DOUBLE BETA DECAY RATES AND THE NEUTRINO MASS

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Abstract

Predictions for 2ν and 0ν double beta decay rates are given for all nuclei with $A \geq 70$, for which double beta decay is energetically allowed. These predictions are based on detailed nuclear structure studies of the beta strength distribution and replace earlier estimates basing mostly on phase space considerations. New and more stringent limits on the Majorana neutrino mass are deduced from existing double beta decay experiments. Since the collective effects arising from spin-isospin as well as quadrupole-quadrupole forces are found to lead to a strong reduction of the nuclear matrix elements for two-neutrino double beta decay, but to have only minor influence on the matrix elements $M^{0\nu}$ for the neutrinoless decay mode, the smaller limits for m_ν result mainly from the fact that the widely used scaling procedure underestimates the 0ν matrix elements.

1. Introduction - The neutrino mass and GUTs

The exploration of the nature of the neutrino at the lower end of the mass hierarchy of elementary particles is a great challenge. Nuclear physics can contribute to the solution of the question of neutrino masses and mixing by the investigation of neutrinoless double beta decay (0ν $\beta\beta$). The latter is a sensitive probe for a Majorana mass of the neutrino [1], which might be a unique property of the neutrino among the elementary fermions.

Neutrinos with Majorana masses would correspond to the breaking of B-L symmetry and should manifest themselves in the appearance of neutrinoless double beta decay. We shall briefly repeat the motivation from grand unified theories (GUTs) for searching for Majorana neutrinos by double beta decay experiments (see also [2]).

It is known that, starting from the standard SU(5) model, the prediction for the neutrino mass is zero, because on one hand in minimal SU(5) the right-handed neutrino ν_R (not to be confused with the normal antineutrino conventionally denoted by $\bar{\nu}$; $\bar{\nu} = (\nu_L)^C$) does not exist at all; hence no Dirac mass is possible, and on the other hand SU(5) invariant Majorana couplings are not possible with the Higgs content of the minimal model. However, it is also well known that this result should not be taken too seriously, because there are many shortcomings of this model, for example it does not naturally include CP violation.

Extending the standard model naturally leads to nonvanishing neutrino masses. In SO(10), for example (finite neutrino mass can, of course, also be generated in models based on the SU(5) gauge group with enlarged particle content) the elementary fermions of one family are placed in a 16-dimensional spinor representation, which consists of the 5- and 10 dimensional SU(5) representations and one additional neutral fermion, which has to interpreted as the right-handed neutrino ν_R. From this arrangement it is already clear that the neutrino mass must be related to the masses of the other elementary fermions of the same family. In this model the neutrino has a Dirac mass proportional to the u-quark mass, both masses being equal in the simplest case. Although there is a possibility of making this Dirac mass term for the neutrino vanish by a very delicate choice of model parameters, the most natural prediction is a neutrino Dirac mass in the MeV range. An attractive model for the generation of fermion masses has recently been considered by Stech [3].

It is the exceptional position of the neutrinos that could solve this apparent conflict with reality. An interplay between the large Dirac mass terms, which can hardly be avoided, and an additional Majorana mass term could lead to the phenomenologically desired small neutrino mass eigenstate. As considered by [4], in the presence of a right-handed Majorana term $m_R^M(\overline{\nu_R})^C\ \nu_R$ much larger than

m^D, related to the breaking of a global B-L symmetry, the neutrino mass matrix simplified to the one-flavor case (for n flavors m^D and m_R^M have to be replaced by n x n matrices) would have the form

$$\left(\bar{\nu}_L \overline{(\nu_R)^C}\right) \begin{pmatrix} 0 & m^D \\ m^D & m_R^M \end{pmatrix} \begin{pmatrix} (\nu_L)^C \\ \nu_R \end{pmatrix} \qquad \begin{matrix} m_R^M >> m^D \\ m^D = O(\mathrm{MeV}) \end{matrix} \tag{1}$$

Diagonalisation yields one very light Majorana neutrino with

$$m_1 \simeq \frac{(m^D)^2}{m_R^M}$$

consisting mostly of ν_L, and one very heavy one with $m_2 \simeq m_R^M$. For $m_R^M \gtrsim 10^3$ GeV $m_1 \lesssim O(\mathrm{eV})$ would result. The simple model illustrates the general mechanism of generating small Majorana neutrino masses by the mixing of the unavoidable (except in SU(5)) large Dirac mass terms with even larger Majorana mass terms.

