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Two-Dimensional Symmetry Breaking at the Event Horizon of Black Holes

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Abstract: This work investigates the combined dynamics of the Yang–Mills and Liouville gravity fields at the event horizon of black holes. To analyze quantum dynamics at the event horizon of black holes existing in a three-dimensional (spatial) universe, a two-dimensional formulation is introduced. The following hypothesis is proposed in this work: there exists a two-dimensional analogue to the Higgs field at the event horizon. This field is then considered as a two-dimensional Yang–Mills field. The interaction and symmetry breaking of the combined two-dimensional Yang–Mills and Liouville gravitational fields are then discussed. The resulting gravitational scalar boson and its implications to the quantum dynamics occurring at the event horizon are presented.

Keywords: Yang–Mills field; two-dimensional Liouville gravity; symmetry breaking; event horizon; gravitational scalar boson

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1. Background

Many recent works have been focused on analyzing the gravitational phenomena within the framework of the Yang–Mills field theory. The Yang–Mills field theory is a type of gauge theory of a special unitary group, SU(N). The theory specifically describes the dynamics of elementary particles using non-abelian Lie groups. Since its conception, Yang–Mills field theory has found many recent implementations across various areas of physics [1]. In [2], Yang–Mills theory was reformulated using lattice gauge theory for construction of simulations. The authors then uncovered numerical evidence that supports the dual superconductivity for quark confinement. The outcome of the simulations was analyzed with respect to the dual Meissner effect where the magnetic monopole currents, chromo-electric flux tube (between quark-antiquark pair) and the type of superconductivity were measured. An interesting research work focusing on the transport coefficients in the Yang–Mills theory and quantum chromodynamics was presented in the work of [3]. In that work, the authors computed the shear-viscosity-over-entropy-density ratio in the Yang–Mills field theory using the Kubo formula. In [3], the author also developed an analytic formula for the temperature dependence of the shear-viscosity-over-entropy-density ratio over a range of temperatures—for glueball resonance gas from low to high temperatures. They then presented an estimate for shear-viscosity-over-entropy-density ratio in quantum chromodynamics.

Theories involving gravity have also been explored using Yang–Mills theories [4,5]. For instance, in Cachazo et al. (2015) [6], the authors employed the Einstein–Yang–Mills theory to theoretically generate tree-level S-matrix elements for a range of theories. The Einstein–Yang–Mills theoretical formulations were able to cater for scenarios with a variety of spins mixed in arbitrary dimensions. An interesting work in this direction is seen in Plefka et al. (2019) [7]. In that work, the authors determined the classical effective action of color charges (along the worldlines). The authors performed this computation by integrating out the Yang–Mills gauge field to next-to-leading-order in the coupling. The validity of the obtained results was checked by designing an effective action in dilaton gravity by redefining the fields and gauge options that significantly simplified the perturbative construction. Further research into the gravitational phenomena using the Einstein–Yang–Mills framework is seen in the work of Liu et al. (2019) [8]. In that research, the authors employed the Einstein–Yang–Mills theory to analyze gravitational lensing phenomena of regular black holes predicted by non-minimal Einstein–Yang–Mills theory. Liu et al. (2019) [8] found that only the apparent radii of the photon spheres are attainable using current technology. Thus, there is still insufficient resolution to differentiate the Reissner–Nordström black hole from the regular Einstein–Yang–Mills black hole. A more recent study on gravitational lensing of black holes using non-minimally coupled Einstein–Yang–Mills theory is seen in Kala et al. (2022) [9]. In that work, the authors studied the effects of the Yang–Mills parameter and the magnetic charge on the radius of photonic orbits. The authors then compared their results against general relativistic Kerr–Newman and Schwarzschild black hole solutions.

