Small x Parton Distributions and Initial Conditions for Ultrarelativistic Nuclear Collisions

At Brookhaven's Relativistic Heavy Ion Collider (RHIC) and at CERN's Large Hadron Collider (LHC) nuclei will be smashed together at energies of 100 GeV and 2.7 TeV per nucleon, respectively, with the expectation of creating an exotic, short-lived state of matter called the quark-gluon plasma. The initial conditions which determine the dynamical evolution of this plasma depend crucially on the small x, or longitudinal momentum, component of the nuclear wavefunction before the collision. I discuss recent work which argues that, for large nuclei, weak coupling techniques in QCD can be used to calculate the distribution of these small x, wee partons. The ramifications of this approach for the dynamics of heavy ion collisions and for the various signatures of quark-gluon plasma are discussed.

Key Words: *high parton density in QCD, screening, non-Abelian Weizsäcker–Williams fields, initial conditions for nuclear collisions, quark-gluon plasma*

1. INTRODUCTION

What does a nucleus look like when it is boosted to relativistic energies? The special theory of relativity tells us that the nucleus must contract a distance R/γ in the direction of its motion, where R is its radius and $\gamma \gg 1$ is the Lorentz factor. If we increase γ indefinitely, do we expect the longitudinal size of the nucleus to shrink to zero? Would this be consistent with the uncertainty principle of quantum mechanics? What would it mean in terms of the underlying parton degrees of freedom? What happens to its transverse size—does it approach a constant at asymptotic energies or does it keep growing?[1]

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With the advent of the Relativistic Heavy Ion Collider (RHIC) at Brookhaven National Laboratory in 1999 and the Large Hadron Collider (LHC) at CERN about five years later, the above questions are not merely academic but are extremely relevant for understanding these collisions. The primary objective of heavy ion collision experiments at these energies is to investigate the dynamics of quarks and gluons at high energy density, often called the quark-gluon plasma, and a transition to more familiar hadronic matter.[2] The formation of the plasma will depend sensitively on the answers to these questions.

In this Comment I will discuss recent work[3–10,28] which seeks to answer the above questions quantitatively by addressing the problem of initial conditions for nuclear collisions within the the framework of Quantum Chromodynamics (QCD). The center of mass energies of the colliding nuclei at RHIC and LHC are 100 GeV and 2.7 TeV per nucleon, respectively. Since these energies are far greater than the mass of a proton or neutron, the appropriate degrees of freedom must be quarks and gluons, whose interactions are described by QCD.

The properties of quarks and gluons in the nuclear wave function at very small values of $x \approx k_t / \sqrt{s}$ turn out to be very relevant for the description of ultrarelativistic nuclear collisions in the center of the momentum frame. (The standard notation is that x is the light cone fraction of the nuclear momentum carried by the quark or gluon, k_t is its transverse momentum, and $\sqrt{s}$ is the center of mass energy). Recently there has been renewed interest in QCD at small x because of the results of the deeply inelastic electron-proton scattering experiments for $Q^2 \gg \Lambda^2_{\mathrm{QCD}}$ at the HERA machine at DESY in Hamburg and the nuclear shadowing experiments at Fermilab and CERN. (For an excellent introduction to the field, see Ref. 11.) The results of the HERA experiments show a very rapid rise in parton density for $x \ll 1$. This has been explained by the conventional, leading twist, double leading log approximation in the operator product expansion known as the DGLAP equation[12,13,15] and by the less conventional BFKL equation[16,43] (both acronyms are named after the initials of their respective authors). However, in the asymptotic limit $x \to 0$, neither of these approximations can be correct because they would both violate the unitarity bound on the growth of cross section at high energies.[14]

At very small x, parton densities become very large and many-body effects become important. Consequences of parton overcrowding are that two soft partons may recombine to form a harder parton or a parton may be screened by a cloud of surrounding wee partons.[17,18] These processes inhibit the growth of parton distributions which saturate at some critical x. Indeed, these processes become important in nuclei at larger values of x than in nucleons. This may explain the strong A-dependent shadowing seen in the deep inelastic scattering (DIS) of leptons off nuclei at Fermilab and CERN.[19]

Gluon distributions extracted from the nuclear structure function F_2^A at small x can be used to determine the dynamics after a nuclear collision.[20] Using the QCD factorization theorem, products of the probabilities of finding a parton in the nucleus may then be convoluted with the elementary parton-parton cross sections to determine parton scattering rates. However, factorization breaks down at small x, when coherence effects become important. Partons from one nucleus, which have the typical transverse momenta relevant for mini-jet processes, of 1–5 GeV, cannot resolve individual partons from the other nucleus. As is usual in the quantum theory of scattering, one needs to take the overlap of the *wavefunctions* for these quanta or, more specifically, the small x Fock component of the nuclear wavefunction to determine the subsequent time evolution.