2. Neutrinoless double beta decay and neutrino mass

The lepton number conserving process

$${}^A_Z X \longrightarrow {}^A_{Z+2} X + 2e^- + 2\bar{\nu} \quad (2\nu \text{ decay}) \tag{2}$$

is already allowed in the standard Fermi theory and, as far as particle physics is concerned, can be parameter-free predicted.

The more interesting lepton number violating neutrinoless (0ν) decay,

$${}^A_Z X \longrightarrow {}^A_{Z+2} X + 2e^- \qquad (0\nu \text{ decay}) \tag{3}$$

not possible in the standard model, becomes allowed in the case of right-handed current admixtures or for a finite Majorana mass of the neutrino (a detailed description of the general formalism can be found in [5]).

We neglect right-handed current contributions in the following. In this case the relation between the 0ν decay rate $w_{0\nu}$ observable in the experiments and the effective neutrino mass

$$\langle m_\nu \rangle = \Sigma\, U_{e_i}^2 m_i$$

is given by (see, e.g. [2,6]):

$$w_{0\nu} = f^{0\nu} |1 - x_F|^2 |R_o M^{0\nu}|^2 \langle m_\nu \rangle^2 / m_e^2 \tag{4}$$

Relation (4) is based on the closure approach, in which case the phase space $f^{0\nu}$ can be factorized from the nuclear structure part $|1 - x_F|^2 |R_0 M^{0\nu}|^2$. The closure treatment, which neglects the structure of the intermediate 1^+ spectrum, is well justified only for the 0ν decay, where the intermediate energy denominator is dominated by the virtual neutrino. In the case of 2ν decay the intermediate states must be treated properly [7]. $M^{0\nu}$ is the closure matrix element for Gamow-Teller transitions (arising from the axial vector coupling) and x_F is the relative fraction of Fermi (vector coupling) to Gamow-Teller contributions:

$$M^{0\nu} = \langle F|\Sigma\vec{\sigma}(i)\tau^-(i)\vec{\sigma}(j)\tau^-(j)H(i,j)|I\rangle \tag{5}$$

$$x_F = \left(\frac{g_V}{g_A}\right)^2 \frac{\langle F|\Sigma\tau^-(i)\tau^-(j)H(i,j)|I\rangle}{M^{0\nu}}$$

$H(i,j)$ is the potential describing the exchange of the virtual neutrino and is dominated by a $1/r_{ij}$ dependence. A major problem in applying eq. (4) to deduce limits on the neutrino mass arises from the fact that such nuclear structure calculations up to now predicted much too high decay rates for the conventional allowed 2ν decay as compared to the geochemical total decay rates for ^{82}Se and $^{128,130}Te$ (these are the only nuclei where positive evidence for $\beta\beta$ decay was found). For the sometimes used calculation by Haxton et al. [8] this discrepancy with experiment is a factor 100 compared to the geochemical result for ^{128}Te and 150 for ^{130}Te [9]. Since the calculated matrix elements for 2ν were too large, up to now it was widely believed that similarly the matrix elements for the 0ν decay $M^{0\nu}$ resulting from the same nuclear structure calculations are too large, too, and cannot be used without modification in analyzing experimental data.

A usual procedure was therefore to scale $M^{0\nu}$ down according to the observed discrepancy on the 2ν sector:

$$M^{0\nu}_{scaled} = M^{0\nu}_{calc}\,(\omega_{2\nu_{exp}}/\omega_{2\nu_{calc}})^{1/2} \tag{6}$$

Implicitly this scaling assumes that the 2ν and 0ν matrix elements are reduced proportional to each other by some unknown reduction mechanism. As will be discussed below, we find that such a scaling procedure underestimates systematically the 0ν matrix element and leads to too large neutrino mass limits.