In [10], the exploration of the higher-dimensional $f(R)$ gravity was carried out by coupling its spinning solution with the nonlinear Yang–Mills field. The authors employed a series of complex transformations (Newman–Janis Algorithm) to achieve this coupling. In the work of [10], the authors conducted two critical analyses:

(a) Local stability analysis: the stability of local solutions improves as the rotation parameter increases.
(b) Thermodynamic analogue: P–V criticality of both non-rotating and rotating black hole solutions in $f(R)$ gravity coupled with the Yang–Mills field.

Another interesting work concerning higher-spin self-dual Yang–Mills and gravity fields is seen in [11]. In that research, the authors proved an analogue of the Ward theorem for higher-spin extension of the self-dual Yang–Mills field. The authors of [11] also uncover a correspondence between holomorphic vector bundles in twistor space and the solutions of the field equations. The data from their analysis defined the complex structure of the integrable twistor space. A more direct application of the Yang–Mills field in gravitational physics is seen in the work of [12]. In that work, the strongly gravitating, static, spherically symmetric, compact stellar distributions were derived using the minimal geometric deformation method. These distributions were resulting solutions of the Yang–Mills–Einstein–Dirac coupled field equations on fluid membranes (with finite tension). These solutions characterize minimal geometric deformation of the Yang–Mills–Dirac stars. In [12], the physical features of these stars are discussed and their and their Arnowitt–Deser–Misner masses were derived. In astrophysics, the Yang–Mills field has also been employed in the study of accretion models. This can be seen in the work of [13], where the authors studied the image of a power-Yang–Mills black hole in terms of its luminosity under various accretion models. Their studies uncovered that the observed specific intensity is directly affected by the type of emission profile of the accretion. In [13], it was also found that as the power parameter increases, the intensity of the black hole grows with it.

The black hole phase transition phenomenon has also been studied in the context of Yang–Mills field theory. In [14], the solutions of the non-abelian Yang–Mills theory were utilized to investigate the thermodynamics and critical behavior of Yang–Mills black holes (quasi topological). The authors demonstrated the possible first-order phase transition (from a small to a large black hole). They also ascertained that the Yang–Mills black hole (pure and quasi topological) does not achieve thermal stability. Further theoretical investigations into the thermodynamics of black holes were carried out in the work of [15]. That work focused on certain observable quantities in nonlinear charged AdS (anti-de Sitter) black holes—its shadow radius and the first-order phase transition. These observables were studied within the framework of the Einstein-power-Yang–Mills theory. The generated thermal profiles were generated at various nonlinear Yang–Mills charge parameters. The main results obtained in [15] were that a first-order phase transition which is dependent on temperature (or pressure) exists from the perspectives of the shadow radius and horizon radius. In [16], the authors explored the Joule–Thomson expansion of a specific black hole in $f(R)$ gravity combined with the Yang–Mills field. They derived the four-dimensional equation of state and Joule–Thomson coefficient from the thermodynamic properties. They then obtained isenthalpic and inversion curves for higher dimensions of the black hole. Comparative analysis was also conducted against van der Waals fluid and other black holes with reducing dimensionality. An interesting work that investigates gravitational lensing as well as thermodynamics of black holes with Yang–Mills fields is seen in [17]. In that work, the authors derived an exact black hole solution in four-dimensional AdS space-time for the Einstein–Gauss–Bonnet gravity with Yang–Mills field. They also found that the solution shows a P-v critical phenomenon which belongs to a universality class of Van der Waals fluids. In [17], the authors found that values of the critical exponents decrease with the Gauss–Bonnet coupling constant and increase along with the Yang–Mills charge.

