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Constant Density Models in Einstein–Gauss–Bonnet Gravity

Sunil D. Maharaj [1,*], Shavani Naicker [1] and Byron P. Brassel [1,2]

[1] Astrophysics Research Centre, School of Mathematics, Statistics and Computer Science, University of KwaZulu-Natal, Private Bag X54001, Durban 4000, South Africa; naickers@ukzn.ac.za (S.N.); byronb@dut.ac.za (B.P.B.)
[2] Department of Mathematics, Faculty of Applied Sciences, Durban University of Technology, P.O. Box 1334, Durban 4000, South Africa
* Correspondence: maharaj@ukzn.ac.za

Abstract

We investigate the influence of the higher-order curvature corrections on a static configuration with constant density in Einstein–Gauss–Bonnet (EGB) gravity. This analysis is applied to both neutral and charged fluid distributions in arbitrary spacetime dimensions. The EGB field equations are generated, and the condition of pressure isotropy is shown to generalise the general relativity equation. The gravitational potentials are unique in all spacetime dimensions for neutral gravitating spheres. Charged gravitating spheres are not unique and depend on the form of the electric field. Our treatment is extended to the particular case of a charged fluid distribution with a constant energy density and constant electric field intensity. The charged EGB field equations are integrated to give exact models in terms of hypergeometric functions which can also be written as a series.

Keywords: EGB gravity; constant energy density; neutral and charged spheres

1. Introduction

The Einstein–Gauss–Bonnet (EGB) gravity theory contains higher-order curvature corrections which extend general relativity. The EGB gravitational field equations satisfy the requirements for a physically acceptable gravity theory, and they produce results which are consistent with observations. There have been a number of recent studies in this direction related to dense compact objects. The stellar objects which have been studied include quark stars, gravastars, neutron stars, anisotropic distributions, polytropic equations of state, and colour–flavour-locked equations of state. The EGB correction terms contribute new geometrical and physical features leading to interesting insights in the development of ultracompact astrophysical bodies. These dynamical and geometrical properties have been studied in several treatments including the analyses in [1–15]. The results obtained can be related to observed astronomical objects. The additional Gauss–Bonnet terms in the gravitational action are therefore physically significant and produce stable compact stellar structures which have the necessary physical attributes.

In EGB gravity, the Gauss–Bonnet coupling constant and the additional higher-order curvature terms play a significant role in the gravitational dynamics of stellar structures. These effects are absent in standard general relativity. Bhar et al. [16] extended the Krori and Barua stellar solution, which models compact objects, to EGB gravity in five dimensions. It was shown that the central pressure and energy density of the resulting compact star model was higher than the general relativity counterpart. This work was extended to include the dynamics of charge: Bhar and Govender [17] showed that the presence of the

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Gauss–Bonnet parameters affected the charge density of the resulting compact star model. As the coupling constant increased, the charge density increased. This implies that the Gauss–Bonnet coupling constant induces a squeezing effect, packing in more charge. This behaviour is not observed in standard general relativity. In the context of gravitational collapse, it was found that collapse in five dimensional EGB gravity ended in a weak and conical singularity that was initially naked [18]. This behaviour is different from the standard five-dimensional general relativistic case for which collapse terminates in a strong singularity obeying the cosmic censorship conjecture.

In addition to the above, recent investigations into compact objects, cosmological fluids, and applications of astrophysical processes have been illustrated in various modified gravity theories. For example, the dynamics of compact objects was studied in $f(R)$ gravity by Ilyas et al. [19]. Compact stars were analysed in the framework of a scalar–tensor theory and an equivalent $f(R)$ gravity using a quadratic scalar potential [20]. Anisotropic spherical models in the presence of charge were investigated in the framework of $f(R, L_m, T)$ gravity [21]. Moreover, the cosmological implications of FLRW spacetime geometry was explored in $f(Q)$ gravity in a non-flat setting using a dynamical systems approach [22]. Similar treatments in this direction were considered and illustrated in the following literature [23–28].

It is necessary to obtain exact solutions to the EGB field equations for stellar modelling in higher spacetime dimensions $N \geq 4$. The constant density solutions are of special significance as they extend the Schwarzschild interior solution. An early study in this direction [29] led to a constant density model solution by applying the potential of Newtonian gravity leading to a class of models with potentials which were independent of spacetime dimension. Interestingly, a particular constant-density EGB solution also arose in the treatment of Maharaj et al. [30], who formulated an algorithm which generated new solutions from a given known seed solution. In general relativity, higher-dimensional constant-density models were studied by Patel et al. [31], Krori et al. [32], Ponce de Leon and Cruz [33], Shen and Tan [34], and Das and DeBenedictis [35], amongst others.