3. Nuclear structure studies of 2ν and 0ν $\beta\beta$ decay

The decay rate for 2ν double beta decay between 0^+ states is given by [2,7]:

$$\omega_{2\nu} = \frac{F_A^4 G^4}{32\pi^7}\int_{m_e}^{t_o+m_e} F(Z,\epsilon_1)|k_1|\epsilon_1 d\epsilon_1 \int_{m_e}^{t_o+2m_e-\epsilon_1} F(Z,\epsilon_2)|k_2|\epsilon_2 d\epsilon_2$$
$$\times \int_0^{t_o-\epsilon_1-\epsilon_2+2m_e} \nu_1^2\nu_1^2 d\nu_1 \sum_{MM'} A_{MM'} \tag{7}$$

with

$$A_{MM'} = \langle F||\sigma\tau^-||M\rangle\langle M||\sigma\tau^-||I\rangle\langle F||\sigma\tau^-||M'\rangle\langle M'||\sigma\tau^-||I\rangle$$

$$\times \frac{1}{3}\,(K_M K_{M'} + L_M L_{M'} + \frac{1}{2} K_M L_{M'} + \frac{1}{2} L_M K_{M'}) \quad .$$

There have been used several approaches to calculate nuclear matrix elements for double beta decay.

The straightforward method is to use conventional shell model techniques [8,10,11] . This method, however, is limited to light double beta emitters. For heavy nuclei the model space and configurations have to be seriously truncated and the collective effects important in double beta decay can only partly be included. It is difficult or even impossible to treat explicitly the intermediate 1^+ spectrum, which could be compared with results from (p,n) experiments, within this approach. This has only been done for ^{48}Ca [12] . In the other calculations [8,10,11] the detailed structure of the intermediate states was neglected and the closure approximation was used. In this approximation eq. (7) is replaced by the analogue of eq. (4).

$$w_{2\nu} = f^{2\nu} \frac{|M^{2\nu}|^2}{\langle E\rangle^2} \text{ with } M^{2\nu} = \langle F|\sum_{ij} \vec{\sigma}(i)\tau^-(i)\vec{\sigma}(j)\tau^-(j)|I\rangle \qquad (8)$$

This greatly simplifies the calculations, but is not justified for 2ν decay-[2,7,12].

Table 1. Nuclear matrix elements for double beta decay calculated a) using only particle number projected BCS wave functions (pairing), and b) with additional spin-isospin and quadrupole-quadrupole forces.

	$R_o\|M^{0\nu}\|$		$\|M^{2\nu}\|$		
	Pairing	Pairing $+\sigma\tau\sigma\tau$ +QQ	Pairing	Pairing $+\sigma\tau\sigma\tau$ +QQ	ξ
^{76}Ge	12.5	10.4	7.1	1.93	5.4
^{82}Se	9.7	8.2	5.5	1.38	5.9
^{128}Te	12.3	10.0	6.3	0.52	19.3
^{130}Te	11.9	9.4	5.9	0.48	19.7
^{134}Xe	14.7	11.2	8.0	0.77	14.6
^{136}Xe	6.0	3.9	3.6	0.08	50.8
^{142}Ce	7.7	6.2	3.3	1.30	4.8

The most important collective effect in double beta decay comes from pairing correlations. They lead to the fact that many shells contribute all coherently to double beta decay. This effect is included in calculations using BCS wave functions [11,13], or in an HFB approach [14]. However, taking into account only pairing forces yields much too large matrix elements for the 2ν decay (see table 1). The next step is to include besides pairing also neutron-proton forces, the most important of these for double beta decay being the spin-isospin part, which leads to the concentration of the GT strength for single β^- decay in the Gamow-Teller giant resonance and also to a reduction of the total β^- strength and more important of the β^+ strength. The spin-isospin (GT) force together with the pairing forces can be included in an RPA approach based on BCS wave functions [15]. It is important to note that in this approach the closure approximation is avoided.

We have performed such RPA calculations using a deformed Nilsson potential for all double beta emitters with $A \geq 70$ [2,6]. The BCS wave functions were separately optimized for parent and daughter nucleus (taking into account the change in proton and neutron numbers). The results are given in table 2. (A somewhat simplified approach of this type containing, however, some questionable points, has been used recently for a few $\beta\beta$-emitters in [16]). Unfortunately this approach is not suitable to calculate explicitly the 0ν matrix elements. One step beyond this RPA treatment is to include besides the spin-isospin part also other neutron-proton forces. We have done this for the strongest part of the neutron-proton interaction, the quadrupole-quadrupole force [2,7]. This force especially in the isotopes $^{128,130}Te$ and $^{128,130}Xe$ leads to low-energetic 2^+ phonons, which are admixed to the 0^+ ground states and give rise to destructive contributions to the 2ν double beta matrix elements. We have performed these calculations for several nuclei using particle number projected BCS wave functions (PBCS) in large model spaces (12 subshells). The strength of the pairing interaction was deduced from existing pickup and stripping data. The neutron-proton Hamiltonian

