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Folded Supersymmetry as a candidate to solve the Hierarchy Problem of the Standard Model

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Resumo

O problema da hierarquia no Modelo Padrão surge devido à presença de divergências quadráticas provenientes de correções quânticas ao parâmetro de massa do bóson de Higgs. O presente trabalho trata sobre um recurso conhecido como Supersimetria Dobrada (*Folded Supersymmetry*), que pode ser usado para construir extensões do Modelo Padrão que estejam livres dessas divergências. Dado que a contribuição do top quark é a mais significativa, este trabalho se propõe centralizar nele demonstrando que o cancelamento é possivel mediante um parceiro do top quark de spin oposto e carga de cor diferente ao da particula top. Deve-se notar a diferencia com as teorias supersimétricas, onde o parceiro, apesar de ter spin oposto, necessariamente possui a mesma carga de cor. Finalmente, construimos uma teoria com uma dimensão espacial extra que serve como *UV Completion* para explicar a origem dos cancelamentos à energias maiores.

Palavras Chaves: Supersimetria Dobrada; Supersimetria, Modelo Padrão; Problema de hierarquia; MSSM; Dimensões extra.

Áreas do conhecimento: Além do Modelo Padrão .

Abstract

The hierarchy problem in the Standard Model arises due to the presence of quadratic divergences coming from loop corrections to the mass parameter of the Higgs boson. The present work reviews a tool known as Folded Supersymmetry that can be used to build Standard Model extensions which are free of those divergences. Since the top quark contribution is the most significant, this dissertation focuses on it showing that it is possible to cancel it out with a top quark partner with opposite spin-statistics and the same color charge as the top particle. We must note the difference with supersymmetric theories where the partner (superpartner), despite having opposite spin-statistics, necessarily has the same color charge. Finally, we construct a suitable UV completion in a 5-dimensional spacetime for the folded supersymmetric theory that explains the origin of the cancellations at higher energies.

Keywords: Folded Supersymmetry; Supersymmetry; Standard Model; Hierarchy problem; MSSM; Extra Dimensions.

Knowledge areas: Beyond the Standard Model.

Contents

Chapter 1

Introduction

There exist four known fundamental interactions in nature: electromagnetism, weak interaction, strong interaction and gravitation. In the search for unification, physicist have achieved a description of the first three of them in one theory known as the Standard Model (SM). It is, so far, the best theory to describe elementary particle interactions neglecting the effects of gravity. Its predictions have been tested with great accuracy including the discovery of the Higgs particle at the LHC in 2012. All currently known elementary particles, along with some of their properties, are listed in Table 1.1.

However, lots of phenomena are still not explained by the SM. Among them we have: the nature of dark matter, the origin of the mass of the neutrinos, the matter-antimatter asymmetry, the strong CP problem and the hierarchy problem. These suggest that the SM is, actually, an effective theory valid up to some high energy, above which new physics is expected. New physics means new particles which must have specific properties that could solve some or all those phenomena.

One of the problems that the SM faces is the Hierarchy problem, which arises when one calculates the quantum corrections to the mass parameter of the Higgs particle. It turns out that these corrections are quadratically sensitive to high energy scales, making the physical mass of the Higgs field extremely large unless an incredible fine tuning is impose, which seems to be unnatural.

Naturalness in the form of the hierarchy problem is one of the basis upon which extended realizations of the SM are constructed. In this dissertation we describe one possible way of extending the Standard Model in order to partially solve it called Folded Supersymmetry which was first introduced by Burdman, Chacko, Goh and Harnik in [1].

Particle	Mass	Electric charge	Spin
e	$0.5109989461 \pm 0000000031$ MeV	-1	$1/2$
ν_e	< 2 eV	0	$1/2$
μ	$105.6583745 \pm 0.0000024$ MeV	-1	$1/2$
ν_μ	< 0.19 MeV	0	$1/2$
τ	1776.86 ± 0.12 MeV	-1	$1/2$
ν_τ	< 18.2 MeV	0	$1/2$
u	$2.2^{+0.6}_{-0.4}$ MeV	$2/3$	$1/2$
d	$4.7^{+0.5}_{-0.4}$ MeV	$-1/3$	$1/2$
s	96^{+8}_{-4} MeV	$-1/3$	$1/2$
c	1.275 ± 0.003 GeV	$2/3$	$1/2$
b	$4.18^{+0.04}_{-0.03}$ GeV ($\overline{MS}$)	$-1/3$	$1/2$
t	173.21 ± 0.51 GeV	$2/3$	$1/2$
γ	$< 1 \times 10^{-18}$ eV	$< 10^{-35}$	1
$W^\pm$	80.385 ± 00151 GeV	± 1	1
Z	91.1876 ± 0.0021 GeV	0	1
g	0 (theoretically)	0	1
H^0	125.09 ± 0.11 GeV	0	0

Table 1.1: Masses, electric charges and spin of the elementary known particles.[13]

1.1 Outline of the dissertation

The present work is structured as follows.

- In Chapter 2, we present the notation and conventions used in the dissertation. We also review some concepts such as Dirac, Weyl and Majorana spinors and Grassmann numbers.

- In Chapter 3, we review the Standard Model. We show how the Higgs mechanism works in order to give mass to fermions and gauge bosons and finally we list some problems within the SM that serve as motivation for looking for extended theories.

- In Chapter 4, we review basic concepts of supersymmetry in 4 dimensions. We study the chiral and the vector supermultiplet and construct supersymmetric

Lagrangians out of them. Then we treat the minimal extension of the SM with supersymmetry, namely, the Minimal Supersymmetric Standard Model (MSSM).

- Chapter 5 deals with the topic of Extra dimensions. We study the particular case of a 5-dimensional spacetime. We introduce the concepts of orbifold and Scherk-Schwarz compactifications as well as the Kaluza-Klein decomposition.
- In Chapter 6, Folded supersymmetry is introduced. We begin by studying two examples of orbifolded theories in order to understand how they overcome the hierarchy problem and elaborate a prescription to build folded supersymmetric theories. We also note the need for a UV completion which is constructed in a 5-dimensional spacetime framework.

Appendix A and Appendix B contain derivations that are useful to understand some implications of folded supersymmetric theories and their UV completions.

Chapter 2

Preliminary concepts, notation and conventions

This chapter is devoted to specify the notation, conventions and preliminary concepts used throughout the dissertation for four dimensions of space-time, one temporal and three spatial dimensions. In chapter 5 and beyond we will work with 5 dimensions. The respective conventions are specified there.

As is usual in high energy physics, we work in natural units where physical constants like the speed of light (c) and the reduced Planck constant ($\hbar$) are set to 1. The space-time metric in 4 dimensions chosen here is $\eta_{\mu\nu} = \text{diag}\,(+1, -1, -1, -1)$. Lorentz indices are represented by greek letters μ, ν, etc. and they run from 0 to 3.

With the chosen conventions, a free complex scalar field ϕ with mass m_ϕ is described by the Lagrangian

$$\mathcal{L}_\phi = \partial_\mu \phi^\dagger \partial^\mu \phi - m_\phi^2 \phi^\dagger \phi, \tag{2.1}$$

a free spin-1 field A_μ with mass m_A is described by

$$\mathcal{L}_A = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + m_A^2 A_\mu A^\mu, \tag{2.2}$$

where the strength field tensor is

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \tag{2.3}$$

and a free four component Dirac spinor field Ψ with mass m_Ψ is described by

$$\mathcal{L}_{\text{Dirac}} = i\bar{\Psi}\gamma^\mu \partial_\mu \Psi - m_\Psi \bar{\Psi}\Psi, \tag{2.4}$$

where

$$\bar{\Psi} \equiv \Psi^\dagger \gamma^0. \tag{2.5}$$

We will work with the Weyl representation of the Gamma matrices γ which is defined by

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}, \tag{2.6}$$

where $\sigma^\mu = (1_{2\times 2}, \overrightarrow{\sigma})$ and $\bar{\sigma}^\mu = (1_{2\times 2}, -\overrightarrow{\sigma})$. Here, $\overrightarrow{\sigma}$ denotes the three Pauli matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{2.7}$$

Gamma matrices satisfy the Clifford algebra

$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}. \tag{2.8}$$

Also, we define the *chirality operator*

$$\gamma_5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} -1_{2\times 2} & 0 \\ 0 & 1_{2\times 2} \end{pmatrix}. \tag{2.9}$$

If we decompose a Dirac spinor in two 2-component form as

$$\Psi = \begin{pmatrix} \chi \\ \psi \end{pmatrix} \tag{2.10}$$

we note that χ and ψ are eigenvectors of the chirality operator in the sense that

$$\gamma_5 \begin{pmatrix} \chi \\ 0 \end{pmatrix} = -\begin{pmatrix} \chi \\ 0 \end{pmatrix}, \qquad \gamma_5 \begin{pmatrix} 0 \\ \psi \end{pmatrix} = \begin{pmatrix} 0 \\ \psi \end{pmatrix}. \tag{2.11}$$

The 2-component spinors χ and ψ are known as *Weyl-spinors*. The chirality eigenvalue of χ is -1 and it is said that it has *left-handed* chirality and the chirality eigenvalue of ψ is $+1$ and it is said that it has *right-handed* chirality. They can be obtained from Ψ by applying the projector operators P_L and P_R defined as

$$P_L = \frac{1}{2}(1 - \gamma_5), \tag{2.12}$$

$$P_R = \frac{1}{2}(1 + \gamma_5). \tag{2.13}$$

Then, indeed

$$\Psi_L \equiv P_L\Psi = \begin{pmatrix} \chi \\ 0 \end{pmatrix}, \qquad \Psi_R \equiv P_R\Psi = \begin{pmatrix} 0 \\ \psi \end{pmatrix}. \tag{2.14}$$

Ψ_L and Ψ_R are the left-handed and right-handed projection of Ψ respectively.