Black holes and other gravitational phenomena have also been a focal point for the implementation of symmetry breaking mechanisms. This research direction is seen in the work of [18] where mass accretion into black holes with massive gravitational fields were investigated using Lorentz symmetry breaking. The authors in that work studied the effects of the scalar charge on the electromagnetic radiation from black holes. In addition, they analyzed the mass accretion rate with various fluids as well as discussions on phase transition and stability of accreting black holes. A similar work on Lorentz symmetry breaking is seen in [19]. In that work, an exact black hole solution is obtained from the gravitational field equations Einstein-bumblebee gravity model. In that model, the Lorentz symmetry is spontaneously broken at the point when the vacuum expectation value is acquired by the vector field. The effects of this symmetry breaking were analyzed by considering the black hole shadow as well as the behavior of the radial of the spherical orbit. Similarly, in [20], the bumblebee vector field with Lorentz symmetry breaking was analyzed. The authors performed this analysis by studying certain features of the black holes' shadow and its influence on the polarization of light. In [21], the authors analyzed the effects of Lorentz symmetry breaking on the thermodynamics of Schwarzschild-like black holes in modified gravity models. The authors in that work found that the Lorentz symmetry breaking changes the thermodynamic properties and it makes it necessary to consider the modification of the first law of thermodynamics. A similar work on Lorentz symmetry violation is seen in [22]. In that research, the characteristics of the quantum tunneling radiation of scalar bosons in black holes were studied. The authors limited their investigations on Lorentz symmetry violation to the stationary Kerr-AdS black hole. Upon deriving and solving the dynamical equations for bosons, the authors showed that the Lorentz symmetry violation influenced the temperature, entropy and the Hawking tunneling radiation rate of the Kerr-AdS black hole. Another interesting work on black hole characteristics in the presence of Lorentz symmetry breaking is found in the work of [23]. In that work, the author investigated spontaneous Lorentz symmetry breaking by considering a massive scalar perturbation on the top of a small spinning-like black hole. The analysis was carried out using the Einstein-bumblebee modified gravity. The results obtained in the work of [23] showed the influence of the degree of Lorentz symmetry violation on the black hole's super-radiance scattering and its subsequent instability.

Studies have also been conducted on the bending angle of massive particles and the light for bumblebee black hole solutions in the context of Lorentz symmetry breaking. This is seen in the work of [24], where such black hole solutions resulting from a non-zero vacuum expectation value of the bumblebee field break the Lorentz symmetry. Using the Ishihara method, the authors investigated the deflection angle of massive particles using the Gauss–Bonnet theorem. The authors further systematized the Ishihara method in their massive particle deflection analysis for applying it to the Jacobi metric. A recent study on the connection between quasinormal modes and Hawking radiation sparsity with Lorentz symmetry breaking in AdS black holes was investigated in Gogoi and Goswami [25]. In that work, the authors explored the power spectrum, greybody factors and the sparsity of black holes with vanishing effective cosmological constant in the presence of perturbations. The results of that work indicate that the black hole area quantization rule undergoes modification during Lorentz symmetry breaking. Phase transition phenomena have also been recently studied in AdS black holes in presence of Lorentz symmetry breaking. In [26], it was found that the first law of black hole thermodynamics and the Smarr formula could be constructed in the presence of Lorentz symmetry breaking. However, in such a scenario, the ideas/conceptualization of black hole area, entropy and volume within the event horizon would need to be modified due to its anisotropy. In that work, two phase transitions were discovered: (a) Hawking–Page phase transition and (b) the small- large black hole phase transition. In [27], the energy extraction and magnetic reconnection of a spinning black hole was studied under the presence of broken Lorentz symmetry. In a series of comparative analyses, Khodadi [27] shows that the efficiency of the plasma energization process and the power of energy extraction through fast magnetic reconnection is more efficient as compared to other black hole solutions. In addition to symmetry breaking, various types of physical analysis have been effectively conducted using modified gravitational theories, [28–30]. These theories have been implemented in several interesting applications related to electromagnetic fields and relativistic fluids in $f(R)$ gravity [31,32].