This question about the nuclear wavefunction is best formulated on the light cone using the method of light cone quantization.[21] The light cone QCD Hamiltonian is separable into a kinetic term and a potential term. Alfred Mueller has shown that for heavy quarkonia, where the scale of the coupling constant is set by the mass of the onium, light cone perturbation theory can be used to construct multiparton eigenstates at small x.[23] Unfortunately, despite many attempts which all go under the label of Light Front QCD, only limited success in the *nonperturbative* sector has been achieved.[25]

One can argue that when the density of partons is extremely large, at very low x in a nucleon or in extremely large nuclei, the density of partons sets the scale for the running of the coupling constant. In other words, if

$$\rho = \frac{1}{\pi R^2} \frac{dN_{\text{part}}}{dy} \gg \Lambda^2_{\text{QCD}}, \tag{1}$$

then $\alpha_s(\rho) \ll 1$. Here I will discuss specifically the application of weak coupling techniques in large nuclei $A \gg 1$ at small values of $x \ll A^{-1/3}$. An intrinsic scale in the problem is set by the quantity $\mu^2 \sim A^{1/3}\ \text{fm}^{-2}$, which is the valence quark color charge squared per unit area. Since it is the only scale in the problem, the coupling constant will run as a function of this scale.[3]

First I will motivate a partition function for the parton distributions at small x in the presence of the valence quarks which play the role of external sources. The background field for this theory is the non-Abelian analogue of the well-known Weizsäcker–Williams field in quantum electrodynamics.[26] The parton distribution functions are formally expressed as correlation functions of a two-dimensional Euclidean field theory with the effective dimensionful coupling $\alpha_s\mu$. The correlation functions are expanded order by order in α_s but involve an infinite resummation to all orders in $\alpha_s\mu$.[4]

Lattice results[27] show that this classical theory does not generate a screening mass of order $\alpha_s\mu$ as anticipated, but is instead infrared divergent. In a recent preprint, J. Jalilian-Marian *et al.*[28] argue that the theory in Ref. 4 is ill-defined in the infrared because the authors did not properly regulate a singular term in the classical equations of motion. When properly regulated, the classical equations can be solved *analytically*. The theory does not generate a screening mass, but the dependence of distribution functions on an infrared scale is only logarithmic. Rather remarkably, the solution of the classical problem by J. Jalilian-Marian *et al.* lends itself to a renormalization group picture of the quantum corrections at small x.

Nuclear collisions are addressed next. Within the above picture, nuclear collisions can be understood as the collision of two Weizsäcker–Williams fields. Since the fields are non-Abelian, the classical gluon field generated after the collision is obtained by solving the nonlinear Yang–Mills equations with boundary conditions specified by the Weizsäcker–Williams field of each nucleus.[9,10] In the central region of the collision one sees the highly non-perturbative (in $\alpha_s\mu$) evolution of the Weizsäcker–Williams glue and sea quarks. The time scale for the dissipation of these nonlinearities is $\sim 1/\alpha_s\mu$. For times much larger than this the evolution of these fields can be described by the hydrodynamic equations proposed by Bjorken.[35] The quantum picture of nuclear collisions is also discussed briefly in the context of the onium picture of Mueller.

Finally I will briefly discuss points of commonality as well as difference between the Weizsäcker–Williams model and other models of nuclear collisions, both in their conceptual foundations and in their predictions for the experiments which will be performed at RHIC and LHC. These include the parton cascade model of Geiger and Müller[29,30] and the HIJING cascade of Wang and Gyulassy,[41] the various string fragmentation models,[31] and the color capacitor models[32] which are the QCD analog of the Schwinger mechanism in quantum electrodynamics.