Weyl fermions are extremely useful when we work with supersymmetric theories. Therefore, it is convenient to develop a manageable notation when dealing with them. In order to distinguish between right and left handed Weyl spinors we will use the 'dot notation'. The components of a left-handed Weyl spinor χ will be denoted by an undotted lower index χ_a where a takes the values $1, 2$. The components of a right-handed Weyl spinor ψ will be denoted by a dotted upstairs index and a bar above as $\bar{\psi}^{\dot{a}}$ with $a = 1, 2$. The last bar notation must not be confused with that which was used in (2.5) since, in that case, it was defined for 4-component spinors, while here it is defined for two-component Weyl spinors.

It is worth emphasizing that, when quantized, the components χ_a and $\psi^{\dot{a}}$ are anticommuting quantities or *Grassmann numbers* (explained later).

Index a can be raised by

$$\chi^a = \epsilon^{ab}\chi_b \tag{2.15}$$

and index $\dot{a}$ can be lowered by

$$\bar{\psi}_{\dot{a}} = \epsilon_{\dot{a}\dot{b}}\bar{\psi}^{\dot{b}}, \tag{2.16}$$

where

$$\epsilon^{11} = \epsilon^{22} = 0, \qquad \epsilon^{12} = +1, \qquad \epsilon^{21} = -1, \tag{2.17}$$

$$\epsilon_{\dot{1}\dot{1}} = \epsilon_{\dot{2}\dot{2}} = 0, \qquad \epsilon_{\dot{1}\dot{2}} = -1, \qquad \epsilon_{\dot{2}\dot{1}} = +1. \tag{2.18}$$

Also, we can define the matrices ϵ_{ab} and $\epsilon^{\dot{a}\dot{b}}$

$$\epsilon_{ab}\epsilon^{bc} = \delta_a^c, \tag{2.19}$$

$$\epsilon_{\dot{a}\dot{b}}\epsilon^{\dot{b}\dot{c}} = \delta_{\dot{a}}^{\dot{c}}, \tag{2.20}$$

which lower undotted indices and raised dotted indices respectively. Their components are

$$\epsilon_{11} = \epsilon_{22} = 0, \qquad \epsilon_{12} = -1, \qquad \epsilon_{21} = +1, \tag{2.21}$$

$$\epsilon^{\dot{1}\dot{1}} = \epsilon^{\dot{2}\dot{2}} = 0, \qquad \epsilon^{\dot{1}\dot{2}} = +1, \qquad \epsilon^{\dot{2}\dot{1}} = -1. \tag{2.22}$$

Left-handed and right-handed Weyl spinors have different but well defined transformations under Lorentz transformations. However, we can construct Lorentz scalars terms out of them. Example of them are

$$\chi_a\xi^a \equiv \epsilon^{ab}\chi_a\xi_b, \tag{2.23}$$

$$\bar{\chi}^{\dot{a}}\bar{\xi}_{\dot{a}} \equiv \epsilon_{\dot{a}\dot{b}}\bar{\chi}^{\dot{a}}\bar{\xi}^{\dot{b}}. \tag{2.24}$$

Also, we can construct four-vectors as

$$\bar{\psi}_{\dot{a}}\left(\bar{\sigma}^{\mu}\right)^{\dot{a}b}\chi_b, \tag{2.25}$$

$$\psi^a \left(\sigma^\mu\right)_{a\dot{b}} \bar{\chi}^{\dot{b}}. \tag{2.26}$$

The transformation properties of these terms as well as that of the Weyl spinors are explored in more detail in Ref. [2]. Terms in (2.23)−(2.26) will be used to construct Lorentz invariant terms in Lagrangians. However, wherever it is understood that we are working with Weyl-spinors, repeated indices contracted as

$$^{a}{}_{a} \quad \text{or} \quad {}_{\dot{a}}{}^{\dot{a}} \tag{2.27}$$

will be omitted to avoid clutter.

Finally, we must know that the hermitian conjugate of any left-handed Weyl spinor is a right-handed Weyl spinor and viceversa. That is, we can write

$$(\chi_a)^\dagger \equiv \bar{\chi}_{\dot{a}}, \tag{2.28}$$

$$(\bar{\chi}^{\dot{a}})^\dagger \equiv \chi^a. \tag{2.29}$$

Here we list some identities involving Weyl spinors that will be useful later

$$\chi\psi = \psi\chi, \tag{2.30}$$

$$\bar{\chi}\bar{\psi} = \bar{\psi}\bar{\chi}, \tag{2.31}$$

$$(\chi\psi)^\dagger = \bar{\chi}\bar{\psi}, \tag{2.32}$$

$$\chi\sigma^\mu\bar{\psi} = -\bar{\psi}\bar{\sigma}^\mu\chi, \tag{2.33}$$

$$\theta\sigma^\mu\bar{\theta}\theta\sigma^\nu\bar{\theta} = \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}\eta^{\mu\nu}, \tag{2.34}$$

$$\epsilon^{ab}\frac{\partial}{\partial\theta^b} = -\frac{\partial}{\partial\theta_a}, \tag{2.35}$$

$$(\theta\chi)(\theta\psi) = -\frac{1}{2}(\theta\psi)(\theta\theta), \tag{2.36}$$

$$(\bar{\theta}\bar{\chi})(\bar{\theta}\bar{\psi}) = -\frac{1}{2}(\bar{\theta}\bar{\psi})(\bar{\theta}\bar{\theta}). \tag{2.37}$$

These and more identities can be found in the appendices section of the references [3] and [4], although care must be taken with some different conventions.

Now, we are able to write the Dirac Lagrangian in (2.4) in terms of Weyl spinors. With Ψ decomposed as in (2.10), it takes the form

$$\mathcal{L}_{\text{Dirac}} = i\bar{\chi}\bar{\sigma}^\mu\partial_\mu\chi + i\psi\sigma^\mu\partial_\mu\bar{\psi} - m_\Psi\left(\bar{\chi}\bar{\psi} + \chi\psi\right). \tag{2.38}$$

Just as we use Weyl spinors we can also use Majorana spinors since they both have the same number of degrees of freedom. A Majorana fermion is defined as being the same as its charge conjugate. The charge conjugate of a 4-component spinor Ψ is given by

$$\Psi^C = C\Psi^*, \tag{2.39}$$

where

$$C = -i\gamma^2. \tag{2.40}$$

Then, Ψ_M is a Majorana fermion if it fulfills the condition

$$\Psi_M^C = \Psi_M. \tag{2.41}$$

Given the left-handed Weyl spinors χ and ξ, we can construct the Majorana spinors

$$\Psi_M^\chi = \begin{pmatrix} \chi_a \\ \bar{\chi}^{\dot{a}} \end{pmatrix}, \qquad \Psi_M^\xi = \begin{pmatrix} \xi_a \\ \bar{\xi}^{\dot{a}} \end{pmatrix}, \tag{2.42}$$

and use the relations

$$\chi\xi = \bar{\Psi}_M^\chi P_L \Psi_M^\xi, \tag{2.43}$$

$$\bar{\chi}\bar{\xi} = \bar{\Psi}_M^\xi P_R \Psi_M^\xi, \tag{2.44}$$

$$\bar{\chi}\bar{\sigma}^\mu\xi = \bar{\Psi}_M^\chi \gamma^\mu P_L \Psi_M^\xi, \tag{2.45}$$

$$\chi\sigma^\mu\bar{\xi} = \bar{\Psi}_M^\chi \gamma^\mu P_R \Psi_M^\xi \tag{2.46}$$

to translate from Weyl to Majorana language.

2.1 Grassmann numbers

Grasmann numbers are defined as being anticommuting quantities, that is, two Grassmann numbers η and ξ satisfy

$$\eta\xi + \xi\eta = 0, \tag{2.47}$$

from where, it follows that

$$(\eta)^2 = (\xi)^2 = 0. \tag{2.48}$$

Then, if we have a general function of, let us say, η, $f(\eta)$, the power expansion in η

$$f(\eta) = f_0 + f_1\eta \tag{2.49}$$

will only contain terms up to the linear order in η because higher order terms will vanish.