The concept of higher spacetime dimensions is of fundamental importance in applications of astrophysics and cosmology. Dimensions profoundly influence the geometry of spacetime itself and the underlying physics of stellar objects. Most notably, the dimensions appear quite explicitly in the field equations and reveal themselves in the form of metric functions. The equations of motion and the resulting gravitational potentials differ significantly from the four-dimensional counterpart. This idea was found in a number of results in general relativity [36–40] and most recently, in EGB gravity, by Naicker et al. [41]. In a pioneering result, Paul [42] demonstrated that the mass-to-radius ratio changed in a higher-dimensional constant-density gravitating sphere. Furthermore, spacetime dimensions play a significant role in the process of gravitational collapse and black hole dynamics. In the case of black hole physics, it was illustrated in [43] that the location of the horizons of the Vaidya-type radiating black hole changed quite dramatically in comparison to the four-dimensional counterpart. Moreover, the collapse dynamics of the Boulware–Deser spacetime metric were analysed in EGB gravity by a number of works [18,44,45]. In the case of neutral matter, the collapse terminated in an initially naked, weak, and conical singularity with a delay in the event horizon formation. However, in the presence of higher spacetime dimensions, collapse was a strong singularity that may or may not be naked.

For matter distributions with isotropic pressures, the field equations reduce to the EGB condition of pressure isotropy. In general, the EGB condition of pressure isotropy is difficult to integrate as it is an Abel differential equation of the second kind. However, some particular exact solutions have been found by Naicker et al. [41,46] and Ismail et al. [47], which contain earlier exact models. In this paper, we perform a general analysis of the

EGB field equations with a constant energy density. The field equations in EGB gravity are more complicated when compared to general relativity and therefore more difficult to solve. In this treatment, we write the condition of pressure isotropy in a form that generalises the corresponding equation in general relativity. An equivalent system of the EGB field equations is generated. We integrate the field equations with a constant energy density for neutral matter containing the EGB curvature corrections in general. Therefore, we can find all constant-density neutral EGB models with isotropic pressures. It is possible to match the neutral interior to the exterior Boulware–Deser metric. Previous results in $N = 5$ spacetime dimensions are regained. Our results arise from the observation that the corresponding gravitational potential gives a form of the condition of pressure isotropy which is similar to general relativity. The potentials are uniquely given for neutral fluids. The case of charged matter distributions with a constant energy density is also considered. The gravitational potentials are not unique in the presence of a charge, in contrast to neutral fluids. Exact solutions in terms of special functions are possible for constant electric field intensity. The functional form of the condition of pressure isotropy is different in the presence of a charge, and a variety of potentials arise depending on the choice of the electric field.

This paper is structured as follows: In Section 2, we provide an outline of the basic equations in EGB gravity and present the EGB field equations for a charged fluid distribution in a spherically symmetric static setting and in arbitrary dimensions. We analyse the condition of pressure isotropy in Section 3. In Section 4, we discuss the formulation of constant density for neutral fluids in EGB gravity and show that our results regain the five-dimensional solution in [29]. In Section 5, we provide a detailed analysis of the charged condition of pressure isotropy by considering a constant density with a constant electric field. We are able to show that the charged condition of pressure can be solved exactly, yielding a series solution. Finally, we present our conclusion in Section 6.

2. Field Equations

The gravitational action for EGB gravity in arbitrary spacetime dimensions is

$$S = \int \sqrt{-g}[R + \alpha L_{GB}]d^N x + S_{matter}, \tag{1}$$

where $\alpha > 0$, and L_{GB} is the Gauss–Bonnet term defined by

$$L_{GB} = R^2 + R_{abcd}R^{abcd} - 4R_{cd}R^{cd}. \tag{2}$$

Note that when the spacetime dimension is $N = 4$, the Gauss–Bonnet term acts a topological invariant. This may lead to a ghost mode that arises for asymptotically flat spacetimes as pointed out by Boulware and Deser [48] for string-generated models. This ghost mode is avoided in our treatment as we consider spacetime dimensions $N \geq 5$.

In this investigation, we consider a spherically symmetric static spacetime in N dimensions given by

$$ds^2 = -e^{2\nu(r)}dt^2 + e^{2\lambda(r)}dr^2 + r^2 d\Omega^2_{N-2}, \tag{3}$$

where λ and ν are gravitational potentials that are functions of r, and the unit $(N-2)$-sphere is denoted by

$$d\Omega^2_{N-2} = \sum_{i=1}^{N-2}\left(\left[\prod_{j=1}^{i-1}\sin^2(\theta_j)\right](d\theta_i)^2\right), \tag{4}$$

generalising the 2-sphere to higher dimensions.

