishing expectation values and they would not be connected to each other by field operators (zero non diagonal field matrix element). This last property is the one that determines the spontaneous symmetry breaking of the φ^4 theory ($\varphi \to -\varphi$ symmetry) in the sense that the two vacua, being completely disconnected, define two orthogonal Hilbert spaces any one of them being equally good. In our case, there is only one Hilbert space. The vacuum does not change structure at Δ_c , what happens is that for $\Delta < \Delta_c$ a zero gap state appears. To understand the reason of the vacuum dynamical instability, is better to start from a single dimensional B space and with $\Delta = 0$, where

$$\Psi_0 = \prod_i \chi^i \tag{12}$$

If we construct a (box) state made up of χ_0^i everywhere but in an interval in which we have χ_1^i (fig.3), i.e., $\prod_{\substack{i<\ell \\ i>m}} \chi_0^i \prod_{\ell \le i \le m} \chi_\ell^i$

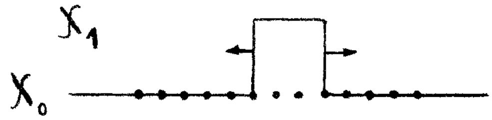

Fig.3

we find that it evolves in time by expanding with a finite velocity. We therefore understand the vacuum instability. If a local field $\bar{\psi}_i(0)$ operator $\bar{\psi}_i(0)$ is applied to the vacuum Φ_0, it gives rise at $y = 0$ the state χ_ℓ^i which gives a single site box state which expands therefore with time (y) . At infinite y, χ_1 will be excited at all sites and Φ_0 would have gone into Ψ_0 . The zero gap state Ψ_0 lies over the vacuum Φ_0 and depletes it dynamically.

The introduction of Δ does not change the picture (as far as $\Delta < \Delta_c$). It is easy to get from these results that $S = 1$ outside the box (disc in two-dimensional b space) this being a basic requirement of any reggeon theory. On the other hand, eq. (11) restricted to the disc gives rise to total cross section increasing as $(\log s)^2$.

$$|\tilde{g}_\ell(s)| \leqslant const \frac{s^{N+1/4}}{\sqrt{|\ell|}} \exp\left\{-\left(Re\,\ell+\frac{1}{2}\right)\ln\left[\varkappa_0+\sqrt{\varkappa_0^2-1}\right]\right\},$$

$$|\tilde{h}_\ell(s)| \leqslant const \frac{s^{N+\frac{1}{4}}}{\sqrt{|\ell|}} \exp\left\{-\left(Re\,\ell+\frac{1}{2}\right)\ln\left[\varkappa_1+\sqrt{\varkappa_1^2-1}\right]\right\}.$$

Используя стандартные аналитические свойства по ℓ для функций

$$f^{\pm}(s,\ell)=\tilde{g}_\ell(s) \pm \tilde{h}_\ell(s)$$

и применяя теорему, доказанную в работе Киношиты, Лоеффеля, Мартена (Phys. Rev. , 135, 1464 (1964)), можно показать, что в физических точках по ℓ для $\tilde{g}_\ell(s)$ и $\tilde{h}_\ell(s)$ справедлива оценка

$$|\tilde{g}_\ell(s)|,\ |\tilde{h}_\ell(s)| < d, \tag{10}$$

если только

$$\ell \geqslant \ell_0(s) = n + \left[const \cdot \ln\left(\frac{s}{s_c}\right)\right]. \tag{11}$$

В (10) d - некоторая не зависящая от s постоянная.

Неравенства (10) мы будем называть "унитарными" ограничениями для коэффициентов $\tilde{g}_\ell(s)$ и $\tilde{h}_\ell(s)$.

После того, как установлено "унитарное" ограничение (10), удобно в амплитуде рассеяния произвести некоторую перегруппировку членов и представить ее в виде: $F(s,z)=G(s,z)+H(s,z)+R(s,z)$,

где
$$G(s,z)=\frac{8\pi\sqrt{s}}{K}\sum_{\ell=\ell_0+1}^{\infty}(2\ell+1)\tilde{g}_\ell(s)P_\ell(z), \tag{12}$$

$$H(s,z)=\frac{8\pi\sqrt{s}}{K}\sum_{\ell=\ell_c+1}(2\ell+1)\tilde{h}_\ell(s)(-1)^\ell P_\ell(z), \tag{13}$$

$$R(s,z)=\frac{8\pi\sqrt{s}}{K}\sum_{\ell=c}^{\ell_c}(2\ell+1)f_\ell(s)P_\ell(z). \tag{14}$$

Следует отметить, что $G(s,z)$ и $H(s,z)$ от $\tilde{G}(s,z)$ и $\tilde{H}(s,z)$, соответственно, отличаются полиномами по z, и поэтому они на конечном расстоянии в z -плоскости имеют одинаковую аналитическую структуру.

Так как для наших целей достаточно ограничиться конечной областью по z, то оценка вкладов функций G, H и R даст ответ о влиянии ближайших особенностей t и u - каналов на амплитуду рассеяния при $s\to\infty$.

Обозначим через E_δ эллипс с фокусами в точках ±1 и большой полуосью $\delta>1$, где δ - фиксированное число.

Ту область, которая получается из E_δ после выбрасывания сегмента $[-\delta,-\varkappa_1]$, обозначим через E_δ^-, а область, которая получается из E_δ выбрасыванием сегмента $[\varkappa_c,\delta]$, через E_δ^+.

Можно показать (доказательство мы опускаем), что имеют место оценки:

$$|G(s,z)|_{z\in E_\delta^+} \leqslant A(\delta)\, s^{N_1(\delta)}, \tag{15}$$

$$|H(s,z)|_{z\in E_\delta^-} \leqslant B(\delta)\, s^{N_2(\delta)}, \tag{16}$$

где $A(\delta)$, $B(\delta)$, $N_1(\delta)$, $N_2(\delta)$ - некоторые конечные постоянные, не зависящие от энергии.

Перейдем теперь к оценкам функций G, H и R в физической области.

В силу условия унитарности и ограничения $|P_\ell(z)|\leqslant1$ из (14) непосредственно следует, что

$$|R(s,\cos\theta)| \leqslant \frac{8\pi\sqrt{s}}{K}\sum_{\ell=c}^{\ell_c}(2\ell+1)=\frac{8\pi\sqrt{s}}{K}(\ell_c+1)^2.$$

Учитывая (11), находим:

$$|R(s,\cos\theta)| \underset{s\to\infty}{\leqslant} const\,\ln^2\left(\frac{s}{s_0}\right). \tag{17}$$

Для того, чтобы оценить $|H(s,\cos\theta)|$, мы воспользуемся методом, предложенным в работах В.В.Ежелы, А.А.Логунова и др. (ТМФ 6, 42 (1971); 9, 3, 153 (1971); 15, 153 (1973)).

Введем ассоциированную по Борелю к $H(s,z)$ функцию вида:

$$\Psi(s,\cos\theta,y)=\frac{8\pi\sqrt{s}}{K}\sum_{\ell=\ell_c+1}^{\infty}(2\ell+1)(-1)^\ell\tilde{h}_\ell(s)P_\ell(\cos\theta)y^\ell. \tag{18}$$

Из (18) очевидно, что

$$\Psi(s,\cos\theta,1)=H(s,\cos\theta). \tag{19}$$

Можно доказать, что если $H(s,z)$ аналитична по z в области E_δ^-, то функция $\Psi(s,\cos\theta,y)$ будет аналитической по y в области, изображенной на рис.1

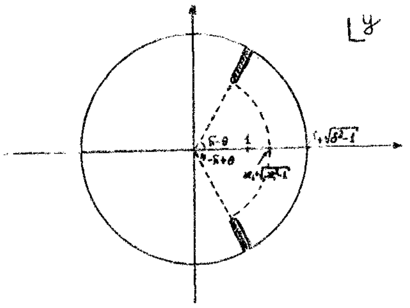

Рис.I.

и представляющей собой круг радиуса $\delta+\sqrt{\delta^2-1}$ с разрезами вдоль лучей (см. рис.I). Положение разрезов зависит от угла рассеяния Θ.

Кроме того, может быть доказано, что если $H(s,z)$ удовлетворяет оценке (I6), то $\Psi(s,\cos\theta,y)$ также полиномиально ограничена по s в области, указанной на рис.I.

Сделаем следующее построение (рис.2).

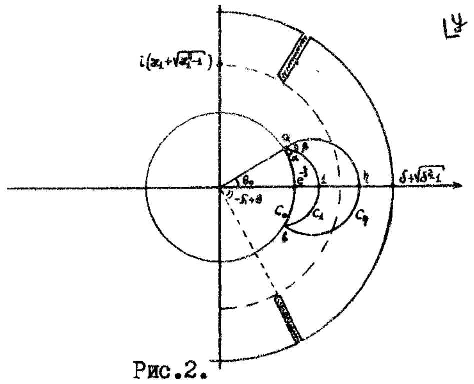

Рис.2.

Проведем круг радиуса $e^{-\xi(s)}$ $(\xi>0)$ с центром в начале. Дугу $\alpha\beta$ этой окружности обозначим через C_0. Через точки a и b, полярные углы которых есть $\pi-\Theta_c$ и $-\pi+\Theta_c$, проведем еще следующие дуги окружностей: дугу C_1, которая проходит через точку $y=1$ и дугу C_n, которая ортогональна к дуге $a b$. Обозначим через α угол между дугами C_0 и C_1, а через β – угол между дугами C_1 и C_n. Очевидно, $\alpha+\beta=\pi/2$ по построению. Область между дугами C_c и C_n обозначим через B. Заметим, что если $\Theta_c>\Theta$, то область B полностью находится в области аналитичности функции $\Psi(s,\cos\theta,y)$.

Применяя теорему о двух постоянных, можно написать

$$|\Psi(s,\cos\theta,1)|\leqslant\left[\max_{y\in C_c}|\Psi(s,\cos\theta,y)|\right]^{\omega}\left[\max_{y\in C_n}|\Psi(s,\cos\theta,y)|\right]^{1-\omega}. \tag{20}$$

Здесь ω является гармонической мерой области B и равна

$$\omega=\frac{2}{\pi}\beta. \tag{21}$$

На окружности $y\in C_0$ легко найти $\max_{y\in C_c}|\Psi|$, если учесть в (I8) "унитарное" ограничение (I0) на коэффициенты $\tilde{h}_\ell(s)$ и неравенство $|P_\ell(\cos\theta)|\leqslant 1$:

$$\max_{y\in C_0}|\Psi(s,\cos\theta,y)|\leqslant\frac{8\pi\sqrt{s}}{K}\cdot\frac{d}{th^2(\xi/2)}. \tag{22}$$

Что касается $\max_{y\in C_n}|\Psi(s,\cos\theta,y)|$, то мы можем использовать полиномиальную ограниченность Ψ по s, и из (20) найдем $|H(s,\cos\theta)|=$

$$=|\Psi(s,\cos\theta,1)|\leqslant\left(\frac{8\pi\sqrt{s}}{K}\frac{d}{th^2(\xi/2)}\right)\left(\frac{s}{s_0}\right)^{\tilde{N}(1-\omega)} \tag{23}$$

Из рис.2 видно, что в нашем распоряжении остается параметр $e^{-\xi(s)}$, т.е. радиус дуги C_c. Если с ростом s, $e^{-\xi(s)}\to 1$, то $\alpha\to 0$ и $\beta\to\pi/2$, поэтому $\omega\to 1$ и в (23) множитель $s^{\tilde{N}(1-\omega)}$ становится не существенным. Чтобы подавить $s^{\tilde{N}(1-\omega)}$, достаточно выбрать $\omega\sim 1-1/\ln\left(\frac{s}{s_c}\right)$. Параметр ξ при этом будет определяться из равенства

$$th\left(\frac{\xi}{2}\right)\simeq\frac{const}{\ln(s/s_c)}$$

Тогда окончательно находим

$$|H(s,\cos\theta)|_{\Theta_0>\theta}=|\Psi(s,\cos\theta,1)|_{\Theta_c>\theta}\leqslant \tag{24}$$

$$\leqslant const\,\ln^2(s/s_c).$$

Рассмотрим теперь поведение $\left.\frac{d\sigma}{dt}\right|_{t=0}$ –

$$\left.\frac{d\sigma}{dt}\right|_{t=0}=\frac{1}{64\pi s K^2}|F|^2_{\theta=0}=\frac{1}{64\pi s K^2}|G+H+R|^2.$$

Так как экспериментальные данные показывают, что $\left.\frac{d\sigma}{dt}\right|_{t=0}\geqslant const\neq 0$, а для R и H справедливы оценки (I7) и (24), видим, что основной вклад при высоких энергиях в $\left.\frac{d\sigma}{dt}\right|_{t=0}$ вносит только функция $G(s,\cos\theta)$, определяемая особенностями t-канала.

ANALYTICAL FORM OF THE FROISSART BOUND AT FINITE ENERGIES

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The work of Indurain [1] initiated investigations of the Froissart bound of finite energies. This study was developed in papers of Common [2], Stewen [3], Roy [4], Blankenbeckler and Sawit [5] and others. However, the most stringent bounds have been obtained only in the numerical form.

It is known [4] that the scattering length α_2 in the t-channel is connected with the amplitude of the elastic scattering in the s-channel in the following form:

$$\alpha_2 = \frac{4}{15\pi}\int_1^\infty \frac{A(s',1)}{s'^3}\,ds' ; \quad 4m_\pi^2 = 1. \tag{1}$$

Here $A(s',1) \equiv \mathrm{Im}\,F(s',1)$, $F(s',1)$ is the amplitude of the elastic scattering, s and t are usual invariant variables. We carry out the calculations for the specific case of the $\pi^\circ\pi^\circ$ scattering. We introduce the following isospin combinations $\alpha_2 = \frac{1}{3}\alpha_2^{(0)} + \frac{2}{3}\alpha_2^{(2)}$; $A(s',1) = \frac{1}{3}A^{(0)}(s',1) + \frac{2}{3}A^{(2)}(s',1)$. From the inequality $A(s',1) \geqslant 0$ the obvious inequality is deduceable

$$\alpha_2 > \frac{4}{15\pi}\int_1^{\nu s} A(s',1)\,ds'/s'^3 . \tag{2}$$

Here s is an arbitrary energy, $\nu > 1$ is an arbitrary number. Making use of the mean-value theorem we obtain

$$A(\bar{s},1) < \frac{15\pi\alpha_2\nu^2}{2(\nu^2-1)}\, s^2 . \tag{3}$$

Here $\bar{s}$ is some point in the interval $(s, \nu s)$. It is known [4], that given the value $A(\bar{s},1)$ $\max A(\bar{s},0)$ can be realized by choosing the following values of partial amplitudes

$$a_\ell(s) = \begin{cases} 1 & \ell < L \\ \eta & \ell = L+2 \\ 0 & \ell > L+2 \end{cases} \tag{4}$$

In our case only partial waves with even ℓ are nonvanishing. Hence, when $A(s,0)$ is maximal, then according to eq. (4) we have

$$A(\bar{s},1) = 4\sqrt{\frac{\bar{s}}{\bar{s}-1}} \sum_{\substack{\ell=0 \\ \ell\text{-even}}}^{L} (2\ell+1) P_\ell\left(1+\frac{2}{\bar{s}-1}\right) + (2L+5)\eta P_{L+2}\left(1+\frac{2}{\bar{s}-1}\right). \tag{5}$$

Considering energies for which $s \gg 1$ and using the known formula (see (A.2) in ref. [4]) we obtain

$$A(\bar{s},1) = 4P'_{L+1}\left(1+\frac{2}{\bar{s}}\right) + (2L+5)\eta P_{L+2}\left(1+\frac{2}{\bar{s}}\right). \tag{6}$$

In order to determine L we calculate $P'_{L+1}\left(1+\frac{2}{s}\right)$. For this purpose we make use of the following integral representation for $P_\ell(\mathrm{ch}\,\alpha)$ [6]:

$$P_\ell(\mathrm{ch}\,\alpha) = \frac{2}{\pi}\int_0^\alpha \frac{\mathrm{ch}(\ell+1/2)\theta}{\sqrt{2\,\mathrm{ch}\,\alpha - 2\,\mathrm{ch}\,\theta}}\,d\theta \tag{7}$$

which by means of the substitution $z = \alpha - \theta$ can be reduced to the form

$$P_\ell(\mathrm{ch}\,\alpha) = I_{\ell+\frac{1}{2}} + I_{\ell-1/2} \tag{8}$$

where

$$I_{\ell+1/2}(\mathrm{ch}\,\alpha) = \frac{e^{(\ell+\frac{1}{2})\alpha}}{\pi\sqrt{2\,\mathrm{sh}\,\alpha}}\int_0^\alpha \frac{e^{-(\ell+\frac{1}{2})z}}{(\mathrm{sh}\,z)^{1/2}}\left(1-\mathrm{th}\frac{z}{2}\,\mathrm{cth}\,\alpha\right)^{-1/2} dz .$$

It is easy noticeable that in our case $\alpha < \frac{2}{\sqrt{3}}$. Taking into account the condition $s \gg 1$ and performing simple manipulations we obtain

$$P'_{L+1}(\mathrm{ch}\,\alpha) = I^*_{L+\frac{3}{2}} + I^*_{-L-\frac{3}{2}} + o(\alpha^2) + o(\alpha^4) \tag{9}$$

where

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$$I^*_{L+3/2} = \frac{L+3/2}{\mathrm{sh}\,\alpha}\frac{\sqrt{\pi}}{2}\left(1-\frac{\alpha^2}{12}-\frac{\alpha}{4(L+3/2)}\right)\times e^{(L+\frac{3}{2})}$$
$$\times\int_0^1 e^{-\alpha(L+3/2)x^2}(1-x^2)\left(1-\frac{x^2}{2}\right)^{-1/2}dx.$$

One can show that $o(\alpha^2) > 0$ and that numerically it is significantly smaller than explicitly given terms. We note that, by neglection of positive terms in $P'_{L+1}(\mathrm{ch}\,\alpha)$ we make only the quantity L larger (see eq. (11) below).

From eq. (9) it follows that at energies of the order of 10 GeV the correction terms with respect to α are small. We will neglect them because errors in determing α_2 are significantly larger than these corrections. We note that they are easily calculable.