Grassmann numbers can be multiplied by ordinary numbers in the usual way, as well as they can be added and subtracted normally. Differentiation is defined by

$$\frac{\partial(a\eta)}{\partial\eta} = a, \tag{2.50}$$

where a is any ordinary number. Also, for the differentiation of the product of two Grassmann numbers, we have

$$\frac{\partial(\eta\xi)}{\partial\eta} = \xi \tag{2.51}$$

and

$$\frac{\partial(\xi\eta)}{\partial\eta} = -\xi. \tag{2.52}$$

Then, for f we have

$$\frac{\partial f}{\partial\eta} = f_1. \tag{2.53}$$

Integration is defined by

$$\int d\eta = 0, \quad \int \eta d\eta = 1 \tag{2.54}$$

and the imposition of linearity. Then

$$\int d\eta f(\eta) = f_1, \tag{2.55}$$

from were we notice that differentiation and integration are equivalent for Grassmann numbers.

Finally, for a Weyl spinor field θ which, as we already said, has anticommuting components, we will use the following definitions

$$d^2\theta \equiv -\frac{1}{4}d\theta^a d\theta_a = -\frac{1}{4}d\theta d\theta, \tag{2.56}$$

$$d^2\bar{\theta} \equiv -\frac{1}{4}d\bar{\theta}_{\dot{a}} d\bar{\theta}^{\dot{a}} = -\frac{1}{4}d\bar{\theta}d\bar{\theta}, \tag{2.57}$$

$$d^4\theta \equiv d^2\theta d^2\bar{\theta}, \tag{2.58}$$

which lead to the following properties

$$\int d^2\theta(\theta\theta) = 1, \quad \int d^2\bar{\theta}\left(\bar{\theta}\bar{\theta}\right) = 1, \tag{2.59}$$

that are widely used in supersymmetry.

Chapter 3

The Standard Model

The Standard Model (SM) is the theory that best describes the electromagnetic, weak and strong interactions of all known elementary particles so far as well as the mechanism by which some of them acquire a mass. It is a gauge field theory build on the local symmetry group $U(1)_Y \times SU(2)_L \times SU(3)_C$, where Y , L, and C denote hypercharge, left-handed chirality and color, respectively. The field content of the theory and their representations under the symmetry group is shown in Table 3.1. There, we describe all three families of fermions. Here, for simplicity, we write the SM Lagrangian just for one family:

$$\begin{aligned}\mathcal{L}_{SM} =& i\bar{l}\gamma^\mu D_\mu l + i\bar{q}\gamma^\mu D_\mu q + i\bar{e}\gamma^\mu D_\mu e + i\bar{u}\gamma^\mu D_\mu u + i\bar{d}\gamma^\mu D_\mu d \\ &- \frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu} - \frac{1}{4}G^A_{\mu\nu}G^{A\mu\nu} \\ &+ (D_\mu h)^\dagger (D^\mu h) - \lambda\left(h^\dagger h - \frac{1}{2}v^2\right)^2 \\ &- y_e\left(\bar{l}he + \bar{e}h^\dagger l\right) \\ &- y_d\left(\bar{q}hd + \bar{d}h^\dagger q\right) - y_u\left(\bar{q}\tilde{h}u + \bar{u}\tilde{h}^\dagger q\right),\end{aligned} \tag{3.1}$$

where covariant derivatives are defined as:

$$D_\mu l = \partial_\mu l + i\frac{g_2}{2}\sigma^a W^a_\mu l - i\frac{g_1}{2}B_\mu l, \tag{3.2}$$

$$D_\mu q = \partial_\mu q + i\frac{g_3}{2}\lambda^A G^A_\mu q + i\frac{g_2}{2}\sigma^a W^a_\mu q + i\frac{g_1}{6}B_\mu q, \tag{3.3}$$

$$D_\mu e = \partial_\mu e - ig_1 B_\mu e, \tag{3.4}$$

$$D_\mu u = \partial_\mu u + i\frac{g_3}{2}\lambda^A G^A_\mu u + i\frac{2g_1}{3}B_\mu u, \tag{3.5}$$

$$D_\mu d = \partial_\mu d + i\frac{g_3}{2}\lambda^A G^A_\mu d - i\frac{g_1}{3}B_\mu d, \tag{3.6}$$

$$D_\mu h = \partial_\mu h + i\frac{g_2}{2}\sigma^a W^a_\mu h + i\frac{g_1}{2}B_\mu h, \tag{3.7}$$

where g_1, g_2 and g_3 are the coupling constants associated with the groups $U(1)_Y$, $SU(2)_L$ and $SU(3)_C$, respectively. Also, in eq.(3.1), $SU(3)_C$ indices for q, d and u were not written explicitly.

Names	**Fields** $(I = 1, 2, 3)$	$\boldsymbol{SU(3)_C}$	$\boldsymbol{SU(2)_L}$	$\boldsymbol{U(1)_Y}$
Lepton doublet	$l_I = \begin{pmatrix} \nu_{IL} \\ l_{IL} \end{pmatrix}$	**1**	**2**	-1/2
Lepton singlet	$e_I \equiv e_{IR}$	**1**	**1**	-1
Quark doublet	$q_I = \begin{pmatrix} u_{IL} \\ d_{IL} \end{pmatrix}$	**3**	**2**	1/6
Up-type quark singlet	$u_I \equiv u_{IR}$	**3**	**1**	2/3
Down-type quark singlet	d_{IR}	**3**	**1**	-1/3
Higgs doublet	$h = \begin{pmatrix} h^+ \\ h^0 \end{pmatrix}$	**1**	**2**	1/2
Gluons	G^A_μ	**8**	**1**	0
W bosons	W^a_μ	**1**	**3**	0
B boson	B_μ	**1**	**1**	0

$$l_1 = \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix},\ l_2 = \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix},\ l_3 = \begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}$$

$$e_1 = e_R,\ e_2 = \mu_R,\ e_3 = \tau_R$$

$$q_1 = \begin{pmatrix} u_L \\ d_L \end{pmatrix},\ q_2 = \begin{pmatrix} c_L \\ s_L \end{pmatrix},\ q_3 = \begin{pmatrix} t_L \\ b_L \end{pmatrix}$$

$$u_1 = u_R,\ u_2 = c_R,\ u_3 = t_R$$

$$d_1 = d_R,\ d_2 = s_R,\ d_3 = b_R$$

Table 3.1: Field content of the Standard Model with corresponding transformations under the gauge group. The index I labels the three generations (or families) of leptons and quarks; index $a\,(= 1, 2, 3)$ label the three $SU(2)_L$ gauge bosons; and index $A\,(= 1, ..., 8)$ label the eight $SU(3)_C$ gauge bosons (gluons). Quantum numbers of fields respect the Gell-Mann Nishijima relation ($Q = T_3 + Y$), where Q, T_3 and Y denotes electric charge, third component of isoespin and hypercharge, respectively.

The terms in the first line of the Lagrangian in eq.(3.1) are the fermionic (matter content) kinetic terms as well as the interaction of these with the gauge bosons contained in the covariant derivatives.

The second line describes the kinetic terms for gauge bosons. The field strength tensors are defined as

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu, \tag{3.8}$$

$$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu - g_2 \epsilon^{abc} W^b_\mu W^c_\nu, \tag{3.9}$$

$$G^A_{\mu\nu} = \partial_\mu G^A_\nu - \partial_\nu G^A_\mu - g_3 f^{ABC} G^B_\mu G^C_\nu, \tag{3.10}$$

where ϵ^{abc} (a, b, c run from 1 to 3) is the Levi Civita symbol and λ^{ABC} (A, B, C run from 1 to 8) are the completely antisymmetric structure constants of the $SU(3)$ algebra. The non-zero values are

$$\begin{aligned} &f^{123} = 1, \quad f^{147} = f^{246} = f^{257} = f^{345} = f^{516} = f^{637} = \frac{1}{2}, \\ &f^{458} = f^{678} = \frac{\sqrt{3}}{2} \end{aligned} \tag{3.11}$$

with their corresponding permutations.

The third line contains the Higgs kinetic term and its potential which will be important for the Higgs mechanism explained later.

Finally, the fourth and fifth lines in eq.(3.1) are the Yukawa interaction terms from which masses of fermion fields and interaction of the Higgs field with fermions will appear after the mentioned Higgs mechanism. $\tilde{h}$ is defined as

$$\tilde{h} \equiv i\sigma^2 h^*. \tag{3.12}$$

3.1 Higgs mechanism

We note that in the Lagrangian in eq.(3.1), there are no mass term neither for the gauge bosons nor for the leptons and quarks. For fermions, a mass term would be proportional to $\bar{f}f = \bar{f}_L f_R + \bar{f}_R f_L$ and this would be invariant only if left and right components of fields transform in the same way under gauge transformations. Since the SM is a chiral theory, that is, left and right components interact in different ways, a term of this form is also forbidden.

However, we know by experimental data, that quarks, leptons and some gauge bosons actually have a mass. So, how is this possible? The answer to this relies on the *Higgs mechanism* [5, 6, 7]. This consists on the spontaneous breaking of a gauge symmetry. We say that a symmetry is spontaneously broken when the Lagrangian of the theory is invariant under the action of the symmetry group but its vacuum does not. Let us see how this works in the SM.

The principal ingredient in the SM that allows the Higgs mechanism is the Higgs potential:

$$V(h) = \lambda \left(h^\dagger h - \frac{1}{2}v^2 \right)^2 . \tag{3.13}$$

In order to have a theory with a bounded from below energy, λ must be positive, and, to allow spontaneous symmetry breaking, v^2 also has to be positive.