It is convenient to define quantities associated with the electromagnetic field. The Faraday tensor $\boldsymbol{F}$ has the form

$$F_{ab} = A_{b;a} - A_{a;b}, \tag{5}$$

and is defined in terms of the electromagnetic potential $\boldsymbol{A}$. The electromagnetic matter tensor $\boldsymbol{E}$ in higher dimensions has the form

$$E_{ab} = \frac{1}{\mathcal{A}_{N-2}}\left(F_{ac}F_b{}^c - \frac{1}{4}g_{ab}F_{cd}F^{cd}\right), \tag{6}$$

where $\mathcal{A}_{N-2}$ is the total surface area of the unit $(N-2)$-sphere defined in an N-dimensional manifold by

$$\mathcal{A}_{N-2} = \frac{2\pi^{\frac{N-1}{2}}}{\Gamma\left(\frac{N-1}{2}\right)}. \tag{7}$$

Note that $\Gamma(\dots)$ defines the complete gamma function. Maxwell's equations are given in covariant form by

$$F_{ab;c} + F_{bc;a} + F_{ca;b} = 0, \tag{8a}$$
$$F^{ab}{}_{;b} = \mathcal{A}_{N-2}J^a. \tag{8b}$$

The divergence of $\boldsymbol{F}$ is related to J^a, which is the current density given by

$$J^a = \sigma u^a, \tag{9}$$

for a non-conducting fluid. Here, σ is the proper charge density.

In the presence of a charge, we choose

$$A_a = (\Phi(r), 0, 0, \dots, 0), \tag{10}$$

the electromagnetic potential $\boldsymbol{A}$. This is consistent with the choice made in general relativity when studying static spheres. We then obtain the Faraday tensor component

$$F^{01} = e^{-2(\nu+\lambda)}\Phi'(r) = e^{-(\nu+\lambda)}E(r). \tag{11}$$

We define the electrostatic field intensity by

$$E(r) = e^{-(\nu+\lambda)}\Phi'(r), \tag{12}$$

in terms of $\Phi(r)$.

The energy momentum tensor for neutral matter is defined by

$$T_{ab} = (\rho + p)u_a u_b + p g_{ab}, \tag{13}$$

in the absence of anisotropy and dissipation. We introduce the following matter variables: ρ is the energy density and p is the isotropic pressure. They are measured relative to $\boldsymbol{u}$, which is the comoving fluid velocity ($u^a u_a = -1, u^a = e^{-\nu}\delta^a{}_0$). For a charged relativistic fluid, the energy momentum tensor $\mathcal{T}$ is the sum

$$\mathcal{T}_{ab} = T_{ab} + E_{ab}, \tag{14}$$

which is the basis of this paper.

The simplest extension to general relativity is the second-order Lovelock class of gravitational theories, EGB gravity. In EGB gravity, an additional curvature tensor arises and is known as the second-order Lovelock tensor. This is described by

$$H_{ab} = g_{ab}L_{GB} - 4RR_{ab} + 8R_{ac}R^{c}{}_{b} + 8R^{cd}R_{acbd} - 4R_{a}{}^{cde}R_{bcde}. \tag{15}$$

Note that the second-order Lovelock tensor (15) and the Gauss–Bonnet term (2) are quadratic in the curvature terms. Varying the action in (1) with respect to the metric tensor yields an expression for the EGB field equations. The EGB field equations are defined by

$$G_{ab} - \frac{\alpha}{2}H_{ab} = \kappa_N \mathcal{T}_{ab}. \tag{16}$$

where α is the Gauss–Bonnet coupling constant and

$$\kappa_N = \frac{2(N-2)\pi^{\frac{N-1}{2}}\mathcal{G}}{c^4(N-3)\Gamma\left(\frac{N-1}{2}\right)}. \tag{17}$$

When $N = 4$, $\kappa_N = \kappa_4 = \frac{8\pi\mathcal{G}}{c^4}$.