We consider the case when $\alpha(L+\frac{3}{2}) \equiv \gamma \gg 1$. Obviously γ is an increasing function of s. The numerical calculations show that the condition $\gamma \gg 1$ for the $\pi^0\pi^0$ scattering is fulfilled already at $s \sim 10\,\mathrm{GeV}^2$. Simple but tedious calculations lead to the formula

$$P'_{L+1}\left(1+\frac{2}{s}\right) = \frac{\sqrt{\gamma}\, e^{\gamma}\bar{s}}{4\sqrt{2\pi}}\left(1-\frac{3}{8\gamma}-\frac{5}{16\gamma^2}\right); \quad (10)$$
$$\gamma = \frac{2(L+3/2)}{\sqrt{s}}$$

On the right hand side of eq. (10) we have neglected the positive terms which are small in comparison with the main terms. The precision of eq. (10) increases with increasing γ, i.e., increasing energy. Returning to ineq.(3) and sibstituting eqs. (6) and (10) into it we obtain the equation for L :

$$\sqrt{\gamma}\, e^{\gamma}\left(1-\frac{3}{8\gamma}-\frac{5}{16\gamma^2}\right) = 30\left(\frac{\pi}{2}\right)^{3/2}\alpha_2\, s\,\frac{\nu^2}{\nu^2-1}, \quad (11)$$

We have omitted the term $(2L+5)\eta P_{L+2}(1+\frac{2}{s})$ which could lead to some reinforcement of the sought for inequality.

Solving eq. (11) by means of the method of subsequent approximations we obtain

$$\gamma(\bar{s}) = \ln\left(\frac{b\alpha_2\nu^2}{\nu^2-1}s\right) - \ln\ln\left(\frac{b\alpha_2\nu^2}{\nu^2-1}s\right)\left(1-\left(2\ln\frac{b\alpha_2\nu^2 s}{\nu^2-1}\right)^{-1}\right)$$
$$+\frac{3}{8\ln(b\alpha_2\nu^2 s/\nu^2-1)}+\frac{5}{16\ln^2\left(\frac{b\alpha_2\nu^2 s}{\nu^2-1}\right)};\; b = 30\left(\frac{\pi}{2}\right)^{3/2}. \quad (12)$$

Formula (12) has been written down with accuracy to the terms which are much smaller than 1. Using the equality

$$\sigma(\bar{s}) = \frac{8\pi}{m_\pi^2 s}\sum_{\substack{\ell=0\\ \ell\text{ - even}}}^{L+2}(2L+1), \quad (13)$$

we obtain with accuracy to the terms $\sim \frac{1}{\sqrt{s}}$ the equality

$$\sigma(\bar{s}) = \frac{\pi}{m_\pi^2}\gamma^2(\bar{s}). \quad (14)$$

One can improve this bound by estimating on the basis of experimental data the integral in eq. (3) over the interval $(1, s)$. Let us note also that the bound on the cross section in the interval $(s, \nu s)$ depends on its behaviour at energies larger than νs .

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DESCRIPTION OF INCLUSIVE REACTION SCALING PROPERTIES IN TERMS OF DENSITY MATRIX

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Description of multiparticle production processes at high energies is complicated by the fact that the number of permitted channels of the reaction is tremendously great and it is a problem to select processes with a fixed number of particles in the final state. In this case one is interested in the momentum distribution for a small number of selected particles getting no information about the remaining secondary hadrons (inclusive reactions). So it is expedient to introduce a density matrix that would describe the behaviour of these picked out particles in self-consistent field of all the remaining particles in the final state [1].

In the framework of the equal-time formulation of quantum field theory [2,3] maniparticle wave function is introduced in the form: $(2\pi)^3\delta(\vec{p}_2+\sum_{i=1}^{n}\vec{k}_i-q_1-q_2)\Psi_n(\vec{k}_1,\dots\vec{k}_n;t)=$

$$=\sqrt{\frac{2p_2^0\,2k_1^0\dots 2k_n^0}{n!\;(2\pi)^{3(n-1)}}}\;\langle 0|T\{\psi(\vec{p}_2t)\varphi(\vec{k}_1t_1)\dots\varphi(\vec{k}_nt_n)\,S\}|\vec{q}_1\vec{q}_2\rangle\Big/\langle 0|S|0\rangle\Big|_{t_1=\dots t} \qquad (1)$$

where ψ and φ are operators of nucleon and meson fields and $|\vec{q}_1\vec{q}_2\rangle$ is initial state vector of meson-nucleon system.

In the quasipotential approach the single-time wave function Ψ_n satisfies the following equation [4-6] :

$$(E_k-E)\Psi_n(t)=V_{n+1}\Psi_n(t) \qquad (2)$$

where $E=q_1^0+q_2^0$, E_k , and V_{n+1} are kinetic energy and interaction quasipotential of $n+1$ particles. The equation (2) is a relativistic generalization of Schrödinger equation for multiparticle system. Likeness of the quasipotential approach with nonrelativistic quantum mechanics is emphasized by explicit three-dimensional form of the quasipotential equation in contrast to other relativistic equations. Notice that the quasipotential is not equal to a sum of interaction quasipotentials for separate pairs of particles.

We shall construct the density matrix for single-particle inclusive reaction of equal-time wave functions by means of usual rules of quantum mechanics [1] :

$$\rho_{\vec{c}}(\vec{k}_1,\vec{\varkappa}_1|t)=\sum_{n=2}^{\infty}\frac{1}{(n-1)!}\int d\vec{k}_2\dots d\vec{k}_n\,\delta(\vec{c}+\sum_{i=2}^{n}\vec{k}_i-\vec{q}_1-\vec{q}_2)\times\Psi_n(\vec{k}_1,\vec{k}_2\dots\vec{k}_n;t)\,\Psi^*(\vec{\varkappa}_1,\vec{k}_2,\vec{k}_n;t) \qquad (3)$$

This density matrix describe a behaviour of the picked out meson in the final state of meson-nucleon collision when the summed momentum of this meson and nucleon is fixed and equal $\vec{c}$. The full density operator is defined as superposition over all $\vec{c}$:

$$\rho(t)=\int d\vec{c}\;\rho_{\vec{c}}(t). \qquad (4)$$

It is not difficult to see that the diagonal element of the density matrix (4) is related with single-particle inclusive cross section:

$$\lim\;\rho(\vec{k};\vec{k}|t)=\delta(0)\frac{I}{(2\pi)^2}\frac{d\sigma}{d\vec{k}} \qquad (5)$$

where $I=4\sqrt{(q_1q_2)^2-\mu^2m^2}$

Starting from the quasipotential equation (2) it is possible to show that the density operator (3) obeys the following equation of the quasipotential type:

$$(E_k-E)\rho_{\vec{c}}(t)-\rho_{\vec{c}}(t)(E_\varkappa-E')= \qquad (6)$$
$$=\vartheta\,\rho_{\vec{c}}(t)-\rho_{\vec{c}}(t)\,\vartheta^{+},$$

where $E_K = K^\circ + \sqrt{(\bar{c}-\bar{K})^2 + m^2}$ and the effective quasipotential is determined by taking an average of maniparticle interaction quasipotentials over all multiparticle wave functions (1). The equation (6) is a relativistic generalization of the equation for density operator in quantum mechanics and in its form coincides with the equation for the density operator of some fictious pure two particle state. However the quasipotential ϑ is not equal to true interaction quasipotential of meson and nucleon but is screened by presence of all the remaining mesons in the final state. In particulary the imaginary part of ϑ is not generally positive.

Solving the equation (6) we obtain with the help of (4) and (5) an expression for inclusive cross section in the fragmentation region $K_\perp \leq K$:

$$2K^\circ \frac{d\sigma}{d\vec{K}} = \frac{(2\pi)^5}{2E^2} \frac{\beta(E^2)}{(2-x)} \Phi(K_\perp, x), \; x = \frac{2K}{E} \quad (7)$$

It is clear that if inelastic cross section and average multiplicity are slowly changing function $\beta(E^2) = \beta_\circ E^2$ where $\beta_\circ$ also slowly varies. Thus we come to radial scaling experimentally observed. Notice that we derived this result from shortrange behaviour of strong interactions and relativistic invariance without using concrete form of the quasipotential ϑ .

In the region of large transverse momenta $K_\perp$ interactions take place mainly at small distances less than nuclear size. So the behaviour of effective quasipotential ϑ at origin plays a significant role. We assume that constituents have point-like character and so the interaction quasipotential is singular. In this case the inclusive cross section has the following form as $X_\perp \to 1$ /7/ :

$$2K^\circ \frac{d\sigma}{d\vec{K}} \cong (2\pi)^5 A \beta_\circ \frac{(1-X_\perp)^\nu}{K_\perp^n} \quad (8)$$

where A is a constant, the power index n is determined by quasipotential singularity and ν is related with behaviour at exclusive threshold.

In this scheme we come to the following interpretation: the initial nucleon is a composite system containing an arbitrary number of mesons. Each constituent (meson) is distributed inside nucleon according to the certain law. The production process is treated as a breakdown of the composite nucleon. It is no problem to extend the scheme for the case of any number of different kinds of particles.

Now we apply the density matrix formalism to the deep inelastic electroproduction of hadrons /8/ . In the one-photon approximation inclusive cross section is reduced to the total cross section of virtual photon absorption. We shall regard the virtual photon as a real particle with mass q^2 . As a result we come to the formula (7) for the inclusive photon--proton scattering where function $\Phi = \Phi(K_\perp, x, \omega')$, $\omega' = 1 - S_q/q^2$. The electroproduction cross section has the following form:

$$2K^\circ \frac{d\sigma}{d\vec{K}} = \frac{(2\pi)^5 \alpha^2 \beta(S_q)}{I \, t^2} \int_0^1 \frac{d\xi}{\xi} \tilde{\phi}(\xi, \omega') \quad (9)$$

where

$$\tilde{\phi}(\xi, \omega') = \frac{1}{2-\xi} \int \varkappa_\perp d\varkappa_\perp \Phi(\varkappa_\perp, \xi, \omega') \quad (10)$$

Thus the structure part of the electroproduction cross section depends only on scaling variable ω' . Notice that the formulas obtained here are more general than in parton model. In particular when $\beta(S_q) \sim S_q^{-2}$ and $\tilde{\Phi}(\xi, \omega') = \delta(\xi\omega' - 1) f(\xi)$ we come to the usual parton model /10/ .

How we consider the general case not restricting ourselves by one-photon approximation. For this purpose we can use directly the formulae (7). Using shortrange character of effective quasipotential we come to the following expression for electroproduction cross section in deep inelastic region

($S_q \to \infty$, ω' is fixed):

$$2K^0 \frac{d\sigma}{d\vec{K}} = \frac{(2\pi)^5 \rho(s)}{2I(2-x)} \left|T(s_c, t_c)\right|^2 \tag{11}$$

where

$$s_c = \frac{2m^2}{2-x}, \quad x = 1 - S_q/s \tag{12}$$

$$t_c = -\frac{m^2 x(1-x)}{2(2-x)(\omega'-1)} \tag{13}$$

The structure part of the cross section depends only upon two scaling variables x and ω' in accordance with the dimensionality analysis and the automodelity principle /9/ .

We assume now that electron interacts with separate constituents of proton through one-photon exchange so that electron interaction with proton as a whole remains multiphoton. Then for the amplitude T describing electron scattering on constituents we have the following standard form:

$$\left|T(s_c, t_c)\right|^2 = \frac{\alpha^2}{t_c(1-t_c/4m^2)} \Big\{ 4\left[(s_c-m^2)^2 + t_c s_c\right]\phi_E(t_c) - \frac{t_c}{m^2}\left[(s_c-m^2)^2 + s_c t_c - 2m^2 t_c + \frac{1}{2}t_c^2\right]\phi_M(t_c)\Big\} \tag{14}$$

where ϕ_E and ϕ_M are some structure functions analogies to the squared proton electromagnetic formfactors. We see that these structure functions depend only on one scaling variable

$$\zeta = \frac{x(1-x)}{(2-x)(\omega'-1)} \tag{15}$$

which is a combination of Bjorken's and Feynman's variables. Comparison of this new scaling law with experimental date is shown in Fig.1. The obtained result points to the fact that account of multiphoton exchange in deep inelastic scattering of electron on proton may lead to new scaling law for the cross section of this process.

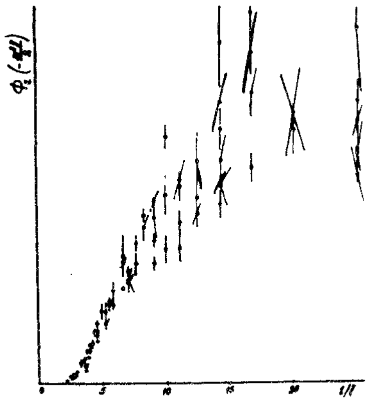

Fig.1

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INVESTIGATION OF THE POWER AUTOMODEL ASYMPTOTIC BEHAVIOUR OF THE LARGE-ANGLE HADRON-HADRON SCATTERING

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The investigation of the power automodel asymptotic behaviour of the large-angle scattering cross sections is, at present, an important problem of high-energy physics.

Here we consider the power decrease of differential cross sections at large angles in the framework of the Logunov-Tavkhelidze quasipotential approach [1].

The power decrease of large-angle differential cross sections discovered in recent experiments [2]

$$\frac{d\sigma}{dt} \sim \frac{1}{s^n} f\left(\frac{t}{s}\right) \qquad (1)$$

was treated on the basis of the principle of automodelity [3], i.e., on the basis of the assumption on absence of essential dimensional parameters defining the dynamics of interaction at small distances. By using the dimensional analysis and considerations on the composite nature of particles one may relate the degree n in (1) to the number of elementary constituents of hadrons. For instance, for the binary reaction $a+b \to a+b$ we have $n = 2(n_a + n_b - 1)$, where n_a and n_b are the numbers of elementary constituents (quarks) of hadrons a and b, respectively. In paper [4] the explicit form of angular dependence of differential cross section of the large-angle high energy scattering has been found for different processes on the basis of the dynamical interpretation of quark diagrams for two-particle scattering amplitudes which results in a generalization of the "quark counting" rules.

On the basis of these investigation results, in papers [5] the problem was raised concerning the structure of the local two-particle quasipotential which produces, in the case of large-angle two-particle scattering, the behaviour of type (1).

In this report the scattering both of spinless particles and of particles with spin is analysed in a unique way for the quasipotentials obeying the representation

$$\hat{g}(E;\vec{\Delta}^2) = g(E)\int_0^\infty dx\, \hat{\rho}(E;x)\, e^{-x\vec{\Delta}^2}, \quad t = -\vec{\Delta}^2 \qquad (2)$$

and being analytic functions of t in the half-plane $Re\, t \leq 0$. In representation (2) $\hat{\rho}(E;x)$ is a matrix defining the nature of interaction at high energies, with a rank dependent on the spin of particles. The main energy dependence in eq. (2) is factorized into $g(E)$ which depends on E by a power low. At x fixed $\hat{\rho}(E;x)$ is assumed to be slowly varying function of energy. Also it is supposed that for the density $\hat{\rho}(E;x)$ there exists the weak limit

$$\lim_{s\to\infty} s^N \hat{\rho}(E; x = z/s) = \hat{\varphi}(z) \qquad (3)$$
$$0 < z < \infty, \quad N > 0 .$$

Note that in describing the interaction of particles with spin we use essentially the requirement of γ_5 invariance of the interaction at high energies and large transfer momenta [6].

Quasipotentials (2) obeying condition (3) are called analytic. Later we obtain a formal representation for the large-angle high energy elastic scattering amplitude for particles with arbitrary spin within the quasipotential approach. For concrete scattering processes it results in the following asymptotic behaviour of the differential cross section of the high energy large-angle scattering

$$\frac{d\sigma}{dt}(s,t) \sim \left|e^{2i\chi(0)}\right|^2 \frac{1}{s^n} f\left(\frac{t}{s}\right),$$

where $\chi(0)$ is the value of the eikonal scattering phase at zeroth impact parameter. Recall that the eikonal function $\chi(\vec{b})$ defines the behaviour of the scattering amplitude at small angles. Thus, the result obtained allows one to establish certain correlation between asymptotic forms of the scattering amplitude at small and large angles.

Consider the quasipotential equation describing the interaction of particles with spin which is written in general form

$$\hat{G}(E;\vec{p},\vec{k}) = \hat{g}(E;\vec{p}-\vec{k}) + \int d^3\vec{q}\, \hat{g}(E;\vec{p}-\vec{q}) \times \frac{\hat{A}(E;\vec{q})}{E^2(\vec{q})-E^2-i0}\, \hat{G}(E;\vec{q},\vec{k}) \tag{4}$$

where $E=\sqrt{m_1^2+\vec{p}^2}+\sqrt{m_2^2+\vec{p}^2}$ is the total energy of particles in the c.m.s., $E(\vec{q})=\sqrt{m_1^2+\vec{q}^2}+\sqrt{m_2^2+\vec{q}^2}$, m_1 and m_2 are the masses of the first and second particles, respectively. $\hat{A}(E;\vec{q})$ is a matrix which form depends on the spin of interacting particles. We do not need here the explicit form of $\hat{A}(E;\vec{q})$.

Assuming the quasipotential is given by representation (2) we will solve eq. (4) by iterations

$$\hat{G}(E;\vec{p},\vec{k}) = \sum_{n=0}^{\infty} \hat{G}_{n+1}(E;\vec{p},\vec{k}), \quad \hat{G}_1 = \hat{g}(E;\vec{p}-\vec{k}).$$

On changing variables, $\hat{G}_{n+1}(E;\vec{p},\vec{k})$ can be transformed to the form

$$\hat{G}_{n+1}(E;\vec{p},\vec{k}) = \int\!\!\ldots\!\!\int dx_1\ldots dx_n \exp\Big\{t/\sum_{i=1}^{n+1} 1/x_i\Big\}\, \hat{\mathcal{J}}_n(x_1,\ldots,x_{n+1}) \tag{5}$$

where $\hat{\mathcal{J}}_n(x_1,\ldots,x_{n+1})$ is a 3n-dimensional integral. By using representation (5) we analyze the behaviour of the scattering amplitude at high energies and large angles, i.e., in the region $s\to\infty$, $|t|/s = \frac{1}{2}(1-z)$ fixed. In this limit the main contribution, to the asymptotic form of integral (5) comes from the region $1/\sum_i 1/x_i \sim 0$ otherwise we have the decreasing exponential function $\exp\{-|t|/\sum_i 1/x_i\}$. If condition (3) holds, the main contribution to the asymptotic form of (5) can be shown to come from the region where only one of $x_i \sim 0$. In this case we find the following formal expression for the scattering amplitude

$$\hat{G}(E;\vec{p},\vec{k}) = e^{i\hat{\chi}(0)\hat{B}(\vec{p})}\, \hat{g}(E;\vec{p}-\vec{k})\, e^{i\hat{B}(\vec{k})\hat{\chi}(0)} \tag{6}$$

with notations

$$\hat{\chi}(0) = \int_{-\infty}^{\infty} dz\, \frac{\hat{g}(E;\sqrt{z^2+\vec{b}^2})}{g(E)}\Big|_{\vec{b}=0}$$

$$\hat{B}(\vec{p}) = \hat{A}(E;\vec{p})\, \frac{g(E)}{16|\vec{p}|}$$

where $\hat{g}(E;\vec{r})$ is the Fourier transform of quasipotential (2).