The minimum of the potential in eq. (3.13) is achieved when

$$h^\dagger h = \frac{1}{2}v^2. \tag{3.14}$$

There exist infinite equivalent configurations for H such that (3.14) is satisfied. Each of them has the same energy and we can go from one to another by an $SU(2)$ trannsformation. So, we can choose any of them without lose of generality. We choose

$$h_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}, \tag{3.15}$$

where v is real. A state φ is invariant under the action of a generator $\mathcal{G}$ if

$$\exp(i\alpha\mathcal{G})\varphi = \varphi \tag{3.16}$$

or, infinitesimally, if

$$(1 + i\alpha\mathcal{G})\varphi = \varphi \quad \Leftrightarrow \quad \mathcal{G}\varphi = 0. \tag{3.17}$$

Let us see how H_0 behaves under the action of the generators of the group $SU(2)_L \times U(1)_Y$

$$\tau_1 h_0 = \frac{\sigma_1}{2} h_0 = \frac{1}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \frac{1}{2\sqrt{2}} \begin{pmatrix} v \\ 0 \end{pmatrix} \neq 0, \tag{3.18}$$

$$\tau_2 h_0 = \frac{\sigma_2}{2} h_0 = \frac{1}{2} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = -\frac{i}{2\sqrt{2}} \begin{pmatrix} v \\ 0 \end{pmatrix} \neq 0, \tag{3.19}$$

$$\tau_3 h_0 = \frac{\sigma_3}{2} h_0 = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = -\frac{1}{2\sqrt{2}} \begin{pmatrix} v \\ 0 \end{pmatrix} \neq 0, \tag{3.20}$$

$$Y_h h_0 = \frac{1}{2} h_0 \neq 0 \tag{3.21}$$

but

$$Qh_0 = (\tau_3 + Y_h)h_0 = \frac{1}{2}(\sigma_3 + I)\Phi_0 = \frac{1}{2\sqrt{2}}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ v \end{pmatrix} = 0. \tag{3.22}$$

Then, the vacuum h_0 remains invariant under gauge transformations of the group $U(1)_Q$, that is

$$\exp(i\theta Q)h_0 = h_0. \tag{3.23}$$

We say that the Higgs vacuum h_0 produce the spontaneous breaking

$$SU(2)_L \times U(1)_Y \to U(1)_Q. \tag{3.24}$$

The gauge field corresponding to this $U(1)_Q$ local group will be interpreted as the photon.

In order to derive the interactions and mass terms after the spontaneous symmetry breaking it is more convenient to write the Higgs field as

$$h(x) = \frac{1}{\sqrt{2}}\exp\left(\frac{i}{v}\theta^a(x)\tau^a\right)\begin{pmatrix} 0 \\ v + \hat{h}(x) \end{pmatrix}, \tag{3.25}$$

where $\theta^a(x)$ and $h(x)$ are four real scalar fields. Since the exponential in (3.25) is an element of the $SU(2)$ gauge group, it can be rotated away by a particular element of the group. This choice is called *unitary gauge*. Then, only the physical field $h(x)$ remains and the Higgs doublet reads

$$h(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + \hat{h}(x) \end{pmatrix}. \tag{3.26}$$

3.2 Gauge bosons masses

Let us see how masses for gauge bosons are generated. In unitary gauge, we have

$$\begin{aligned} D_\mu(x)h(x) &= \left[\partial_\mu + \frac{i}{2}g_2 W_\mu^a(x)\sigma^a + \frac{i}{2}g_1 B_\mu(x)\right]h(x) \\ &= \begin{pmatrix} \partial_\mu + \frac{i}{2}g_2 W_\mu^3 + \frac{i}{2}g_1 B_\mu & \frac{i}{2}g_2(W_\mu^1 - iW_\mu^2) \\ \frac{i}{2}g_2(W_\mu^1 + iW_\mu^2) & \partial_\mu - \frac{i}{2}(g_2 W_\mu^3 + g' B_\mu) \end{pmatrix}\begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v + \hat{h}) \end{pmatrix} \\ &= \frac{1}{\sqrt{2}}\begin{pmatrix} i\frac{g_2}{\sqrt{2}}W_\mu^+(v + \hat{h}(x)) \\ \partial_\mu \hat{h}(x) - \frac{ig_2}{2\cos\theta_W}Z_\mu(x)(v + \hat{h}(x)) \end{pmatrix}, \end{aligned} \tag{3.27}$$

where we used the definitions

$$W_\mu^\pm \equiv \frac{1}{\sqrt{2}}(W_\mu^1 \mp iW_\mu^2), \tag{3.28}$$

$$A_\mu \equiv \sin\theta_W W_\mu^3 + \cos\theta_W B_\mu, \tag{3.29}$$

$$Z_\mu \equiv \cos\theta_W W_\mu^3 - \sin\theta_W B_\mu. \tag{3.30}$$

Replacing this on the second and third line in eq. (3.1), we obtain

$$\begin{aligned}
\mathcal{L}_{SM} \supset & -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu} + (D_\mu h)^\dagger (D^\mu h) - \lambda\left(h^\dagger h - \frac{1}{2}v^2\right)^2 \\
= & -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \left(-\frac{1}{2}W^+_{\mu\nu}W^{-\mu\nu} + \frac{g_2^2 v^2}{4}W^-_\mu W^{+\mu}\right) + \\
& \left(-\frac{1}{4}Z_{\mu\nu}Z^{\mu\nu} + \frac{g_2^2 v^2}{8\cos^2\theta_W}Z_\mu Z^\mu\right) \\
& + \frac{1}{2}(\partial_\mu \hat{h})(\partial^\mu \hat{h}) - \lambda v^2 \hat{h}^2 - \lambda v \hat{h}^3 - \frac{\lambda}{4}\hat{h}^4 \\
& + \frac{g_2^2 v}{2}W^-_\mu W^{+\mu}\hat{h} + \frac{g_2^2 v}{4\cos^2\theta_W}Z_\mu Z^\mu \hat{h} \\
& + \frac{g_2^2}{4}W^-_\mu W^{+\mu}\hat{h}^2 + \frac{g_2^2}{8\cos^2\theta_W}Z_\mu Z^\mu \hat{h}^2,
\end{aligned} \tag{3.31}$$

where

$$W^\pm_{\mu\nu} = \partial_\mu W^\pm_\nu - \partial_\nu W^\pm_\mu, \tag{3.32}$$

$$Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu, \tag{3.33}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \tag{3.34}$$

Then, the Lagrangian (3.31) describes a massless gauge field A_μ (photon), a complex vector field W_μ^+ (W bosons) with mass $m_W = g_2 v/2$, a real vector field Z_μ (Z boson) with mass $m_Z = g_2 v/2cos\theta_W$ and a real scalar field $\hat{h}$ (Higgs boson) with mass $m_h = \sqrt{2\lambda v^2}$.

We note that the masses of the W and Z bosons are related by

$$m_Z = \frac{m_W}{cos\theta_W}. \tag{3.35}$$

In 1973, in the Gargamelle bubble chamber located at CERN, it was observed for the first time the effects of neutral current interactions as was predicted by the electroweak theory [8]. The W and Z vector bosons were experimentally detected in 1983 in a series of experiments led by Carlo Rubbia and Simon van der Meer [9, 10, 11, 12]. Their measured masses are $m_W = 80.385 \pm 0.015$ GeV and $m_Z = 91.1876 \pm 0.0021$ GeV [13]. The experimental measure of $\sin\theta_W$ is 0.23176 ± 0.00060. Then, we can see that relation (3.35) is satisfied with great precision in nature.

3.3 Fermion masses

Fermions acquire their masses after electroweak symmetry breaking due to the Higgs mechanism and it occurs through the Yukawa interactions between the fermionic fields and the Higgs field.

As we explained before, a fermionic mass term must contain the left and right handed components of the field. Since in the SM, neutrino fields do not have a right handed component, they will not acquire a mass *. However, charged fermionic fields, having both chiralities, will posses a mass term. Let us consider the fourth and fifth lines of the SM Lagrangian written in (3.1)

$$\mathcal{L}_{SM} \supset -y_e \left(\bar{l}he + \bar{e}h^\dagger l\right) - y_d \left(\bar{q}hd + \bar{d}h^\dagger q\right) - y_u \left(\bar{q}\tilde{h}u + \bar{u}\tilde{h}^\dagger q\right). \tag{3.36}$$

We emphasize that here we are considering just one family of fields for simplicity. After spontaneous symmetry breaking, H takes the form in (3.26), and the previous sector of $\mathcal{L}_{SM}$ converts into

$$\begin{aligned} \mathcal{L}_{SM} \supset &- \frac{y_e}{\sqrt{2}}(v+\hat{h})\left(\bar{e}_L e_R + \bar{e}_R e_L\right) - \frac{y_d}{\sqrt{2}}(v+\hat{h})\left(\bar{d}_L d_R + \bar{d}_R d_L\right) \\ &- \frac{y_u}{\sqrt{2}}(v+\hat{h})\left(\bar{u}_L u_R + \bar{u}_R u_L\right) \\ = &- \frac{y_e v}{\sqrt{2}}\bar{e}e - \frac{y_d v}{\sqrt{2}}\bar{d}d - \frac{y_u v}{\sqrt{2}}\bar{u}u - \frac{y_e}{\sqrt{2}}\hat{h}\bar{e}e - \frac{y_d}{\sqrt{2}}\hat{h}\bar{e}e - \frac{y_u}{\sqrt{2}}\hat{h}\bar{e}e, \end{aligned} \tag{3.37}$$

which shows mass terms for the electron, down quark and up quark fields with masses $m_e = y_e v/\sqrt{2}$, $m_d = y_d v/\sqrt{2}$ and $m_u = y_u v/\sqrt{2}$, respectively. Also, it shows Yukawa interaction terms between the fermions and the Higgs boson. In this way, we managed to generate mass for fermion fields through the Higgs mechanism.

