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Generalization of the Coleman-Mandula theorem and the ring paradigm

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Abstract. One of the biggest open problems in cosmology is to explain the accelerated expansion of the Universe in the last 5 billion years. There were attempts to solve it classically, but one could also use quantum gravity. We introduce a new approach to the quantization of gravity, called the ring paradigm, according to which graviton is a phonon on a dynamical grid. We obtain, after the application to cosmology, a new model of dark energy, inside which we solve the old problem of the cosmological constant. The only way how to conceptually verify the ring paradigm is to show that there exists a certain generalization of the Coleman-Mandula theorem.

It has already been 119 years since the special theory of relativity was formulated, and we have learned one big lesson from it: no particle or field could travel with a velocity bigger than the velocity of light in vacuum, c; Physicists have tried many times to work with so-called tachyons in modern physical theories, which travel with a velocity bigger than c. But without much success. The objections are usually connected with the fact that we obtain a violation of causality. The tachyon vacuum is unstable, and we get the collapse of unitarity. As it could be shown, [1], each of these arguments considered separately looks relevant, but when we take them together, they are probably wrong. However, we still need a concrete procedure on how to overcome the difficulties. And so the question always remains: if superluminal signalling exists in Nature, what object is the mediator of it?

Actually, the two biggest open questions in modern cosmology are connected with the existence of dark matter and the accelerated expansion epochs in the development of the universe. We don't find it too surprising that we don't know the whole composition of the matter around us therefore, the problem of dark matter is not so much burning. But the problem of dark energy (DE), [2], will probably have a different character. First of all, it could be seen from all the main approaches to quantum gravity (QG), the late-time accelerated expansion of the universe does not seem to be a phenomenon of classical physics, [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19]. And many more puzzling things remained to be discovered. Already, the name DE looks misleading. As was recently stressed by Roger Penrose: 'It is not dark, and it is not energy.' So, what would DE be if it were really the cause of the late-time accelerated expansion? Couldn't we find a sign that we don't know phenomenologically everything concerning the gravity interaction?

Already, the notion of graviton is somehow shrouded in mystery. Freeman Dyson posed the question of whether graviton was actually detectable in the measurement apparatus, [20]. But even when we confirm it in some very futuristic experiment, one needs to find out the velocity c_g, by which the Newtonian gravitational interaction is mediated. Most of the people would explain at this moment that we know the bound on the velocity of gravitational waves c_{gw}, $|c_{gw} - c| < 10^{-15}$. However, we could still suggest that $c_g \neq c_{gw}$ and $c_g > c$ on the full non-perturbative QG level of physics. Then we need to immediately explain what graviton is. This should be exactly the point where the story of a new QG paradigm - ring paradigm (RP) - begins, [21].

Imagine the set of Hopf-linked loops in $\mathbb{R}^3$, Figure 1. They could not be knotted or twisted, and they must not create the Brunnian type of configuration. Every such loop called the gravitational ring will have a length from approximately 1 Planck length to Mpc distances (the whole construction is attached to some boundary). Then, all the elementary particles in the standard model, instead of gravitons, can move just around these rings, and we will use open string (field) theory to model them, [22].

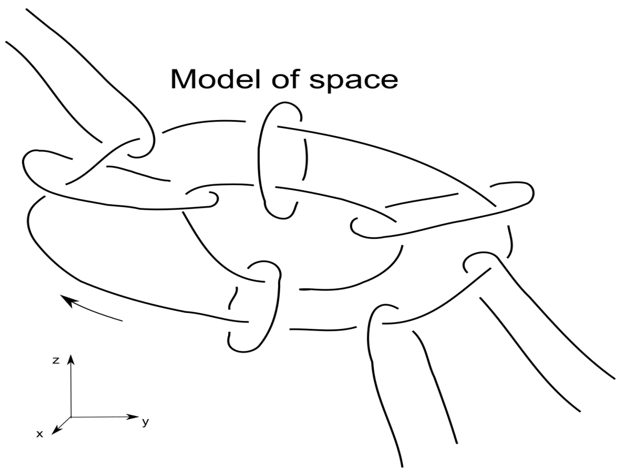

Figure 1:
We start our construction with a discretization of $\mathbb{R}^3$ to the set of Hopf-linked rings, which mediate gravity and create trajectories for other particles and fields. This set is also known from loop quantum gravity, but with a different usage.

