28. F. Gursey. Ann. of Phys. *12*, 91 (1961).
29. N. Kemmer. Helv. Phys. Acta. *33*, 829 (1961).
30. P. A. Dirac. Can. J. Phys. *33*, 650 (1955).
31. I. Goldberg. Phys. Rev. *112*, 1361 (1958).
32. R. Arnowitt, S. Deser. Phys. Rev. *113*, 745 (1959).
33. C. L. Hammer, R. H. Good, Jr., Ann. Phys. *12*, 463 (1961).
34. B. S. De Witt. Journ. Math. Phys. *2*, 151 (1961).
35. S. Mandelstam. Quantum electrodynamics without potentials. Preprint (1961).
36. B. S. De Witt. Quantum theory without electromagnetic potentials. Preprint (1961).
37. В. И. Огиевецкий, И. В. Полубаринов. Квантовая электродинамика в терминах напряженностей электромагнитного поля. ЖЭТФ, 1962, в печати.
38. Y. Aharonov, D. Bohm. Phys. Rev. *115*, 485 (1959); *123*, 1511 (1961).

DISCUSSION

BLUDMAN: I would just like to comment that perhaps the role of gauge invariance has recently been overstressed, and that we have seen recently that gauge invariance need not imply masslessness of the electromagnetic field. My interpretation of Schwinger's recent paper in the Physical Review on mass and gauge invariance is that if photons are non-elementary particles to begin with and do not appear in the Lagrangian, then there is certainly no question of gauge transformations on the photon field. Then gauge invariance on the electromagnetic potential plays no role at all, and implies nothing concerning the mass of the photon.

NE'EMAN: Just to add that for the other vector mesons besides the electromagnetic field, we also get too strong conditions through gauge invariance because all couplings would then be F couplings, in the case of a gauge, whereas we have just seen that it is a mixture of F and D in the case of SU_3, for instance.

FEINBERG: Since we now agree that gauge invariance has nothing to do with zero mass, perhaps someone can answer why the photon *does* have zero mass?

YAMAGUCHI: It is a " miserable " experimental fact.

BLUDMAN: How can you know that it really does have zero mass?

FEINBERG: The experimental limit on the photon mass is 10^{-49} g. Historically in physics whenever one has had a thing very close to zero, it has usually actually been zero. Therefore I would say that until you do an experiment to find the photon mass it is safe enough to ask the question I did.

ρ-MESONS AND THE YANG-MILLS FIELD

J. C. Taylor

University of Cambridge, Cambridge

(presented by J. C. Taylor)

Ever since the formulation by Yang, Mills [1)] and Shaw [2)] of a field permitting local isotopic spin rotations, it has been widely held to be in doubt [1, 3, 4)] whether such a field must have zero mass quanta or not. Schwinger [5)] has even questioned whether there is any kinematical argument for the masslessness of the photon.

This question has gained importance since the discovery of the ρ-meson, because many people have speculated whether this might not be connected with the Yang-Mills-Shaw field [3, 6)].

It is the purpose of this note to point out a simple argument, which seems to have been overlooked, that the Yang-Mills-Shaw field (or the electromagnetic field or any field generated by local gauge transformations on internal [(*)] variables) must carry zero mass quanta, at least if current ideas about the analytic properties of Green's functions are correct.

We first observe that the Yang-Mills-Shaw field must have long range components, $\underset{\sim}{f}_{io}$, coupled to the total isotopic spin of any spatially bounded system. This follows by applying Gauss' theorem in 3-dimensions to the equation of constraint [(**)]

$$\partial_i \underset{\sim}{f}_{i0}(x, t) = \underset{\sim}{\mathscr{J}}_0(x, t) \, .$$

We now look for the effect of this observation on the quantized theory. Let $|A\rangle$ be any normalizable state vector [(***)] in which the total isotopic spin, $\underset{\sim}{T}$, has non-vanishing expectation value. Then, using 4-dimensional momentum-space transforms,

$$k_i \langle A | \underset{\sim}{f}_{i0}(\mathbf{k}, k_0) | A \rangle = \langle A | \underset{\sim}{\mathscr{J}}_0(\mathbf{k}, k_0) | A \rangle$$

whence

$$[k_i \langle A | \underset{\sim}{f}_{i0}(\mathbf{k}, k_0) | A \rangle]_{\mathbf{k}=0} = 2\pi\delta(k_0) \langle A | \underset{\sim}{T} | A \rangle \, .$$

It follows that the matrix element $\langle A | \underset{\sim}{f}_{io}(\mathbf{k}, k_0) | A \rangle$ has a singularity at $\mathbf{k} = 0$, $k_0 = 0$. According to current ideas about analyticity, the presence of such a singularity implies the existence of a state $|B\rangle$ with vanishing energy for which $\langle 0 | \underset{\sim}{f}_{i0} | B \rangle \neq 0$. For there to be such a state, massless particles must exist with all the properties of Yang-Mills quanta.