3.4 Problems of the SM

The Standard Model predictions have been tested with great accuracy over the last years, being the discovery of the Higgs particle in 2012 its greatest recent triumph [14, 15]. However, it is far from being the complete theory of nature. Here we list some of the problems that the SM faces:

Neutrino mass: Neutrinos are massless in the Standard Model. However, it was discovered in the late 1960s that neutrinos can change its lepton flavour (electron, muon or tau) during its travel through space; and this phenomenon, known

*Actually, it is an experimental fact that neutrinos do have a mass but it is not explained in the SM.

as *Neutrino oscillation*, is only possible if neutrinos are massive. We do not know how neutrinos acquire its mass and even more we do not know how they are ordered (*Neutrino mass hierarchy problem*).

Dark Matter: In 1975, the astronomer Vera Rubin, after taking measurements of *rotation curves of galaxies* (the plot of orbital speeds of stars and gas as a function of their distances from the center of the galaxy), announced the discovery that stars in spiral galaxies at large distances from the center rotate around it with almost the same speed [16]. Theoretically, taking into account the visible matter, it was expected that the orbital speed of stars in these galaxies decrease at large radial distances. The conflict between what we expect and what we see is known as the *galaxy rotation problem.* In order to solve this problem, it is considered that some kind of *dark matter* † which is forming a giant but invisible halo around the galaxy must exist in addition to the normal matter.

Apart from the galaxy rotation problem there exist various other observations that reinforce the existence of the dark matter (DM). For instance, a galaxy with great amount of mass can distort the images of other galaxies. This is because it interacts gravitationally and bends the light that pass near it (gravitational lensing). The effect of DM on gravitational lensing can be measured and in fact, it is possible to map the distribution of DM in a galaxy through these measurements ‡. Also, dark matter has notable effects on structure formation, redshift space distortions and the distribution of the cosmic microwave background.

Cosmological measurements shows that ordinary known matter (which is composed by the particles in the SM) constitutes just a 5% of the mass-energy of the universe; 27% is dark matter and 68%, dark energy. Then, extensions of the SM that incorporate new particles with the DM characteristics are sought.

Strong CP Problem: Besides the known terms in the SM Lagrangian, there could be an additional natural one of the form

$$\frac{g^2}{32\pi^2}\overline{\theta}\, G^A_{\mu\nu}\tilde{G}^{A\mu\nu}, \tag{3.38}$$

where $\overline{\theta}$ is an arbitrary dimensionless parameter and $\tilde{G}^{A\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}G^A_{\rho\sigma}$ is the dual

† The term "dark" comes from the fact that it does not interact with electromagnetism and thus it is not visible.

‡ The most detailed map of dark matter recently released was made around the MACS *J0416* galaxy cluster [17].

strength tensor corresponding to the gluon fields. This term would violate CP symmetry in the strong sector (C corresponds to charge conjugation under which particle and antiparticle states are interchanged and P refers to parity symmetry under which left and right component fields are interchanged). However, there is no experimental detection of violation of CP in quantum chromodynamics. This is why we called this apparent contradiction as the *Strong CP problem*. Furthermore, the existence of the θ-term would contribute with the electric dipole moment of the neutron estimated by $d_E(n) \sim 10^{-16}\,\overline{\theta}\, e\,\mathrm{cm}$. The current experimental measurements put an upper limit of $d_E(n) < 2.9 \times 10^{-26} e\,\mathrm{cm}$ [18]. Then, $\overline{\theta} \lesssim 10^{-10}$, which is an extremely small number if we compare it with the values of the other SM parameters. Even, if we start with $\overline{\theta} = 0$ in the SM Lagrangian, quantum corrections will induce a term like (3.38).

A possible solution to this problem was proposed by Peccei and Quinn in 1977 [19], in which a new global chiral symmetry $U(1)_{PC}$ is introduced under which both quarks and Higgs doublets transform non-trivially allowing the parameter $\overline{\theta}$ to be promoted to a dynamical field. The $U(1)_{PC}$ symmetry is spontaneously broken by the vacuum, generating a Goldstone boson, named axion, with bare mass equals to zero but that acquires a small mass due to anomalous quantum corrections. This particle has been searched without success. It also can be a constituent or the whole of dark matter.

The interested reader can find more information about the strong CP problem and other solution proposals in references [20, 21, 22].

Hierarchy problem: It happens to be that the Higgs mass parameter receives quadratically divergent quantum corrections due to loop contributions with large momentum. The most relevant of them are due to the diagrams in Figure 3.1. Their contributions are [23]:

$$-\frac{3}{8\pi^2} y_t \Lambda^2 \sim -(2\mathrm{TeV})^2 \tag{3.39}$$

for the top loop,

$$\frac{1}{16\pi^2} g_2 \Lambda^2 \sim (700\mathrm{GeV})^2 \tag{3.40}$$

for the gauge loop, and

$$\frac{1}{16\pi^2} \lambda^2 \Lambda^2 \sim (500\mathrm{GeV})^2 \tag{3.41}$$

for the Higgs loop.

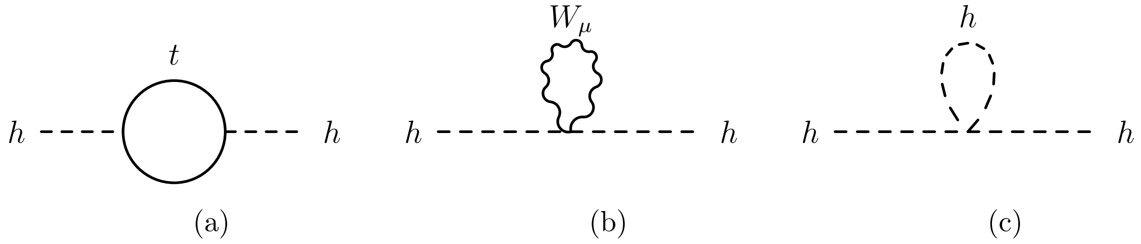


Figure 3.1: One loop contributions to the Higgs mass parameter due to (a) top quark, (b) gauge boson and (c) Higgs autointeractions.

In the last equations Λ is a parameter that regulates the divergent loop integrals resulting from the diagrams. It is interpreted as the energy until which the theory is still valid. For the sake of illustration it was assumed a cutoff energy $\Lambda = 10$ TeV. Then, the physical Higgs mass is approximately

$$m^2_{h,\text{phy}} = m^2_{\text{bare}} - (100 - 10 - 5)(200\text{GeV})^2. \tag{3.42}$$

The measured Higgs mass is $m_{h,phy} = 125$ GeV; so, in order to keep it to that value, we need to finely tune the bare mass. This is even worse if we consider Λ to be the Planck scale $m_P = 1.2 \times 10^{19}$ GeV, the energy at which it is believed that the effects of gravity becomes important.

Among possible solutions to this problem are the Composite Higgs models [24], in which the Higgs boson is not anymore considered a fundamental particle but it is a bound state of new strong interactions. Other candidate theories to solve the hierarchy problem add new particles that interact with the Higgs such that the total corrections cancel out stabilizing the mass of the Higgs boson. Examples of these are Little Higgs theories, twin Higgs theories and weak scale supersymmetry.

Chapter 4

Supersymmetry and the Minimal Supersymmetric Standard Model

An interesting way to solve the hierarchy problem is based on a new type of symmetry called *supersymmmetry* (SUSY) which is discussed in the present chapter. This transformations convert fermionic into bosonic states and viceversa by the action of elements of the group generated by spinor operators Q:

$$Q\,|\text{fermion}\rangle = |\text{boson}\rangle\,, \qquad\qquad Q\,|\text{boson}\rangle = |\text{fermion}\rangle\,. \tag{4.1}$$

The history of supersymmetry dates back to the late 1960's. In 1967, Coleman and Mandula, based on certain assumptions, proved a no-go theorem which states that the biggest symmetry any 4-dimensional quantum field theory could have is always a direct product of the Poincaré group and an internal group, that is, spacetime and internal symmetries cannot mix in any but a trivial way [25]. The first supersymmetric theories were constructed around 1971 in the context of the string theory as 2-dimensional theories where fermionic and bosonic fields can be interchanged leaving the action invariant [26, 27, 28, 29]. The first $4D$ realization of supersymmetry was done by Golfand and Likhtman [30] at approximately the same time but, surprisingly, their work did not attract so much attention [31]. About three years later, Wess and Zumino published a series of articles in the same direction [32, 33, 34] where they established the foundations of the construction of $4D$ supersymmetric theories. They also pointed out that the Coleman-Mandula theorem does not apply to supersymmetric theories because one of the assumptions of the theorem is violated, namely, that only generators that satisfy commutation relations are allowed. In supersymmmetry, the generators are fermionic operators, satisfying anticommutation relations. This condition enlarges the most general symmetry that a theory can have to be a direct product of the so-called superPoincaré group and an internal symmetry, as shown by Haag, Łopuszański and Sohnius in 1975 [35].