The gravitational ring (a more appropriate name for it would be some quasi-field or meta-field, but these terms are already taken) is actually the mediator of the Newtonian gravitational force between all the particles. It could be created over large distances, which means that gravity is an extremely non-local interaction. We can zoom in on the given situation, when the massive objects sit in place in $\mathbb{R}^3$, and they get impulses from the gravitational rings. Furthermore, these entities re-create Planck time later for two fixed objects, so there appears to be some material scattered on the connecting line between them.

If the rings are responsible for the gravitational attraction, we need to reproduce "gravitons" with some computation because gravitational waves would be composed of them. We already know that the ring appears between the other two rings in Planck time, and as the two objects begin to feel the Newtonian force, they start to vibrate. We can imagine that there is effectively a spring between the two bodies. Now we quantize the longitudinal vibrations of the ring (spring), and we identify the quanta with "gravitons", Figure 2. They are called graviton-phonons for obvious reasons, [23]. It is evident that we actually return to the idea of some form of aether in physics, since the graviton-phonons should actually be longitudinal vibrations of it.

The following toy-model illustrates the propagation of graviton-phonons in the crystal, Figure 3. We take a row of galaxies (two for concreteness). They are connected with a bunch of rings, which we substitute with springs. Let's denote the spring constants inside the row by k_3 and on the boundary by k. The masses of the galaxies are fixed to some value m. The total Hamiltonian is constructed for a system of harmonic oscillators tied with springs, and we take over the procedure of the quantization according to [24] (operators will be denoted as other functions, so without a hat):

$$H = \sum_{i=1}^{2} \frac{1}{2m} P_i^2 + \sum_{i,j=1}^{2} V_{ij} Q_i Q_j, \tag{1}$$

where

$$V = \begin{pmatrix} \frac{1}{2}k + \frac{1}{2}k_3 & -\frac{1}{2}k_3 \\ -\frac{1}{2}k_3 & \frac{1}{2}k + \frac{1}{2}k_3 \end{pmatrix},$$

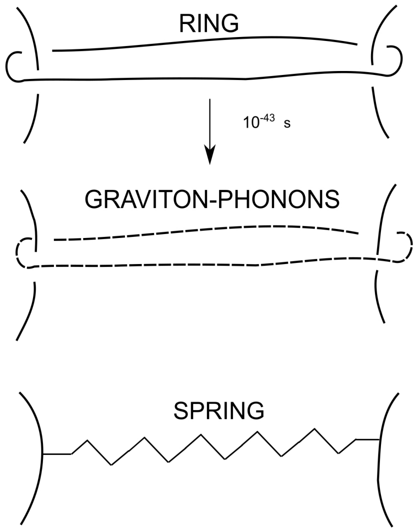

Figure 2:
Here we illustrate the basic idea of RP. Graviton is not a true particle, as are all other particles in the standard model. But the gravitational waves should be composed of graviton-phonons. Another illustrative view of the given situation would be that the gravitational ring (a new phenomenological object in RP, similar to a brane in string theory) is "decaying" in Planck time to the graviton-phonons. Let's imagine we play a dart with an arrow that has a thread - made from ring material - at the end. We give the impulse by the arrow to the target, and there is running a phonon on the thread later.

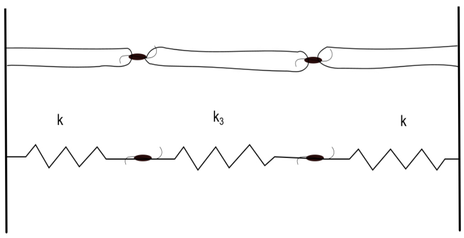

Figure 3: The creation of rings in Planck time effectively gives rise to springs between the galaxies. We quantize their longitudinal vibrations and obtain the graviton-phonons, which mediate the Newtonian force.

