An approach to Grand Unification

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Abstract. Around the time of the completion of the Standard Model of particle physics in the 1970s, schemes were put forward for unifying the three gauge interactions (electric, strong and weak) using the renormalization equations at an energy approaching the Planck mass. Though these looked promising, the exact unification never materialised, and doubts have been raised about whether this Grand Unification can be achieved. It may be possible, however, to create Grand Unification at the Planck mass if we start with a radical examination of the nature of the colour model of quarks.

1. Introduction

The Standard Model is the most successful theory ever devised. It was completed about 1973 and very little has happened since to change it in any significant way. If we want to calculate the results of an experiment in a particle collider we have an equation which can be solved iteratively to take it to the order of magnitude desired. We know how all the fundamental fields work and we know all the interacting particles (Table 1). We know how everything happens. We just don't know *why*. We have four interactions, gravity and the three gauge interactions (strong, weak and electric), which have seemingly different force laws or gauge symmetries, and we don't know why they are different and how they all connect.

Table 1. The Standard Model particles.

	Quarks		*Gauge Bosons*	*Higgs Boson*
u	c	t	g	H
d	s	b	γ	
	Leptons			
e	μ	τ	Z	
ν_e	ν_μ	ν_τ	W	

Two things that happened in the 1960s and 1970s seemed to give us an inkling of a way forward. The first was the Glashow-Weinberg-Salam theory 'combining' the weak and electric interactions (1967). The second was the possibility of a 'Grand Unification' of the three gauge forces at a high enough energy regime [1]. In the electroweak theory the electric and weak interactions are combined mathematically in an $SU(2) \times U(1)$ gauge group, which combines the original weak isospin fields W_1, W_2, and W_3, and the weak hypercharge field B, into the familiar weak W^+, W^-, Z^0 and the massless γ (photon) for the electric component. The mixing is specifically between W_3 and B, which creates the new fields Z^0 and γ. It is quantified by the weak mixing angle θ_W, whose sine squared fixes the ratio of the charge squared for the weak and electric interactions, and whose cosine defines the mass ratio of the W and Z bosons. Current measurements suggest that $\sin^2\theta_W = 0.231$ at energies close to the values of the W and Z masses (80 to 91.2 GeV). The non-unit value means that we should be careful about implying that the electric and weak forces have become a single force, as some authors have been inclined to do. We have shown that they are connected, but they have different sources, and don't have the same coupling constant. Connected does not mean interchangeable, and weak and electric sources are not interchangeable.

The second potential development, grand unification, became possible because, though charges are conserved as units, their value in terms of energy of interaction varies with the strength of the field. This is because a particle such as an electron will polarize the vacuum around it to produce virtual electron-positron pairs, which then produce further pairs, and so on. All the gauge forces act in this way, but, fortunately, the potentially infinite series of interactions can be shown to yield a finite sum, using the process we call renormalization. The parameter that we use is called the 'fine structure constant' for the interaction. This is the ratio of the charge squared to the square of the Planck charge ($\hbar c$). So the electric fine structure constant is given by

$$\alpha = e^2 / \hbar c = 1/137.0359 \ldots$$

(Here, and elsewhere, it will be assumed that e^2 automatically includes the factor $1/4\pi\varepsilon_0$ introduced with SI units.)

The charge responsible for each interaction is given by its own fine structure constant a_n. At any given energy m it has a value determined by its renormalizaton, according to the gauge symmetry which governs the interaction. The first order equations for the $SU(2)$ weak and $SU(3)$ strong interactions are as follows:

$$\frac{1}{\alpha_2(\mu)} = \frac{1}{\alpha_G} - \frac{5}{6\pi} ln \frac{M_X^2}{\mu^2}$$

$$\frac{1}{\alpha_3(\mu)} = \frac{1}{\alpha_G} - \frac{7}{4\pi} ln \frac{M_X^2}{\mu^2}.$$

Here M_X is the GU energy scale, α_G is the fine structure constant at this energy and μ is the energy scale of measurement. These two fine structure constants decrease with increasing energy, but the remaining one, for the electric interaction, will increase.

$$\frac{1}{\alpha(\mu)} = \frac{1}{\alpha_G} + \frac{5}{3\pi} ln \frac{M_X^2}{\mu^2}.$$

So, there is a possibility that they could converge, and this was put forward by Georgi and Glashow in 1974. One of the problems is that the weak coupling cannot be measured directly in the same way as the strong and electric couplings. It can, however, be determined through calculation of $\sin^2\theta_W$, which can be found on the assumption that the interactions can be accommodated within a single grand unified gauge group. The simplest group accommodating each of $SU(3)$, $SU(2)$ and $U(1)$ is $SU(5)$.