It might be thought that we have used arguments from analyticity too cavalierly, and that the singularity found might correspond to an anomalous threshold or something like that. By specializing to the case where $|A\rangle$ represents a one-particle state, say a proton, one is concerned with the singularities of a 3-point Green's function, two of whose legs represent protons. For this example there is no reason to suppose any anomalous type of singularity should exist.

Departures from charge independence, due to electromagnetic and weak interactions, will presumably give the Yang-Mills particles some mass [(†)]. But it would be surprising if this should be as large as the observed mass of the ρ-mesons. Moreover there is no reason why the (electromagnetic) mass of the charged ρ-mesons should be similar to the mass of the neutral ρ-meson, which is generated entirely from weak interactions.

This note arose out of conversations with C.Troebel and I. Biriela who are responsible for any truth it may contain. It resembles somewhat a forthcoming, much deeper, paper by Goldstone, Salam and Weinberg.

LIST OF REFERENCES

1. C. N. Yang and R. Mills. Phys. Rev. *96*, 191 (1954).
2. R. Shaw. Cambridge University Thesis, unpublished (1954).
3. J. Schwinger, Phys. Rev. *125*, 1043 (1962).
4. See also some of the authors in reference 6.
5. J. Schwinger, Phys. Rev. *125*, 397 (1962).
6. J. J. Sakurai, Phys. Rev. Letters, *7*, 426 (1961). M. Gell-Mann. Phys. Rev. *125*, 1067 (1962). A. Salam, Nuovo Cimento, *23*, 455 (1962). Y. Ne'eman, Nuclear Physics, *26*, 222, (1961).
7. K. M. Case and S. G. Gasiorowicz, Phys. Rev. *125*, 1055 (1962).

DISCUSSION

WEINBERG: I would like to offer a different criticism of this proof which also applies completely to the proof presented by Goldstone, Salam and myself. This proof really does prove that there is a singularity at $k = 0$. However it does not prove that if k is one volt or one millionth of a volt, the amplitude is large. In no sense do you show that there is a pole with some reasonable residue. Now, most proofs in field theory always make one very natural approximation, and that is that they regard the matrix elements to be evaluated in a vacuum and somehow the walls in the room we are in are ignored in these

---

(*) Gravitation is not so simple!

(**) Heavy type denotes isotopic vectors, **x** denotes a spatial vector. In general the notation is that of reference 1.

(***) The argument might fail if no such states existed.

(†) In view of a recently proved theorem, electrically charged spin 1 particles *must* have non-zero mass. See K. M. Case and S. G. Gasiorowicz,[7] Phys. Rev. *125*, 1055 (1962).

calculations. This is reasonable because the walls of the room are a few metres away and a few metres correspond to very small momenta, so as long as you are not concentrating on these very tiny momenta, this is an excellent approximation. However, this proof is not in that position; in this proof you are talking strictly about what happens at $k = 0$ and therefore you must take into consideration the effect of the state A, not only in so far as it consists of one proton here in the room, but in so far as it consists of the room and the rest of the universe. (I have never thought of this in detail for Taylor's proof but I've thought about it a little bit more for ours.) One of the reasons the vector particle coupled to baryon number does not have zero mass, might be that the universe has baryon number. Perhaps the reason the particle coupled to isotopic spin, the Yang-Mills field, does not have zero mass, is that the universe does have a definite isotopic spin orientation in the sense that there are about seven times as many protons as neutrons. There is one case, however, of a well-known conserved quantum number which the universe does *not* have and that is charge. The universe is certainly electrically neutral to a very high degree so my remarks don't apply to the photon. Perhaps this may lead to an answer to Feinberg's question, about why the photon has zero mass.

TAYLOR: May I just make one remark about the purpose of my proof. It was not to answer any very subtle points about theories that are gauge invariant, but it was to find whether the situation of the Yang-Mills field was on a footing with the situation of the electromagnetic field or whether it was on a different footing.

BLUDMAN: Can I ask about the relation between your theory and the Goldstone, Salam and Weinberg one? In what you have said is there any question of a degenerate vacuum? Should I conclude that the proofs are rather different?

WEINBERG: The similarities between the proofs are mathematical. The techniques used are extremely similar; the physical ideas that they deal with are quite different.

NISHIJIMA: I have a question to Dr. Taylor. If this proof applies, does it apply only to the longitudinal wave or does it also apply to the transverse waves?

TAYLOR: Well naturally I hope it applies to the transverse waves but if you can show me that it does not, then of course it breaks down.

ROLLNIK: I would like to ask Dr. Taylor how he could avoid in the frame of his argument that any theory of vector particles must contain mass zero particles; in fact if you look at these arguments, you can formally carry through this proof even if you start with a bare mass m_0 different from zero for the vector field. You have only to redefine your current J_0 by adding a mass term m_0 times the vector potential. So it seems to me that if this proof is correct, then in any theory of vector particles one must have some particles with zero mass.

TAYLOR: I think the answer to this question is that the hypothesis, that there exists a state $|A\rangle$ such that the expectation value of the charge (as you would define it) is non-zero, is no longer true when there is a mass term. The term you add to J, which contains the field we are talking about, exactly cancels out the normal part of the current in any state that you can think of.