$k, k_3 > 0$. Because we are building a field theory concept, it is necessary to do the second quantization. We suppose that there is spreading longitudinal rippling inside the rings. It effectively means some material, which has very special properties, should be present in the space between the galaxies. We start with a quantization of a real scalar field ψ described by this Lagrangian, [24]:

$$L(\psi, \dot{\psi}) = \frac{1}{2} \int \left[\dot{\psi}(x)\right]^2 \, dx - \frac{1}{2} \iint K(x - x')\psi(x)\psi(x') \, dx dx', \tag{2}$$

where $K(x - x') = K(x' - x)$ denotes a potential, and we keep only one dimension. It is obtained directly from the Euler-Lagrange equations that

$$0 = \frac{\partial}{\partial t}\frac{\delta L}{\delta\dot{\psi}(x)} - \frac{\delta L}{\delta\psi(x)} = \ddot{\psi}(x) + \int K(x-x')\psi(x')\ dx'. \tag{3}$$

Let's do an important case. We put $K(x-x') = -c_p^2 \frac{\partial^2}{\partial x^2}\delta(x-x')$ and the Lagrangian is according to (2) the following:

$$L = \frac{1}{2}\int [\dot{\psi}(x)]^2 - c_p^2\big[\frac{d}{dx}\psi(x)\big]^2\ dx \tag{4}$$

Therefore, we get a normal wave equation with the velocity of propagation c_p:

$$\frac{d^2}{dx^2}\psi(x) - \frac{1}{c_p^2}\ddot{\psi}(x) = 0 \tag{5}$$

Now, when the Lagrangian is constructed, we obtain the Hamiltonian and later the momentum of the phonon by following the standard procedures. We compute $\frac{dP}{dt}$ in the Heisenberg representation. The ultimate goal is to reproduce with the bouncing of graviton-phonons the Newtonian gravitational law, and there will be present all three basic constants of Nature, c, G and $\hbar$, in the resulting equation. Actually, it is necessary to equate the impulse of the phonon with the impulse of graviton obtained from the modified theory of gravity with the field equations

$$\mathscr{R}_{\mu\nu} - \frac{1}{2}\mathscr{R}\mathscr{G}_{\mu\nu} + \Lambda_r\mathscr{G}_{\mu\nu} = \frac{8\pi G\mathscr{T}_{\mu\nu}}{c_g^4} = \frac{8\pi G}{c_g^4}(T^m_{\mu\nu} + \mathscr{T}^r_{\mu\nu}), \tag{6}$$

where $\mathscr{G}_{\mu\nu}$ is the metric and also all the other quantities have an analogous meaning as in general theory of relativity. (There are included both the energy-momentum tensor for the ordinary matter $T^m_{\mu\nu}$, as well as the energy-momentum tensor for the gravitational ring, $\mathscr{T}^r_{\mu\nu}$. The cosmological constant Λ_r could be computed from quantum field theory.) We do the identification of the phonon with the graviton by splitting the metric $\mathscr{G}_{\mu\nu} = \eta_{\mu\nu} + \mathscr{H}_{\mu\nu}$ ($\mathscr{H}_{\mu\nu}$ is a small perturbation) and using the standard methods of quantum field theory. The further step is to equate the impulse of the graviton in this theory with the impulse of graviton-phonon in the standard theory with Einstein field equations containing in the energy-momentum tensor $T_{\mu\nu}$ some phantom scalar field. In other words, the gravitational ring will be unstable, and it decays to this phantom field in Planck time, 10^{-43}s, [21].

What is extremely interesting, RP could give an explanation for two facts in classical physics. We know from experiments that gravitational waves are travelling by velocity c_{gw}, which is very close to the velocity of light c with an amazing precision, $|c - c_{gw}| < 10^{-15}$, [25]. But c_{gw} should be exactly $c = c_p$ according to RP (it was supposed in general relativity that $c_{gw} = c$, but there had been no reason for that). Further, the similarity of the formulas for the Coulomb and Newton laws is not accidental, but it is necessary for building the parallelism between photons and graviton-phonons. As we will now see, these are only the first little surprises that RP brings us.

As we already mentioned, the gravitational ring creates a trajectory for all particles and fields, which means that we need to embed the crystal made of rings into the space $\mathbb{R}^3$. Therefore, we obtain a self-evident explanation for the problem of DE because there is present some extra energy (mass) in our model, which will be identified with the missing energy (about 68 %).

Next, let's take two objects connected by a ring. We start to stretch them, and we feel resistance. The situation resembles a quasi-mechanical model when we obtain a Hook law for the dependence of the deformation on the stress. Then the elasticity limit is reached, and finally the spring breaks after crossing the yield point (the detailed model concerns condensed matter physics), [26], which corresponds to the moment in the history of the evolution of the Universe 5 billion years ago, when the galaxies were remoted from each other approximately $8.\bar{3}$ Mpc.