2. Grand Unification

A Grand Unified Theory (GUT), to unite electric, weak and strong interactions, based on the $SU(5)$ group, was first proposed by Georgi and Glashow in 1974, following on from the GWS $SU(2) \times U(1)$ unification of the electric and weak interactions, governed by the weak mixing angle parameter $\sin^2\theta_W$, which is effectively the ratio (α / α_2) between the weak and electric couplings (α_2 and α). Georgi and Glashow showed that, in any GU scheme determined by a single GU gauge group, not necessarily $SU(5)$, $\sin^2\theta_W$ would be given by the ratio of the sum of all the squared units of weak isospin ($t_3 = \pm$ ½) for the fermions of the SM to the sum of all their squared units of electric charge (Q).

$$sin^2\theta_W = \frac{Tr\left(t_3^2\right)}{Tr\left(Q^2\right)}$$

Taking the weak components with only left-handed contributions to weak isospin, for the first generation of quarks and leptons, that is, for 3 colours of u, 3 colours of d, and the leptons e and ν, we obtain:

$$Tr\left(t_3^2\right) = \frac{1}{4}\times 8 = 2\,.$$

For, the fractional electric charges of the Gell-Mann-Zweig quark model (1964), with both LH and RH contributions in the first generation, we obtain

$$Tr\left(Q^2\right) = 2\times\left(\frac{4}{9}\times 3+\frac{1}{9}\times 3+1+0\right) = \frac{16}{3}\,,$$

from which

$$\sin^2\theta_W = 0.375.$$

The SU (5) theory raised many hopes because of the seeming ‘near convergence’ at an energy of about 10^{15} GeV, which was only a few orders of magnitude below the magical ‘Planck mass’, the key value for quantum gravity at 1.22×10^{19} GeV, and suggesting a route to a four-force unification.

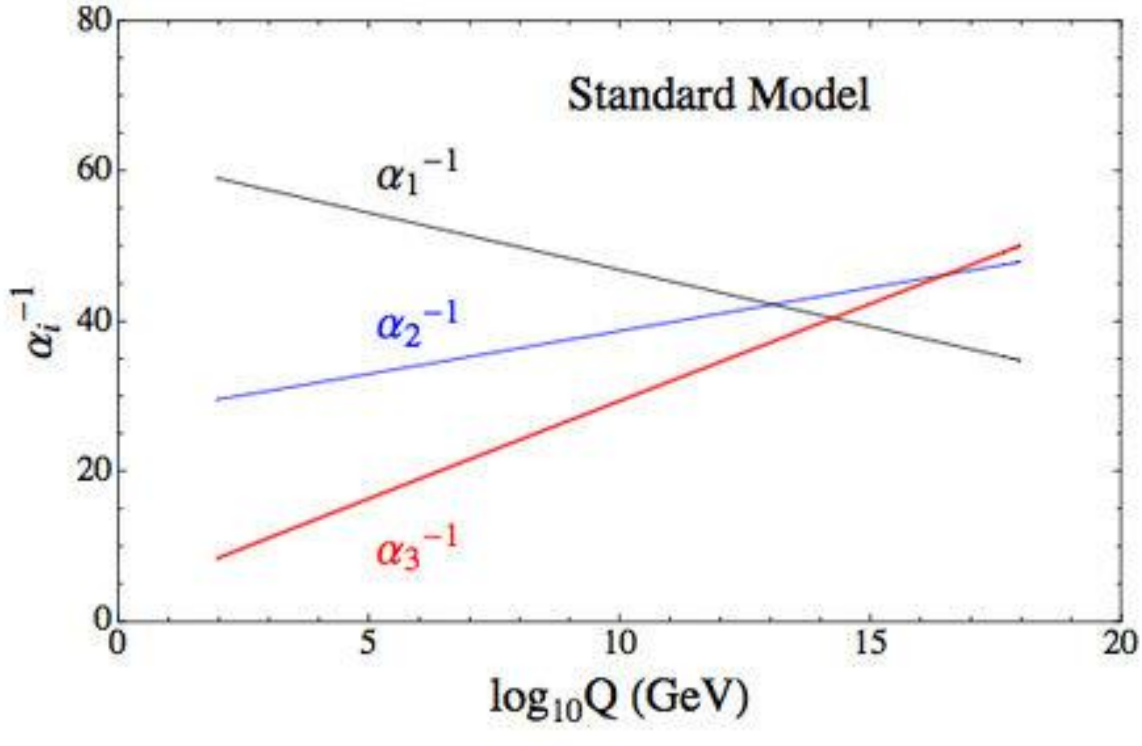

Figure 1. The SU(5) Grand Unification (from [2]).