Consider now the contribution of exchange forces for the scattering of different particles. The quasipotential is written in the form

$$\hat{V}(E;\vec{p},\vec{k}) = \hat{g}(E;\vec{p},\vec{k}) + \hat{h}(E;\vec{p},\vec{k}). \tag{7}$$

The quasipotentials $\hat{g}$ and $\hat{h}$ are called direct and exchange parts of the quasipotential $\hat{V}$. We assume for $\hat{g}$ representation (2) and for $\hat{h}$ the following representation

$$\hat{h}(E;\vec{p},\vec{k}) = h(E)\int_0^{\infty} dy\, \hat{\sigma}(E;y)\, e^{-y(\vec{p}+\vec{k})^2}. \tag{8}$$

We assume also that the densities $\hat{\rho}$ and $\hat{\sigma}$ obey the weak limits of the type (3) with identical N.

Furthermore, let the scattering amplitude $\hat{T}$ be represented as a sum of two quantities

$$\hat{T} = \hat{G} + \hat{H}. \tag{9}$$

Then inserting (7) and (9) into (4) we arrive at the system of quasipotential equations

$$\hat{G} = \hat{g} + \hat{g}\otimes\hat{G} + \hat{h}\otimes\hat{H} \tag{10}$$
$$\hat{H} = \hat{h} + \hat{h}\otimes\hat{G} + \hat{g}\otimes\hat{H}. \tag{11}$$

For the meson-nucleon scattering the

backward scattering peak is suppressed, i.e.,

$$\left| \frac{\hat{h}(E; |\vec{\Delta}_u| \sim 0)}{\hat{g}(E; |\vec{\Delta}_t| \sim 0)} \right| \to 0 \quad \text{at} \quad E \to \infty .$$

Therefore, when the quasipotentials $\hat{g}$ and $\hat{h}$ obey conditions (2),(8) and (3), the last term in eq. (10) may be neglected. In this way the system of eqs. (10) and (11) becomes uncoupled. The contribution of $\hat{g}$ has been analysed above, next, we consider only the contribution from the exchange part $\hat{h}$.

By using the method developed it can be shown that the main contribution to the asymptotic form of $\hat{H}$ at large $|u = -(\vec{p}+\vec{k})^2| \sim s$ comes from the region $y \sim 0$. Then

$$\hat{H}(E;\vec{p},\vec{k}) \simeq e^{i\hat{\chi}(0)\hat{B}(\vec{p})} \hat{h}(E;\vec{p}+\vec{k})\, e^{i\hat{B}(\vec{k})\hat{\chi}(0)}$$

Thus, with (6) taken into account, for the scattering of nonidentical particles the total scattering amplitude at large angles has the form

$$\hat{T}(E;\vec{p},\vec{k}) = e^{i\hat{\chi}(0)\hat{B}(\vec{p})} \left[\hat{g}(E;\vec{p}-\vec{k}) + \hat{h}(E;\vec{p}+\vec{k})\right] e^{i\hat{B}(\vec{k})\hat{\chi}(0)}$$

Now we proceed to calculate the scattering cross sections for concrete processes. First consider the scattering of two identical spinless particles. The quasipotential being a scalar function, with the exchange forces taken into account, is written in form (6), and the scattering amplitude can be represented by (9). With crossing symmetry, we have

$$T(E;\vec{p},\vec{k}) = e^{2i\chi(0)} \left[g(E;t) + g(E;u) \right]$$

and

$$\frac{d\sigma}{dt} \sim \frac{1}{s^2} \left| e^{2i\chi(0)} \right|^2 \left| g(E;t) + g(E;u) \right|^2 .$$

Consider now the scattering of a scalar particle on a spinor one. The quasipotential is taken in the simple γ_5 - invariant form with the densities

$$\hat{\rho}(E;x) = \gamma_0 \tilde{\rho}(E;x),$$

$$\hat{\sigma}(E;x) = \gamma_0 \tilde{\sigma}(E;x).$$

Passing to the differential cross section

$$\frac{d\sigma}{dt} \sim \frac{1}{s} \sum_{spin} \langle \bar{\psi}(\vec{p}) | T | \psi(\vec{k}) \rangle \langle \bar{\psi}(\vec{k}) | T^{+} | \psi(\vec{p}) \rangle$$

we get

$$\frac{d\sigma}{dt} \sim \frac{(1+z)}{s} \left| e^{2i\tilde{\chi}(0)} \right|^2 \left| \tilde{g}(E;\vec{p}-\vec{k}) + \tilde{h}(E;\vec{p}+\vec{k}) \right|^2$$

Here $\tilde{\chi}$ is eikonal meson-nucleon scattering phase

$$2i\tilde{\chi}(\vec{b}) = -\frac{1}{2ip} \int_{-\infty}^{\infty} dz\, V(E;\sqrt{z^2+\vec{b}^2}).$$

Analogously, one may consider the large-angle NN scattering. However, in this case we choose the eikonal phase as follows

$$\hat{\chi}(0) = \gamma_0^{(1)} \gamma_0^{(2)} \tilde{\chi}(0)$$

that implies the spin-flip amplitudes are small at high energies and fixed transfer momenta. The quasipotential is taken in the γ_5 - invariant form

$$\hat{V}(s,t) = \gamma_\mu^{(1)} \gamma^{\mu(2)} C(s,t) + \gamma_5^{(1)} \gamma_5^{(2)} \gamma_\mu^{(1)} \gamma^{\mu(2)} D(s,t).$$

As its expression is combersome, we omit it here (see /5/).

Now let us give a simple example of the use of the above formulae. Let the densities have zero of a finite order at $x \to 0$. Then one can find the following expressions for the differential cross sections of different processes:

A. For the scalar particles

$$\frac{d\sigma}{dt} \simeq \frac{A}{s^{2N+2}} \left| e^{2i\chi(0)} \right|^2 \left[\frac{1}{(1-z)^m} + \frac{1}{(1+z)^m} \right]^2$$

B. For the meson-nucleon scattering

$$\frac{d\sigma}{dt} \simeq \frac{(1+z)}{s^{2N+2}} \left| e^{2i\chi(0)} \right|^2 \left[\frac{A}{(1-z)^m} + \frac{B}{(1+z)^k} \right]^2$$

C. For the nucleon-nucleon scattering

$$\frac{d\sigma}{dt} \simeq \frac{\left| e^{2i\chi(0)} \right|^2}{s^{2N+2}(1-z^2)^{2m}} \left\{ |\alpha+\beta|^2 \left[(1+z)^{2m+2} + \right.\right.$$

$$+(1-z)^{2m+2}+4\left((1+z)^{m}+(1-z)^{m}\right)^{2}\Big]-$$

$$-2(\alpha\beta^{*}+\beta\alpha^{*})\left[(1+z)^{2m+2}+(1-z)^{2m+2}\right]\Big\}$$

The parameter in these formulae can be found from comparison with experimental data that has been performed for the $\pi^{\pm}p$ and pp scattering at energies $P_L \gtrsim$ 7 GeV/c and large angles.

The results of calculation are drawn in Figs. 1,2,3 and are in good agreement with experiment.

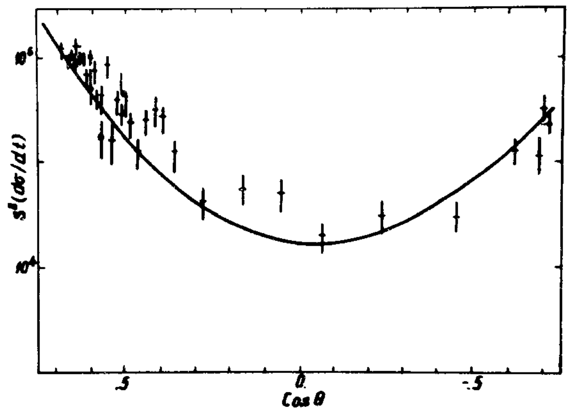

Fig.1. $S^8\left(\frac{d\sigma}{dt}\right)$ for π^+p reaction

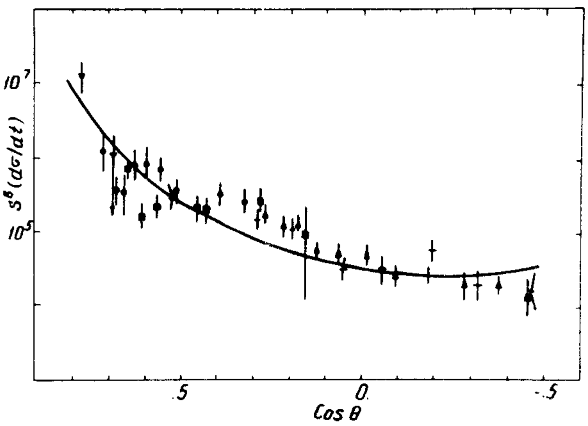

Fig.2. $S^8\left(\frac{d\sigma}{dt}\right)$ for π^-p reaction

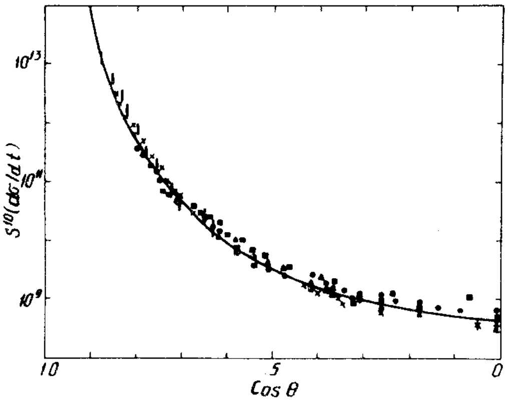

Fig.3. $S^{10}\left(\frac{d\sigma}{dt}\right)$ **for** pp **reaction.**

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HIGH ENERGY BEHAVIOUR OF GAUGE THEORIES

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In this report we discuss the high energy behaviour of gauge field theories with emphasis on its application to hadronic processes at the energies available at ISR and Fermilab. We shall be concerned only with the limit

$$s \to \infty$$

with all transverse momenta fixed. Here as usual s is the square of the center-of-mass energy. We shall discuss the following cases:

1. Abelian gauge theory (electrodynamics);

   A. Pomeron exchange

   B. Quantum number exchange

2. Non-Abelian gauge theory (Yang-Mills-theory /1/).

The present status in these cases is as follows:

		Result from gauge field theories	Phenomenology and Comparison with experiments
Abelian gauge theories	Pomeron exchange	V	V
	Quantum number exchange	V	
Non-Abelian gauge theories		V	

Here a check mark V means that a good deal of information is now known while a blank space means the almost total absence of useful information. It is clear from this table that an urgent problem is to develop a phenomenology for quantum number exchange on the basis of the high-energy behaviour of gauge theories.

The gauge theories are all studied theoretically on the assumption of summing the leading terms. We emphasize that this is a pure assumption and may well be misleading.

1A. Abelian gauge theory - Pomeron exchange.

The leading contributions in this case come from the tower diagrams. The result was first given by Cheng and Wu /2/ for scalar electrodynamics (SED) and slightly later by Frolov, Gribov and Lipatov /3/ for QED /4/ . In both cases, the result is, give or take a few $\ln s$,

$$i\, s^{1+A\alpha^2}$$

where A is positive, but different for SED and QED. This is the origin of "Pomeron above 1" (or $\alpha(0)>1$) extensively discussed at this Conference.

This result is interpreted as the saturation of Froissart bound /5/ and applied to hadron physics by Cheng and Wu /6/ to predict the infinitely rising total cross sections. Quantitative results are obtained by Cheng, Walker and Wu /7/ by an optical model with potential increasing as s increasing (now sometimes referred to as geometrical scaling). Subsequent measurements /8,9/ at ISR show that the actual increase in pp cross section is about 50% higher than this first phenomenomogical model. Using the ISR measurements on pp, the parameters of the model are improved /10/ and used to predict the total cross sections for $\pi^{\pm}p$, $K^{\pm}p$ and $\bar{p}p$. Fig.1 shows the theoretical prediction together with all the measurements available at that time.

In fig.2-4 we show the comparison of theoretical and experimental /11/ total cross sections /12/ . We emphasize that in the cases of fig. 3-4 the data were obtained much later than the theoretical prediction and the comparison supports the theme that results from quantum field theory are useful in the understanding of hadron physics.

There are many other theoretical predictions. Those that are verified include the existence of pionization plateau /13/ ;

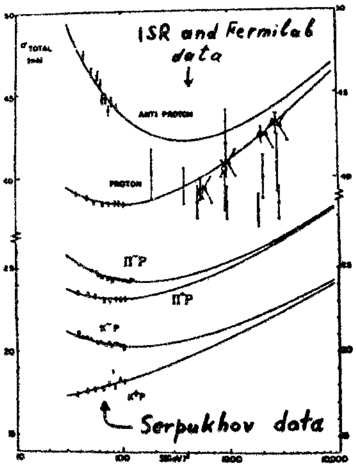

Fig.1. Experimental data from Serpukhov, Fermilab and ISR together with phenomenological results

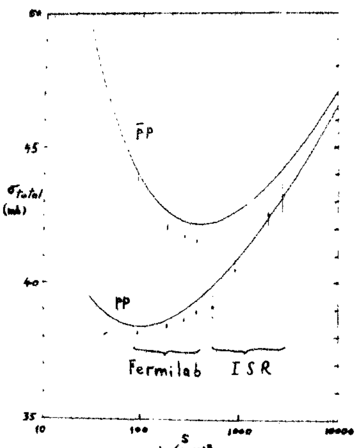

Fig.2. Comparison of theory and experiment for pp and pp total cross sections. The ISR data for pp were obtained before theory, while the Fermilab data for pp and pp are after theory.

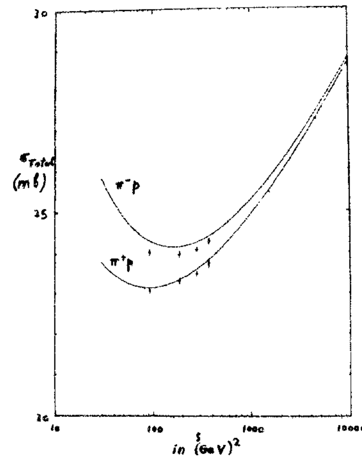

Fig.3. Comparison of theoretical predictions for $\pi^{\pm} p$ total cross sections with later experimental data from Fermilab.

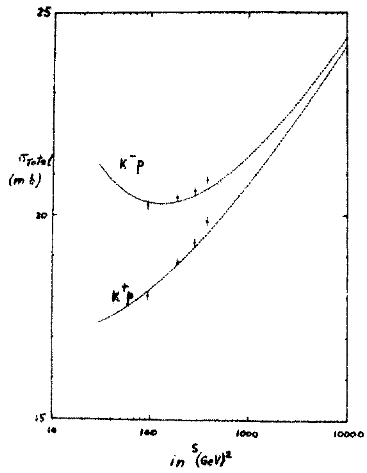

Fig.4. Comparison of theoretical predictions for $K^{\pm} p$ total cross sections with later data from Fermilab.

Real/Imaginary for forward scattering /14/; rising total elastic cross section /6,8/ ; movement of the dip inelastic scattering to smaller momentum transfer /6,15/ ; and most recently the rising pionization plateau discussed this morning by Prof.A.Bialaz. Predictions that remain to be tested include the photoproduction of ϕ and diffractive production of ρ^{*} /17/ .

1B. Abelian gauge theory – Quantum number exchange

Fermion exchange in QED was first studied by Gell-Mann et al. [18]. Under the assumption of summing the leading terms their result for the positive signature amplitude is correct, but they missed a sixth-order diagram in the negative-signature amplitude. The corrected result for this negative-signature amplitude shows a fixed cut with a simple pole coming out at a physical value of momentum transfer [19,20]. Therefore there is great similarity between quantum number exchange and Pomeron exchange [19].

2. Non-Abelian gauge theory

The rule of computation for Yang-Mills theory [1] was first given by Faddeev and Popov [21]. To avoid the complicated problem of infrared divergence, we introduce as usual Higg's particles [22]. The calculations are most conveniently carried in the R gauge of 't Hooft [23]. The study of the high energy behaviour was initiated by Nieh and Yao [24], but their result is qualitatively incorrect. The correct result in 6th order comes from about 20 diagrams and was first given by McCoy and Wu [25] and Lipatov [26]. Fadin, Kuraev and Lipatov [27], using an assumption of the multi-Regge form that is explicity verified in the eight order [28], obtains the sum of leading terms for Pomeron exchange. This result for SU(2) is

$$s^{1+g^2(\ln 4)/\pi^2}$$

again a Pomeron above 1. Therefore the similarity between the Abelian and non-Abelian cases is most striking. Fadin and Sherman [29] have treated the case of Fermion exchange in Yang-Mills theory. Both topics will be discussed by Professor Lipatov in the following talk.

I thank Professor A.A. Anselm and K.G. Chetyrkin, M.I.Sakvarelidze, M.A.Smondyrev for their most kind help in preparing this report.