We are now coming to the point, what is the extent of the nonlocality of gravity. The phenomenologically new feature is that the rings are created by some finite velocity c_g, where $c_g > c$. We can obtain a bound on this velocity when we realize that the fibre of rings is prepared in Planck time out and out the whole Universe, so the velocity of the spreading of gravitational interaction is minimally 10^{70}m/s (

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the radius of the observable universe is approximately $46.5.10^9$ light years). This is simultaneously a new limit to the maximal velocity for the spreading of information in our Universe!

It is possible to find a solution to the old problem of the cosmological constant through the application of RP. We assume that general relativity is a plausible theory up to very high energies, and therefore, shortly before the onset of inflation, we can approximate the correct field equations by (7). The cosmological constant Λ_r could be computed from quantum field theory. We neglect the RHS with respect to the LHS, so

$$\mathscr{R}_{\mu\nu} - \frac{1}{2}\mathscr{R}\mathscr{G}_{\mu\nu} + \Lambda_r \mathscr{G}_{\mu\nu} = \frac{8\pi G}{c_g^4}\mathscr{T}^r_{\mu\nu}. \tag{7}$$

The picture seems to be correct because there were only gravitational rings at the beginning of the cosmological inflation. These equations transformed soon into the classical equations of general relativity (in other words, $\mathscr{G}_{\mu\nu}$ is unstable and decays to $g_{\mu\nu}$ Planck time later, details will be delivered by the field equations of RP):

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu} = \frac{8\pi G}{c^4}(T^m_{\mu\nu} + T^{pf}_{\mu\nu}), \tag{8}$$

where $T_{\mu\nu}$ is composed from energy momentum tensor of casual matter $T^m_{\mu\nu}$ and energy momentum tensor of phantom field $T^{pf}_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}\Big(\frac{1}{2}\partial^\sigma\phi\partial_\sigma\phi + V(\phi)\Big)$. A new effective cosmological constant term (in the classical limit caused by the phantom field), Λ, appeared approximately 8 billion years after the Big Bang due to the QG phenomenon:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T^m_{\mu\nu} \tag{9}$$

It should not be surprising that the value of the cosmological constant from the cosmological inflation, Λ_r, possesses many orders of magnitude difference from the value of the effective constant responsible for the late-time cosmic acceleration because they have a different origin. Λ_r should be connected with the energy in the false vacuum (a pure crystal made of rings), well-known from quantum field theory, and the other is a manifestation of the limit of the firmness of the DE material.

We want to stress that the limit of RP to general relativity is immediate because we don't change the shape of Einstein equations even in the high-energy sector. The paradigm also reduces in the corresponding limiting process to standard quantum field theory, which is based on the observation that it contains the standard model of particle physics. RP is a theory within which we will find a clue on how to solve many other open problems in physics. First of all, we wish to mention that it gives a resolution to the black hole information paradox on the full non-perturbative level because the information leaks out from the black hole superluminally in the final part of the evaporation process, [27]. This has been an open problem since the 1970s. Further, the theory is formulated on a dynamical grid. It could shrink, but the lattice is everpresent even in the center of a black hole, so the theory will be non-singular, and we should search for null energy conditions violations in the singularity theorems of R. Penrose and S. Hawking, [28]. But what seems to be completely striking is that we find no singularity in the early Universe, which motivates us in the quest for the classical cosmological limit, and it would be interesting to show whether the cyclic universes of P. Steinhardt and N. Turok (containing probably some inflationary period in the early Universe) fit well into our scheme, [29, 30]. Here we can also see that the presence of the phantom field in this model is often connected with superluminal signaling, which should be in concordance with the postulates of RP.

All these issues are already enough motivation for the formulation of a new physical theory, but RP contributes to further problems. We could speak for the first time about an explanation for the positive space curvature in the Universe based on a quantum model, which follows from the finite velocity of the spreading of the information during the finite time of the travel. Also, worth mentioning, rings are 1+1 dimensional quantum objects, therefore, it looks like the occurrence of the dimensional reduction (mostly to the dimension of spacetime 2) in the other approaches to QG was not accidental, [31]. The paradigm gives new impulse to the problem of determinism in physical theories, and finally, it is related to very interesting mathematical problems. For example, we can construct a well-defined Feynman path integral in full generality because the theory is built on a lattice with a finite size, [32].