Many decades later, these hopes have not been met. First, the hoped for convergence never materialised. Second, the SU (5) predicted a new range of X and Y bosons, carrying all three gauge interactions, which seemed to require a decay of the proton, which again has not materialised. Finally, the value for $\sin^2\theta_W$, calculated from the theory at 0.375, was nothing like the experimental value of 0.231, leading to a claim that the parameter would be ‘renormalized’ at higher energies to ~0.2. Another attempt to save the situation was to invoke supersymmetry [3]. If exactly the right amount of the right kind of supersymmetry was included, then the values of the *a* terms would be reduced, leading to an exact unification at ~10^{16} GeV. It was even claimed – and still is – that this was *evidence* for supersymmetry – a claim beyond all reasonable interpretations of the scientific method. The fact that no such evidence has been found suggests that we should abandon this idea as well. Ethan Siegel, an influential and incisive commentator on many aspects of modern physics, suggests that GU 'may be a dead end for physics' [4].

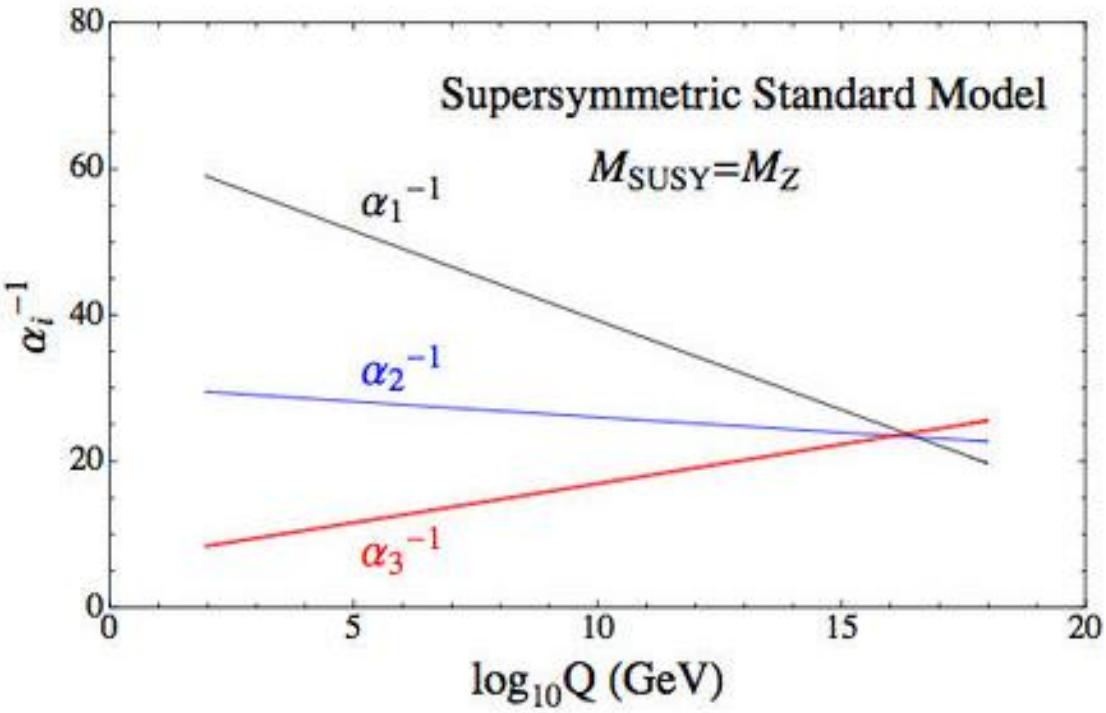

Figure 2. Grand Unification including supersymmetry (from [2]).

3. Two versions of quark theory

Does this mean that we should abandon all hope of GU? When I first started seriously investigating the subject in the 1990s, partly with my colleague John Cullerne, I put down the equations for renormalization in the three gauge theories, and could not reconcile them [5-7]. If I put in the value of 10^{15} or 10^{16} GeV for the GU mass I got nonsense for the three individual equations. They simply didn't work at all. Nor did $\sin^2\theta_W$ 'renormalize' in the way suggested. Instead of decreasing with energy it *increased*, becoming 0.6 at GU, not 0.2! It looked like the supposed convergence was merely a result of compensating errors. But, even more significant, was the fact that the graphs did not represent the convergence of the three gauge interactions at all, for the electric α was replaced by an 'electroweak' α_1.