2. COMPUTING PARTON DISTRIBUTIONS FOR A LARGE NUCLEUS

In this section the problem of calculating parton distributions in the nuclear wavefunction is formulated as a many-body problem in the infinite momentum frame using the technique of light cone quantization. For an excellent discussion of the advantages of light cone quantization we refer the reader to Ref. 21. In light cone quantization and light cone gauge, the electromagnetic form factor of the hadron F_2 measured in deep inelastic scattering experiments is simply related to parton distributions by the formula[22]

$$F_2(x,Q^2) = \int^{Q^2} d^2k_t dk^+ x\delta\left(x - \frac{k^+}{P^+}\right)\sum_{\lambda=\pm 1}\langle a_\lambda^\dagger a_\lambda\rangle. \tag{2}$$

Here we use natural light cone coordinates: P^+ is the momentum of the nucleus, k^+ and k_t are the parton longitudinal and transverse momenta, respectively, x is the light cone momentum fraction, Q^2 is the momentum transfer squared from the projectile, and $a^\dagger a$ is the number density of partons in momentum space. One only need integrate the distributions up to the scale Q^2 to make comparison with experiment.

2.1. A Partition Function for Wee Partons in a Large Nucleus

In QED the infinite momentum frame wavefunction of a system in an external source is a coherent state.[3] Failing to do the same in QCD we compute ground state expectation values instead. The partition function for the ground state of the low x partons in the presence of the valence quarks, treated as an external source, is

$$Z = \left\langle 0\left| e^{iTP^-} \right| 0\right\rangle = \lim_{T\to i\infty}\sum_N \left\langle N\left| e^{iTP^-} \right| N\right\rangle_Q. \tag{3}$$

Here P^- (the generator of translations in light cone time $x^+ = (t+x)/\sqrt{2}$) is the light cone QCD Hamiltonian. It can be split separately into kinetic and potential pieces. The sum above implicitly includes a sum over the color labels of the sources of color charge (denoted Q) generated by the valence quarks.

Valence quarks are predominantly found at large values of x. It is therefore reasonable to assume that they constitute the sources of the external charge seen by the wee partons. The current due to the valence quarks is taken to be

$$J_a^\mu = \delta^{\mu +}\rho_a(x^+,\mathbf{x}_\perp)\delta(x^-). \tag{4}$$

In the gauge $A^+ = 0$, the static component J^+ is the only large component of the valence quark current. The transverse and minus components are proportional to $1/P^+$ (P^+ is the light cone momentum of the nucleus) and are therefore small. The current seen by the wee partons is proportional to $\delta(x^-)$ if the valence quarks are Lorentz contracted to a size which is much smaller than a co-moving wee parton's wavelength. This is satisfied if $x \ll 1/Rm \sim A^{-1/3}$, where R is the nuclear radius.

Evaluating the trace in the partition function for quantized sources of color charge is difficult. Resolving the transverse space into a grid of boxes of size $d^2x_t \gg 1/\rho$ (or parton transverse momenta $q_t^2 \ll \rho$) which contain a large number of color charges, the sum over color configurations above can be performed classically.[3] We average over the color charges by introducing in the path integral representation of the partition function the Gaussian weight

$$\exp\left\{-\frac{1}{2\mu^2}\int d^2x_t\rho^2(x)\right\}, \tag{5}$$

where ρ is the color charge density (per unit area) and the parameter μ^2 is the average color charge density squared (per unit area) in units of the coupling constant g. It can be written as

$$\mu^2 = \rho_{\text{val}}\langle Q^2\rangle \equiv \frac{3A}{\pi R^2}\frac{4}{3}g^2 \sim 1.1A^{1/3}\,\text{fm}^{-2}, \tag{6}$$

where $Q^2 = 4g^2/3$ is the average color charge squared of a quark.

We can now write the partition function Z in the light cone gauge $A^+ = 0$ as

$$Z = \int[dA_t\, dA_+][d\psi^\dagger d\psi][d\rho]$$
$$\times \exp\left(iS + ig\int d^4xA_+(x)\delta(x^-)\rho(x) - \frac{1}{2\mu^2}\int d^2x_t\rho^2(0,x_t)\right). \tag{7}$$

The result of our manipulations is to introduce a dimensionful parameter $\mu^2 \approx 1.1\ A^{1/3}\ \text{fm}^{-2}$ in the theory. For an alternative justification of our Gaussian averaging procedure and the A dependence of μ^2, see Ref. 45.

2.2. The Classical Background Field of a Nucleus

The procedure followed in Ref. 4 to compute classical gluon distributions from the above partition function was to find the classical background field in the presence of external sources, compute correlation functions in this background field, and then integrate over the Gaussian random sources.