In the present work, the two-dimensional Liouville gravity field is considered. In many recent works, the study of Liouville gravity is primarily motivated by the need to develop a stable and consistent theory of quantum gravity. For instance, in the work of Mertens and Turiaci [33], the minimal string theory and the two-dimensional Liouville gravity was studied on spaces with fixed length boundaries. In that work, the authors arrived at explicit relations that provide descriptions of gravitational dressing of bulk and boundary correlators in the disk. They found initial evidence of that the overall theory could be considered as a two-dimensional dilaton gravity model with a sinh Φ dilaton potential. Another example of research focused on developing a quantum theory of gravity using ideas from Liouville gravity is seen in the work of Li [34]. In that work, using the holographic dual of the conformal Liouville gravity field, a consistent and unitary three-dimensional quantum theory of gravity was defined. Some unique properties of the theory are (a) the gravitational model contains black holes with no spin, (b) the theory has no normalizable AdS_3 vacuum and (c) there exists a unique universal interaction between states in the theory. A more recent work in this direction is seen in [35]. In that work, the author studied dilaton gravity models having negative and constant curvature: Liouville gravity and Jackiw–Teitelboim gravity models. In addition to proposing boundary conditions, the consistency of the asymptotic conditions was verified by calculating the entropy of their black hole solution. In Suzuki and Takayanagi [36], the authors investigated the connection between Jackiw–Teitelboim (JT) gravity on two-dimensional anti de-Sitter spaces and a semiclassical limit of a two-dimensional string theory. The world sheet theory of the two-dimensional string theory has a coupling between a space-like Liouville and non-rational conformal field theories (defined by a time-like Liouville conformal field theory). In addition to identifying the matrix dual of the two-dimensional field theory, the boundary Schwarzian theory was reproduced from the description of the Liouville theory.

In this work, the dynamics and the symmetry breaking of the combined Yang–Mills and Liouville gravity fields at the event horizon of black holes are investigated. This paper is organized as follows: Section 2 presents some background on the two-dimensional Liouville gravity fields. In Section 3, the interaction and symmetry breaking of the combined two-dimensional Yang–Mills and Liouville gravitational fields resulting in the existence of the gravitational scalar boson are described. This paper ends with the Section 4—summarizing key concepts and research findings presented in this work.

2. Two-Dimensional Liouville Gravity Field

Liouville gravity (or Liouville quantum gravity) arises from the Liouville field theory which is a two-dimensional conformal field theory. In the Liouville field theory, the classical equations of motion are a generalization of Liouville's equation in differential geometry: $\Delta_0 \, log \, f = -Kf^2$ with the flat Laplace operator: $\Delta_0 = \partial_x{}^2 + \partial_y{}^2 = \frac{4\partial^2}{\partial z \partial \bar{z}}$. The parameter f can be considered as a conformal factor relative to the flat metric and K is the Gaussian curvature. It has been shown that Liouville gravity could be obtained directly from the Einstein field equations in two-dimensions [37]. In this work, the Liouville gravity field is employed to describe the two-dimensional gravitational field at the event horizon of black holes. Consider the dynamics of a two-dimensional Liouville gravitational field, ϕ with the presence of an external potential: $V(\phi) = exp(2b\phi)$. Thus, the Liouville gravitational field, ϕ, is not a free field and the parameter, b, is the field coupling constant. It is important to note that since the Liouville gravitational field considered in this work is not a free field, the momentum is not conserved. $exp(2b\phi)$. The definition of the Liouville theory covers all complex values of the central charge, c_0 for $1 < c_0 < +\infty$. The term $exp(2b\phi)$ in the external potential is the energy eigenvectors. The central charge, c_0, gives rise to the Virasoro symmetry algebra of the Liouville field theory. The background charge Q is defined in terms of the coupling constant as: $Q = \frac{1}{b} + b$. Although the exponential potential violates momentum conservation, the conformal symmetry is preserved. Since the Liouville gravity field theory is a conformal field theory, the central charge is represented in terms of its background charge: $c_0 = 6Q^2 + 1$. Under the duality: $b \to \frac{1}{b}$, the Liouville gravity field's conformal dimensions and central charge, c_0 are invariant—while its correlation functions are covariant. In the Lagrangian formulation, the exponential potential is not invariant under the duality and the quantum symmetries do not become apparent.