There are used two basic tricks in RP. We go beyond the limit of the maximal velocity of the spreading of information in our Universe, which is considered to be c. But this would not yet be enough. We simultaneously postulate the existence of non-local objects - gravitational rings. This simply enables us to avoid the presence of casual tachyons in our theory, RP, but another question immediately appears. It is necessary to violate the Lorentz invariance. Such theories were already considered in the literature, and we have one example in the Hořava-Lifshitz gravities, [33, 34, 35]. But we must be very careful for two main reasons: the Lorentz invariance is one of the fundamental principles that we use in modern physics; It is supported by all the experiments that have been performed so far. There is no evidence that such symmetry has to be broken at high energies. And further, the violation of the Lorentz invariance could have a significant effect on the low-energy physics through interactions between gravity and matter, [36].

Both of these objections can be answered inside RP. First of all, the stringent limits that we have for the possible breaking of the Lorentz invariance are basically bounds on the velocity of graviton-phonons. But these "mediators" travel with the velocity c, so there is no discrepancy with the result of our measurements. We need to remember the details of our construction of gravitational rings for the analysis of the second point. We built our theory in the space $\mathbb{R}^3$, where the rings are created by the velocity c_g. Then the particles move on these objects with the maximal velocity c, which means that we could fully use the design of the "Lorentz invariant" theory with a limiting velocity c_g. To be concrete, we come up with the following generalization of the Lorentz transformations (in 2 dimensions):

$$t' = \frac{t - \frac{x}{v}\frac{v^2}{c^2}\epsilon - \frac{x}{v}\frac{v^2}{c_g^2}}{\sqrt{1 - \frac{v^2}{c^2}\epsilon - \frac{v^2}{c_g^2}}},$$
$$x' = \frac{x - tv}{\sqrt{1 - \frac{v^2}{c^2}\epsilon - \frac{v^2}{c_g^2}}}, \tag{10}$$

where $\epsilon = \epsilon(v)$ denotes some step function defined by the prescription

$$\epsilon(v) = \begin{cases} 1 & \text{for } v \leq c, \\ 0 & \text{for } v > c. \end{cases} \tag{11}$$

The expression $\frac{v^2}{c_g^2}$ is very small for velocities $v < c < c_g$, and we obtain the standard Lorentz transformations in this case. But the given transformations have again the Lorentzian form for $c < v < c_g$. Only gravitational rings could be created by a velocity higher than c, and the ultimate goal is to glue the quantum field theory with the limiting velocity c_g to the standard quantum field theory.

Now we come to the key question. RP is not falsifiable by any experiment in the near future. So, how do we want to verify its validity? "No-Go" theorems in quantum field theory, concretely the McGlinn theorem, LOR theorem, and mainly the Coleman-Mandula theorem, are results proving the impossibility of non-trivially combining Lorentz invariance and internal symmetry for physical theory. If we look at the transformations (10), it is clear these theorems must be generalized. All of them hold if one considers Lie groups as symmetry groups of the theory (the LOR theorem only holds for finite-order Lie algebras, while the CM theorem holds also for the infinite-dimensional case) and are of local nature. However, if one relaxes the assumption of using only standard Lie groups, for example, by allowing for graded structures, then the negative-type of conclusions no longer hold. The most surprising feature of these graded algebras is the occurrence of transformations mixing particles differing by spin: this is the birth of supersymmetry; In [37] the most general supersymmetric algebra of the S matrix was introduced, and its representations were extensively studied, closing the era of the "no-go" theorems with the "let's go" theorem: the Haag-Łopuszánski-Sohnius theorem;

The proofs of these theorems rely heavily on the use of mathematical tools that are peculiar to flat space. It is not immediately obvious how to generalize it to other situations. Further, for RP will be important that the Lorentz group is a non-compact group. And we need to ask what type of theorems we will obtain for the case of compact groups. Then the analogy of the Haag-Łopuszánski-Sohnious theorem would confirm that these types of transformations (10) are really realized in Nature. Especially, we wish to see how the gravitational ring is mathematically represented, because it could not be classified according to an irreducible representation of the group.

We state the main result of this article: there is no exchange of casual gravitons between two bodies; We need to come up with new phenomenological objects that are mediating gravity, and the space around us is filled with them. The classical limit of our theory should be Einsteinian relativity, which contains

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two types of cosmological constants, one true constant and one effective constant. And if confirmed, the theory could have countless technical applications in the far future. We conclude the paper with a quote from R. P. Feynman:

'Everything is made of atoms.'

We agree with this statement. It is only necessary to add that we are composed of elementary particles as well. This is all? Probably not. The next few years will give us the answer as to whether we are also made of rings.

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