The solution of the classical Yang–Mills equations in the presence of the external current of Eq. (4) is given by the background field: $A^{\pm} = 0$, $A_i(x) = \theta(x^-)\ \alpha_i(x_t)$. The two-dimensional gauge field α satisfies the physical gauge condition $\nabla \cdot \alpha = g\rho(x_t)$, where $\rho(x_t)$ is the color charge density of the valence quarks. Also, because the field strength $F_{12} = 0$, α_i is a pure gauge: $\tau \cdot \alpha_i = -(1/ig)U\nabla_i U^{\dagger}$, where U is an $SU(N_c)$ compact gauge field. Combining the two equations results in the highly non-linear stochastic differential equation for the U's:

$$\nabla \cdot U \nabla U^{\dagger} = -ig^2 \rho(x_t). \tag{8}$$

To compute the correlation functions $\langle \alpha_i^a(x_t) \alpha_j^b(y_t) \rangle$ associated with our classical solutions, we must solve the above equation and integrate the rho-dependent gauge fields with the Gaussian weight in Eq. (5) over all color orientations of the external sheet of color charge. It was argued in Ref. 4, on dimensional grounds, that the gluon distribution function has the functional form

$$\frac{1}{\pi R^2} \frac{dN}{dx\, d^2k_t} = \frac{(N_c^2 - 1)}{\pi^2} \frac{1}{x} \frac{1}{\alpha_s} H(k_t^2/\alpha_s^2 \mu^2), \tag{9}$$

where $H(k_t^2/\alpha_s^2\mu^2)$ is a non-trivial function of the effective coupling $k_t/\alpha_s\mu$. In the weak coupling limit $k_t \gg g^2\mu$, $H(y) \to 1/y$ and one obtains the Weizsäcker–Williams result,

$$\frac{1}{\pi R^2} \frac{dN}{dx\, d^2q_t} = \frac{\alpha_s \mu^2 (N_c^2 - 1)}{\pi^2} \frac{1}{xq_t^2}, \tag{10}$$

scaled by μ^2. In the strong coupling region $k_t \ll g^2\mu$, where the classical correlation functions have to be solved for numerically, it was conjectured that $H(k_t^2/\alpha_s^2\mu^2) \propto \alpha_s^2\mu^2/(k_t^2 + M_s^2)$. Here $M_s \sim \alpha_s\mu$ is a screening mass which regulates the divergence of the distribution function at small k_t. If such a screening mass did exist, it would provide a simple understanding of saturation already at the classical level.

Unfortunately, this turns out *not* to be the case. R. Gavai and I have computed the classical correlation functions numerically. This required solving stochastic difference equations on a two-dimensional lattice using the conjugate gradient method.[27] Our results suggest the following. Weak coupling on the lattice holds when $0.2g^2\mu L \ll 1$, where L is the lattice size, and strong coupling holds when $0.2g^2\mu L \gg 1$. In the weak coupling limit, our results indicate a discrete transverse momentum dependence which is of the $1/k_t^2$ Weizsäcker–Williams form. As one increases $g^2\mu L \sim 5$, the lattice results still agree reasonably well with the analytical lattice expression, albeit one notices an increasing trend of fewer solutions to the lattice equations at the a^4 level (a is the lattice spacing). For larger values of $g^2\mu L > 10$, no solutions exist at the a^4 level. This result shows that the classical theory in Ref. 4 is ill-defined in the infrared.

J. Jalilian-Marian *et al.*[28] have pointed out that the classical theory in Ref. 4 is flawed because the authors failed to properly solve the Yang–Mills equations for the transverse components A^i of the classical field. The problem originates with the delta function singularity (in the longitudinal light cone coordinate x^-) assumed for the valence quark current in Eq. (4). If one regulates the source such that instead of a δ-function in x^-, the color charge density ρ depends on the spacetime rapidity $y = -\log(x^-)$, the equation for the transverse fields A^i can be re-written as

$$D_i \frac{dA^i}{dy} = g\rho(y, x_t), \tag{11}$$

where D_i is the covariant derivative. The Gaussian weight in Eq. (5) is simultaneously modified to

$$\exp\left\{-\frac{1}{2\chi}\int d^2x_t \mathrm{Tr}\rho^2(y, x_t)\right\}, \tag{12}$$

where $\chi = \int_y^\infty \mu^2(y, Q^2)$ is the charge squared per unit area at rapidities greater than y and transverse momenta of the order Q^2 at which we measure the distribution function.