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MULTI-REGGE PROCESSES AND THE POMERANCHUK SINGULARITY IN NON ABELIAN GAUGE THEORIES

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1. My talk is based on the results obtained by V.S.Fadin, E.A.Kuraev and muself /1/. I would like to tell you about our calculations of the high energy asymptotics in non-Abelian gauge theories. The papers of other authors on the subject are enumerated in ref. /2/. Our final expression for the elastic scattering amplitudes is obtained as a generalization of our results valid up to the eight order of perturbation theory. The result of the tenth order calculation by B.M.McCoy and T.T.Wu /3/ coincides with the corresponding term in the perturbative expansion of our final expression and therefore it supports indirectly our conjecture of a multi-Regge form of production amplitudes.

To study the Pomeranchuk singularity at $j = 1$ in perturbation theory one is obliged to involve vector particles. Renormalizable field models containing vector particles are based on gauge theories. In the simplest gauge theory quantum electrodynamics (QED) - the leading j plane singularity turns out to be a fixed branch point at $j = 1 + \frac{11}{32}\pi\alpha^2$ /4/. The violation of the Froissart bound here is a result of the fact that the photon is not reggeized in QED. More realistic models for the strong interaction may be based on the Yang-Mills fields with the Higgs phenomenon. In these theories the vector boson is reggeized as it was shown by many authors by studying several orders of the perturbation theory /2/ (the fermion is also reggeized - see ref. /5/).

Therefore the main reason for the violation of the Froissart bound in the leading logarithmic approximation is absent. But our result is a disappointing one: the leading plane singularity turns out to be to the right of the point $j=1$.

2. The field model we have used contains an isotriplet of vector fields with mass M and a scalar isosinglet φ . This model is obtained owing to the Higgs phenomenon from the theory with the Yang-Mills fields interacting with a doublet of scalar particles with negative m^2 .

We have calculated the asymptotics of elastic scattering amplitudes in different channels in the kinematical region:

$$s \gg M^2, \quad -t \sim M^2 \tag{Ia}$$

provided that

$$q^2 \ll 1, \quad q^2 \ln\frac{s}{M^2} \sim 1. \tag{Ib}$$

The calculation method is based on using the dispersion relations in s- and t-channels and unitarity conditions. Therefore it is necessary to know inelastic amplitudes $A_{2\to 2+n}$ to find the elastic one. We have verified in lowest orders of the perturbation theory that in the multi-Regge kinematical region the following expression holds for inelastic vector-vector scattering amplitudes:

$$A_{2\to 2+n} = s\,\Gamma^{C_1}_{A\to\mathcal{D}_0}\,\frac{\left(\frac{s_{12}}{m^2}\right)^{\alpha(t_1)-1}}{t_1-m^2}\,\gamma^{\mathcal{D}_1}_{C_2C_1}\,\frac{\left(\frac{s_{23}}{m^2}\right)^{\alpha(t_2)-1}}{t_2-m^2} \tag{2}$$

$$\ldots\frac{\left(s_{n+1,n+2}/m^2\right)^{\alpha(t_{n+1})-1}}{t_{n+1}-m^2}\,\Gamma^{C_{n+1}}_{B\to\mathcal{D}_n},$$

$$s_{i,i+1} \gg m^2, \quad -t_i \sim m^2,$$

where $s_{i,i+1}$ is the squared sum of the energies of the produced particles $\mathcal{D}_i, \mathcal{D}_{i-1}$ in their c.m. system, $C_i=1,2,3$ are the isotopic states of the virtual vector bosons with mass $\sqrt{t_i}$ and

$$\alpha(t) = 1 + \frac{g^2}{(2\pi)^3}(t-M^2)\int\frac{d^2K}{(K^2+m^2)\left[(q-K)^2-m^2\right]}, \tag{3}$$

$$q^2 = t.$$

For $n=0$ the relation (2) demonstrates the reggeization of the vector boson $(\alpha(M^2)=1)$. For $n \geqslant 1$ we checked the relation (2) in the Born approximation and by calculating first radiative corrections. In Eq. (2) the vertex functions Γ and γ have the following form:

$$\Gamma^{C_1}_{A\to\mathcal{D}_0} = g\frac{\sqrt{2}}{2}\,\delta_{AC_1}\,\delta_{\lambda_A 3}, \quad \gamma^{\mathcal{D}_1}_{C_2C_1} = \delta_{C_2C_1}\cdot g\cdot m \tag{4a}$$

the case of scalar particle production and

$$\Gamma^{C_1}_{A\to\mathcal{D}_0} = +i\,\varepsilon_{C_1 A\mathcal{D}_0}\sqrt{2}\,g\,\alpha_{\lambda_A}\,\delta_{\lambda_A\mathcal{D}_0}, \quad \alpha_{\lambda_A} = \begin{Bmatrix}-1, & \lambda_A=1,2\\ -1/2, & \lambda_A=3\end{Bmatrix} \quad \gamma^{\mathcal{D}_1}_{C_2C_1} = -i\,\varepsilon_{\mathcal{D}_1C_2C_1}\,g\Big[-(q_2+q_1) \tag{4b}$$

$$+p_A\left(\frac{p_{\mathcal{D}_1}p_B}{p_Ap_B}+\frac{t_1-M^2}{p_{\mathcal{D}_1}p_A}\right)-p_B\left(\frac{p_{\mathcal{D}_1}p_A}{p_Bp_A}+\frac{t_2-M^2}{p_{\mathcal{D}_1}p_B}\right)\Big]_\mu e_\mu^{\lambda\mathcal{D}_1}$$

in the case of vector meson production. Here $\mathcal{D}_i$ are the isotopic indices of the produced particles and $\lambda_{\mathcal{D}} = 1,2,3$ are their S channel polarizations ($\lambda_{\mathcal{D}}=3$ corresponds to a longitudinally polarized vector particle).

The multi-Regge region

$$s_{i,i+1} \gg m^2, \quad \prod s_{i,i+1} \sim s, \quad q_i^\perp \sim M \tag{5}$$

gives the main contribution to the s and u channel discontinuities corresponding to $(n+2)$ particle intermediate states. By using the dispersion relation in s channel and by summing contributions from all intermediate states we obtain:

$$f^{A'B'}_{AB}(s,t) = \sum_{T=0,1,2} s\int_{a-i\infty}^{a+i\infty}\frac{d\omega}{i}\left(\frac{s}{m^2}\right)^{\omega}\frac{(-1)^T-e^{-i\pi\omega}}{\sin\pi\omega}\,f^T_\omega(t)\,\gamma^{A'B'}_{AB}(T), \quad \gamma^{A'B'}_{AB}(T) = \left[\alpha^2_{\lambda_A}\cdot\delta_{\lambda_A\lambda_{A'}}C_T+\delta_{\lambda_A3}\delta_{\lambda_{A'}3}\right]\left[\alpha^2_{\lambda_B}\delta_{\lambda_B\lambda_{B'}}C_T+\frac{\delta_{\lambda_B3}\delta_{\lambda_{B'}3}}{4}\right]$$

$$\times C^{Tm}_{A'A}\,C^{Tm}_{B'B}, \quad C_T = 2-\frac{1}{2}T(T+1), \tag{6}$$

$$B_T = -\frac{5}{2}+\frac{3}{4}(T+1)T,$$

where T is the total isospin in the t channel and $C^{Tm}_{A'A}$ are the Clebsch-Gordan coefficients. t -channel partial waves $f^T_\omega(t)$ satisfy the following equation:

$$\left[2+\omega-\alpha(-K^2)-\alpha(-(q-K)^2)\right]\cdot f^T_\omega(K,q-K) = \frac{\omega g^2}{-A_Tq^2+B_TM^2}+\left(-A_Tq^2+B_TM^2\right)\frac{g^2}{2\pi^3}\times\int d^2K'\,\frac{f^T_\omega(K',q-K')}{(K'^2+m^2)((q-K')^2+m^2)}+A_T\frac{g^2}{(2\pi)^3}\int\frac{d^2K'\,f^T_\omega(K',q-K')}{\left[(K-K')^2+m^2\right]}\times$$

$$\times\left[\frac{K^2+m^2}{K'^2+m^2}+\frac{(q-K)^2+m^2}{(q-K')^2+m^2}\right]. \tag{7}$$

The solution of this equation for $T=1$ is

$$f_{\omega}^{T}(k,q-k)\Big|_{T=1} = -\frac{\omega g^2}{(q^2+m^2)(\omega+1-\alpha(q^2))} \quad (8)$$

which shows that the assumption (2) is self-consistent: we have a bootstrap scheme in which a multi-Regge equation gives in the j plane the assumed Regge behaviour.

For the cases $T=0$ and $T=2$ Eq. (5) leads to the j plane cut singularities resulting from two reggeized vector meson exchange. In the vacuum channel $T=0$ the Equation can be solved exactly in the region $k_{\perp}^2 \gg M^2$. The leading j plane singularity turns out to be a square root branch point at $j=1+\frac{2g^2}{\pi^2}\ln 2$. (In the general case of the gauge group $SU(N)$ the branch point is located at $j=1+N\frac{g^2}{\pi^2}\ln 2$).

Thus, the total cross section in the non-Abelian gauge theory increases as a power of s although the cross sections for the production of any finite number of particles decrease rapidly with energy owing to the multi-Regge form of inelastic amplitudes (2). The reason for the violation of the Froissart theorem is that the s channel elastic unitarity is not fulfilled in the leading logarithmic approximation (in Eq. (2) the contribution of the vacuum t-channel state should be taken into account when $g^2\ln\frac{s}{m^2} \gg 1$).

There is an interesting phenomenon valid in the leading logarithmic approximation. It can be shown that at large $g^2\ln\frac{s}{m^2} \gg 1$ the essential region of integration over $k_{\perp}^i$ grows with energy. If one modificates Eq. (7) by introducing an invariant charge $g^2(k_{\perp}^2)=\frac{g^2}{1+g^2\ln\frac{k_{\perp}^2}{m^2}}$ instead of the renormalized charge g^2 it results in the decay of the fixed branch point into moving poles (their total number is of the order of $1/g^2$) which, unfortunately, remain to the right of $j=1$.

It should be stressed that in our leading logarithmic approximation the vacuum singularities emerge as bound stated on only two Reggeons with $T=1$. One can hope, however, that an appropriate Reggeon field theory similar to the Gribov calculus with Reggeon vertices which can be computated by perturbative methods would lead to a selfconsistent theory compatible with the Froissart restriction.

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MULTIPARTICLE REGGE POLES AND THEIR TRAJECTORIES

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This talk is based on the three papers by A.G.Sedrakian and author /1-3/ .

It became usual to relate to the reggeon a sum of ladder diagrams though the $g\varphi^3$-theory gives its intercept near $j = 1$.

In this connection it always seemed attractive to provide the positive intercept by the inclusion of multiparticle states in the t-channel. Well known Mandelstam diagrams corresponding to multi-particle states in t-channel and giving the branch points in j-plane, of course, are not enough, as they refer to the multiparticle configurations where only "two-body bound" states are present. However, in the theory with attraction it is possible the formation of many-particle bound states. This problem was first considered in the field theory by McCoy and Wu /4/ . They showed, that the contribution of fig.1b type diagrams to the asymptotics dominates over the contribution of fig.1a type Mandelstam diagrams corresponding to the reggeon-particle cut.

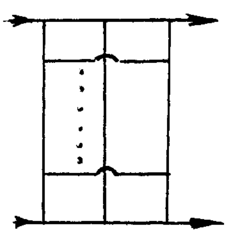

Fig.1a

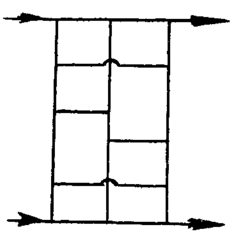

Fig.1b

We generalize here this important result for the case of an arbitrary number of particles in t-channel.

Let there be n_1+n_2 particles in t-channel, of which n_1 emit and n_2 absorb an arbitrary number of particles. Fig.2 gives the example of the considered diagrams.

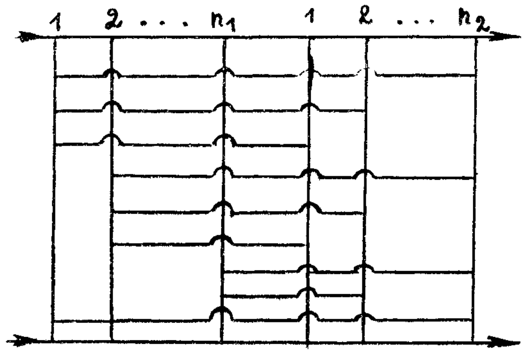

Fig.2

The detailed analysis shows that any line from n_1 group can be connected with any line of the n_2 group, giving thus the additional factor $g^2 \ln\frac{s}{m^2}$. The connections between one group lines are not essential giving the factor $\sim g^2$. Leading logarithmic asymptotics of the diagram of an elastic scattering with $n=n_1+n_2$ particles in the t-channel and L horizontal lines has a form ($\vec{q}$ is the transverse momentum):

$$\frac{-i\,(s/m^2)^{-n+1}}{(L+m_1)!}\, g^{2(L+n)} \left(\ln\frac{s}{m^2}\right)^{L+m_1} \mathcal{K}(\vec{q}),$$

$$\left(m_{1(2)} = \max_{(\min)}\,(n_1,n_2)\right). \qquad (1)$$

First, consider small $\vec{q}$ values $(|\vec{q}| < nm)$. Summing all the topologically different diagrams of the $2(L+n)$ order obtained by means of the rearrangement of horizontal lines as well as of all the possible crossings of vertical lines both within n_1 and n_2 group and the crossings of n_1 group lines with n_2 group ones, and summing over L we find the following asymptotics (for $\vec{q}$ =0):

$$-i\left(\frac{g^2}{m^2}\right)\left[K_0(2\gamma)\right]^{m_2-m_1}\left(\frac{2}{\gamma^2}\right)^{m_1-1}\left(\frac{1}{8\pi^2}\right)^{m_2-1}\times$$

$$\times\left(\frac{s}{m^2}\right)^{\alpha^{(n_1,n_2)}(0)}\delta_{n_1 n_2}, \qquad (2)$$

where

$$\alpha^{(n_1,n_2)}(0) = -n+1+n_1 n_2 \frac{g^2}{16\pi^2}\left(\frac{\gamma}{m}\right)^2 K_0^2(2\gamma) \qquad (3)$$

(γ - is an arbitrary constant connected with our inability to solve n - body problem exactly),

and

$$\xi_{n_1 n_2} = \left[1+\exp\left(i\pi\left(-1+\frac{g^2}{16\pi^2}\,\frac{\gamma^2}{m^2}\,K_0^2(2\gamma)\right)\right)\right]^{n_1 n_2} \qquad (4)$$

is a signature factor.

The most essential result here is the appearance in (3) of the term with $n_1 n_2$ factor, which means that, in principle, multiparticle states in t-channel may lead to a positive intercept in the field theory with spinless particles.

Formulae (2)-(4) show that we deal with $n_1 n_2$ effective ladders coalesced along all the length of their ribs. In spite of this coalescence each ladder preserves its individuality as it seem from structure of signature factor.

The siggularity (3) is a pole.

For $\vec{q} \neq 0$ $\left(|\vec{q}| \ll nm\right)$ we have:

$$\alpha^{(n_1 n_2)}(\vec{q}) = -n+1+\frac{g^2}{16\pi^2}\,\frac{\gamma^2}{m^2}\,K_0^2(2\gamma)\,n_1 n_2 \left[1-\frac{\beta\,\vec{q}^{\,2}\gamma^2}{n^2 m^2}\right] \qquad (5)$$

where β is constant of order of unity.

For the slope of multiparticle Regge pole trajectory it is easy to obtain:

$$\alpha'^{(n_1,n_2)} = \frac{4 n_1 n_2}{(n_1+n_2)^2}\,\alpha'^{(1,1)} \qquad (6)$$

where $\alpha'^{(1,1)}$ is the slope for simple ladder with two particles in t-channel.

For $|\vec{q}| \gg nm$ we have:

$$\mathrm{Re}\,\alpha^{(n_1,n_2)}(\vec{q}) \approx -n+1+n_1 n_2 n^2 \times \qquad (7)$$

$$\times \frac{g^2}{16\pi^2}\,\frac{1}{q^2}\,\ln^2\left(\frac{2|\vec{q}|}{nm}\right).$$

Thus, the observed multiparticle Regge poles realize the Mandelstam conjecture /5/ that multiparticle states in the t-channel is to provide the linearity of Regge trajectories. Figure 3 illustrates this fact.

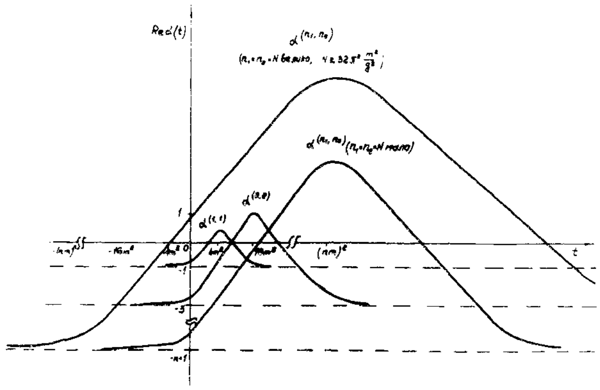

Fig.3

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UNIVERSALITY OF HADRON MULTIPLICITY IN COLOR-GAUGE THEORY MODELS

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We are accustomed to the idea that in e^+e^- annihilation the final state begins as a fast separating $q-\bar{q}$ pair, and that the final state hadrons are produced in order to prevent the appearance of quark quantum numbers. The multiplicity intuitively must increase as the relative rapidity of the q and $\bar{q}$ increases, This report describes briefly a model in which these ideas can be quantified and extended to other interactions such as deep inelastic and hadron hadron scattering.

First we recall the remarkable multiplicity universality studied by Albini et al./2/. They fit $pp\to X$ average multiplicity by

hadron hadron;

$$\langle n\rangle_{pp\to X} = 2.50 + .28\ln E_r + .53\ln^2 E_r \equiv f(s_r) \qquad (1)$$

where $E_r = \sqrt{s_r} = \sqrt{s} - m_a - m_b = \sqrt{s} - 2m_p$

(Note that a $\ln^2$ term is required to describe the full range of data). They then note that this same function, f, describes multiplicities in many other processes. In particular

e^+e^- annihilation;

$$\langle n\rangle_{e^+e^-\to X} = f(s),\ s = E^2_{CM}; \qquad (2)$$

and

deep inelastic;

$$\langle n\rangle_{ep\to e'X} = f(W^2),\ W^2 = 2M\nu - Q^2 \qquad (3)$$

among others.