$$H_{np} = 2\chi\Sigma\vec{\sigma}(i)\tau^-(i)\vec{\sigma}(j)\tau^+(j) - \kappa QQ \qquad (9)$$

was then diagonalized in this model space. Ground state correlations involving four quasiparticles arising from the spin-isospin part were included by exact diagonalization. The spin-isospin force was also applied to the intermediate 1^+ spectrum and the force strength χ was adjusted to reproduce the known position of the GTGR. The quadrupole term κQQ was treated by RPA (lowest order ground state correlations = four quasiparticle excitations are not sufficient for this strongly attractive interaction). The parameter κ was adjusted to reproduce the position of the collective 2^+ states (one-phonon states). A consistency test of

Table 2. Calculated matrix elements and half-lives in the quasiparticle RPA (QRPA) treatment for $\beta\beta$ emitters with A $\geq$ 70. Results from the more refined treatment including quadrupole correlations [2,7] are marked with an asterisk. T_0 denotes the Q-value for double beta decay, δ the deformation parameter used in the calculation, $M^{2\nu}$ is defined in eq. (8). The 0ν half-lives are neutrino-mass dependent and the given figures correspond to $\langle m_\nu \rangle$ = 1 eV (from [6]).

	QRPA					
	T_0 (MeV)	δ	$M^{2\nu}$	$T^{2\nu}_{1/2}$ (years)	$T^{2\nu *}_{1/2}$ (years)	$T^{0\nu}_{1/2} \times \langle m_\nu \rangle^2$ (years x eV^2)
^{70}Zn	1.00	0	6.79	1.9×10^{22}		7.6×10^{23}
^{76}Ge	2.04	0.2	5.54	1.1×10^{20}	2.2×10^{20}	2.6×10^{23}*
^{80}Se	0.136	0.2	5.33	9.0×10^{28}		6.7×10^{25}
^{82}Se	3.01	0.2	4.47	4.5×10^{18}	1.5×10^{19}	9.5×10^{22}*
^{86}Kr	1.25	0	1.63	3.5×10^{22}		5.0×10^{24}
^{94}Kr	1.15	-0.1	4.63	4.1×10^{21}		6.2×10^{23}
^{96}Zr	3.35	-0.12	4.83	5.2×10^{17}		1.6×10^{22}
^{98}Mo	0.111	-0.19	4.33	1.6×10^{29}		7.3×10^{25}
^{100}Mo	3.03	-0.24	3.81	1.8×10^{18}		3.3×10^{22}
^{104}Ru	1.30	-0.26	3.81	1.8×10^{21}		5.0×10^{23}
^{110}Pd	2.01	-0.23	3.56	5.0×10^{19}		1.3×10^{23}
^{114}Cd	0.54	0.14	1.80	2.7×10^{24}		1.7×10^{25}
^{116}Cd	2.81	0	1.63	8.3×10^{18}		1.7×10^{23}
^{122}Sn	0.36	0	1.75	1.4×10^{26}		3.6×10^{25}
^{124}Sn	2.28	0	1.65	9.3×10^{19}		3.1×10^{23}
^{128}Te	0.87	0.15	2.36	1.2×10^{23}	5.7×10^{23}	9.8×10^{23}*
^{130}Te	2.53	0.10	2.55	1.9×10^{19}	1.2×10^{20}	4.6×10^{22}*
^{134}Xe	0.84	0	3.18	5.1×10^{22}	2.5×10^{23}	8.7×10^{23}*
^{136}Xe	2.48	0	1.22	6.0×10^{19}	3.3×10^{19}	3.0×10^{23}*
^{142}Ce	1.41	0	1.21	2.8×10^{21}	4.1×10^{20}	4.7×10^{23}*
^{146}Nd	0.061	0	3.38	2.9×10^{30}		5.6×10^{25}
^{148}Nd	1.93	0.18	2.63	2.5×10^{19}		1.1×10^{23}
^{150}Nd	3.37	0.24	2.17	4.8×10^{17}		2.4×10^{22}
^{154}Sm	1.25	0.28	3.10	9.5×10^{20}		2.4×10^{23}
^{160}Gd	1.73	0.29	3.46	4.4×10^{19}		6.4×10^{22}
^{170}Er	0.66	0.27	2.80	6.6×10^{22}		9.1×10^{23}
^{176}Yb	1.08	0.26	2.69	1.1×10^{21}		2.5×10^{23}
^{186}W	0.49	0.20	2.46	3.2×10^{23}		1.2×10^{24}
^{192}Os	0.41	-0.15	2.32	1.7×10^{24}		1.6×10^{24}
^{198}Pt	1.04	-0.10	0.78	1.2×10^{22}		1.6×10^{24}
^{204}Hg	0.41	0	0.47	4.6×10^{25}		2.6×10^{25}
^{232}Th	0.85	0.23	3.57	1.6×10^{20}		3.8×10^{22}
^{238}U	1.15	0.24	2.87	2.2×10^{19}		2.4×10^{22}