With these modified source distributions, Jalilian-Marian *et al.* find that the classical equations can now be solved and an analytic solution found for the classical correlation functions.[28] They find that the classical gluon distribution function at large momentum retains the Weizsäcker–Williams form, while at small momenta, $dN/dk_t^2 \sim \log(k_t^2/\chi(y,k_t^2))$. At low momenta the distribution is constant up to logarithmic corrections—the dependence on the strong interaction scale Λ_{QCD} is weak. As we shall see in the next section, this solution of the classical problem naturally lends itself to a renormalization group picture of the quantum corrections at small x. Note that a similar approach to that of Ref. 28 is advocated in interesting recent work by Balitskii.[34]

2.3. Quantum Corrections to Background Field

Quantum corrections to the classical distributions can be computed systematically using the Dyson–Schwinger expansion.[36] In Ref. 6, the small fluctuations propagator for the non-Abelian Weizsäcker–Williams fields was computed in light cone gauge. This expression was then used in Ref. 7 to compute the one loop corrections to the background field and the gluon distribution function.

One finds that the modifications to the background field introduced by quantum fluctuations do not induce extra terms in the expression for the distribution function.[7] This is consistent with the theorem of Dokshitzer, Diakonov and Troyan.[39] The effect of quantum corrections to the background field can be included entirely by replacing the coupling constant g by the renormalized coupling constant g_R which runs as a function of μ^2. The structure of the background field at one loop remains unchanged.

The perturbative expression for the gluon distribution function to second order in α_s is[7]

$$\frac{1}{\pi R^2}\frac{dN}{dx\, d^2k_t} = \frac{\alpha_s \mu^2 (N_c^2-1)}{\pi^2}\frac{1}{xk_t^2}\left\{1+\frac{2\alpha_s N_c}{\pi}\ln\left(\frac{k_t}{\alpha_s \mu}\right)\ln\left(\frac{1}{x}\right)\right\}. \quad (13)$$

The above equation contains both $\ln(1/x)$ and $\ln(k_t)$ corrections to the $1/(xk_t^2)$ distribution and they represent the leading first order contributions to the perturbative expansion for the distribution function. These terms are large in the kinematic range of validity. This signals that in order to properly account for the perturbative corrections one has to devise a mechanism to isolate and sum up these leading corrections.

In the original version of our model (discussed in Refs. 7 and 8), the only sources of color charge were the valence quarks localized in x^- on the light cone. The derivation of the effective action in Eq. (7) for the wee partons followed simply from this *ansatz*. However, this distinction is questionable since one may expect that *hard* gluons at rapidities larger than a particular rapidity value will act as sources of color charge to gluons at that rapidity. Furthermore, since gluons at this rapidity will act as sources to *soft* gluons at lower rapidities, the "back-reaction" of these soft fields must also be accounted for.

In their recent work, Jalilian-Marian *et al.* have shown that both these effects can be included in a Wilson renormalization group approach.[28] The approach is as follows. One defines the spacetime rapidity $y = y_{\rm proj} + \ln(x^-/x_0^-)$ where $y_{\rm proj}$ is the rapidity of the valence quarks and $x_0 = R/\gamma$, where γ is the Lorentz factor (we expect that $y_{\rm spacetime} \approx y_{\rm mom}$). Divide the parton rapidities between $y_{\rm proj}$ and minus infinity into rapidity slices and consider the wee partons in a particular rapidity slice, say between y_N and y_{N+1}. Assume as in our previous *ansatz* that partons at all rapidities above y_N act as classical sources and the classical background field at y_N is as described in the previous section. The partons in the rapidity slice are also coupled to the soft fields at rapidities less than y_{N+1}. One then obtains the effective action for the partons at the rapidity y_{N+1} by integrating over the small fluctuations in the rapidity interval $y_{N+1} < y < y_N$ (or $P^+_{N+1} < P^+ < P^+_N$).

Since the longitudinal momenta in the slice $P^+_{N+1} < P^+ < P^+_N$ are not arbitrarily small, $\alpha_s \ln(1/x_N) \ll 1$. Therefore the quantum fluctuations integrated over are not large. The new effective action for the $N+1$th rapidity slice has exactly the same structure as the original effective action, and one can show that the charge squared per unit area $\chi = \int_y^\infty \mu^2(y, Q^2)$ obeys the evolution equation

$$d\chi(y, Q^2)/dy\, dQ^2 = \frac{N_c}{N_c^2 - 1}\frac{1}{(2\pi)^2}\frac{1}{g^2\pi}\langle \alpha^2(y, Q^2)\rangle, \qquad (14)$$

where $\langle \alpha(y, Q^2)\rangle$ is the classical background field. When the above equation is integrated over y, one obtains a DGLAP-like equation; when integrated over Q^2, one obtains a BFKL-like equation. Within this unified approach it is now possible to systematically incorporate the non-linearities which eventually lead to the saturation of parton distributions. Much work remains to be done in this direction.