Secondly we remind the reader that in all these cases, (1)-(3) the produced particles form jets. The collection of particles of limited p_T along the beam target axis in (1) is just as much a jet as similar collections observed in (2) and (3). We shall see that both the universality summarized in (1)-(3) and the jet structure is expected in a model where separation of color quantum numbers results in radiation of soft colored vector gluons, which in turn materialize into hadrons /2/.

We begin by abstracting the basic features of such a model and return to specific details momentarily. The multiplicity in such a model is a function of: a) the color quantum #'s of the separating objects; b) the relative rapidity $Y_{12} \approx 2p_1p_2/m_1m_2$ of the objects (labeled 1 and 2)

$$\langle n\rangle = f(\text{color } Q\ \#\text{'s; relative } Y_{12}) \qquad (4)$$

Flavour quantum numbers, etc., of the objects which are moving apart are irrelevant. In e^+e^- we know of course that one makes a 3 and $\bar{3}$ of color moving in opposite directions in the center of mass. Since $2p_3p_{\bar{3}} = s$ we have

$$\langle n\rangle_{e^+e^-} = f(3-\bar{3};\ s). \qquad (5)$$

In deep inelastic scattering the massive photon strikes quark in the incoming color singlet hadron forcing it to separate from the rest of the quarks in the hadron (the collection of which we call the "core"). If the hadron has momentum P, the struck quark has final momentum $xP+q$ $\left(\omega = \frac{2M\nu}{Q^2} = \frac{1}{x}\right)$ the core has momentum $(1-x)P$. Thus we establish immediately after the interaction a separating 3 and 3 of color with $2p_3p_{\bar{3}} = 2(xP+q)(1-xP) = W^2$. Hence

$$\langle n\rangle_{eP\to e'X} = f(3-\bar{3};\ W^2) \qquad (6)$$

is indepent of Q^2 at fixed W^2 as experimentally varified. Note that we have not needed to refer the "hole"-quark communication or equality of e^+e^- and hadron plateau heights in order to derive (6)[3]. Our one assumption is that the 3 or "core" state can be treated as a single radiating particle with a mass near that of the original bound state proton. As the momentum fraction $x \to 0$ the quark probed comes, in general from "sea" multi- $q\bar{q}$ pair states of the struck hadron's Fock space. Thus our core will consist of many quarks but all, we presume, with the same velocity and thus part of coherent state. This assumption may hold in a strong coupling limit but is quite orthogonal to standard multiperipheral approaches to the quark Fock space states. Additional arguments in support of the $x \to 0$ continuation appear in Ref.2.

Turning now to hadron-hadron collisions we may visualize (in the center of mass,e.g.) their interaction as occuring by annihilation of a low momentum quark in one hadron with a low momentum antiquark in the second hadron. Immediately after this annihilation the $\bar{3}$ and 3 cores at the respective hadrons are travelling in opposite directions along the collision axis, with essentially the full momenta of the hadrons. Thus $2P_3P_{\bar{3}} = s$ and the multiplicity of hadrons will be

$$\langle n \rangle_{HH \to X} = f(3\text{-}\bar{3};\, s) \tag{7}$$

In the case of hadron scattering it is clear that only $q\bar{q}$ annihilation leads simply to universality of multiplicity. For instance in the Low-Nussinov model of the Pomeron [4] the interaction between incoming hadron color singlets occurs via exchange of a colored (octet) gluon so that the incoming hadrons are transformed into rapidly separating color octets. Simple group theory yields

$$\langle n \rangle_{HH \to X} (\text{gluon exchange}) = f(8\text{-}8;\, s) \simeq \frac{9}{4} f(3\text{-}\bar{3};\, s), \tag{8}$$

so that at the same s (corrected for mass effects as below (1)) $\langle n \rangle_{HH \to X} \neq \langle n \rangle_{e^+e^- \to X}$ contrary to experiment. In the same model one expects the multiplicity in deep inelastic processes to change by a factor of $\frac{9}{4}$ at fixed W^2 as one goes from the valence dominated region $(x > \frac{1}{2})$, where $3\text{-}\bar{3}$ separation chearly occurs, to the "Pomeron" region $(x < .1)$, in which the gluon exchange (present in HH) should dominate.

Turning now to specific details we wish to calculate the multiplicity of colored gluons radiated from a separating 3 and 3 of color. This problem is entirely analogous to soft photon radiation from separating charges [5]; if we presume that the n gluon cross section follows a poisson, as in the usual QED soft photon-problem, then we may calculate $\langle n \rangle_{\text{gluons}}$ from lowest order diagrams. The result is (in the c.m. frame with cos angle of gluon w.r.p. to the 3)

$$\langle n \rangle_{\text{gluons}} \propto \int \frac{dk}{k} \int \frac{d\cos\theta\, (\beta_3 + \beta_{\bar{3}})(\cos^2\theta - 1)}{(1-\beta_3\cos\theta)^2 (1+\beta_{\bar{3}}\cos\theta)^2}. \tag{9}$$

Eq. (9) has two logarithmic singularities; one arises from $\cos\theta = \pm 1$ and the second from the $\frac{dk}{k}$ photon momentum integral. The $d\cos\theta$ integral in fact gives directly the relative $3\bar{3}$ rapidity (i.e a plateau in $\frac{dn}{dY_{3\bar{3}}}$) while the $\frac{dk}{k}$ integral can cause the plateau to rise as indicated by (1). The result is (for $P_3 P_{\bar{3}} \to \infty$)

$$\langle n \rangle_{\text{gluons}} = \frac{4}{3} \frac{\alpha_s}{\pi} \left\{ \ln \frac{2P_3P_{\bar{3}}}{m_3 m_{\bar{3}}} \left(\ln \frac{2P_3P_{\bar{3}}}{k^2_{min}} + C_1 \right) + C_2 \right\} \tag{10}$$

where the infrared cutoff k_{min} is a natural result of the fact that long wave length gluons cannot give rise to color singlet hadrons if $\lambda > R_{\text{hadron}}$. For $\lambda > R_{\text{hadron}}$ all the final hadrons appear to be "neutral" and the gluon coupling vanishes.

We, of course, imagine that the emitted gluons turn into quark-antiquark pairs which then match up to form hadrons. Thus

$$\langle n \rangle_{hadrons} \approx \langle n \rangle_{gluons} \tag{11}$$

and by using the fit (1) and the theoretical form (10) we obtain $\alpha_g \approx$ 0.46. There are several other interesting features of this theory but we mention only one. Consider the fragmentation regions in hadron hadron collisions. In our approach particles with substantial x_F do not arise from the soft gluon emission subsequent to the $q\bar{q}$ annihilation (which defines the interaction). Rather they are most probably fragments of the fast quark cores which appear immediately after the annihilation. The dimensional counting rules then make a specific prediction for the x_F dependence as $x_F \to 1$. In general

$$F_{H_1 \to H; H_2}(x_F) \propto g_{H/H_1 \, core} \propto (1-x_F)^{2n_s - 1} \tag{12}$$

where $g_{H/H_1 \, core}(x)$ is the probability for the core of H_1 to emit a secondary H with fraction x of its momentum and n_s is the minimal number of spectator quarks left over after the emission. For example consider $pp \to \pi^+ + X$. The minimal core following interaction capable of emitting a π^+ consists of a (pn) quark pair, Upon emitting a π^+ the minimal spectator system is a pair of (nn) quarks. Thus $n_s = 2$ and $F_{p \to \pi^+; p}(x_F) \propto (1-x_F)^3$ independent of t and p_T . Experimentally the power is $\approx$ 3.5 and is in fact independent of t and p_T . The tripls Pomeron theory does not yield this result in any natural way. Note also that in Low's Pomeron model the pp interaction leaves a minimum of three quarks continuing in opposite directions along the beam target axis. Fragmentation into a π^+ then leaves behind a least 3 spectators for which $F_{p \to \pi^+; p}(x_F) \propto (1-x_F)^5$ a somewhat high power for agreement with existing data.

Thus while no formal field theory has been solved which realizes all of our key assumptions, the basic picture provides both a qualitative and quantitative explanation of all the basic features and relations between hadron production in the full variety of experimentally accessible processes.

References

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P L E N A R Y   R E P O R T

HIGH ENERGY THEORY OF STRONG INTERACTIONS

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Development of strong interaction theory at high energies can be overlooked by following two supplementary directions: surveying the current theoretical approaches and models- on the one hand,and analysing recent experimental results which give us information about the dynamics of strong interaction- on the other hand.This formidable task can be solved only by all the speakers summarizing their efforts in discussions at parallel sessions AI-A5.

The contents of this talk are arranged as follows:

I. General results in QFT
II.Theory of diffraction scattering and the vacuum exchange
III.Regge/quark analysis
IV. High energy scattering in models of QFT
V. Constituent theory of hadron interactions
VI. Power laws
VII. Scaling and similarity laws
VIII. Concluding remarks

I.GENERAL RESULTS IN QFT

I.QFT as a guide to the theory of strong interactions.

The general principles and results of the local QFT are the basis of the most of theoretical constructions and phenomenological analysis in studying strong interaction phenomena at high energies.

One of the most successful methods through the years, was the method of dispersion relations introduced into the QFT by Gell-Mann,Goldberger and Thirring.

In the papers by Bogolubov on the theory of dispersion relations the fundamental idea of scattering amplitude as a unique analytic function of its kinematical variables has been introduced for all the physical channels connected by crossing symmetry relations. This concept proved to be very useful in attempts to understand the existing theoretical schemes and phenomenological approaches as possible approximations to the theory of strong interactions.

Use of the analyticity of amplitudes as functions of momentum transfer related to the short range character of nuclear forces has been particularly fruitful in the study of strong interactions at high energies. First of all, this has led to the derivation of a number of fundamental asymptotic relations and bounds on the cross sections. This concept has also served as an adequate tool for introducing the Regge ideas to QFT and for developing the quasi-optical picture of the high energy hadron scattering processes.

The most crucial problem of the field theory approach to the theory of strong interactions nowdays is the problem of consistent incorporation of the idea of the composite quark structure of hadrons.

2. Asymptotic bounds and theorems

As is well known, analytic properties of scattering amplitudes in transferred momentum lead to the ultimate relation between high energy behaviour and t-channel singularities. A number of new results have been recently found in that direction.

In the paper by Logunov,Mestvirishvili et al.[I] submitted to this conference, the question has arisen:"Do u-channel singularities in the complex t-plane influence the high energy behaviour of the forward scattering cross sections?" The basic assumptions made in this paper are as follows:

I. Existence of dispersion relations in t at $s > s_{thr.}$ with a finite number of subtractions.

2. Polynomial boundness at large s.

The answer given by the authors is no! Namely,the relative contribution of the u-channel singularities is bounded asymptotically by

$$\left| \frac{T_u(s,\theta=0)}{T_t(s,\theta=0)} \right| \lesssim \frac{\ln^2 s}{s}$$

in the case of the non-decreasing total cross sections.

This result which is based essentially on the dispersion relations and unitarity gives a rigorous extension of the principle of nearest singularity dominance from the high energy behaviour of absorptive part:

$$\mathrm{Im}\, T(s,t=0)= s\,\sigma_{tot}.$$

to the complete amplitude

$$T(s,t=0)=s \left(\frac{d\sigma}{dt}\right)^{\frac{1}{2}}_{t=0} .$$

The result has been improved in the work by Rcheulishvili and Samokhin[2] where the more restricted domain of analyticity in the complex Cosθ-plane was considered (see Fig.I).

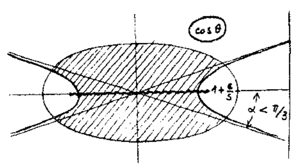


Fig.1.The analyticity domain in CoS -plane as being assumed in the paper[2] (dashed region).

It is argued that scattering in the forward (backward) cones at high energies is determined by singularities which lie in the right(left) half-planes of $\cos\theta$.

Interesting results are found in the recent paper[3] titled as "Why is the diffraction peak a peak?" An attempt is made here to find rigorous and general (as much as possible) answers to a number of questions concerning the asymptotic characteristics of the diffraction peak, namely, whether:

(?) the high energy differential cross section has a maximum exactly in the forward direction;

(??) the slope of the diffraction peak is determined by the absorptive part of scattering amplitude;

(???) the slope parameter is really bounded by $\log^2 s$, etc.

The positive answers to all the questions listed above were found in this paper. Moreover, it was pointed out that the oscillating cross sections are allowed, that the real part of scattering amplitude should not be small and so on.

In two papers by Mnatsakanova and Vernov(jr.)[4] submitted to this conference, a justification of a number of rigorous asymptotic bounds on scattering amplitudes and cross sections is given which are effective on finite energy intervals.

An analysis of analytic properties of scattering amplitudes in local QFT and a review of available axiomatic restrictions on high energy hadron scattering are presented in papers[5].

2. Quasi-local dispersion relations

Evaluation of the real part of a scattering amplitude in terms of an integral over the imaginary part by means of the dispersion relations is a formidable task because of the essentially non-local character of these relations. The quasi-local form of these relations or the derivative analyticity relations have attracted general attention in the last few years[6]. It was found that the DAR provides a simple and effective approximation for the conventional dispersion relations of the real part calculations at high energies if the threshold and resonance effects are unimportant. Moreover, it was proposed to use the DAR as a tool for complete reconstruction of a scattering amplitude from experimental data at asymptotic energies.

In the paper by V.A.Mescheryakov et al.[7] submitted to this conference, another form of quasi-local dispersion relations was presented which is based on the method of uniformization of scattering amplitude at high energies (or UM). It was proposed to use as a uniformization variable the eq.

$$w(\nu)=\frac{I}{\pi}\arcsin\nu=\frac{I}{2}+\frac{i}{\pi}\ln|\nu+\sqrt{\nu^2-I}|,$$

where $\nu=(s-u)/4m$ and $\mu=I$. At $\nu\gg I$, $w(\nu)\sim-\frac{I}{i\pi}(\ln s-i\pi/2)$ which points to complex logarithmic structure of the Riemann surface of a forward scattering amplitude as a function of the energy variable ν.

The function $w(\nu)$ gives a mapping of the whole Riemann surface of a scattering amplitude on the complex w-plane by a system of strips (Fig.2). The image of the physical sheet is the strip: $-\frac{1}{2}<\mathrm{Re}\,w<\frac{1}{2}$. The forward scattering amplitude determined by the formula $\mathrm{Im}F_\pm=(\sigma\pm\bar{\sigma})$ has the kinematical poles at $w=n+\frac{1}{2}$, so as $F_\pm=f_\pm/\sqrt{\nu^2-I}=\frac{f\pm(w)}{i\cos\pi w}$

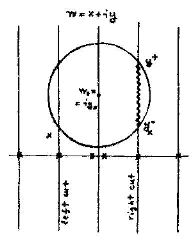


Fig.2. The map of the Riemann surface of the forward scattering amplitude. x - position of poles.

The analyticity and the symmetry relations

$$F_\pm(w)=\pm F_\pm(-w)$$
$$F_\pm^*(w)=-F_\pm(w^*)$$

lead to the generalization of DAR:

$$F_\pm^*(w)=\mp e^{-2x\frac{d}{dw}}F_\pm(w),\quad(w=x+iy).$$

The method has used as a tool for analysis of experimental data. Assume that $F=u+iv$ is a holomorphic function within the circle $|w-w_0|\leqslant R$ (see Fig.2) so that $V(x,y)$ is harmonic there, i.e. $\Delta V=(\partial_x^2+\partial_y^2)V=0$. By the theory of analytic function[8]

$$F(w)=2iV\left(\frac{w+w_0}{2},\frac{w-w_0}{2i}\right)+F^*(w_0).$$

Determining $V(x,y)$ by its decomposition in the convergent (in the circle) series

$$V(x,y)=\sum_{n=I,..}\sum_{m\geq 2n-1}a_m\frac{(-)^{n+I}(m-I)!}{(m+I-2n)!}(y-y_0)^{m+I-2n}\times$$

$$\times\begin{cases}\dfrac{x^{2n-2}}{(2n-2)!} & (\text{for } F_+)\\[2ex] \dfrac{x^{2n-I}}{(2n-I)!} & (\text{for } F_-)\end{cases}$$

one gets the decomposition of a forward scattering amplitude in powers of $y(\approx\log{}^{s}/_{\pi})$ convergent in some finite interval of energies:

$$y^-\leqslant y\leqslant y^+\ ;\ y^\pm\simeq\frac{I}{\pi}\log 2\nu_\pm=y_0\pm(R^2-\tfrac{1}{4})^{\frac{1}{2}}.$$

By fitting experimental data on πp, Kp and pp-scattering (with the only parameters $a^{+}_{I,2,3}$ and a^{-}_{I} and $c=F_{-}(iy_0)$ the authors have tested a number of quark relations among the total cross sections. It was found that Lipkin's sum rule

$$^{2}/_{3}\sigma_{tot}(\pi^{-}p)+{}^{1}/_{3}\sigma_{tot}(\pi^{+}p) = {}^{1}/_{2}\sigma_{tot}(K^{+}p)+{}^{1}/_{3}\sigma_{tot}(pp)$$

has to be hold with 3% accuracy from P=I0 GeV/c up to $I0^3$ GeV/c .

T A B L E

Test of Lipkin's sum rule(L and R are the left- and right-hand sides of the relation)

	(L)	(R)
p_L (GeV/c)	L (mb)	R (mb)
200	24.14±0.05	23.69±0.07
400	25.05±0.05	24.45±0.08
600	25.83±0.06	25.13±0.10
800	26.48±0.07	25.71±0.12
1000	27.04±0.08	26.23±0.14
1200	27.54±0.09	26.69±0.16
1400	27.99±0.09	27.10±0.18
1600	28.40±0.10	27.48±0.20
1800	28.78±0.11	27.83±0.21
2000	29.13±0.12	28.15±0.23

Works with 3% accuracy from P=I0 up to $I0^3$GeV/c

II. THEORY OF DIFFRACTION SCATTERING AND THE VACUUM EXCHANGE.

I. Models of the fast growth

Five years ago the flattering out of the hadron total cross sections and the growth of σ_{K^+p} with energy have been observed in experiments at IHEP and named "Serpukhov's effect". The foregoing experiments at CERN and FNAL have shown universality of the total cross section energy growth[9].