the parameters is given by the fact that only with the pairing strength used, κ varies smoothly between neighbouring isotopes, and B(E2) values are in reasonable agreement with experiment.

The 2ν decay rates resulting from this more refined (compared to the RPA) treatment are also given in table 2. It is seen that inclusion of the κQQ term in some cases leads to a further strong reduction of the decay rates (especially for $^{128,130}Te$). The discrepancy with the experiments on the 2ν sector is considerably reduced compared to the calculations of [8]. While in the detailed treatment leading to the half-lives given in table 2 we include the intermediate 1^+ spectrum explicitly and do not use the closure approximation, the closure matrix elements are given in table 1 for demonstration of the collective effects on the nuclear matrix elements. It is seen that $M^{2\nu}$ is strongly reduced by the spin-isospin and quadrupole-quadrupole correlations compared to the value obtained with pairing wave functions only.

The influence of these long-range correlations on the 0ν matrix elements $M^{0\nu}$, which we have calculated using the same wave functions described above on the other hand is found to be rather weak [6]. 0ν and 2ν matrix elements are, therefore, not proportionally reduced as assumed by the scaling procedure, but $M^{0\nu}$ remains large. The reason for this is the neutrino potential H, which favors the decay of neutrons close to each other. Long-range correlations are therefore less important. One can express this result also by the ratio of closure matrix elements $\xi = R_0 M^{0\nu}/M^{2\nu}$. As can be seen from table 1 we find for ξ a strong dependence on the considered nucleus and generally large values. Using these results for $M^{0\nu}$ we obtain for the 0ν half-lives given in table 2. For the nuclei which we have treated only in RPA and for which we did not calculate $M^{0\nu}$ explicitly, a mean value for ξ was used, deduced from the calculated $M^{0\nu}$ values (see [6]).

Table 3. Limits for the neutrino mass deduced directly from the matrix elements $M^{0\nu}$ of table 1 (unscaled) and from matrix elements reduced according to the discrepancy between calculation and experiment on the 2ν sector (scaled).

	$T^{0\nu}_{1/2}$ experiment (years)			$\langle m_\nu \rangle$ (eV) This work unscaled	scaled	Haxton et al.+) scaled
^{76}Ge	$>2.5 \times 10^{23}$		[17]	<1.0	<3.1	<6.3
^{82}Se	$>3.1 \times 10^{21}$	(1σ)	[18]	<5.5	<17	<32
^{128}Te	$>8 \times 10^{24}$	(2σ)	[9]	<0.35	<1.6	<8.8
^{130}Te	$>2.2 \times 10^{21}$	(2σ)	[9]	<4.5	<21	<108

+) Deduced from the calculated matrix elements of [8].

4. Results for the neutrino mass

Using the matrix elements of table 1, we get the reduced limits on the neutrino mass $\langle m_\nu \rangle$ shown in table 3. The sharpest limits are obtained at present from the direct measurements of ^{76}Ge and from the geochemical result for ^{128}Te. Depending on whether we use directly the calculated $M^{0\nu}$ or scale according to the remaining discrepancy on the 2ν sector, we obtain a limit of 0.35 eV or 1.6 eV for $\langle m_\nu \rangle$. In the case that the still-remaining discrepancy for the 2ν rates could be explained by some refined treatment of long-range nuclear structure correlations, we would expect the lower limit of 0.35 eV to be the more realistic one. The deduced values correspond to effective neutrino masses. It is discussed in detail in [2], that considering possible interference effects between light and heavy neutrinos, consistent with these effective masses the true mass of the lightest (electron) neutrino in the case of mixing with a heavy neutrino of mass larger than 100 MeV could still be as large as 25 eV.

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