Now we proceed with the technical treatment of supersymmetry.

4.1 Algebra of supersymmetry

As we said, supersymmetry generators are fermionic operators which we will denote by Q. Furthermore, they are Weyl-spinors carrying two spinor indices. Then, there exist in total four fermionic supersymmetric generators Q_a and $\bar{Q}_{\dot{a}}$ with a and $\dot{a}$ both taking the values 1 and 2. We can extend the number of generators having N distinct copies of these four generators giving rise to an extended $\mathcal{N} = N$ supersymmetry.

Here we will focus on $\mathcal{N} = 1$ supersymmetry. Its algebra is defined by the relations

$$[Q_a, P_\mu] = [\bar{Q}_{\dot{a}}, P_\mu] = [P_\mu, P_\nu] = 0, \tag{4.2}$$

$$\{Q_a, Q_b\} = \{\bar{Q}_{\dot{a}}, \bar{Q}_{\dot{b}}\} = 0, \tag{4.3}$$

$$\{Q_a, \bar{Q}_{\dot{a}}\} = 2\sigma^\mu_{a\dot{a}} P_\mu, \tag{4.4}$$

where P_μ is the usual translation generator. Relation (4.4) is an important one because therein lies the direct connection between the supersymmetry transformations and space-time translations. Therefore, we can regard SUSY as an extended space-time symmetry.

The supersymmetry algebra can be seen as a Lie algebra with anticommutating elements. So, we can define a finite supersymmetric transformation (element of the algebra) as:

$$G(x^\mu, \theta, \bar{\theta}) = e^{i(\theta Q + \bar{\theta}\bar{Q} - x_\mu P^\mu)}. \tag{4.5}$$

Here, θ, $\bar{\theta}$ and x_μ are real constant parameters associated with each generator. In order to find out how the successive action of supersymmetric transformations change the coordinates, we multiply two of this group elements with different parameters $(a_\mu, \xi, \bar{\xi})$ and $(x_\mu, \theta, \bar{\theta})$. Then, we obtain

$$G(a^\mu, \xi, \bar{\xi})G(x^\mu, \theta, \bar{\theta}) = G(x^\mu + a^\mu - i\theta\sigma^\mu\bar{\xi} + i\xi\sigma^\mu\bar{\theta}, \theta + \xi, \bar{\theta} + \bar{\xi}), \tag{4.6}$$

that is, if we set the initial values of the spacetime and fermionic coordinates to 0, after the two successive transformations defined by $G(x^\mu, \theta, \bar{\theta})$ and $G(a^\mu, \xi, \bar{\xi})$, they will transform as

$$\begin{aligned} &0 \longrightarrow x^\mu \longrightarrow x^\mu + a^\mu - i\theta\sigma^\mu\bar{\xi} + i\xi\sigma^\mu\bar{\theta}, \\ &0 \longrightarrow \theta \longrightarrow \theta + \xi, \\ &0 \longrightarrow \bar{\theta} \longrightarrow \bar{\theta} + \bar{\xi}. \end{aligned} \tag{4.7}$$

The first line of the last equation reinforces what was said lines above, that SUSY transformations affect space-time translations and, moreover, shows how they do it.

Now we introduce a concept that will help us in dealing with supersymmetric theories: the *superfield*. Any function of the coordinates x^μ, θ and $\bar{\theta}$ will be called a superfield and will be denoted by S. Then, if we start from a superfield $S(0,0,0)$ and we make two successive SUSY transformations on it, by the previous analysis, we will end up with the superfield

$$S(x^\mu + a^\mu - i\theta\sigma^\mu\bar{\xi} + i\xi\sigma^\mu\bar{\theta},\ \ \theta+\xi,\ \ \bar{\theta}+\bar{\xi}). \tag{4.8}$$

With the aim to find how infinitesimal SUSY transformations act on superfields we make an expansion of this field considering the parameters a_μ, ξ and $\bar{\xi}$ as infinitesimal ones:

$$\begin{aligned} S(x^\mu &+ a^\mu - i\theta\sigma^\mu\bar{\xi} + i\xi\sigma^\mu\bar{\theta},\ \ \theta+\xi,\ \ \bar{\theta}+\bar{\xi}) \\ &= S(x^\mu,\theta,\bar{\theta}) + (a^\mu - i\theta\sigma^\mu\bar{\xi} + i\xi\sigma^\mu\bar{\theta})\partial_\mu S + \xi^a\partial_a S + \bar{\xi}_{\dot{a}}\partial^{\dot{a}} S \\ &= S(x^\mu,\theta,\bar{\theta}) + a^\mu\partial_\mu S + \xi^a(\partial_a S + i\sigma^\mu\bar{\theta}\partial_\mu S) + \bar{\xi}_{\dot{a}}(\partial^{\dot{a}} S + i\sigma^\mu\bar{\theta}\partial_\mu S) \\ &= S(x^\mu,\theta,\bar{\theta}) + (-ia^\mu P_\mu + i\xi^a Q_a + i\bar{\xi}_{\dot{a}}\bar{Q}^{\dot{a}})S, \end{aligned} \tag{4.9}$$

where

$$P_\mu = i\partial_\mu, \tag{4.10}$$

$$Q_a = -i\left(\frac{\partial}{\partial\theta^a} + i\sigma^\mu_{a\dot{a}}\bar{\theta}^{\dot{a}}\frac{\partial}{x^\mu}\right), \tag{4.11}$$

$$Q_{\dot{a}} = -i\left(-\frac{\partial}{\partial\theta^{\dot{a}}} - i\theta^a\sigma^\mu_{a\dot{a}}\frac{\partial}{x^\mu}\right). \tag{4.12}$$

Then, the expressions in (4.10), (4.11) and (4.12) are the supersymmetry generators expressed in field representation, i.e. the supersymmetric transformations act on superfields as:

$$S(x^\mu,\theta,\bar{\theta}) \to e^{i(\xi Q + \bar{\xi}\bar{Q} - a^\mu P_\mu)} S(x^\mu,\theta,\bar{\theta}). \tag{4.13}$$

Any given superfield S can be expanded in power series in the coordinates θ and $\bar{\theta}$ as

$$\begin{aligned} S(x,\theta,\bar{\theta}) =& A(x) + B(x)\theta + \bar{C}(x)\bar{\theta} + D(x)\theta\theta + E(x)\bar{\theta}\bar{\theta} \\ &+ F_\mu(x)\theta\sigma^\mu\bar{\theta} + G(x)\theta\bar{\theta}\bar{\theta} + \bar{H}(x)\bar{\theta}\theta\theta + G\theta\theta\bar{\theta}\bar{\theta}. \end{aligned} \tag{4.14}$$

Every field that is a function of x^μ in the previous expansion is known as *component fields*. We must say that Lorentz invariance was imposed in (4.14). With the help of the generators in field representation found in (4.10), (4.11) and (4.12), we can know how each field component changes under a SUSY transformation. There are some group of field components that form irreducible representations of the SUSY algebra, that is, they transform into each other. These irreducible representations are described by special superfields that can be obtained by imposing constraints to a general superfield $S(x,\theta,\bar{\theta})$. Here we study two of them: the chiral and the vector superfields.

4.2 Chiral superfield

A chiral superfield (a.k.a. matter or scalar superfield) is defined by the constraint

$$\bar{D}_{\dot{a}} S(x,\theta,\bar{\theta}) = 0, \tag{4.15}$$

where

$$\bar{D}_{\dot{a}} = -\frac{\partial}{\partial \bar{\theta}^{\dot{a}}} + i\theta^a \sigma^\mu_{a\dot{a}} \frac{\partial}{\partial x^\mu}, \tag{4.16}$$

$$D_a = \frac{\partial}{\partial \theta^a} - i\sigma^\mu_{a\dot{a}} \bar{\theta}^{\dot{a}} \frac{\partial}{x^\mu}. \tag{4.17}$$

We normally denote a chiral superfield by Φ. The general solution for eq. (4.15) is of the form

$$\Phi(y,\theta) = \phi(y) + \sqrt{2}\theta\chi(y) + \theta^2 F(y) \tag{4.18}$$

with y^μ given by

$$y^\mu = x^\mu - i\theta\sigma^\mu\bar{\theta}. \tag{4.19}$$

$\phi(y)$, $\chi(y)$ and $F(y)$ are the component fields of the chiral superfield Φ and it is said that they form a *chiral supermultiplet*. ϕ and F are complex scalar fields while χ is a left-handed Weyl spinor field.