We now consider using the transformation

$$e^{2\nu(r)} = y^2(x), \quad e^{-2\lambda(r)} = Z(x), \quad x = r^2, \tag{18}$$

first introduced by Durgapal and Bannerji [49] in general relativity. This transformation was first applied to a neutron star model. The line element (3) in terms of the above variables is then given by

$$ds^2 = -y^2(x)dt^2 + \frac{1}{4xZ(x)}dx^2 + xd\Omega^2_{N-2}. \tag{19}$$

This form has proved useful in generating exact solutions to the field equations. The EGB field equations can be expressed as the system

$$\kappa_N\left(\rho + \frac{E^2}{2\mathcal{A}_{N-2}}\right) = (N-2)\left[\frac{(N-3)(1-Z) - 2x\dot{Z}}{2x} + \frac{\hat{\alpha}(1-Z)}{2x^2}\left(-4x\dot{Z} + (N-5)(1-Z)\right)\right], \tag{20a}$$

$$\kappa_N\left(p - \frac{E^2}{2\mathcal{A}_{N-2}}\right) = (N-2)\left[\frac{2Z\dot{y}}{y} + \frac{(N-3)(Z-1)}{2x} + \frac{\hat{\alpha}(1-Z)}{x^2}\left(\frac{4xZ\dot{y}}{y} - \frac{(N-5)(1-Z)}{2}\right)\right], \tag{20b}$$

$$\begin{aligned}\kappa_N\left(p + \frac{E^2}{2\mathcal{A}_{N-2}}\right) = {} & \frac{2}{y}\left[2xZ\ddot{y} + x\dot{Z}\dot{y} + (N-2)\dot{y}Z\right] + (N-3)\left[\dot{Z} + \frac{(N-4)(Z-1)}{2x}\right] \\ & + \hat{\alpha}\left[\frac{4(N-4)Z(1-Z)\dot{y}}{xy} + \frac{8Z(1-Z)\ddot{y}}{y} + \frac{4\dot{Z}\dot{y}(1-3Z)}{y} + \frac{2(N-5)\dot{Z}(1-Z)}{x} - \frac{(N-5)(N-6)(1-Z)^2}{2x^2}\right],\end{aligned} \tag{20c}$$

$$\sigma^2 = \frac{Z\left[2x^{\frac{(N-1)}{2}}\dot{E} + (N-2)x^{\frac{(N-3)}{2}}E\right]^2}{(\mathcal{A}_{N-2})^2\, x^{(N-2)}}. \tag{20d}$$

Note that we have set $\hat{\alpha} = \alpha(N-3)(N-4)$. When $E = 0$, we obtain the field equations for a neutral fluid in EGB gravity. In the case where $\alpha = 0$, then N-dimensional general relativity is regained.

At this point, we mention some general issues related to matching an interior spacetime to an exterior spacetime in EGB gravity to describe a static star. The boundary conditions in EGB gravity were first considered by Davis [50] in the braneworld context, which can be adapted to describe a relativistic star in EGB gravity. The Davis boundary conditions are complicated as they contain the trace of the extrinsic curvature, the divergence-free part of the Riemann tensor, and the Gauss–Bonnet coupling constant. A detailed study of the Davis boundary conditions was performed by Brassel et al. [51], which reduced the Davis boundary conditions to a simpler form. If the Israel–Darmois conditions hold, then the Davis boundary conditions are satisfied. Therefore, it is correct to utilize the continuity of the first and second fundamental forms to model a bound self-gravitating body in EGB gravity. In many instances, particularly in spacetimes with symmetry, the continuity of the second fundamental form is equivalent to imposing the continuity of the first derivatives in the metric functions, which simplifies the calculations [52,53]. For a charged fluid, the exterior spacetime is given by the Wiltshire [54] metric

$$\begin{aligned} ds^2 &= -\left[1+\frac{x}{2\hat{\alpha}}\left(1-\left(1+\frac{4\hat{\alpha}}{(N-3)}\left(\frac{2M}{x^{\frac{1}{2}(N-1)}}-\frac{\kappa_N Q^2}{(N-2)\mathcal{A}_{N-2}x^{N-2}}\right)\right)^{\frac{1}{2}}\right)\right]dt^2 \\ &\quad +\frac{1}{4x\left[1+\frac{x}{2\hat{\alpha}}\left(1-\left(1+\frac{4\hat{\alpha}}{(N-3)}\left(\frac{2M}{x^{\frac{1}{2}(N-1)}}-\frac{\kappa_N Q^2}{(N-2)\mathcal{A}_{N-2}x^{N-2}}\right)\right)^{\frac{1}{2}}\right)\right]}dx^2 \\ &\quad +x d\Omega^2_{N-2}. \end{aligned} \tag{21}$$