Instead of using

$$\frac{1}{\alpha(\mu)} = \frac{1}{\alpha_G} + \frac{5}{3\pi} ln \frac{M_X^2}{\mu^2}$$

we now have

$$\frac{1}{\alpha_1(\mu)} = \frac{1}{\alpha_G} + \frac{1}{6\pi} ln \frac{M_X^2}{\mu^2} ,$$

where

$$\frac{5}{3\alpha_1(\mu)} + \frac{1}{\alpha_2} = \frac{1}{\alpha}$$

is introduced from the electroweak theory. It doesn't seem to make sense to unify electroweak, weak and strong rather than electric, weak and strong. But we can see why this was done. If we use *a* rather than a_1, we get nothing resembling convergence at all.

But I also came into the problem with a more flexible attitude to the nature of quarks than has been usual since about 1980, and this may lead us to a remarkable solution of the problem. have no doubt at all that experimental QED studies using deep inelastic scattering tell us that up quarks behave as though they have +2/3 of the value of the fundamental unit of charge (that of the electron), while down quarks behave as though they have –1/3. This is true at all energies and there is no evidence of any transition at higher energies. So the quark charge quantum numbers would be as follows:

Table 2. Quark charges: Gell-Mann Zweig (GMZ).

	Blue	Green	Red
up	$2e / 3$	$2e / 3$	$2e / 3$
	$B / 3$	$B / 3$	$B / 3$
down	$-e / 3$	$-e / 3$	$-e / 3$
	$B / 3$	$B / 3$	$B / 3$

Here, B = baryon number, effectively strong charge.

Experiment shows that QED correctly uses fractional charges. But we don't know, in principle, if these are *fundamental* or if they stem from a broken symmetry. Experimental measurements of fractional charges may be a result of *competing* symmetries, and we will show later how this could be possible. Certainly, there are problems with fractional charges. Do we, for example, have to redefine the fundamental charge unit as e / 3, instead of e? If so, does this mean that the electron is not really a fundamental particle? How does it manage to incorporate 3 repelling units in a point-like structure, and how does an 'elementary' up quark incorporate 2?

But the original and now standard Gell-Mann-Zweig version was not the only quark model proposed. As O. W. Greenberg, one of the major players, wrote in 2015: 'In 1965 Moo-Young Han and Yoichiro Nambu introduced a model in which an *SU*(3) color group is explicit. They avoided fractional electric charges by introducing nine quarks with integer charges …' [8, 9] (The origin of quark color. *Physics Today*, 1 January 2015, p. 33) He continued, 'but they paid the price that color was not an exact symmetry. (Moreover, the notion of integer quarks ended up conflicting with later experimental evidence.)' (Both these claims are wrong.) The Han-Nambu model, the first to introduce colour as a gauge symmetry, can be summarised as follows:

Table 3. Quark charges: Han-Nambu (HN).

	Blue	Green	Red
up	e	e	0
	B	0	0
down	0	0	$-e$
	B	B	B

We may note here that *B* (strong charge) is on only one quark at a time.

Han and Nambu may originally have thought that colour singlets or separated quarks might emerge at higher energies, but it seems that the colour symmetry is never broken. The symmetry is exact because the strong interaction is perfectly gauge invariant and has massless bosons. Greenberg's claim that in their model 'color was not an exact symmetry' is utterly incorrect. There is no problem with colour because the symmetry is exact, and it is entirely due to the strong force or 'colour charge', and has nothing to do with what other charges are present. There is also no problem with experiment because QED and QCD (the theory of the strong interaction) are totally independent.

QED phenomenology clearly doesn't decide the question of which basic structure of charges to use in other areas such as GU, or the *gauge relations between the interactions*. Our fundamental methodology suggests that we go for the best fit with the symmetry, and, if that leads to modifications in our usual practice, see if this has any interesting consequences. In fact, as we shall see, there is one of exceptional interest.

A result of immense significance came with the remarkable discovery, by Laughlin in 1983 [10], of the fractional quantum Hall effect, where electrons assumed effective charges of $e / 3$, $e / 5$ and many other fractional values, by forming pseudobosonic combinations with odd numbers of magnetic flux lines and so became shared out between them. As long as the total number of 'fermions' (i.e. electrons and flux lines) is even, then collective bosonic states can form. For example, 3 electrons can combine with 5 flux lines to produce effective charges on each flux line of $3\ e / 5$. This discovery could have made a difference to the understanding of the quark model, but it didn't.