Expressing (4.18) in terms of x^μ, θ and $\bar{\theta}$, we have

$$\begin{aligned} \Phi(x,\theta,\bar{\theta}) =& \phi(x) + \sqrt{2}\theta\chi(x) + \theta^2 F(x) - i\theta\sigma^\mu\bar{\theta}\partial_\mu\phi(x) \\ &+ \frac{i}{\sqrt{2}}\theta^2\partial_\mu\chi\sigma^\mu\bar{\theta} - \frac{1}{4}\theta^2\bar{\theta}^2\partial_\mu\partial^\mu\phi(x). \end{aligned} \tag{4.20}$$

The adjoint

$$\begin{aligned} \Phi^\dagger(x,\theta,\bar{\theta}) =& \phi^\dagger(x) + \sqrt{2}\bar{\theta}\bar{\chi}(x) + \bar{\theta}^2 F^\dagger(x) + i\theta\sigma^\mu\bar{\theta}\partial_\mu\phi^\dagger(x) \\ &- \frac{i}{\sqrt{2}}\bar{\theta}^2\theta\sigma^\mu\partial_\mu\bar{\chi} - \frac{1}{4}\theta^2\bar{\theta}^2\partial_\mu\partial^\mu\phi^\dagger(x) \end{aligned}. \tag{4.21}$$

satisfies the constraint

$$D_a\Phi^\dagger = 0. \tag{4.22}$$

These type of superfields are known as *antichiral superfields.*

We now see how the component fields vary under a supersymmetric transformation. The supersymmetric infinitesimal transformation of the chiral supermultiplet Φ is given by:

$$\delta\Phi = i(\xi Q + \bar{\xi}\bar{Q})\Phi, \tag{4.23}$$

where Q and $\bar{Q}$ are defined in (4.11) and (4.12). Then, by comparison with

$$\delta\Phi = \delta\phi + \sqrt{2}\theta\delta\chi + \theta^2\delta F, \tag{4.24}$$

we can infer the supersymmetric infinitesimal variations for the component fields ϕ, χ and F:

$$\delta\phi = \sqrt{2}\xi\chi, \tag{4.25}$$

$$\delta\chi_a = \sqrt{2}\xi_a F - i\sqrt{2}\left(\sigma^\mu\bar{\xi}\right)_a \partial_\mu\phi, \tag{4.26}$$

$$\delta F = i\sqrt{2}\partial_\mu\chi\sigma^\mu\bar{\xi} \tag{4.27}$$

from where we notice that, indeed, ϕ, χ and F form an irreducible representation of the SUSY algebra.

We eventually will want to construct realistic supersymmetric theories. For that, we have to be able to construct supersymmetric Lagrangians out of superfields. The most general supersymmetric and renormalizable Lagrangian including just chiral superfields Φ_i can be written in the form:

$$\mathcal{L} = \int d^4\theta\Phi_i^\dagger\Phi_i + \left[\int d^2\theta W(\Phi_i) + \text{h.c.}\right], \tag{4.28}$$

where $W(\Phi)$, called *superpotential* has the form

$$W(\Phi_i) = \frac{1}{2}m_{ij}\Phi_i\Phi_j + \frac{1}{3}\lambda_{ijk}\Phi_i\Phi_j\Phi_k. \tag{4.29}$$

An important feature of the superpotential is that it is holomorphic, meaning that it cannot contain adjoints of Φ. The presence of $\Phi^\dagger$ in $W(\Phi_i)$ would spoil supersymmetry.

In terms of component fields, the Lagrangian in eq.(4.28) is:

$$\begin{aligned}\mathcal{L} =& \partial_\mu\phi_i^\dagger\partial^\mu\phi_i + i\bar{\chi}_i\bar{\sigma}^\mu\partial_\mu\chi_i + F_i^\dagger F_i \\ &+ (m_{ij}\phi_i F_j + \lambda_{ijk}\phi_i\phi_j F_k - \frac{1}{2}m_{ij}\chi_i\chi_j - \lambda_{ijk}\chi_i\chi_j\phi_k + \text{h.c.}).\end{aligned} \tag{4.30}$$

Since there are no derivatives of F in the Lagrangian, this is an auxiliary field and can be removed by replacing its EOM:

$$F_i^\dagger = -m_{ij}\phi - \lambda_{ijk}\phi_j\phi_k. \tag{4.31}$$

Using this into eq.(4.30) we obtain:

$$\begin{aligned}\mathcal{L} =& \partial_\mu\phi_i^\dagger\partial^\mu\phi_i + i\bar{\chi}_i\bar{\sigma}^\mu\partial_\mu\chi_i - |m_{ij}\phi_j + \lambda_{ijk}\phi_j\phi_k|^2 \\ &- \left(\frac{1}{2}m_{ij}\chi_i\chi_j + \lambda_{ijk}\chi_i\chi_j\phi_k + \text{h.c}\right).\end{aligned} \tag{4.32}$$

Let us analyze the case for just one chiral superfield Φ. In this case the superpotential becomes

$$W(\Phi) = \frac{1}{2}m\Phi^2 + \frac{1}{3}\lambda\Phi^3 \tag{4.33}$$

and the Lagrangian after integrating out the auxiliary field F is

$$\mathcal{L} = \partial_\mu\phi^\dagger\partial^\mu\phi + i\bar{\chi}\bar{\sigma}^\mu\partial_\mu\chi - |m\phi + \lambda\phi^2|^2 - (m\chi\chi + \lambda\chi\chi\phi + \text{h.c.}) \tag{4.34}$$

from which we see that the theory describes a complex scalar field ϕ and a Weyl spinor χ, both with the same mass m.

Let us analyze the matching between degrees of freedom (d.o.f.) of fields with opposite statistic in a chiral supermultiplet. An off-shell chiral supermultiplet (without using EOM's) contains three fields: two complex scalar fields ϕ and F, each of them with 2 d.o.f., and one off-shell Weyl spinor with 4 d.o.f.; then we note that the number of fermionic and bosonic degrees of freedom is equal. Also, an on-shell chiral multiplet contains a complex scalar ϕ with 2 d.o.f. which match with the 2 d.o.f. of the on-shell Weyl spinor χ. This must not be surprising, since we are working in the SUSY framework where transformations change bosons into fermions and viceversa and we cannot loose or gain degrees of freedom after the transformations.

Finally, let us see how SUSY theories can solve the hierarchy problem. Let us expand the Lagrangian in (4.34):

$$\begin{aligned}\mathcal{L} =& \partial_\mu\phi^\dagger\partial^\mu\phi + i\bar{\chi}\bar{\sigma}^\mu\partial_\mu\chi - m^2\phi^\dagger\phi - \lambda^2(\phi^\dagger\phi)^2 - m\lambda\phi^\dagger\phi^2 - m\lambda\phi^{\dagger 2}\phi \\ &- m\chi\chi - m\bar{\chi}\bar{\chi} - \lambda\chi\chi\phi - \lambda\bar{\chi}\bar{\chi}\phi^\dagger,\end{aligned} \tag{4.35}$$

and find the quadratic one loop contributions to the mass of the scalar field ϕ. These are given by the diagrams in Figure 4.1.

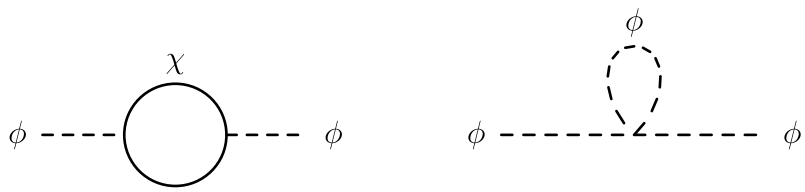


Figure 4.1: Quadratically divergent one loop contributions to the mass of the field ϕ due to (a) a χ loop and (b) a ϕ loop.

with quadratic contributions

$$-4\lambda \int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2} \tag{4.36}$$

for the χ loop, and

$$+4\lambda \int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2} \tag{4.37}$$

for the ϕ loop. Then, both divergences cancel each other making the supersymmetric theory free of quadratic divergences. We see that the existence of the fermionic counterpart χ of the scalar ϕ is crucial for the cancellation to take place, since its contribution have the same value as the bosonic one but with opposite sign. Also, the parameters of the boson-fermion and boson-boson interaction terms must be related.

4.3 Vector superfield

The second type of irreducible representation of the SUSY algebra to be studied is the vector superfield. A vector superfield, which we will denote by V, is defined by the constraint

$$V(x,\theta,\bar{\theta}) = V^{\dagger}(x,\theta,\bar{\theta}). \tag{4.38}$$

The most general solution to 4.38 can be written in the form:

$$\begin{aligned} V(x,\theta,\bar{\theta}) =& C(x) + i\theta\psi(x) - i\bar{\theta}\bar{\psi}(x) + \frac{i}{2}\theta^2\left[M(x)+iN(x)\right] \\ &- \frac{i}{2}\bar{\theta}^2[M(x)-iN(x)] + \theta\sigma^{\mu}\bar{\theta}A_{\mu}(x) \\ &+ i\theta^2\bar{\theta}\left[\bar{\lambda} - \frac{i}{2}\bar{\sigma}^{\mu}\partial_{\mu}\psi(x)\right] - i\bar{\theta}^2\theta\left[\lambda - \frac{i}{2}\sigma^{\mu}\partial_{\mu}\bar{\psi}(x)\right] \\ &+ \frac{1}{2}\theta^2\bar{\theta}^2\left[D - \frac{1}{2}\partial_{\mu}\partial^{\mu}C\right], \end{aligned} \tag{4.39}$$

where C, M, N and D are real scalar fields; V_μ is a real vector field and χ, λ are Weyl spinor fields.