This effect which is not quite unexpected due to the Froissart theorem known from I96I, did not explain nevetherless, by the popular at that time, the theory of complex angular momenta in its original form. So many different models have appeared, namely: field theory models, quasi-optical models, regge-eikonal models and others, where different approaches in the description of the total cross section energy growth as well as of the whole range of diffraction phenomena were developed.

Notice that a simple and physically lucid description of the cross section energy growth is given in the framework of the hadron scattering quasi-optical picture[I0].

It was known for a long time that the energy dependent smooth local quasi-potential allows one to reproduce the main features of the high energy hadron scattering. The polynomial growth of the strength of interaction depending on the energy leads to the maximal energy growth of a total cross section permitted by the s-channel unitarity [II].

In the papers by Khrustalev and Tyurin [I2] and Tyurin et al. submitted to this Conference, the quantitative description of the total cross section energy growth as well as of other observable quantities of the high energy πN-scattering was given on the basis of the U-matrix method. The U-matrix is related to the scattering amplitude T by the relativistic one-time dynamical equations

$$T= U + \int\limits_{\text{phase volume of real particles}} U \cdot T \, dw$$

proposed by Logunov, Khrustalev, Savrin and Tyurin in I970[I3]. By analytic continuation into the t-channel $U\to\tilde{U}$ and using the Regge-pole expansion for $\tilde{U}_-$, the authors of the paper[I2] developed a method for description of the high energy scattering which allows one to take into account the unitarity in the direct s-channel (by construction). Thus, a good fit to the experimental data is found with polynomially growing $\tilde{U} \sim s^{\alpha}$ where $\alpha > I$ without contradicting the Froissart theorem (see Fig.3).

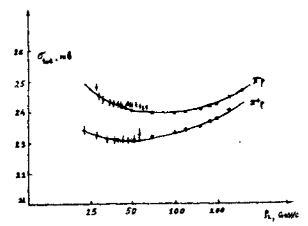


Fig.3.
The total cross-section of $\pi^{\pm}$ p scattering in the U-matrix approach.

One of the first detaild analyses of an assumption that the high energy hadron scattering observed at Serpukhov and also at CERN and FNAL realize the maximal energy growth of the scattering amplitudes, was made in the papers by L.D.Soloviev [I4] and Soloviev and Schelkachev[I5]. The authors introduced there the term "model of the fast growth". Determining the complete scattering amplitude as a sum

$$T = T_{Froissart} + T_{Regge} \, ,$$

where the "Froissart" part at finite energies has the form

$$T_{Froissart} \sim i\, sR^2 \exp(xt/2) \cdot J_1(Rq)/_{Rq}; \; q=\sqrt{-t}$$

$$R^2 = \rho^2 + R_0^2(s) \; ; \; R_0^2(s) = \frac{I}{M^2} \log^2 s/_{s_0} \; ,$$

the authors have found a good description of existing experimental data with the universal energy growth for all the hadronic total cross sections (excluding pp-scattering cross sections)

$$\sigma_{tot} \to 2\pi R_0^2 = c \log^2 s/s_0$$

The fit gives c (0.2 ÷ 0.4)mb or for an effective mass of exchange states M≈(I.3 ÷ 3.0) Gev/c^2 (too high as compared with the pion mass). The authors state that B is too small to be observed at available energies due to the kinematical suppressing the $\log^2 s$ term (see Fig.4 a,b- illustrating the fit)

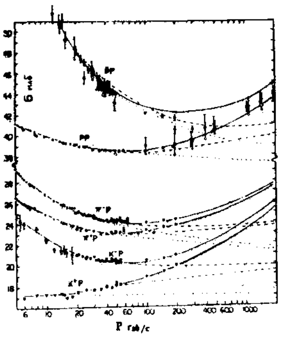


Fig.4a. Total cross sections in the model of "fast growth".

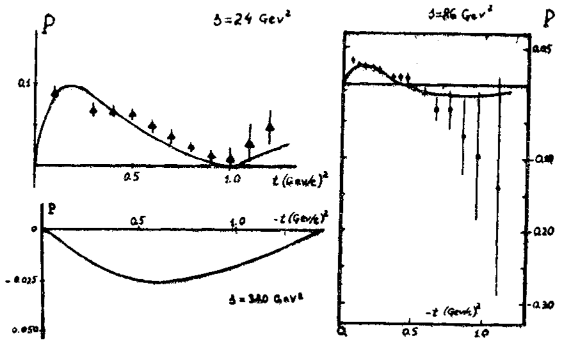


Fig.4b. Polarization in pp-scattering.

An analysis of the high energy hadron scattering in terms of j-plane leading singularity with $\alpha(0)>$ I was performed in the papers by Dubovikov and Ter-Martirossian[I6], Kopeliovich and Lapidus[I7] and others[I8], who used s-channel unitarization by means of eikonal representation.

2.Nature of the vacuum exchange

Idea of the Regge-poles proved to be very fruitful in the analysis of the high energy hadron reactions. There is a clue of complex problems of the Regge-approach still waiting for their solution. One of them is the nature of the vacuum exchange(the Pomeranchuk singularity). From the time when it was postulated in order to explain the gross features of diffraction scattering,especially the miraculous constancy of total cross sections the term "Pomeron" is still an enigma.

This notion is surrounded by a constellation of different effects, such as: self-consistent j-plane singularity or Reggeon field theory(RFT), critical phenomenon in RFT,Goldstone particle, "cylinder"topology contribution in dual theory, multiparton fluctuations, "bag" model (with colored gluon exchange), "glue balls" and many others [I9-26].

Some of these approaches were discussed at parallel session A5. This rich circle of questions is beyond of scope in our talk and will be discussed by the following speakers(see Kaidalov's talk) Here we only briefly mention some points. In the paper by Sugar et al[20], the Reggeon field theory is studied in zero transverse dimensions,i.e.,at d=0. By means of renormalization group the existence of the infrared stable fixed point is found. It is pointed out that the solution has no bona fide second order phase transition.[2I]

In the paper by Dyatlov[24] a theory of the weak-coupled Pomeron as a Goldstone particle was presented. The bound states of two Pomerons are found in this theory which can be useful in explanation of the diffraction peak structure. Unfortunately,this leads to the non-analyticity in transferred momenta at t=0.

The RFT on a lattice for $\alpha(o)>$I is studied in the paper by Amati et al.[25] Using the low-lying structure of an excitation spectrum the existence of the critical phenomena(second order phase transition) is found which is characterized by the value of α_c more than unity. The final asymptotic picture is that of a grey disc expanding like logs so that the total cross sections grow like $(\log s)^2$, if $\alpha(o)>\alpha_c$. For $\alpha(o)<\alpha_c$ the total cross sections drop as the universe powers of s.

In the paper by Matinyan et al.[26] the Regge-pole singularities associated with the multiparticle states in t-channel are studied and their role in solving the Mandelstam's program of the linear Regge trajectories is discussed.

III.REGGE/QUARK ANALYSIS

I. Regge behaviour vrs.SU_3/quark structure

Merger of the idea of the Regge(power-like) behaviour with the quark constituent theory,dual model and internal symmetries gives an effective tool for analysis of experimental data.

In the last years a considerable attention has been attracted to the study of the energy dependence of hadron scattering versus SU_3/quark structure of the scattering amplitudes.

Differents authors give various results for the dominant power term $s^{\alpha_p(0)}$ (which might be the case either $\alpha_p(0)=1$ or $\alpha_p(0)>1$ or even $\alpha_p(0)<1$) as well as for the SU_3 structure of the Pomeron couplings (different mixtures of the singlet and octet components).

The reason for such wide variations of possible values of the leading singularity position can be illustrated by Fig.5 from the paper [27].

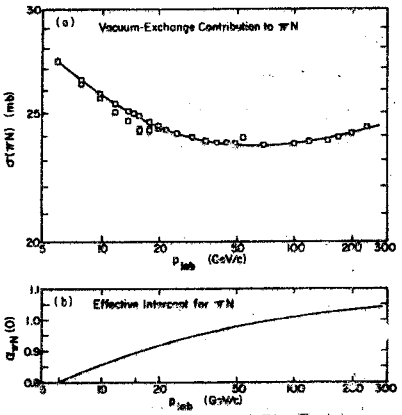


Fig.5(a). The vacuum-exchange contribution to πN total cross sections $\frac{1}{2}(\sigma_t(\pi^+p)+\sigma_t(\pi^-p))$

$\alpha_{\pi N}(0)>1$ at $P_{lab}>70$ GeV/c.

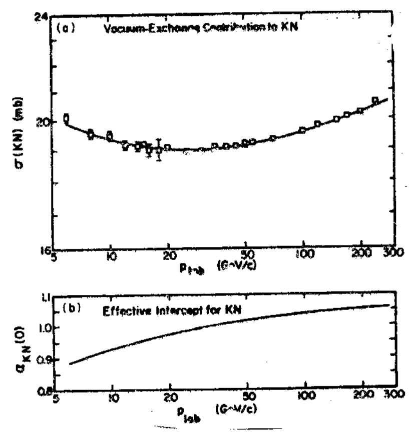


Fig.5(b). The vacuum-exchange contribution to KN total cross sections $\frac{1}{4}[\sigma_t(K^+p)+\sigma_t(K^+n)+\sigma_t(K^-p)+\sigma_t(K^-n)]$

$\alpha_{KN}(0)>1$ at $P_{lab}>30$ GeV/c.

An effective power of energy dependence is determined by

$$\alpha_{eff}(P_{lab}) = \frac{\partial}{\partial \log P_{lab}}[\mathrm{Im}\,T(P_{lab},t=0)]$$

which gives different values of $\alpha_p(0)$ for various energy intervals. This is in the qualitative accordance with DAR which relates the effective exponent α_{eff} to the ratio of the imaginary-to-real parts of scattering amplitude, i.e.

$$\alpha_{eff} \approx 1 + \frac{2}{\pi}\rho(P_{lab},t=0), \qquad \text{(DAR)}$$

So, α_{eff} exceeds unity where ρ is negative. The detailed analysis shows the dependence of a spectrum of Regge singularities with the vacuum quantum numbers on the assumed SU_3 or quark structure of hadron scattering amplitude.

2. SU_3- structure of the vacuum exchange

One usually describes the vacuum exchange amplitude as the sum of the Pomeron(diffraction) part and the f-pole(resonance-like or dual) part

$$\text{V.E.} = s^{\alpha_p}\hat{p} + s^{\alpha_f}\cdot\hat{f}$$

The SU_3 assignment is determined in the TABLE

		SU_3 - content: 1	8
Regge terms	p	p_1	p_8
	f	f_1	f_8

Different possibilities for the SU_3-structure and the spectrum of Regge singularities in the vacuum exchange amplitude were considered [27-33]:

* Harari & Freund - P + R(f,f');
* Carlitz, Green, Zee - (1971) P coupled trough f,f' $\alpha_p(0)=1$;
* Bali, Dash (1974) Chew, Rosenzweig (1975) $p=f_0$, f'; $\alpha_p(0)\approx 0.85$
* Lipkin (1974) (p_1,p_2) + R, $\alpha_{p_1}\approx 1.13$: $\alpha_{p_2}\approx 0.8$
* Quigg, Rabinovici (1975) p($\hat{1}+\hat{8}$) + f_0(ideally mixed) $\alpha_p \approx 1.075$
* Others.

No unique solution exists!

We mention here general trend, however. The SU_3 structure of the vacuum exchange depends on the transferred momentum (the Pomeron as a "chameleonic" object[31]).

For t-large and positive, p ideally mixed Regge pole: associated with planar diagrams (in terms of Veneziano's topological expansion[34]). The notion of asymptotic planarity[35] arises related closely to the problem of OZI-rule[36] (Freund, Nambu, 1975[37]).

For t-negative and large enough ($-t \geq 0.5$ GeV/c^2) p is dominated by SU_3-singlet. This can be illustrated by Fig.6 from ref.[27], where the singlet (s) and octet(0) components of the non-forward scattering amplitudes for the πp- and Kp-systems are determined by

$$\frac{d\sigma}{dt}(\pi p) \approx |s+20|^2 \approx |s|^2 + 4\mathrm{Re}(s^*0),$$

$$\frac{d\sigma}{dt}(Kp) \approx |s-0|^2 \approx |s|^2 - 2\mathrm{Re}(s^*0).$$

An interesting question arises: at how large t will p be dominated by SU_4-singlet term(if any)? A simple estimation gives the comparatively low value:

$$-t_{cr} \sim \ln \frac{\sigma^2_{tot}(pp)}{\sigma^2_{tot}(p\psi)} \Big/ (b_{pp} - b_{p\psi}) \sim \mathrm{I\ GeV/c^2}!$$

Fig.6 (a)(b) The singlet- octet contributions to $d\sigma/dt$

(a) SU_3 - singlet part of vacuum exchange

(b) $Re(S^*O)/|s|$ octet part

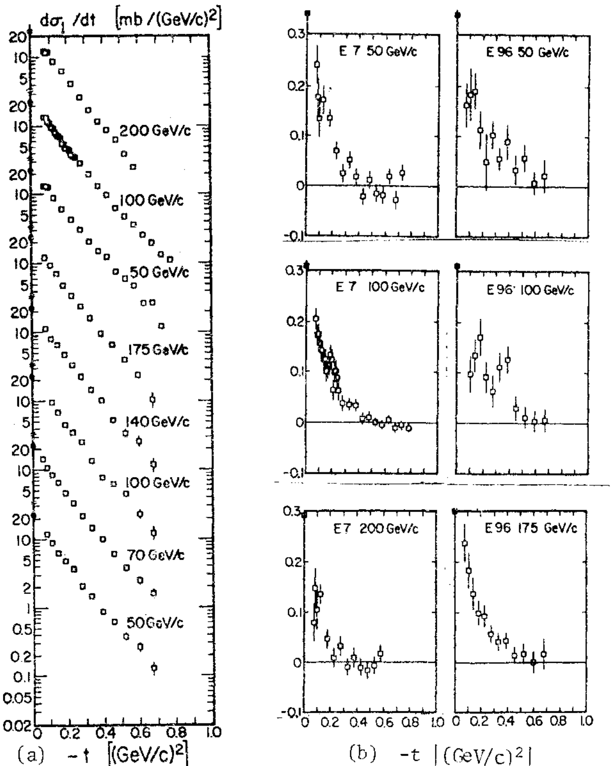


Fig. 6

3. Quark analysis of total cross sections

During the years the quark model was very successful in analysis of regularities in the asymptotic values and variations of the hadron scattering total cross sections. It is now time to ask the following questions:

* Why do some combinations of total cross sections grow, some not?
* Why $\sigma_{tot}(K^+p)$, $\sigma_{tot}(pp)$ and $\sigma_{tot}(\phi p) = 2\sigma_{tot}(Kp) - \sigma_{tot}(\pi p)$,all being pure Pomeronic in two-component duality, have different energy behaviour?
* Why does the Levin-Frankfurt rule [38] $\sigma_{\pi N}/\sigma_{NN} = 2/3$ work only with I0% accuracy and $(\sigma_{\pi N} - \sigma_{KN})$ decreases too slowly?

By the way, it has been noticed in I97I by Prokoshkin and analysed in details by Denisov et al. [39] on the basis of the Serpukhov experimental data that the ratio

$$R = \frac{2\sigma_{\pi N}}{\sigma_{KN} + \frac{2}{3}\sigma_{NN}} = \mathrm{I.020} \pm 0.0\mathrm{I5} \qquad (p_{lab} = 20 \div 60)$$

is consistent with the quark model predictions with much more accuracy than the others.

One of the possible answers to these questions was given by Lipkin who has proposed the three-component picture for the high energy hadron scattering (the first two components for the vaccum exchange, and the third one- for the Regge-pole contribution [40]).

It was assumed that the second vacuum component contributes to πN, KN and NN forward scattering as respectively $2:\mathrm{I}:9/2$. Thus, the following sum rule is satisfied:

$$\sigma_{\pi N} = \tfrac{1}{2}\sigma_{KN} + \tfrac{1}{3}\sigma_{NN} \qquad \text{or } R = \mathrm{I}.$$

The model can be formulated in terms of the specific total cross sections $Y_a = \sigma_a/n_a$ (per single quark of a projectile for nucleon as a target:

$$Y = \alpha + \beta \cdot n_{\text{non-strange quarks}}$$

Two terms in this expression belong to the first (p_1) and second (p_2) components of the vacuum exchange. In these terms the famous ratio R=I corresponds to the equidistance rule $Y_\pi - Y_K = Y_N - Y_\pi$. Notice that while the differences $Y_\pi - Y_K$ and $Y_N - Y_\pi$ pick out the second p_2-component, the differences $Y_\phi = 2Y_K - Y_\pi$ and $Y'_\phi = Y_\pi + Y_K - Y_N$ separate the first p_1 - component with an energy dependence different from that of the previous one: p_1 is slowly increasing ($\sigma \sim s^{0.07}$ or powers of log's); p_2 is slowly decreasing ($\sigma \sim s^{-0.2}$).

A number of interesting generalizations was proposed in [4I-42]. The simplest is given by $Y = \alpha + \beta n_{\text{non-strange}} + \gamma n_{\text{str}}$, which leads to the relations between total cross sections of baryons and hyperons:

$$Y_\pi - Y_K = Y_K - Y_\phi = Y_N - Y_{\Lambda/\Sigma} = Y_{\Lambda/\Sigma} - Y_\Xi, \text{etc.}$$

In the paper by Joynson and Nicolescu [4I] submitted to the Conference, the new interesting relation between total cross sections is proposed

$$2\sigma_{\pi^+p} - \sigma_{\pi^-p} = \frac{7}{\mathrm{I}6}\sigma_{K^+p} + \frac{3}{8}\sigma_{pp},$$

which is in a surprising agreement with experimental data (see Fig.7).The authors interpret this by introducing the special form of the second vacuum component for the total cross sections

$$\sigma_{p_2} \sim \frac{\mathrm{I}}{n_a n_b} \sum_{i \in a,b} y_i^2;$$

where y_i are hypercharges of quarks in the colliding hadrons a and b.