3. NUCLEAR COLLISIONS OF WEIZSÄCKER–WILLIAMS FIELDS

In the previous section, we discussed the properties of the Weizsäcker–Williams field of a single nucleus. Recently, A. Kovner, L. McLerran and H. Weigert[9,10] have made significant progress in solving the *classical* problem of the evolution of these fields after the nuclear collision.

Before the two nuclei collide ($t < 0$) the Yang–Mills equations for the background field of two nuclei on the light cone are $A^{\pm} = 0$ and

$$A^i = \theta(x^-)\theta(-x^+)\alpha_1^i(x_\perp) + \theta(x^+)\theta(-x^-)\alpha_2^i(x_\perp). \tag{15}$$

The two-dimensional vector potentials are pure gauges, as in the single nucleus problem, and for $t < 0$ solve $\nabla \cdot \alpha_{1,2} = g\rho_{1,2}(x_\perp)$. The interesting aspect of this solution is that the classical field configuration does not evolve in time for $t < 0$! This is a consequence of the highly coherent character of the wee parton clouds in the nuclei.

The above solution for $t < 0$ is a fairly straightforward deduction from the single nucleus case. The above-mentioned authors find a non-trivial solution to the field equations after the nuclear collision ($t > 0$). It is given by

$$A^{\pm} = \pm x^{\pm}\alpha(\tau, x_\perp); \quad A^i = \alpha_\perp^i(\tau, x_\perp), \tag{16}$$

where $\tau = \sqrt{t^2 - z^2} = \sqrt{2x^+x^-}$. The relation between $A^{\pm}$ follows from the gauge condition $x^+A^- + x^-A^+ = 0$. This solution only depends on longitudinal boost invariant variable τ and has no dependence on the spacetime rapidity variable $y = (1/2)\ln(x^+/x^-)$. Therefore the parton distributions will be boost invariant for all later times. This result supports Bjorken's *ansatz*[35] for the subsequent hydrodynamic evolution of the system.

The above *ansatz* for the background field can be substituted in the Yang–Mills equations to obtain highly non-linear equations for $\alpha(\tau, x_\perp)$ and $\alpha_\perp^i(\tau, x_\perp)$. The detailed expressions are given in Ref. 10. The initial conditions for the evolution of these equations will depend on the single nucleus solutions.

The Yang–Mills equations with the Weizsäcker–Williams boundary conditions are solved in Ref. 10 perturbatively by expanding the fields in powers of the valence quark charge density ρ. Since one now has the complete classical solution for a single nucleus, it is likely that a complete solution to all orders can be obtained for the field of two nuclei.

For asymptotically large τ, Kovner *et al.* find that a gauge transform of the fields α and $\alpha_\perp^i$ (denoted here by ϵ and $\epsilon_\perp^i$, respectively) have the form

$$\epsilon^a(\tau, x_\perp) = \int \frac{d^2k_\perp}{(2\pi)^2}\frac{1}{\sqrt{2\omega}}\left\{a_1^a(\mathbf{k}_\perp)\frac{1}{\tau^{3/2}}e^{ik_\perp x_\perp - i\omega\tau} + \text{c.c.}\right\},$$

$$\epsilon^{a,i}(\tau, x_\perp) = \int \frac{d^2k_\perp}{(2\pi)^2}\kappa^i\frac{1}{\sqrt{2\omega}}\left\{a_2^a(k_\perp)\frac{1}{\tau^{1/2}}e^{ik_\perp x_\perp - i\omega\tau} + \text{c.c.}\right\}, \tag{17}$$

where a_1 and a_2 can be expressed in terms of the ρ fields. In this equation, the frequency is $\omega = | k_\perp |$ and the vector $\kappa^i = \epsilon^{ij} k^j / \omega$.