For a neutral fluid, we can take the exterior spacetime to be the Boulware–Deser spacetime [48]

$$\begin{aligned} ds^2 &= -\left[1+\frac{x}{2\hat{\alpha}}\left(1-\left(1+\frac{8\hat{\alpha}M}{(N-3)x^{\frac{1}{2}(N-1)}}\right)^{\frac{1}{2}}\right)\right]dt^2 \\ &\quad +\frac{1}{4x\left[1+\frac{x}{2\hat{\alpha}}\left(1-\left(1+\frac{8\hat{\alpha}M}{(N-3)x^{\frac{1}{2}(N-1)}}\right)^{\frac{1}{2}}\right)\right]}dx^2 \\ &\quad +x d\Omega^2_{N-2}. \end{aligned} \tag{22}$$

The metrics (21) and (22) generalise the Reissner–Nordström and exterior Schwarzschild solutions, respectively, in EGB gravity. It is necessary to find matter distributions in the interior spacetime that match the exterior metrics (21) and (22).

3. Condition of Pressure Isotropy

The fundamental equation determining the dynamics of the charged fluid is the condition of pressure isotropy. Equating Equation (20b,c), we obtain the (charged) condition of pressure isotropy

$$\begin{aligned}
&4x^2Z\left[x+2\hat{\alpha}(1-Z)\right]\ddot{y} \\
&+2x\left[x^2\dot{Z}-2\hat{\alpha}\left(2Z(1-Z)-x(1-3Z)\dot{Z}\right)\right]\dot{y} \\
&+(x\dot{Z}-Z+1)\left((N-3)x+2\hat{\alpha}(N-5)(1-Z)\right)y-\frac{(N-2)}{(N-3)}x^2E^2y=0. \qquad (23)
\end{aligned}$$

When $E^2 = 0$, the neutral case arises. Exact solutions of (23) in EGB gravity are difficult to find. However, we can write (23) in a form which is revealing in our subsequent analysis. It is interesting to note that (23) can be written in the form

$$(x+2\hat{\alpha}(1-Z))\mathcal{I}_{GR}+4\hat{\alpha}x^2[2xZ\dot{y}+(1-Z)y]\frac{d}{dx}\left(\frac{1-Z}{x}\right)-\frac{(N-2)}{(N-3)}x^2E^2y=0, \qquad (24)$$

where

$$\mathcal{I}_{GR} = 4x^2Z\ddot{y}+2x^2\dot{Z}\dot{y}+(N-3)(x\dot{Z}-Z+1)y, \qquad (25)$$

is related to N-dimensional general relativity. Clearly, $\mathcal{I}_{GR}$ does not contain α. When $\alpha = 0$, (24) gives the condition

$$\mathcal{I}_{GR}-\frac{N-2}{N-3}xE^2y = 0, \qquad (26)$$

for charged fluids in general relativity. If $E^2 = 0$, then we obtain

$$\mathcal{I}_{GR} = 0, \qquad (27)$$

and we regain the condition of pressure isotropy in N-dimensional general relativity for a neutral fluid.

The form of the condition of pressure isotropy in (24) shows that it is possible to have $\mathcal{I}_{GR} = 0$ and $\alpha \neq 0$. It is important to observe that if $1 - Z \sim x$ and $E^2 = 0$, then we have $\mathcal{I}_{GR} = 0$. In this special case, $\alpha \neq 0$ is possible. This is precisely the case that arises for neutral spheres with constant density in EGB gravity.

4. Constant Density: Neutral Fluids

We first study neutral fluids with constant density. In this case, the energy density is constant, and the electrostatic field intensity is zero. These have the form

$$\rho = \text{constant}\,(\neq 0), \qquad (28a)$$

$$E^2 = 0. \qquad (28b)$$

The case of constant density in the stellar interior in general relativity has been considered by several researchers, and metrics are listed in the review by Kumar and Bharti [55]. The resulting solutions do produce stellar models which are physically reasonable. In EGB gravity, Dadhich et al. [29] showed that static spheres with constant density existed and produced masses that contained the Gauss–Bonnet parameter. Even though constant density is a limiting case, astrophysically reasonable models are permitted. Clearly variable density models allow for a wider range of physical features.