In this work, the Lagrangian formulation of the two-dimensional Liouville field is utilized and combined with that of the Yang–Mills field. The local action and the respective equation of motion for the two-dimensional Liouville field are given as follows:

$$\begin{gathered} S(\phi) = \frac{1}{4\pi} \int d^2x \left(g^{\mu\nu} \partial_\mu \phi \, \partial_\nu \phi + QR\phi + \lambda' \, exp(2b\phi) \right) \sqrt{g} \\ \Delta\phi(x) = \frac{1}{2} QR(x) + \lambda' b \, exp[2b\phi(x)] \end{gathered} \tag{1}$$

where R is the Ricci scalar curvature, $g^{\mu\nu}$ is the metric tensor, λ' is the cosmological constant and ϕ is the Liouville field. The operator Δ in Equation (1) is the Laplace-Beltrami operator: $\Delta = \partial_\mu \left(\sqrt{|g|} g^{\mu\nu} \, \partial_\nu \right) / \sqrt{|g|}$. The cosmological constant, λ', is represented as: $\lambda' = 4 \, \Gamma\left(1 - b^2\right) \lambda^b / \Gamma\left(b^2\right)$.

3. Gauge Symmetry Breaking at the Event Horizon

The existence of the two-dimensional Yang–Mills field at the event horizon provides a potential avenue to explore its interactions with another mathematically well-developed two-dimensional field: the Liouville gravity field [38,39]. The central hypothesis in this work is that there exists a two-dimensional analogue to the Higgs field at the event horizon, presented as a two-dimensional Yang–Mills field. The Lagrangian of the combined two-dimensional Yang–Mills and the Liouville fields is as follows:

$$\begin{gathered} L = L_{YM} + L_G \\ L_{YM} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} \\ L_G = \frac{1}{4} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi + QR\varphi + 4\left(\frac{\Gamma\left(1-b^2\right)}{\Gamma\left(b^2\right)} \lambda^b \right) e^{2b\varphi} \text{ where } Q = \left(b + \frac{1}{b} \right) \end{gathered} \tag{2}$$

where $F^{\mu\nu}$ are the Yang–Mills field strengths, $\Gamma()$ is the gamma function, $b \in (0,1)$ is the Liouville field coupling strength, φ is the Liouville gravitational real scalar field, Q is the background charge, R is the Ricci scalar curvature and λ is the parameter that appears in the Liouville field correlation function. Expanding $e^{2b\varphi}$ to the second order via the Maclaurin series and defining $\alpha = 4\Gamma\left(1 - b^2\right) / \Gamma\left(b^2\right)$, the combined Lagrangian, L is as follows:

$$\begin{aligned} L &= -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{1}{4} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi + QR\varphi + \alpha\lambda^b \left(1 + 2b\varphi + 2b^2\varphi^2\right) \\ &= -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{1}{4} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi + V(\varphi) \end{aligned} \tag{3}$$

The field potential $V(\varphi)$ is influenced by the balancing of terms $QR\varphi$ and $\alpha\lambda^b\left(1 + 2b\varphi + 2b^2\varphi^2\right)$. As the Liouville gravity field coupling strength, $b \to 0$, the background charge, $Q \to \infty$ and $\alpha \to 0$. On the other hand, if the Liouville gravity field coupling strength, $b \to 1$, the background charge, $Q \to 2$ and $\alpha \to \infty$. The maximum potential at $R > 0$ and $\varphi = 0$ is $V(\varphi = 0) = \alpha\lambda^b$. To access the lower unstable equilibria at

$R < 0$ via symmetry breaking, the Liouville gravitational real scalar field is defined with respect to its complex components with a new parameter v defined as $v^2 = R/\lambda^b$:

$$\begin{gathered} \varphi = \varphi^* \varphi' \\ \varphi' = \frac{1}{\sqrt{2}}(v + \phi_1 + i\phi_2) \text{ and } \varphi^* = \frac{1}{\sqrt{2}}(v + \phi_1 - i\phi_2) \end{gathered} \tag{4}$$

where $\phi_1, \phi_2 \in R$. The field potential $V\left(\varphi = \varphi^* \varphi'\right) \to V(\phi_1, \phi_2)$ is obtained as follows:

$$\begin{gathered} V(\phi_1, \phi_2) = V_1(\phi_1, \phi_2) + V_2(\phi_1, \phi_2) + V_3(\phi_1, \phi_2) \\ V_1(\phi_1, \phi_2) = \left(\left(\frac{1}{2}Q + 3b^2\alpha\right)\left(\lambda^b v^2\right) + b\alpha\lambda^b\right)\phi_1^2 + \left(\left(\frac{1}{2}Q + b^2\alpha\right)\left(\lambda^b v^2\right) + b\alpha\lambda^b\right)\phi_2^2 \\ V_2(\phi_1, \phi_2) = \left(\left(Q + 2b^2\alpha\right)\left(\lambda^b v^3\right) + (2b\alpha)\left(\lambda^b v\right)\right)\phi_1 + (2b^2\alpha)\left(\lambda^b v\right)\phi_1^3 + (2b^2\alpha)\left(\lambda^b v\right)\phi_1\phi_2^2 \\ + \left(b^2\alpha\lambda^b\right)\phi_1^2\phi_2^2 + \left(\frac{1}{2}b^2\alpha\lambda^b\right)\phi_1^4 + \left(\frac{1}{2}b^2\alpha\lambda^b\right)\phi_2^4 \\ V_3(\phi_1, \phi_2) = \frac{1}{2}Q\lambda^b v^4 + \lambda^b\alpha + \frac{1}{2}b^2\alpha\lambda^b v^4 + b\alpha\lambda^b v^2 \end{gathered} \tag{5}$$

The field potential in Equation (5) could be grouped into three terms: $V_1(\phi_1, \phi_2)$ consists of the scalar gravitational bosonic (dilaton) terms, $V_2(\phi_1, \phi_2)$ consists of the bosonic coupling terms and $V_3(\phi_1, \phi_2)$ represents the constants. The field potential $V(\phi_1, \phi_2)$ describes two bosonic quanta ϕ_1 and ϕ_2. At Liouville field coupling strength, $b \to 0$ (resulting in $Q \to \infty$ and $\alpha \to 0$) the following occurs in $V_1(\phi_1, \phi_2)$:

$$V_1(\phi_1, \phi_2) \to \left(\left(\frac{1}{2}Q\right)\lambda^b v^2\right)\phi_1^2 + \left(\left(\frac{1}{2}Q\right)\left(\lambda^b v^2\right)\right)\phi_2^2 \tag{6}$$

Hence, when the Liouville field coupling strength, $b \to 0$ the bosons ϕ_1 and ϕ_2 have Ricci scalar curvature of the magnitude $\frac{1}{2}Q$. Therefore, in this formulation, a two-dimensional analogue to the three-dimensional Higgs mechanism is presented—where instead of mass, the scalar bosons (or dilatons) ϕ_1 and ϕ_2 have Ricci scalar curvature, $R = v^2\lambda^b$ during symmetry breaking. This occurs due to the non-existence of the property of three-dimensional mass in two-dimensional space. Thus, the property of mass is replaced by the two-dimensional Ricci scalar curvature. At $b \to 1$, the bosons s ϕ_1 and ϕ_2 have the a mixed property of Ricci scalar curvature and λ^b with the magnitude of $\left(\frac{1}{2}Q + 3b^2\alpha\right)R + b\alpha\lambda^b$ and $\left(\frac{1}{2}Q + b^2\alpha\right)R + b\alpha\lambda^b$ respectively. This mechanism could be extended further by introducing a local gauge field with the potential, A_μ and rewriting the Lagrangian as follows:

$$L' = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{4}g^{\mu\nu}D_\mu\varphi D_\nu\varphi + V(\varphi)$$

where

$$\begin{gathered} D_\mu\varphi = \partial_\mu\varphi + eA_\mu\varphi \text{ and } D_\nu\varphi = \partial_\nu\varphi + eA_\nu\varphi \\ F_{\mu\nu} = \partial_\mu A_\nu + \partial_\nu A_\mu \end{gathered} \tag{7}$$