With the above form for the fields, the expressions for the parton number densities is straightforward. For late times, near $z = 0$, one obtains[42]

$$\frac{dN}{dy d^2 k_\perp} = \frac{1}{(2\pi)^3} \sum_{i,a} |a_i^a(k_\perp)|^2. \tag{18}$$

Averaging over the ρ fields with the Gaussian weight in Eq. (5), one obtains the following result for the gluon distribution at late times after the nuclear collision:

$$\frac{1}{\pi R^2} \frac{dN}{dy d^2 k_\perp} = \frac{16\alpha_s^3}{\pi^2} N_c \left(N_c^2 - 1\right) \frac{\mu^4}{k_t^4} \ln\left(\frac{k_t}{\alpha_s \mu}\right). \tag{19}$$

As suggested by the logarithm, the transverse momentum integrals are infrared divergent. They are cut off by a mass scale $\alpha_s\mu$. However, as discussed earlier, the theory does not contain such a mass scale in the infrared, and a weak logarithmic dependence on the QCD scale will persist. How this may be regulated is an interesting question which should be addressed in future works.

Thus far we have only discussed the dynamical evolution of the classical fields. What about quantum effects? One way to include these is to do what we did for a single nucleus: look at small fluctuations around the background field of two nuclei.[38] The background field in this case is much more complicated than in the single nucleus case and the quantum problem is significantly more difficult. Another approach is to consider what quantum effects do to the coherence of the initial wavepacket.

In this regard, A. H. Mueller's[23] formulation of the low x problem is relevant. For an onium (heavy quark–anti-quark) state, the coupling is weak, and it is shown that the the n-gluon component of the onium wavefunction obeys an integral equation whose kernel in the leading logarithmic and large N_c limit is precisely the BFKL kernel.[16] The derivation relies on a picture in which the onium state produces a cascade of soft gluons strongly ordered in their longitudinal momentum; the ith emitted gluon has a longitudinal momentum much smaller than the $i-1$th.

In the large N_c limit the n gluons can be represented as a collection of n dipoles. Hence, in high energy onium-onium scattering, the cross section is proportional to the product of the number of dipoles in each onium state times the dipole-dipole scattering cross section.[24] This cross section is given by two gluon exchange, the pomeron. More complicated exchanges involving multi-pomerons have been studied recently by Salam.[37] However, despite the mathematical elegance and simple interpretation of the onium approach, it is unclear whether it can be extended to nuclei.

4. PARTON CASCADES AND COLOR CAPACITORS

In this section we will discuss the relation of the present model to some other approaches to model the initial conditions for ultrarelativistic heavy ion collisions. They may be broadly, and somewhat imprecisely, classified as follows: (a) perturbative QCD based models which assume the factorization theorem and incoherent multiple scattering to construct a spacetime picture of the nuclear collision, and (b) non-perturbative models where particle production is based on string fragmentation or pair creation in strong color fields.

Among perturbative QCD based models, the parton cascade model of Geiger and Müller[29,30] has been applied extensively to study various features of heavy ion collisions. The evolution of *classical* phase space distributions of the partons is specified by a transport equation of the form

$$\left[\frac{\partial}{\partial t} - \mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}}\right] F_a(\mathbf{p},\mathbf{r},t) = C_a(\mathbf{p},\mathbf{r},t), \tag{20}$$

where F_a are the *classical* phase space distributions for particle type a and C_a is the corresponding collision integral. The matrix elements in the collision integral are computed from the relevant tree level diagrams in perturbative QCD.

The initial conditions in the parton cascade model are specified at some initial time $t = t_0$ by the distribution $F_a(\mathbf{p}, \mathbf{r}, t = t_0) = P_a(\mathbf{p}, \mathbf{P})\, R_a(\mathbf{r},\mathbf{R})$. The momentum distribution $P_a(\mathbf{p}, \mathbf{P}) = f_a(x, Q_0^2) g(p_t)$ is decomposed into an uncorrelated product of longitudinal and transverse momentum distributions, respectively, where $f_a(x, Q_0^2)$ is the nuclear parton distribution which is taken from deep inelastic scattering experiments on nuclei at the relevant Q_0^2 and $g(p_t)$ is parametrized by a Gaussian fit to proton-proton scattering data. The spatial distribution of the partons is described by a convolution of a Woods–Saxon distribution of nucleons in the nucleus and an exponential distribution of individual partons within each nucleon.

Another model which takes as input the perturbative QCD cross sections is the HIJING model[40,41] which describes nuclear scattering in an eikonal formalism which convolves binary nucleon collisions. In both models, detailed predictions have been made for various observables at RHIC, particularly for mini-jet production.