After some calculations, we can show that the field Equation (20a) can be written as

$$\frac{d}{dx}\left[x^{\frac{(N-3)}{2}}(1-Z)+\hat{\alpha}\,x^{\frac{(N-5)}{2}}(1-Z)^2\right] = \frac{\kappa_N\rho}{(N-2)}x^{\frac{(N-3)}{2}}. \tag{29}$$

Equation (29) cannot be solved in general when ρ is a variable depending on r. However, constant density allows us to make progress. Integrating gives

$$x^{\frac{(N-3)}{2}}(1-Z)+\hat{\alpha}\,x^{\frac{(N-5)}{2}}(1-Z)^2 = \frac{2\kappa_N\rho x^{\frac{(N-1)}{2}}}{(N-1)(N-2)}. \tag{30}$$

We have set the constant of integration to vanish to avoid a singularity at $x=0$. The above is a quadratic equation in Z that can be solved to give

$$Z = 1-ax, \tag{31}$$

where a is given by

$$a = \frac{-1+\sqrt{1+4\hat{\alpha}\tilde{\rho}\kappa_N}}{2\hat{\alpha}}, \tag{32}$$

and we let

$$\tilde{\rho} = \frac{2\rho}{(N-1)(N-2)}. \tag{33}$$

With the potential Z in (31) and $E^2=0$, the condition of pressure isotropy (24) reduces to (27). The condition $\mathcal{I}_{GR}=0$ can be solved to give the general solution in the form

$$y = A+BZ^{\frac{1}{2}}, \tag{34}$$

where A and B are constants. The condition $\mathcal{I}_{GR}=0$ is the same as in N-dimensional general relativity; Krori et al. [32] were the first to integrate the full field equations in this context to find (34). The pressure then has the form

$$\begin{aligned}\kappa_N p = & -\frac{(N-2)a}{2}[(N-3)+\hat{\alpha}(N-5)a] \\ & -\frac{(N-2)a}{2}\left[\frac{2(1+2\hat{\alpha}a)B\sqrt{1-ax}}{A+B\sqrt{1-ax}}\right],\end{aligned} \tag{35}$$

and the field equations have been integrated. The line element for neutral constant-energy-density spheres in EGB gravity is then given by

$$ds^2 = -\left(A+B\sqrt{1-ax}\right)^2dt^2+\frac{1}{4x(1-ax)}dx^2+xd\Omega^2_{N-2}, \tag{36}$$

where a is given by (32) and contains the energy density ρ. The potentials in EGB gravity are therefore uniquely determined for a constant energy density for spacetime dimensions $N\geq 5$.

For a physical theory, we need to specify the parameters A and B. We can find the constants A and B by matching the interior spacetime (36) and the exterior Boulware–Deser spacetime (22) at the boundary $r = R$. The free parameters A and B and the masses M and a are determined as follows

$$A = (1 - B)\sqrt{(1 - aR^2)}, \tag{37a}$$

$$B = -\left(1 + \frac{8\hat{\alpha}M}{(N-3)R^{(N-1)}}\right)^{-\frac{1}{2}}\left[1 + \frac{(N-5)M}{a(N-3)R^{(N-1)}}\right], \tag{37b}$$

$$M = \frac{\kappa_N}{2}(N-3)\tilde{\rho}R^{(N-1)}, \tag{37c}$$

$$a = \frac{-1 + \sqrt{1 + 4\hat{\alpha}\tilde{\rho}\kappa_N}}{2\hat{\alpha}}. \tag{37d}$$

Hence, the spacetime dimension N, the stellar radius R, the Gauss–Bonnet parameter α, and the energy density ρ fully specify the constant-density sphere in EGB gravity. The spacetime dimension $N = 5$ is a special case yielding

$$A = (1 - B)\sqrt{(1 - aR^2)}, \tag{38a}$$

$$B = -\left(1 + \frac{8\alpha M}{R^4}\right)^{-\frac{1}{2}}, \tag{38b}$$

$$M = \frac{\pi^2}{2}\rho R^4, \tag{38c}$$

$$a = \frac{-1 + \sqrt{1 + 4\alpha\pi^2\rho}}{4\alpha}. \tag{38d}$$

These regain the results first presented by Dadhich et al. [29].

5. Constant Density: Charged Fluids

We now consider charged fluids with a constant density. This case is more difficult to study as the charged condition of pressure isotropy (24) has to be solved with $E^2 \neq 0$. The simplification for neutral spheres considered previously is not possible, and $\mathcal{I}_{GR} \neq 0$. To make progress, a choice for the electric field E^2 is necessary. We follow an approach first suggested by Wilson [56] in general relativity. Other choices of E^2 are possible but lead to additional complications. This distinguishes the charged case from neutral matter where a unique solution exists. In the case of a charge, the energy density and the electrostatic field intensity are taken to be constants and have the forms

$$\rho = \text{constant}\,(\neq 0), \tag{39a}$$

$$E^2 = \text{constant}\,(\neq 0). \tag{39b}$$

The field Equation (20a) can then be written as

$$\frac{d}{dx}\left[x^{\frac{(N-3)}{2}}(1-Z) + \hat{\alpha}\,x^{\frac{(N-5)}{2}}(1-Z)^2\right] = \frac{\kappa_N\left(\rho + \frac{E^2}{2\mathcal{A}_{N-2}}\right)}{(N-2)}x^{\frac{(N-3)}{2}}. \tag{40}$$