The Han-Nambu quarks and the fractional quantum Hall electrons form a remarkable comparison. In each case the QED phenomenology is defined by a totally external agent. In the first case a totally gauge invariant *strong* interaction acts independently of any electric charges. This is the interaction which makes the composite baryon (e.g. neutron or proton) by combining quarks into a collective structure, which then determines its QED. In the second case a gauge invariant *weak* interaction (fermion / boson converter) acts independently of any electric charges. The collective state of electrons and flux lines governed by this weak interaction then determines its QED.

4. The effect of the HN model on GU

How would the HN model affect the quantities observed in gauge interactions? First of all, it would make no difference to the squared weak components with only left-handed contributions to weak isospin, which, for the first generation of quarks and leptons, would still be:

$$Tr\left(t_3^2\right) = \frac{1}{4} \times 8 = 2 .$$

But for the electric charge, the summation would be different. For, fractional charges, with both LH and RH contributions in the first generation, we would now obtain

$$Tr\,(Q^2) = 2 \times (1 + 1 + 0 + 0 + 0 + 1 + 1 + 0) = 8$$

and

$$sin^2\theta_W = \frac{Tr\left(t_3^2\right)}{Tr\left(Q^2\right)} = 0.25 .$$

While, for Steven Weinberg, 0.375 is in 'gross disagreement' with the experimental value for $\sin^2\theta_W$ of 0.231 at $M_Z = 91.2$ GeV (a sure sign that there is something intrinsically wrong) [11], 0.25 is relatively close to this value, and the true value may well be 0.25 at the vacuum expectation value of the Higgs field (246 GeV), and close to this at M_W or M_Z. There are also small second order corrections, and the effect of the direct production of *W* and *Z* bosons, producing a distinct dip in the measured value [12], to be taken into account. 0.25 is also the value that would be obtained purely from the leptonic contribution, and it would be somewhat strange if the value for a purely electroweak parameter should be different in the quark and lepton sectors. The HN model, unlike GMZ, makes quarks leptonlike, allowing a real unification of these sectors.

Note that this is not dependent on the GU gauge group used. So the Georgi-Glashow group $SU(5)$ is not the only one for which it is valid. In fact, Georgi and Glashow's use of the simple group $SU(5)$ was preceded by Pati and Salam's semisimple $SU(4)_L \times SU(2)_L \times SU(2)_R$, in which the $SU(4)_L$ requires an extension of the colour symmetry of $SU(3)$ from 3 to 4 'colours', representing another route to combining of quarks and leptons [13]. Significantly $SO(10)$, which has been favoured by many, incorporates both $SU(5)$ and $SU(4)_L \times SU(2)_L \times SU(2)_R$ as components. In this sense, the Pati-Salam theory, though not directly based on a simple group, is based on a larger simple group. In this context, the Georgi-Glashow argument for $\sin^2\theta_W$ would still exist for Pati-Salam (which provides an alternative argument for 0.25), but the real unification would be through $SO(10)$, which could similarly be applied to $SU(5)$.

There are two significant consequences of the calculation of an independent and credible value for $\sin^2\theta_W = \alpha / \alpha_2 = 0.25$ at the electroweak scale. The first is that we can now produce a much simpler calculation for M_X without making assumptions about the group structure, by avoiding the problematic running coupling constant equation for $1 / \alpha_1$, using only the more secure equations for $1 / \alpha_2$ and $1 / \alpha_3$. In addition, the hypercharge numbers for the $U(1)$ electromagnetic running coupling equation will now be no longer identical to those for a quark model based purely on QED phenomenology. Looking at the original three graphs for the fine structure constants, we see that those for α_2 and α_3 cross at a distinctively higher value of energy m than when combined with that for α_1. Because α_2 depends on $\sin^2\theta_W$, when this is reduced from 0.375 to 0.25, the crossing point is pushed to an even higher energy. We can use this to find M_X or the GU energy scale, and α_G, which is the fine structure constant at this energy. Let us look again at the two key equations:

$$\frac{1}{\alpha_2(\mu)} = \frac{1}{\alpha_G} - \frac{5}{6\pi} ln \frac{M_X^2}{\mu^2} ,$$

$$\frac{1}{\alpha_3(\mu)} = \frac{1}{\alpha_G} - \frac{7}{4\pi} ln \frac{M_X^2}{\mu^2} .$$