Where these approaches differ significantly from the Weizsäcker–Williams approach is in the factorization assumption, namely, that partons from one nucleus resolve individual partons of the other in each hard scattering. We have argued that the small x partons which dominate the physics of the central region instead have highly coherent wave-like interactions. This results in a very different spacetime picture for the nuclear collision—at least for the primordial stage of the nuclear collision. For instance, because of the intrinsic $p_t \sim \mu$ carried by the Weizsäcker–Williams (or equivalent) gluons, gluon production is enhanced by a factor α_s relative to the lowest order $gg \rightarrow gg$ mini-jet process in a cascade. A simple explanation for this enhancement is that because the valence quarks absorb the recoil, two off-shell equivalent gluons can combine to produce an on-shell gluon. This will impact significantly the many signatures to be studied at RHIC and LHC, such as jet, dilepton and photon production. Further, the intrinsic p_t of the gluons ensures that intrinsic charm and strangeness production is significantly greater in the Weizsäcker–Williams model.[5]

The non-perturbative stringy models[31] primarily attempt to describe the soft physics in ultrarelativistic nuclear collisions and so it is not clear that there is much overlap with the Weizsäcker–Williams model. However, the latter does provide some insight into one of these approaches, which we shall dub the color capacitor approach. Here it is assumed that the nuclei generate a homogeneous chromo-electric field which produces particles non-perturbatively by a mechanism analogous to the Schwinger mechanism for strong electromagnetic fields. The evolution of these fields, including back-reaction, is determined by a Boltzmann-like equation where the source term now is given by the pair production rate.[32,33]

An important assumption in these color capacitor models is that of homogeneity of the initial field configurations. However, the results discussed in the previous section suggest that the Yang–Mills fields are highly non-linear and inhomogeneous. The time scale for the dissipation of the non-linearities in the fields is $\tau \gg 1/\alpha_s\mu$. It would be interesting to see how the solutions to the transport equations are modified for initial conditions given by the inhomogeneous Weizsäcker–Williams field configurations.

5. CONCLUSIONS

I have outlined in this Comment a QCD based approach to describe the initial conditions for ultrarelativistic nuclear collisions. The central region of these collisions is dominated by wee partons which carry only a small fraction of the nuclear momentum. We have argued that for very large nuclei these partons are only weakly coupled to each other. However, due to their large density, many-body effects are important. The classical behavior of these quanta, which is the QCD analogue of the Weizsäcker–Williams equivalent photons, can be described by an effective two-dimensional field theory. Quantum effects are treated by constructing the small fluctuations propagator in the background field of these quanta and by applying a Wilson renormalization group approach to compute the effective charge which is the source for these quanta.

An important objective of this approach is to understand if there is a Lipatov region in nuclei where the parton densities grow rapidly and if the shadowing of parton distributions in nuclei can be understood to result from the precocious onset of parton screening. It is probable that deep inelastic scattering experiments off large nuclei will be performed at HERA in the near future.[44] If so, one may expect unprecedented high parton densities and interesting and perhaps unexpected phenomena in these experiments.

These experiments at HERA would nicely complement the heavy ion program at RHIC and especially LHC since they probe the same range of Bjorken x. The results of these experiments would therefore place strong bounds on mini-jet multiplicities and other signatures of nuclear collisions. Note that these observables are extremely sensitive to the initial parton distributions (for a discussion, see Ref. 20). However, to fully understand the dynamics of nuclear collisions at central rapidities, we have to understand the initial conditions ab initio—preferably in a QCD based approach like the one discussed in this paper.

At the moment there are still many open questions which remain unresolved. An empirical question regards the applicability of weak coupling methods to large nuclei. Obviously the bare parameter $\mu^2 \sim A^{1/3}$ fm^{-2} is not large enough for realistic nuclei. However, we have argued on the basis of the renormalization group approach that this parameter should effectively be larger and should grow with the increasing parton density at small x. These arguments must be made more quantitative. We would also like to understand better how non-linear effects may be computed self-consistently in this approach.

Despite the many technical problems that remain, there is much cause for optimism since it appears now that the problem of initial conditions in ultrarelativistic nuclear collisions can be treated systematically in a QCD based approach. Because the various empirical signatures depend sensitively on the initial conditions, one may hope to identify and interpret the elusive quark-gluon plasma in ultrarelativistic nuclear collisions at RHIC and LHC early in the next millenium.

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