The choice (39) allows us to integrate this equation to give

$$x^{\frac{(N-3)}{2}}(1-Z) + \hat{\alpha}\,x^{\frac{(N-5)}{2}}(1-Z)^2 = \frac{2\kappa_N\left(\rho + \frac{E^2}{2\mathcal{A}_{N-2}}\right)x^{\frac{(N-1)}{2}}}{(N-1)(N-2)}. \tag{41}$$

Note that we have set the constant of integration to zero. Equation (41) is a quadratic equation in Z, which gives the potential

$$Z = 1 - bx, \tag{42}$$

where b is given by

$$b = \frac{-1 + \sqrt{1 + 4\hat{\alpha}\tilde{\rho}\kappa_N}}{2\hat{\alpha}}, \tag{43}$$

and

$$\tilde{\rho} = \frac{2\left(\rho + \frac{E^2}{2\mathcal{A}_{N-2}}\right)}{(N-1)(N-2)}. \tag{44}$$

When $E^2 = 0$, we regain the neutral constant-density models. Note that $b > 0$ always, since $\hat{\alpha}, \tilde{\rho}, \kappa_N > 0$.

To obtain the potential y, we must solve the charged condition of pressure isotropy. The charged condition of pressure isotropy (23) (or equivalently (24)), with the potential Z in (42), then takes the form

$$4x(1 - bx)(1 + 2\hat{\alpha}b)\ddot{y} - 2bx(1 + 2\hat{\alpha}b)\dot{y} - \frac{(N-2)E^2 y}{(N-3)} = 0. \tag{45}$$

Note that when $\hat{\alpha} = 0$ and $N = 4$, we regain the general relativity case of Wilson [56]. We can express (45) as

$$2x(1 - bx)\ddot{y} - bx\dot{y} + Cy = 0, \tag{46}$$

where the constant C is given by

$$C = -\frac{(N-2)E^2}{2(N-3)(1 + 2\hat{\alpha}b)}. \tag{47}$$

Equation (46) governs the gravitational behaviour of y. It is a hypergeometric differential equation, and its general solution can be given in terms of special functions. Using the software package Maple 2024, we can write the potential

$$\begin{aligned} y = {} & C_1 \cdot {}_2F_1\left[\frac{\sqrt{8C+b} - \sqrt{b}}{4\sqrt{b}}, -\frac{\sqrt{8C+b} + \sqrt{b}}{4\sqrt{b}}; 0; bx\right] \\ & + 2C_2 bx \cdot {}_2F_1\left[\frac{\sqrt{8C+b} + 3\sqrt{b}}{4\sqrt{b}}, -\frac{\sqrt{8C+b} - 3\sqrt{b}}{4\sqrt{b}}; 2; bx\right], \end{aligned} \tag{48}$$

in terms of hypergeometric functions where $b \geq -8C$, since $C < 0$ from (47). Hence, the potentials y and Z can be obtained exactly for both constant energy density and electric field intensity.

The form of the solution in (48) is not useful in physical applications. Following the approach of Wilson [56], we can obtain the leading behaviour by performing a series expansion. Equation (46) is a second-order linear differential equation that can be solved using the method of Frobenius, since $x = 0$ is a regular singular point. The series solution is then expressed by

$$y = C_1 y_1 + C_2 y_2, \tag{49}$$

where

$$y_1 = \sum_{n=0}^{\infty} c_n x^{n+1}, \tag{50}$$

and

$$y_2 \quad = \quad \tilde{C} y_1 \ln x + \sum_{n=0}^{\infty} d_n x^n. \tag{51}$$