We have good experimental values of

$$\alpha_3(M_Z^2) = 0.118 \text{ (or 0.12)}$$

and

$$\alpha(M_Z^2) = 1 / 128$$

at the electroweak energy $\mu = M_Z = 91.2$ GeV. We also have a calculated $\sin^2\theta_W = 0.25$ or close, which allows us to convert $\alpha M_Z^2)$ to $\alpha_2(M_Z^2)$. If we solve the two equations, we obtain a value for the GU energy scale ($M_X = 2.8 \times 10^{19}$ GeV) which is extraordinarily close to the Planck value (1.22×10^{19} GeV), and may well be exactly so, as purely first-order calculations overestimate the value of M_X by more than 50 % [12]. Assuming that M_X *is* the Planck mass, we obtain

$$\alpha_G \text{ (the GU value for all interactions)} = 1 / 52.4$$

and

$$\alpha_2(M_Z^2) = 1 / 31.5,$$

which is exactly the kind of value we would expect for the weak coupling with $\sin^2\theta_W = 0.25$ close to M_Z.

5. Including the electric force

We have shown that the weak and strong coupling constants suggests GU at the Planck mass. Can we include the electric force? The original GU of 1974 used a combined electroweak parameter which made assumptions about group structure, and relied on a particular value for the squared 'Clebsch-Gordan coefficient' of the group, $C^2 = 1 / \sin^2\theta_W - 1 = 5 / 3$, that had no experimental or theoretical justification. However, the fermionic contribution to QED vacuum polarization is, for *fundamental* fractional charges,

$$\frac{4}{3}\times\frac{1}{2}\times\left(\frac{1}{36}\times3+\frac{1}{36}\times3+\frac{1}{9}\times3+\frac{4}{9}\times3+\frac{1}{4}\times1+\frac{1}{4}\times1+1\right)\frac{n_g}{4\pi}=\frac{5}{3\pi},$$

where $n_g = 3$ is the number of fermion generations, and the terms in the bracket represent, respectively, the squared average charge in the isospin quark doublet, the squared charges of the quarks, the squared average charge of the isospin lepton doublet, and the squared charges of the leptons, all for both left- and right-handed states. Modifying this for fundamental integral charges, we obtain:

$$\frac{4}{3}\times\frac{1}{2}\times\left(\frac{1}{4}\times3+\frac{1}{4}\times3+1+1+0+0+0+\frac{1}{4}\times1+\frac{1}{4}\times1+1\right)\frac{n_g}{4\pi}=\frac{3}{\pi}.$$

This result corresponds to a change in the squared Clebsch-Gordan coefficient from $C^2 = 5 / 3$ to $C^2 = 3$, when $\sin^2\theta_W = 1 / (1 + C^2)$ changes from 0.375 to 0.25. With the new values we have obtained for the hypercharge numbers, the running coupling of the pure electromagnetic interaction, will be:

$$\frac{1}{\alpha(\mu)}=\frac{1}{\alpha_G}+\frac{3}{\pi}ln\frac{M_X^2}{\mu^2}.$$

Using just the well-established equations for $1 / \alpha_2$ and $1 / a_3$, and $\sin^2\theta_W = a / \alpha_2$, we obtain

$$sin^2\theta_W(\mu)=\alpha(\mu)\left(\frac{1}{\alpha_3(\mu)}+\frac{11}{6\pi}ln\frac{M_x}{\mu}\right).$$

We can *now* make direct use of the equation we have derived for $1 / \alpha$, with the new hypercharge numbers and GU at the Planck mass, to obtain $1 / \alpha(M_Z^2) = 128$, which is, of course, exactly the value obtained experimentally at energies corresponding to $\mu = M_Z$. Conversely, using $1 / \alpha(M_Z^2) = 128$ gives $M_X = 1.42 \times 10^{19}$ GeV. This appears to be a striking confirmation of the assumptions made in the first calculation, leading to M_X, as coincidental agreements are most unlikely for equations involving logarithmic terms, and it is also potentially very significant, for it would now appear that the unification which occurs at M_X might well involve a direct numerical equalization of the strengths of the three, or even four, physical force manifestations, without reference to the exact unification structure.

We can now summarise the Grand Unification theory in four equations:

$$\frac{1}{\alpha(\mu)}=\frac{1}{\alpha_G}+\frac{3}{\pi}ln\frac{M_X^2}{\mu^2}$$

$$\frac{1}{\alpha_2(\mu)}=\frac{1}{\alpha_G}-\frac{5}{6\pi}ln\frac{M_X^2}{\mu^2}$$

$$\frac{1}{\alpha_3(\mu)}=\frac{1}{\alpha_G}-\frac{7}{4\pi}ln\frac{M_X^2}{\mu^2}$$

$$sin^2\theta_W(\mu)=\alpha(\mu)\left(\frac{1}{\alpha_3(\mu)}+\frac{11}{6\pi}ln\frac{M_x}{\mu}\right).$$

The results are represented in graphical form in Figure 3 for the coupling constants and in Figure 4 for $\sin^2\theta_W$. In this representation, the electroweak parameter $\sin^2\theta_W$ notably becomes 1 at Grand Unification, the only true unification of electric and weak.