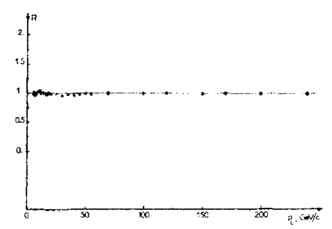


Fig.7. Test of the Joynson - Nicolescu sum-rule. The quantity plotted is $R = (2\sigma_{\pi^+p} - \sigma_{\pi^-p})/(\frac{7}{\mathrm{I}6}\sigma_{K^+p} + \frac{3}{8}\sigma_{pp})$.

IV. HIGH ENERGY SCATTERING IN MODELS OF QFT. QED & QCD.

I. QED.

Investigation of the high energy asymptotic behaviour in different models of QFT has a long history. Most of these studies are based on the order-by-order calculations as well as on the non-perturbative methods like path-integral methods which were devoted to the problems of leading singularities and the eikonal approximation.

As was shown by Gribov, Frolov and Lipatov and by Cheng and Wu in I969 in the framework of the massive QED, the main singularity in J-plane is not pole but a fixed branch point above unity, i.e.[43]

$$T \sim s^{I+g^2 C}, \quad s \sim \infty$$

where $C > 0$ is the main logarithm approximation and different for the scalar and vector variants of QED. The evident contradiction with the Froissart theorem is interpreted as being due to the non-reggeization of the massive photon.

In a series of recent papers [44] by Cheng and Wu, the asymptotic behaviour of the fermion-exchange processes and the related question of the fermion reggeization are studied in the framework of QED up to the record of I2-th order in coupling constant.[45]

There are few new results on the eikonal approximation in the models of field theory.[46-49] The asymptotics of "twisted" graphs (obtained by an interchange of two nucleons in the ladder-type graphs being usually studied in works on the eikonal problem) have been found. This gave rise to an effect of a linkage of large nucleon momenta to the mesons, on the one hand, and to the violation of the simple eikonal formula with Yukawa-type interaction potential-on the other.[46] The corresponding corrections to an effective high energy potential were calculated which have more singular behaviour in origin that the Yukawa term, e.g. $\sim \log {}^{r}/_{r_0}$. The relativistic eikonal representation for the scattering on composite particles was studied in papers [48] by means of the path-integration method.

2. QCD (quantum chromodynamics)

Study of the high energy scattering in non-abelian gauge theories like QCD has recently attracted a great interest.[50] In a number of papers[5I-64] the following problems were attacked:

* What is the leading singularity in J-plane?
* Whether the gauge bosons reggeizes?
* Does the large angle elementary fermion scattering pass the point-like scaling?
* Whether there is a sign of the quark confinement in the high energy scattering amplitudes?

The last question is closely related to the whole problem of the infrared mechanism ("infrared slavery") for quark confinement in QCD which was studued in paper [64]. It was shown by means of the order by order calculations that there is no quark confinement, and the Kinoshita-Lee-Nauenberg theorem[65] is true in the perturbation theory for both the QED and QCD.

The result of the non-perturbative analysis developed in papers [6I-62] on the basis of some renormalization group equations still preserve the general belief in quark confinement in the QCD. The most of results on the high energy scattering in the QCD are made by the order by order calculations up to 6-th (or up to the 8-th order [54]) in coupling constant. The spontaneous breakdown of the gauge group is introduced by the Higgs mechanism [66] into the 't-Hooft's gauge[68] (by the Faddeev's and Popov's receipt [67]) to avoid the infrared problems.

All the necessary elements of Feynman rules to calculate the high energy asymptotic graphs are listed below:

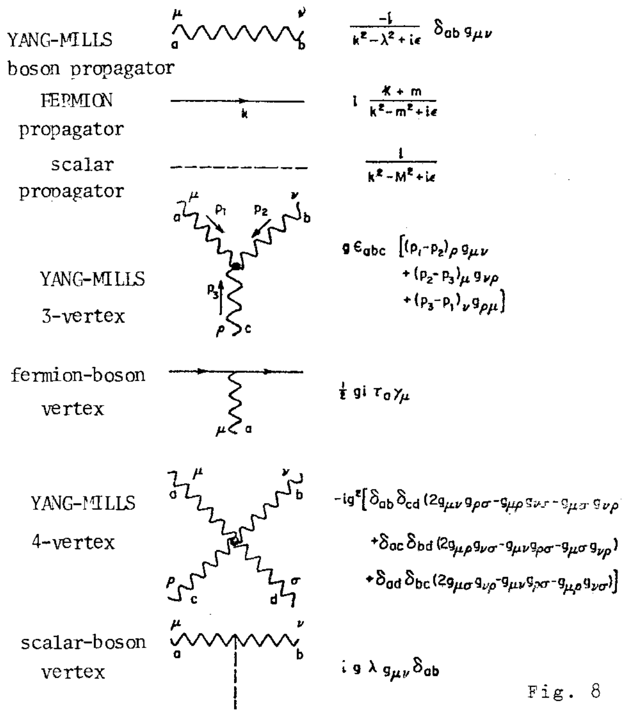


Fig. 8

These include the dimensional coupling of the Higgs scalar with the gauge bosons which goes to zero together with the mass of the vector boson λ (when the symmetry breaking switches off). The results of the calculations for the non-spin - flip amplitudes of the (ff), (fb) and (bb) processes up to the 8-th order can be summarized in the following way:

$$T(\text{non-spin-flip}) = \sum_{i=0,I,\dots} T_i\, \Pi^{(i)} c_i$$

Here, T_i are the universal amplitudes with the t-channel isospin "i"; c_i are some normalization factors depending on channels, and $\Pi^{(i)}$ are the (color) isospin projection operators.

$$T_1 = I - g^2\left(\xi - \frac{i\pi}{2}\right)f_1 + \frac{g^4}{2!}(\xi^2 - i\pi\xi)f_1^2 - \frac{g^6}{3!}\left(\xi^3 - \frac{3\pi i}{2}\xi^2\right)f_1^3 + \ldots$$

that coincides with the decomposition in the leading log's approximation of the cross-odd Regge term.

$$s^{\alpha-1}\left(\frac{I - e^{-i\pi\alpha}}{2}\right), \quad \text{where } \alpha = I - g^2 f,$$

$$f_1 = (\Delta^2+\lambda^2)D = (\Delta^2+\lambda^2)\cdot\int\frac{d^2k_\perp}{(2\pi)^3}\,\frac{I}{(k_\perp^2+\lambda^2)\left[(k_\perp-\Delta)^2+\lambda^2\right]},$$

$$T_0 = \frac{a}{\Delta^2+\frac{5}{4}\lambda^2} + \left(-\frac{a^2}{\Delta^2+\frac{5}{4}\lambda^2} + 4T\right)g^2\xi + \left(\frac{a^3}{\Delta^2+\frac{5}{4}\lambda^2} - 8aT + 4Q\right)g^4\frac{\xi^2}{2!} + \ldots$$

where $a = (2\Delta^2 + \frac{5}{2}\lambda^2)D$

$$T = \int\frac{d^2k_{1\perp}}{(2\pi)^3}\,\frac{d^2k_{2\perp}}{(2\pi)^3}\,\frac{I}{(k_{1\perp}^2+\lambda^2)(k_{2\perp}^2+\lambda^2)\left[(k_{1\perp}+k_{2\perp}-\Delta)^2+\lambda^2\right]}$$

$$Q = \int\frac{d^2k_{1\perp}}{(2\pi)^3}\,\frac{d^2k_{2\perp}}{(2\pi)^3}\,\frac{d^2k_{3\perp}}{(2\pi)^3}\,\frac{I}{(k_{1\perp}^2+\lambda^2)(k_{2\perp}^2+\lambda^2)(k_{3\perp}^2+\lambda^2)}\, x$$
$$x\left\{\frac{}{(k_{1\perp}+k_{2\perp}+k_{3\perp}-\Delta)^2+\lambda^2} + \frac{(q_{1\perp}+q_{3\perp})^2+\lambda^2}{\left[(k_{1\perp}+k_{2\perp}+k_{3\perp})^2+\lambda^2\right]\left[(k_{1\perp}+k_{3\perp}+\Delta)^2+\lambda^2\right]}\right\}$$

This formula coincides with the decomposition of the expression

$$2s^{-a}(D + 2g^2\xi T + 2g^4\xi^2 Q + \ldots)$$

$$T_2 = \frac{b}{\Delta^2+2\lambda^2} + \left(\frac{b^2}{\Delta^2+2\lambda^2} - 4T\right)g^2\xi + \left(\frac{b^3}{\Delta^2+2\lambda^2} - 8bT + 8Q\right)g^4\frac{\xi^2}{2!} + \ldots$$

Here, $b = (\Delta^2+2\lambda^2)D$

Summing up the first terms in each perturbation order one gets s^b in violation of unitarity (no reggeization).

In terms of the reggeization properties the results consist of the following:

(color)- isospin in the t-channel	Reggeization
$I_t = I$	Yes
$I_t = 0$	?
$I_t = 2$	No

The problem of reggeization is not completely clear for the case of $I_t = 0$, because the following normalization factors found in the paper [54/]

$$\text{ff:}\quad c_1 = -\frac{g^2}{4m^2}\,\frac{s}{\Delta^2+\lambda^2};\quad c_0 = 2^{-5}\frac{g^4}{m^2}\,3\pi i s;$$

$$\text{fb:}\quad c_1 = -\frac{g^2}{m}\,\frac{s}{\Delta^2+\lambda^2};\quad c_0 = 2^{-2}\frac{g^4}{m}\,i\pi\sqrt{6s};$$

$$\text{bb:}\quad c_1 = -4g^2\frac{s}{\Delta^2+\lambda^2};\quad c_0 = g^4\,4\pi i s;\quad c_2 = g^4\,i\pi s;$$

satisfy the factorization condition

$$c(ff)\cdot c(bb) = c^2(fb)$$

for the channel $I_t=0$ as well as well as for the channel $I_t = I$. However, as it has been found in the paper [55/], the leading singularity which belong to $I_t=0$ turns out to be a square root branch point at $J = I + \frac{g^2}{\pi^2}\,2\ln 2$ (for SU_2- gauge group) and results from two reggeized vector exchange.

This indicates to a violation of the Froissart bound in the main logarithmic approximation in QCD as well as in QED.

Thus, the situation is puzzling.Before answering these questions let us make up the balance:

I. There is a place for the reggeization of basic particles;
2. The "superconvergence" of integrals over the transverse momenta is observed; so, the automatic transverse cutoff in $P_\perp$ appears, and one can think the QCD is a candidate for the theory of strong interactions;
3. The results under study correspond to a nonphysical world with the free (non-confined) quarks, antiquarks and gluons.

It seems that all found indications to a quark confinement are illusive,and the whole problem of high energy behaviour in hadronic phase of QCD is still unclear.

In conclusion we shall mention the paper [59/] in which some regularities in behaviour of the hadron multiplicities are considered in the framework of QCD. The increase of the hadron multiplycity in the high energy reactions is attributed here to the necessity of confining the quark quantum numbers("color neutralization").

V. CONSTITUENT THEORY OF THE HADRON INTERACTIONS

I. Dynamical equations in QFT

The constituent picture of the high energy hadron interactions leads to the predictive and consistent theory when combined with a dynamical equation approach in QFT.

The relativistic quasipotential approach was successful in studying the high energy hadron scattering at small and large angles,the form factors of composite particles and many other problems.

There is a number of new results in this direction:

First, I shall mention the density matrix method to describe the multiparticle production processes (see Logunov et al. [70/]) which we shall discuss in the following chapter.

Second, there is a development of the null-plane description of composite particles in the framework of the "one-time" quasipotential approach (Tavkhelidze et al.[73]).

In a number of papers[74] the great efforts are made to obtain as much as possible rigorous results on the properties of the n-particle Green functions projected on the null-plane. The spectral and explicit properties of the"two-time" Green functions projected on the null plane are proved. Some of these results were previously found only in the lowest order of the perturbation theory [71-72].

In the paper by Kvinikhidze,Sissakian,Slepchenko and Tavkhelidze [75] submitted to the Conference, the null-plane approach was used in order to obtain a number of general integral representations for the inclusive cross sections which are very useful in the dynamical description of multiparticle production. These results provide the field theoretical grounds for the parton-quark model or constituent- interchange model used by Blanckenbeckler,Brodsky and Gunion to describe the large $P_\perp$ processes[76].

In the paper by Atakishiev,Mir-Kasimov and Nagiev [77] the relativistic Hamiltonian theory with an auxiliary massless spurion field is developed that allows one to exclude all ultraviolet singularities of QFT on the null plane.

The cross-symmetric Bethe-Salpeter equations are studied in the paper by Yaes[78].

2. The Density Matrix Method

The new development of the dynamical approach is made in the paper by Arkhipov,Logunov,Savrin[70] submitted to the Conference, where the method of the density matrix is proposed to describe the multiparticle production processes at high energies

The density matrix of a system of two outcoming nucleons + one additional meson (as an example) is represented by

$$\rho^{(2)}_{\vec c}(2,I/2',I') \sim \sum_{\text{others}} T_{2\to 2'+I'+\text{others}}\, T^*_{2\to 2'+I'+\text{others}}$$

as is illustrated in Fig.9:

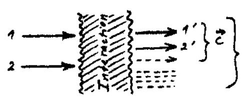


Fig. 9

Quantum statistical ensemble

The total space momentum of all three particles $\vec c = \vec 2 + \vec I = \vec 2' + \vec I'$ is fixed in the relativistic invariant one-time description.

The following dynamical equation is true on the general grounds of QFT:

$$(E_k - E)\rho^{(2)}_{\vec c}(t) - \rho^{(2)}_{\vec c}(t)(E_{k'} - E') = V_{\vec c}\,\rho^{(2)}_{\vec c}(t) - \rho^{(2)}_{\vec c}(t)\,V^{+}_{\vec c}.$$

Here, $V_{\vec c}$ is the integral operator of an effective quasipotential for the three-particle system (2+I or 2'+I') affected by screening of the rest part of multiparticle systems:

$$E_k = (m^2 + \vec p^2)^{\frac12} + (\mu^2 + \vec k^2)^{\frac12} + |(\vec c - \vec p - \vec k)^2 + m^2|^{\frac12}$$

The two-particle correlation function is given by

$$\rho^{(2)}(t) = \int d\vec c\, \rho^{(2)}_{\vec c}(t),$$

and the one-particle distribution function by

$$\rho^{(1)}(t) = \int \rho^{(2)}_{\vec c}(t)\, d\vec c\, d\vec p.$$

The observable inclusive cross section is determined by the adiabatic limit

$$\lim_{\substack{t\to\infty \\ E=E'}} \rho^{(1)}(\vec k, \vec k/t) = \delta^3(0)\frac{I}{(2\pi)^2}\frac{d\sigma}{d\vec k}$$

where $I = 4\sqrt{(q_1q_2)^2 - m^2}$ is the initial flux factor.

The authors look for solutions of the eq. for the $\rho^{(2)}$ of the following form

$$\rho^{(2)}_{\vec c}(2,I/2',I') \sim \psi_{\vec c}(2,I)\cdot \psi^*_{\vec c}(2',I')$$

as the pure three-particle state density matrix. Thus the $\psi_{\vec c}(t)$ obeys the quasipotential eq. for the (pseudo) three-particle system

$$(E_k - E)\,\psi_{\vec c}(t) = V_{\vec c}\,\psi_{\vec c}(t)$$

where $V_{\vec c}$ takes into account the interaction among three particles screened by other particles in the final states of an inclusive reaction.

$$\frac{d\sigma}{d\vec k}(I) \sim \int d\vec c\; d2\, |\psi_{\vec c}(2,I)|^2$$

The authors have derived the automodel solutions which possess the scaling properties predicted in the Feynman parton model.

Thus, the density matrix method gives a dynamical basis for consistent description of multiparticle reactions in the framework of the constituent models(like Chou-Yang or Van Hove & Pokorski models).

The similar dynamical approach to a multiparticle production has been recently formulated by P.Carruthers and F.Zachariasen[79]. They have shown that the field theoretical description of multiparticle production processes can be cast in a form analogous to the ordinary transport theory where the inclusive differential cross sections are given by integrals of the covariant phase-shift distributions.

VI. POWER LAWS

I. Dimensional Quark Counting for Large Angle Scattering.

Power-like asymptotic behaviour of the large scattering processes and hadron form factors was interpreted in papers[70-72] in terms of the Dimensional Quark Counting (DQC).

As was shown in paper [70], DQC is based on the following general assumptions:

I. Quark structure of hadrons with the confinement.

2. Scale invariance (automodelity) of quark interactions at small distances.

3. Finitness of the quark local densities within hadrons.

In papers[81-83] DQC is derived from a perturbation theory analysis of the renormalized field theoretical models with quarks and gluons as fundamental fields and hadrons as bound states of quarks.

The DQC leads to the following well-known results:

$$F_a(t) \sim \left(\frac{I}{t}\right)^{n_a-I}; \quad \frac{d\sigma}{dt}(ab\to cd) \sim \left(\frac{I}{s}\right)^{n_a+n_b+n_c+n_d-2} \cdot f_{ab\to cd}(z) \qquad (z=\cos\theta)$$

where n_a is the minimal number of basic point-like constituents (valent quarks, leptons, etc.).

If one chooses $n_{meson}=2$, $n_{baryon}=3$, $n_{lepton}=I$, he can get

$$F_\pi \sim \frac{I}{t}\text{(monopole)}; \quad F_p \sim \frac{I}{t^2}\text{ (dipole)}; \quad F_e \sim I \text{ (point-like) etc.}$$

and

$$\frac{d\sigma}{dt}(pp\to pp) \sim \frac{d\sigma}{dt}(p\bar{p}\to p\bar{p}) \sim s^{-10},$$

$$\frac{d\sigma}{dt}(\pi p\to \pi p) \sim \frac{d\sigma}{dt}(Kp\to Kp) \sim s^{-8},$$

$$\frac{d\sigma}{dt}(\pi\pi\to\pi\pi) \sim \frac{d\sigma}{dt}(ep\to ep) \sim s^{-6}, \quad \text{etc.}$$

This hierarchy of powers is in a qualitative accordance with observation[83].

In addition to the próblem of power-like automodel asymptotics in QFT and justification of DQC the following questions still remain:

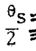
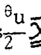
* Angular dependence of $d\sigma/dt$;
* Hadron polarization in large angle processes;
* Absolute scales of $d\sigma/dt$, $F(t)$.