Using standard methods, we can find difference equations for the coefficients c_n and d_n. These coefficients can be determined by conditions

$$c_n \quad = \quad -\frac{(C - bn(2n-1))c_{n-1}}{2(n+1)n}, \tag{52}$$

for all $n \geq 1$, and

$$d_n \quad = \quad \frac{[b(n-1)(2n-3) - C]d_{n-1} - 4\tilde{C}(n-1)c_{n-1} + b\tilde{C}(4n-7)c_{n-2}}{2n(n-1)}, \tag{53}$$

for all $n \geq 2$. Note that we can expand the two series to obtain

$$y_1 \quad = \quad c_0 x + c_1 x^2 + c_2 x^3 + c_3 x^4 + c_4 x^5 + \dots, \tag{54}$$

$$\begin{aligned} y_2 \quad &= \quad \tilde{C} \ln x \Big(c_0 x + c_1 x^2 + c_2 x^3 + c_3 x^4 + c_4 x^5 + \dots \Big) \\ &\quad + d_1 x + d_2 x^2 + d_3 x^3 + d_4 x^4 + \dots \end{aligned} \tag{55}$$

where the constants are given by

$$c_0 \quad \neq \quad 0, \tag{56}$$

$$c_1 \quad = \quad \frac{(b-C)c_0}{4}, \tag{57}$$

$$c_2 \quad = \quad \frac{(C-6b)(b-C)c_0}{48}, \tag{58}$$

$$c_3 \quad = \quad \frac{(15b-C)(C-6b)(b-C)c_0}{1152}, \tag{59}$$

$$c_4 \quad = \quad \frac{(28b-C)(15b-C)(C-6b)(b-C)c_0}{40}, \tag{60}$$

$$d_0 \quad = \quad 0, \tag{61}$$

$$d_1 \quad \neq \quad 0, \tag{62}$$

$$d_2 \quad = \quad -\frac{\tilde{C}(b-C)c_0}{4} + \frac{b\tilde{C}c_0}{4} + \frac{(b-C)d_1}{4}, \tag{63}$$

$$\begin{aligned} d_3 \quad &= \quad -\frac{5\tilde{C}(C-6b)(b-C)c_0}{144} + \frac{b\tilde{C}(b-C)c_0}{48} \\ &\quad + \frac{(6b-C)}{12}\left(\frac{b\tilde{C}c_0}{4} + \frac{(b-C)d_1}{4}\right), \end{aligned} \tag{64}$$

$$\begin{aligned} d_4 \quad &= \quad \frac{b\tilde{C}(C-6b)(b-C)\tilde{a}_0}{128} - \frac{\tilde{C}(15b-C)(C-6b)(b-C)c_0}{2304} \\ &\quad + \frac{(15b-C)}{24}\left(-\frac{5\tilde{C}(C-6b)(b-C)c_0}{144} + \frac{b\tilde{C}(b-C)c_0}{48}\right. \\ &\quad \left. + \frac{(6b-C)}{12}\left(\frac{b\tilde{C}c_0}{4} + \frac{(b-C)d_1}{4}\right)\right). \end{aligned} \tag{65}$$

Hence, we write the potential y in terms of the leading coefficients from the series expansion. This should be helpful in a physical analysis of the behaviour of a charged compact sphere in EGB gravity. The presence of a charge clearly leads to more complex behaviour.

6. Conclusions

We established a general framework to study constant-density models in EGB gravity for both neutral and charged distributions. The EGB field equations for a charged fluid distribution were generated in N spacetime dimensions for a static spherically symmetric spacetime. The assumption of constant energy density allowed for the integration of the field equations for both neutral and charged fluids. Our approach resulted in a linear form for the gravitational potential Z in both cases. To determine the second potential y, we presented the condition of pressure isotropy in a form which generalised the corresponding equation in general relativity to EGB gravity. In the neutral case, the potential y was found explicitly in terms of elementary functions of x and a complete constant density model was obtained. This uniquely described the neutral gravitating sphere in EGB gravity for $N \geq 5$. Matching at the surface of the star to the exterior Boulware–Deser spacetime was possible. When $N = 5$, we regained the matching conditions in [29]. The charged case was treated differently to the neutral case as a unique solution was not possible; the electric field intensity had to be specified to find a particular model. The charged condition of pressure isotropy, which was a hypergeometric differential equation, contained an additional term involving the electric field intensity leading to more complex behaviour. To make progress, we considered the electric field intensity to be constant, as in Wilson [56] in the general relativity case. We demonstrated that the EGB charged condition of pressure isotropy could be solved exactly in terms of special functions. Also, the method of Frobenius allowed for the potential y to be written in terms of a series. It is important to determine the physics and stability of the resulting gravitational potentials to determine if such solutions describe compact objects. As such, it is then useful to perform a perturbative analysis to determine the stability of model. This has been demonstrated in models in general relativity, and we will pursue this in future work in EGB gravity. We expect the EGB case to be more complicated due to the effects of the higher-order curvature terms. Furthermore, it would be desirable to select different forms of the electric field to generate exact solutions to study charged gravitating spheres. This is also a possible future avenue.

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