At GU, we may suppose, all four forces are reduced to scalar phases, with $U(1)$ symmetry and purely Coulombic interaction, all distinguishing aspects of the weak and strong interactions having diminished to zero. One of the most significant aspects of the calculation is that it leads to completely testable predictions, as the values of the three coupling constants can be calculated for any energy with relative precision from the known values of a_G and M_X. In particular, the value of α changes rapidly in a way that can be determined at energies now available to us experimentally. At 14 TeV, for instance, it would have the value of 1 / 118, compared to 1 / 125 from the minimal *SU*(5) theory of Georgi and Glashow. This is a very definite prediction which can be tested with current experimental facilities.

The GU calculation reduces the number of free parameters. The GU energy at the Planck value is purely a consequence of fundamental constants – it's a pure number. It suggests a unification with gravity, something like gravity-gauge theory correspondence, with a quantum aspect. $\sin^2\theta_W$ at the electroweak scale is a result of pure numerical calculation and it rises to 1 at GU. Here, we see that, at the electroweak scale, the parameter is a measure of the *lack of unification* of the two forces, as true unification requires the value 1. There is no electroweak term at GU. At GU, the logarithmic terms zero in the renormalization equations and only the coupling constant, equivalent to the Coulomb component, remains.

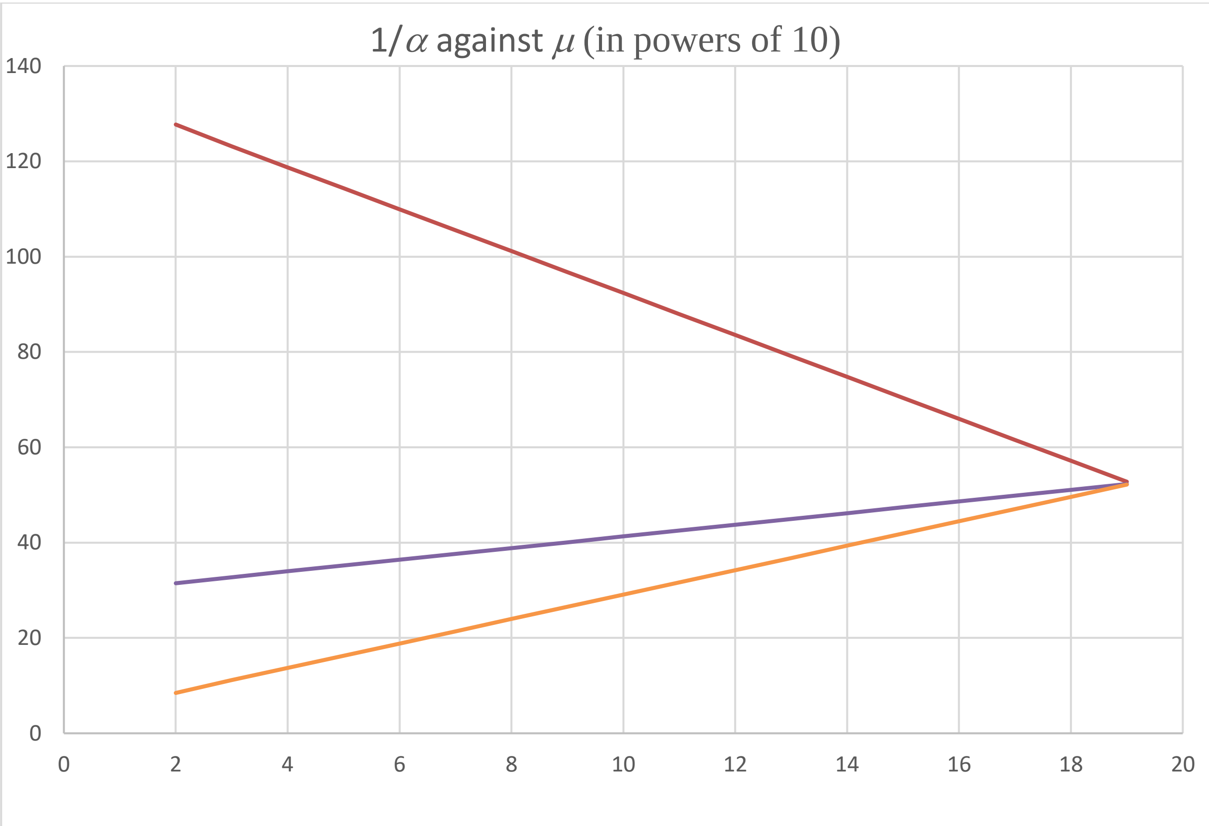

Figure 3. The coupling constants as a function of energy of interaction.