where the parameter, $e = \lambda^b$. Consider Lagrangian invariance under the following local gauge transformations:

$$\begin{gathered} \varphi(x) \to \varphi(x)e^{\beta(x)} \text{ with} \\ A_\mu(x) \to A_\mu(x) - \frac{1}{e}\partial_\mu\beta(x) \text{ and } A_\nu(x) \to A_\nu(x) - \frac{1}{e}\partial_\nu\beta(x) \end{gathered} \tag{8}$$

The terms $D_\mu(\phi_1,\phi_2)D_\nu(\phi_1,\phi_2)$ in the Lagrangian, $L' \to L'(\phi_1,\phi_2)$ would then become:

$$\begin{aligned} L' &= -\tfrac{1}{4}F^{\mu\mu}F_{\mu\mu} + \tfrac{1}{4}g^{\mu\mu}D_\mu(\phi_1,\phi_2)D_\nu(\phi_1,\phi_2) + V(\phi_1,\phi_2) \\ & D_\mu(\phi_1,\phi_2)D_\nu(\phi_1,\phi_2) \\ &= \left(\tfrac{1}{2}\partial_\mu\phi_1^2 + \tfrac{1}{2}\partial_\mu\phi_2^2 + \tfrac{1}{2}ev^2A_\mu + evA_\mu\phi_1 + \tfrac{1}{2}eA_\mu\phi_1^2 + \tfrac{1}{2}eA_\mu\phi_2^2 + v\partial_\mu\phi_1\right) \\ & \left(\tfrac{1}{2}\partial_\nu\phi_1^2 + \tfrac{1}{2}\partial_\nu\phi_2^2 + \tfrac{1}{2}ev^2A_\nu + evA_\nu\phi_1 + \tfrac{1}{2}eA_\nu\phi_1^2 + \tfrac{1}{2}eA_\nu\phi_2^2 + v\partial_\nu\phi_1\right) \\ & \left(v^2\partial_\mu\phi_1\partial_\nu\phi_1 + \tfrac{e^2v^4}{4}A_\mu A_\nu\right) + \ldots \end{aligned} \tag{9}$$

The final term above gives Ricci scalar curvature to the gauge fields A_1 and A_2.

4. Conclusions

In this work, the existence of a two-dimensional analogue to the three-dimensional Higgs field is hypothesized. Considering this field as a two-dimensional Yang–Mills field, the symmetry breaking mechanism of the combined two-dimensional Yang–Mills and Liouville gravity fields is explored. The application of this mechanism focuses on the quantum field dynamics occurring at a black hole's event horizon. Black holes grow in mass by merging with other black holes and/or by accretion. In this work, we focus on the possible mechanism of black hole growth from accretion of mass. By considering the event horizon as a two-dimensional sphere, a two-dimensional analogue to the Higgs mechanism is described using the combined two-dimensional Yang–Mills and Liouville gravity fields. In Equations (2)–(9), the lower unstable equilibria at $R < 0$ is accessed via symmetry breaking. In Equation (6), the scalar bosons (or dilatons) ϕ_1 and ϕ_2 are shown to have Ricci scalar curvature, $R = v^2\lambda^b$ during symmetry breaking as the Liouville field coupling strength vanishes, $b \to 0$. These Liouville gravity scalar bosons give Ricci scalar curvature to the gauge fields A_1 and A_2. These gauge fields A_1 and A_2 represent matter or massive particles entering the black hole's event horizon (see Equations (7)–(9)). Therefore, as matter accretion takes place in the black hole, the Ricci scalar curvature of the two-dimensional event horizon increases causing black hole growth. This growth mechanism is reflected by the increase in its Ricci scalar curvature (at the event horizon), which further warps the surrounding fabric of space-time, increasing the black hole's mass. With further empirical observations on black hole growth phenomena, the gravitational mechanisms described in this work could be further verified.

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