The problem of angular dependence of the large angle scattering cross sections was considered in recent papers [84-92].

In paper[84] the generalized DQC has been proposed based on a specific dynamical interpretation of quark diagrams describing the quark rearrangement processes at small distances. The generalized DQC states that an asymptotic contribution to the large angle scattering amplitude is determined by the topology of quark diagram as a homogenious function of the large kinematic variables s, t and u.

The analysis of angular dependence of a number of large binary reactions in terms of the helicity amplitudes, under an additional assumption of the γ_5- invariance was performed in [93].

For the meson-baryon scattering it gives, e.g., the general result which can be written in the following symbolic form:

$$\frac{d\sigma}{dt} \sim \frac{I}{s^2}|f_{tt}|^2 \sim \frac{I}{s^2}\left|\cos\frac{\theta_s}{2}\right.$$  $$+ \cos\frac{\theta_u}{2}$$  $$\Big|^2$$

where θ_s, θ_u are the scattering angles in the s- and u-channels, respectively, and the asymptotics of two quark diagrams are

 $$\sim \frac{I}{st^2},$$  $$\sim \frac{I}{ut^2}$$

The Table of results is given below.

TABLE: Angular dependence of the large angle binary reactions.

Reaction	$d\sigma/dt \sim \left(\frac{I}{s}\right)^{2(n_a+n_b-I)} \cdot f(\cos\theta = z)$
$\pi^+p\to\pi^+p$	$s^{-8}\frac{(I+z)}{(I-z)^4}\left\|\beta + \frac{4\alpha}{(I+z)^2}\right\|^2$
$\pi^-p\to\pi^-p$	$s^{-8}\frac{(I+z)}{(I-z)^4}\left\|\alpha + \frac{4\beta}{(I+z)^2}\right\|^2$
$\pi^-p\to\pi^0 n$	$s^{-8}\frac{(I+z)}{(I-z)^4}\left\|\frac{\alpha+\beta}{\sqrt{2}}\right\|^2\left(I+\frac{4}{(I+z)^2}\right)^2$
$K^+p\to K^+p$	$s^{-8}\cdot I6\,\frac{(I+z)}{(I-z^2)^4}$
$K^-p\to K^-p$	$s^{-8}\frac{(I+z)}{(I-z)^4}$
$p\bar{p}\to\pi^+\pi^-$	$s^{-8}\,\frac{1}{2}\,\frac{(I+z)}{(I-z)^3}\left\|\alpha+\beta\left(\frac{I-z}{I+z}\right)^2\right\|^2$
$p\bar{p}\to\pi^0\pi^0$	$s^{-8}\,\frac{1}{2}\,\frac{(I+z)}{(I-z)^3}\left\|\frac{\alpha-\beta}{\sqrt{2}}\right\|^2\left(I+\left(\frac{I-z}{I+z}\right)^2\right)^2$
$p\bar{p}\to K^+K^-$	$s^{-8}\,\frac{1}{2}\,\frac{(I+z)}{(I-z)^3}$
$pp\to pp$	$s^{-10}(I-z^2)^{-6}\{[(\alpha(I+z)^2+\beta(I-z)^2)^2+(z\to -z)] + [\frac{2}{I+z}(\bar{\alpha}(I+z)^2+\bar{\beta}(I-z)^2)^2+(z\to -z)]\}$.
$p\bar{p}\to p\bar{p}$	$s^{-10}\frac{(I-z)^{-6}}{64}\{[4\bar{\alpha}+\bar{\beta}(I-z)^2)^2 + +(4\bar{\beta}+\bar{\alpha}(I-z)^2)^2+[\frac{(I+z)}{2}(4\alpha+\beta(I-z)^2) - -\frac{I+z}{I-z}(4\beta+\alpha(I-z)^2)]^2\}$

On the basis of these formulas as well as on the basis of the quasipotential equation with the analytic interaction potentials a new analysis of experimental data has been performed by Kuleshov, Goloskokov, Smondyrev et al. in the paper submitted to the Conference [92] (see Figs.10,11). The authors have shown that

I. $V \approx A$, or the vector-vector and axial-axial vector couplings in $pp/p\bar{p}$ scattering approximately coincide.

So, only same/opposite helicities couple in the proton/antiproton scattering on the nucleon.

2. The double-quark pair exchange is suppressed by two orders of magnitude as compare with a single-quark model.

This analysis shows that the study of angular dependence can give a rich and important information on the dynamics of large angle scattering and quark processes going at short distances.

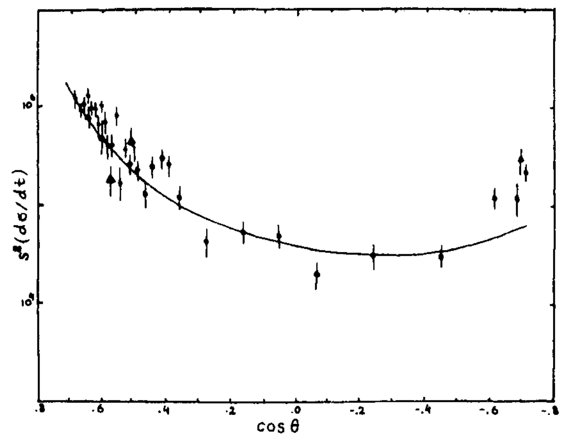


Fig.10. $s^8 \frac{d\sigma}{dt}$ for $\pi^+ p$ scattering.

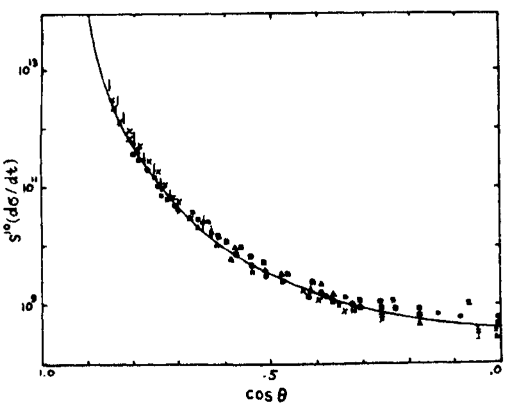


Fig.11. $s^{10} \frac{d\sigma}{dt}$ for pp scattering.

2. Asymptotics of nuclear form factors and structure function.

The dimensional quark counting gives a very simple and direct relationship between asymptotic form of the hadron form factors and complexity of hadrons.

There is an interesting question whether this relationship can be brought into the nuclear physics where fortunately one deals with objects whose complexity varies in wide limits. This question as well as the whole problem of the quark degrees of freedom for nuclei was apparently first arised by A.M.Baldin [94] and discussed in a number of recent papers.

An analysis of recent experimental data on the deuteron electromagnetic form factor as well as the new results of measurements of e^-D elastic and inelastic scattering from $q^2=0.8$ up to $6.0\ GeV^2$ were presented by Arnold et al.[97]. An evidence on the absence of the meson exchange currents is found in these experiments accordingly to the quark picture of the processes.

The test of the dimensional quark counting predictions for the form factors of hadrons and a deuteron can be illustrated in Figs.12,13 from the paper by Brodsky and Chertok[98].

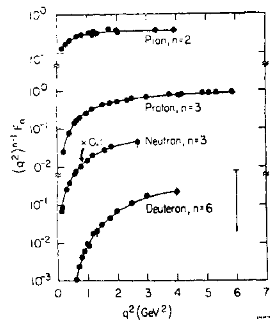


Fig.12. Test of the Dimensional Quark Counting predictions with the experimental data for the pion (electroproduction data) proton /neutron and elastic deuteron form factors.

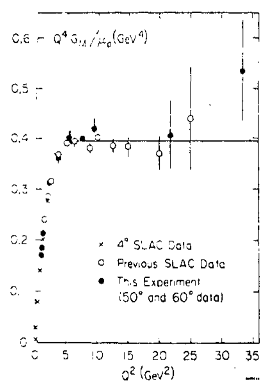


Fig.13. Comparison of the Dimensional Counting prediction with the SLAC experimental data.

One of the most important problems which have to be understood is the value of the level of "flattening out" of the deuteron form factors.

It seems that this is connected to the relative time which the deuteron spends as a six-quark system (not as a losely bound two-nucleon system). To answer such a question one needs a more sophisticated theory of a nucleus in terms of quark degrees of freedom (like quark bag model of nucleus).

VII SCALING AND SIMILARITY LAWS

I. Automodelity in Strong Interactions

Study of the scaling and similarity properties of strong interactions at high energies is interesting from two points of view. First, it is a question of structure of elementary particles and of interaction dynamics at short distances; second, it is a search for a new energy scale (or fundamental length) which could change (if exists) considerably the picture of strong interactions at superhigh energies.

A great number of papers has appeared as a result of an observation of scaling effects in spectra of secondary particles in the experiments carried out at the Serpukhov accelerator and following the Feynman formulation of scaling phenomena in terms of the quark-parton model[100]. Many interesting experimental and theoretical results were obtained in that direction.

In papers[101] a general approach to the description of scaling and similarity properties has been developed based on the idea of the automodelity.

The term "automodelity", (or "selfsimilarity") itself, belongs to the theory of the gazo-and hydrodynamic phenomena. An analogy of these phenomena to the scaling properties of deep-inelatic processes has been noticed by N.N.Bogolubov in 1969. It is worth to be mentioned, particularly, in connection to the physics of deep-inelastic lepton-hadron scattering, where the character of scaling laws indicates to the analogy with the so-called "point-like explosion" in gazo/hydrodynamics. The scaling regularities in the processes of strong interactions of hadrons at high energies lead to an analogy with the "plane-like" explosion. We emphasize, however, that the analogy with the "plane-like" explosion corresponds only to the processes with bounded momentum transfers, the dynamics of which is determined by the finitness of effective interaction radius of strong interactions [99]. In these processes hadrons look as extended objects with infinitely small longitudinal sizes (due to the Lorentz contaction in a system with infinite momentum, i.e. "$P_z \to \infty$"). This allows one to consider the scaling properties of observable quantities under the transformation (the delatations of the longitudinal scale of momenta) as: $P_z \to \lambda P_z$; $P_\perp \to P_\perp$

A separation of the transverse and longitudinal degrees of freedom in description of the interaction processes at bounded momentum transfers which is specific in this approach, leads to a whole number of scaling laws for the amplitudes and cross sections of inclusive as well as of binary reactions (note results of the Mueller-Kancheli approach and quark-parton model).

The automodelity principle leads, particularly, to the requirement for all the quantities of the type

$$\sigma_{tot},\ \sigma_{el},\ B=(\log\sigma')_{t=0}\ \text{etc.}$$

with the same dimensionalty T^2 (in terms of scale of the transverse length T), to have a weak and universal energy dependence on the same footing. This phenomenon known as the law of geometrical scaling is in good accordance with the existing experimental data.

We have noticed in a number of papers[103] that the geometrical scaling laws were effectively mearged with the quark analysis of the hadron interaction total cross sections. It has to be stressed that a weak energy dependence of the transverse length effective scale (say $R^2 \sim \sigma_{tot}$) points to an approximate character of scaling laws related to the similarity transformations,

$$P_\perp \to \lambda P_\perp,\quad P_z \to P_z$$

the separation of the longitudinal and transverse degrees of freedom being suggested for an ideal case.

2. Study of scaling in QFT.

A number of new interesting results was obtained in studying the scaling properties of hadron interactions in the framework of the local QFT. First of all, we shall mention the axiomatic studies of large angle processes which were initiated by the rigorous applications of the Dyson -Lehmann-Jost representation to an investigation of the automodel (scale invariant) asymptotics in the deep inelastic scattering (see Bogolubov, Tavkhelidze and Vladimirov [104].

An extension of this approach to the large $P_\perp$ inclusive reactions on the basis of the DJL-representation was made by Logunov, Mestvirishvili et al. in 1974[105].

The main problems of the axiomatic approach are:

* Consistency of the power-like automodel asymptotics with the general requirements of the local QFT;

** Relations between asymptotic forms in different channels and regions;

*** Connection to the space-time picture of processes.

In the paper by Geyger et al.[I06] submitted to the Conference, the results of simultaneous study of three processes (on the basis of the DLJ-representation) are presented:

$p_1 + q_1$ (on shell) $\to p_2 + q_2$ (on shell)

$p_1 + q_1$ (on shell) $\to p_2 + q_2$ (off shell)

$p_1 + q_1$ (off shell) $\to p_2 + q_2$ (off shell)

It is shown that

- on - and off-shell amplitudes can have different asymptotic behaviour at large momenta;
- absorptive and dispersive parts of the amplitudes on-mass shell can have different asymptotics at large angles.

For instance, the automodel (power-like) asymptotics of the process $\gamma(q^2) + p \to \pi/_\rho + p$ could have different character at $q^2=0$ and $q^2 \to -\infty$.

The consistent description of scaling properties in terms of the density matrix method was given in paper[70]. Starting from the dynamical equation for a density matrix of a subsystem of particles in final states of the inelastic multiparticle reaction, under rather general assumption on the character of interaction quasipotential, the authors[I07] have derived the radial type scaling for the inclusive cross sections. The result is in good agreement with existing experimental data on the inclusive production at high energies[I08].

In the paper mentioned above an interesting application of the DM-method to the deep-inelastic scattering is given. The well known Bjorken scaling is found in the one-photon approximation. In general case (for all orders in the coupling constant) the cross section scales in terms of two dimensionless variables are: $\omega = I - w^2/_{q^2}$ (Bjorken) and $x = I - w^2/_s$ (Feynman). In the intermediate case (multiphoton with the one-photon approximation for the scattering on a single "constituent"), the new scaling has been found with the dimensional variable

$$\zeta = \frac{x(I-x)}{(2-x)(\omega-I)}$$

Moreover, it was shown that the DM-method gives a clarification of the correlation between the elastic and inelastic characteristics of particle interactions, as it can be illustrated by the solution

$$\frac{d\sigma}{d\vec{k}_\perp} = c(E)\int d\vec{b}\sigma(\vec{b})\,\rho(\vec{b},\vec{k}_\perp)$$

which was found and discussed in the papers by Savrin, Semenov, Tyurin, Khrustalev[I09] submitted to the Conference. Here, $d\sigma/_{d\vec{k}_\perp}$ is an inclusive cross section of "soft" mesons, and $\sigma(\vec{b})$ is determined by overlapping of the elastic profiles of initial particles

$$\sigma(\vec{b}) = \int d\,\vec{b}'t_{el}(\vec{b}')t^*_{el}(\vec{b}-\vec{b}')$$

and $\rho(\vec{b},\vec{k}_\perp) \sim \int d\,\vec{\Delta} e^{-i\vec{b}\cdot\vec{\Delta}} g(\vec{k}-\vec{\Delta})\times g(\vec{k}+\vec{\Delta})$,

$g(\vec{k}-\vec{\Delta}) = |\phi_{\vec{k}+\vec{\Delta}}(\vec{k})|^2$ is "a distribution of mesonic constituents" within a screened nucleon with a total momentum $\vec{k}+\vec{\Delta}$.

In the paper by D.Crewther and G.C.Joshi[II0] submitted to the Confereence, the scaling properties of the dual resonance model with the Mandelstam analyticity were studied. Non-linear trajectories are used when calculating the scaling functions which are in reasonable agreement with the experiments.

Factorization properties of the exclusive and inclusive cross sections were discussed in papers[III].

The detailed discussion of scaling in the inclusive multiparticle production processes is given in a review talk by Chliapnikov.

SEARCH FOR THE FUNDAMENTAL LENGTH

The success of the most scaling and similarity relations depends on the absence of an energy scale which will be essential at available energies.

What do we expect from the very high energies?

In the invited talk given by Kadyshevsky[II2] there is an interesting discussion about the existence of the fundamental length l which should be a key to understand the phenomena at very high energies.

This discussion is based on the version of QFT with a fundamental length[II3] which is an alternative to the local QFT in the same sense as the non-Euclidean geometry is a unique alternative to the Euclidean one. Some implications of this theory are listed below:

I. Masses of elementary particles are bounded from above, i.e. $m \lesssim I/_l$;
2. Modification of the usual uncertainty relation between the momentum transfer q and size of interaction r:
$r \sim 1/_{l\log(lq)}$ instead of $\sim I/_q$;
3. Geometrical interpretation of the power laws such as $d\sigma/_{dt} \sim (l^2 s)^{-\nu} f(t/_s)$;
4. Parity violation at very high energies $E \gtrsim I/_l$ owing to the modification of the fermion propagator
$$|\,\not p + (I/_l - \sqrt{I/_{l^2} - }$$
$$l \sim l_{Fermi} \sim I0^{-17}\text{ cm?}$$

5. Sharp fall of the $\sigma_{tot}(e^+e^- \to \text{hadrons})$ at $s > l^{-2}$ and a slower decrease with $Q^2 < l^{-2}$ of the deep-inelastic e^-p scattering cross sections.

This analysis shows that a search for the fundamental length could be one of the intriguing tasks for super-high energy physics at the accelerators of next generation.

VIII. SOME CONCLUDING REMARKS

I would like to conclude this talk with some remarks. Obviously, the list of interesting questions written below is no way complete and it is based mainly on the author's impressions.

I. Asymptotic energies are not yet reached, e.g. s^α behaviour for the total cross sections with $\alpha > I$ is allowed by experimental data; polarization in pp-scattering is still nonzero, etc. Apparently, these questions are problems to be studied at the accelerators of next generation.

2. Nature of the vacuum exchange (Pomeranchuk singularity) is still an enigma and waits for its explanation.

3. A close relationship between the t-dependent SU_3-structure of the vacuum exchange and the OZI-rule regularities in the heavy meson decays, apparently, exists. This makes deeper the Regge idea on relationship of high energy behaviour and particle properties with the relevant quantum numbers.

4. Experiments with the high energy hyperon beams and study of $\psi/_J$ -hadron interactions should be very desirable to understand the quark regularities in the total cross sections.

5. Situation with the quark confinement in QCD is still puzzling. Its understanding is important for high energy physics on the whole.

6. Measuring of angular dependence, polarizations and phases of scattering amplitudes of the binary reactions at large angles is important to understand how far are really good predictions of the Dimensional Quark Counting.

7. Quark degrees of freedom seem to be important for a description of nucleus interactions at high energies and large transferred momenta.

8. Presence of the fundamental length or new energy scale could change drastically the picture of particle interactions at extremal energies.

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