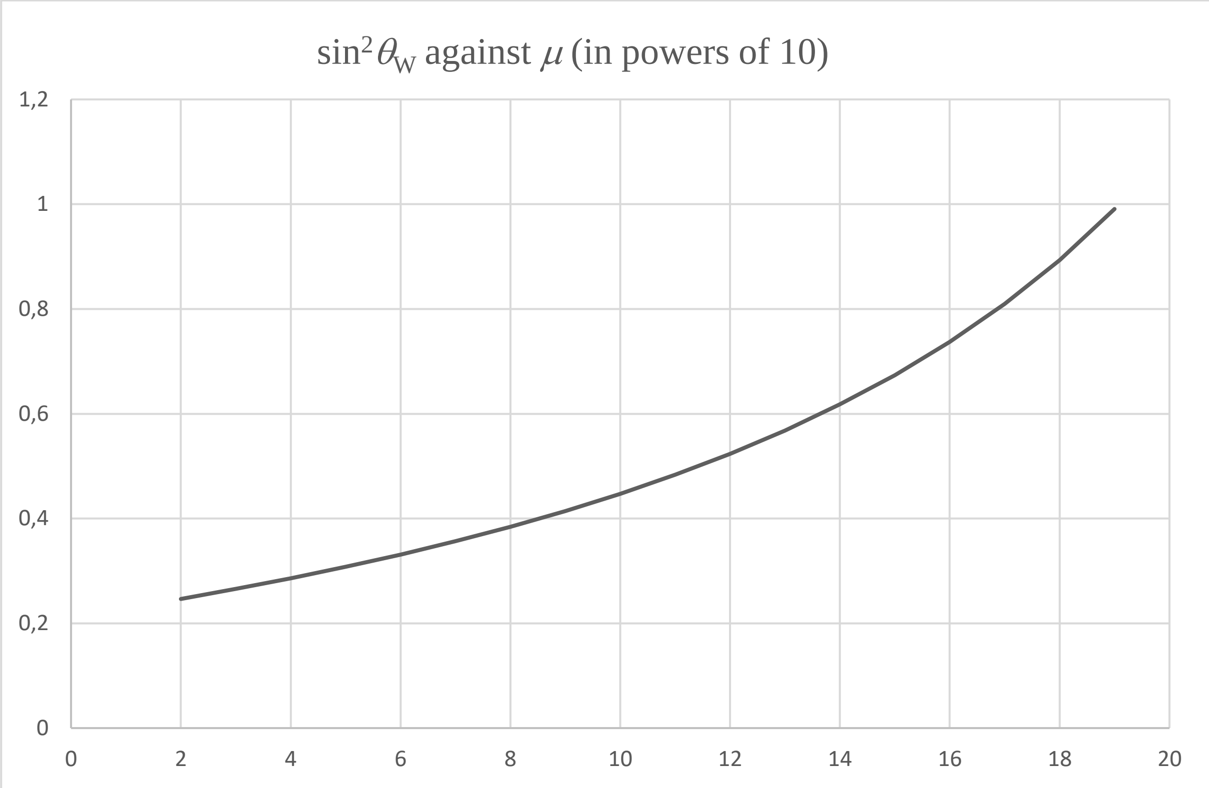

Figure 4. The electroweak parameter as a function of energy of interaction.

6. Conclusion

Grand unification was one of those powerful ideas, like supersymmetry, which seemed destined never quite to live up to its promise. I think the problem stemmed from the lack of understanding of the differences between reality seen as the result of observation (1) and reality seen as underlying structure (2). Broken symmetries, in my view, often stem from clashes between these modes of explanation because they develop from hidden symmetries (reality 2) which are not directly concerned with the observations being made (reality 1). This is what I believe has happened with quarks, where the underlying group structure needed for GU operates on different rules from the QED of observation.

We can overcome this problem by returning to a model that was proposed almost as soon as the quark theory was created and which has never been disproved, but simply fallen out of fashion because more immediate needs have taken precedence. As soon as we do, we obtain remarkable results: a credible value for the electroweak mixing parameter, GU at the Planck mass; pure Coulombic forces at this energy; a connection with gravity, and a meaningful connection between quarks and leptons. And we have predictions that can be tested with realisable experiments.

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