Using the definitions 4.16 and 4.17 we can obtain

$$V| = C \tag{4.40}$$

$$D_a V| = i\chi_a, \ \bar{D}_{\dot{a}} V| = -i\bar{\chi}_a, \tag{4.41}$$

$$D^2 \bar{D}_{\dot{a}} V| = -4i\bar{\lambda}_{\dot{a}}, \ \bar{D}^2 D_a V| = 4i\lambda_a, \tag{4.42}$$

$$D^b \bar{D}^2 D_a V| = 4D\delta_a^b - 2i(\sigma^\mu\bar{\sigma}^\nu)_a^b F_{\mu\nu}, \tag{4.43}$$

$$D^a \bar{D}^2 D_a V| = 8D, \tag{4.44}$$

where the vertical line $|$ means evaluation at $\theta = \bar{\theta} = 0$.

The infinitesimal supersymmetric variations of the superfield V is:

$$\delta_\xi V = i(\xi Q + \bar{\xi}\bar{Q})V, \tag{4.45}$$

with Q and $\bar{Q}$ defined in (4.11) and (4.12). Then, with the help of the identities (4.40)-(4.44), we obtain the infinitesimal supersymmetric variations of the field components:

$$\begin{aligned}
\delta C =& i(\xi Q + \bar{\xi}\bar{Q})V| = (\xi D + \bar{\xi}\bar{D})V| = i(\xi\chi - \bar{\xi}\bar{\chi}), &(4.46)\\
\delta\lambda_a =& \frac{i}{4}(\xi D + \bar{\chi}\bar{D})\bar{D}^2 D_a V| = iD\xi_a + \frac{1}{2}(\sigma^\mu\bar{\sigma}^\nu)_a^b \xi_b F_{\mu\nu}, &(4.47)\\
\delta A^\mu =& - i\bar{\lambda}\bar{\sigma}^\mu\xi + i\bar{\xi}\bar{\sigma}^\mu\lambda - \partial^\mu(\xi\chi + \bar{\xi}\bar{\chi}), &(4.48)\\
\delta D =& \xi\sigma^\mu\partial_\mu\bar{\lambda} + \partial_\mu\lambda\sigma^\mu\bar{\xi} &(4.49)
\end{aligned}$$

Also, for $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$:

$$\delta F^{\mu\nu} = i\left[\partial^\mu\left(\xi\sigma^\nu\bar{\lambda} - \lambda\sigma^\nu\bar{\xi}\right) - \partial^\nu(\xi\sigma^\mu\bar{\lambda} - \lambda\sigma^\mu\bar{\xi})\right]. \tag{4.50}$$

It is easy to construct a vector superfield out of a chiral superfield. It just needs to fulfill the constraint in eq.(4.38). This is:

$$\begin{aligned}
\Phi + \Phi^\dagger =& \phi + \phi^\dagger + \sqrt{2}(\theta\chi + \bar{\theta}\bar{\chi}) + \theta^2 F + \bar{\theta}^2 F^\dagger + i\theta\sigma^\mu\bar{\theta}\partial_\mu(\phi^\dagger - \phi)\\
&+ \frac{i}{\sqrt{2}}\theta^2\partial_\mu\chi\sigma^\mu\bar{\theta} - \frac{i}{\sqrt{2}}\bar{\theta}^2\theta\sigma^\mu\partial_\mu\bar{\chi} - \frac{1}{4}\theta^2\bar{\theta}^2\partial_\mu\partial^\mu(\phi + \phi^\dagger).
\end{aligned} \tag{4.51}$$

Now, we define the generalization of a gauge transformation for superfields as

$$V \to V + \Phi + \Phi^\dagger, \tag{4.52}$$

under which the component fields transform as:

$$\begin{aligned} C &\to C + \Phi + \Phi^\dagger, \\ \psi &\to \psi - i\sqrt{2}\chi, \\ M + iN &\to M + iN - 2iF, \\ A_\mu &\to A_\mu + i\partial_\mu(\phi^\dagger - \phi), \\ \lambda &\to \lambda, \\ D &\to D. \end{aligned} \tag{4.53}$$

We see that λ and D remain invariant under this gauge transformation, while A_μ varies as expected for a massless spin-1 field , that is, $A_\mu \to A_\mu + \partial_\mu \alpha$, where α is a real arbitrary x-dependent parameter. Also, we note that there exists a special gauge where C, ψ, M and N are all zero. This is known as the Wess-Zumino (WZ) gauge. In this gauge, V in eq.(4.39) becomes

$$V = \theta\sigma^\mu\bar{\theta}A_\mu + i\theta^2\bar{\theta}\bar{\lambda} - i\bar{\theta}^2\theta\lambda + \frac{1}{2}\theta^2\bar{\theta}^2 D. \tag{4.54}$$

In order to write a Lagrangian for a vector superfield, we define:

$$W_a = -\frac{1}{4}\bar{D}_{\dot{a}}\bar{D}^{\dot{a}}D_a V, \tag{4.55}$$

which is a chiral superfield since

$$\bar{D}_{\dot{b}}W_a = 0. \tag{4.56}$$

Computing W_a in Wess-Zumino gauge, we have

$$W_a = -i\lambda_a(y) + \left[\delta_a^b D(y) + \frac{i}{2}(\sigma^\mu\bar{\sigma}^\nu)_a^b F_{\mu\nu}\right]\theta_b - \theta^2\sigma^\mu_{a\dot{a}}\partial_\mu\bar{\lambda}^{\dot{a}}(y), \tag{4.57}$$

from where

$$W^a W_a|_{\theta^2} = 2i\lambda\sigma^\mu\partial_\mu\bar{\lambda} + D^2 - \frac{1}{2}F_{\mu\nu}F^{\mu\nu} + \frac{i}{4}F_{\mu\nu}F_{\rho\sigma}\epsilon^{\mu\nu\rho\sigma}. \tag{4.58}$$

So, now, we are able to write a supersymmetric action for a vector superfield. That is,

$$S = \int d^4x \int d^2\theta \frac{1}{4}\left[W^a W_a + \bar{W}_{\dot{a}}\bar{W}^{\dot{a}}\right]. \tag{4.59}$$

Expanding it in field components we obtain

$$S = \int d^4x \mathcal{L} = \int d^4x \left[i\bar{\lambda}\bar{\sigma}^\mu\partial_\mu\lambda - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}D^2\right], \tag{4.60}$$

which describes a massless spinor λ, a massless spin-1 field A_μ and a real scalar D. We say that λ, A_μ and D form a *vector supermultiplet*. The spinor λ in this context is known as *gaugino*.

There are no derivative terms of D in the Lagrangian. Then, this is an auxiliary field and can be integrated out by its EOM $D = 0$ leaving just the fields λ and A_μ.

Let us analyze the degrees of freedom as in the chiral supermultiplet case. An off-shell vector superfield contains an off-shell gaugino with 4 d.o.f., an off-shell spin-1 vector A_μ with 3 d.o.f. and a real scalar D with 1 d.o.f. Then, the number of bosonic and fermionic degrees of freedom is equal. For the on-shell vector supermultiplet we have an on-shell gaugino with 2 d.o.f. that matches with the 2 degrees of freedom of the massless on-shell spin-1 vector field A_μ.

4.4 Supersymmetric gauge invariant Lagrangian

In this part we treat supersymmetric gauge theories.

4.4.1 Abelian case

Let us start with a global $U(1)$ theory. Consider a chiral field Φ that transforms under $U(1)$ global transformations as

$$\Phi \longrightarrow e^{-2i\theta}\Phi, \tag{4.61}$$

where θ is real and constant. Since any constant parameter c satisfies the constraint $\bar{D}_{\dot{a}}c = 0$, θ can be considered as a chiral superfield. Moreover, a product of chiral superfields is also a chiral superfield. Then, the superfield Φ transform into another chiral superfield.

When the theory is promoted to a local $U(1)$ invariant theory, θ is not anymore constant but it depends on x. Then, it has to be promoted to a chiral superfield, such that Φ transforms into another one as

$$\Phi \longrightarrow e^{-2ig\Lambda}\Phi, \tag{4.62}$$

$$\Phi^\dagger \longrightarrow e^{2ig\Lambda^\dagger}\Phi^\dagger, \tag{4.63}$$

where g is a dimensionless parameter which is identified as the gauge coupling that corresponds to the $U(1)$ symmetry group. Now, writing a kinetic term for Φ of the form

$$\int d^4\theta \Phi^\dagger \Phi$$

as we did in section 4.2 would not respect gauge symmetry. Therefore, in order to fix this we introduce a vector superfield V that transforms as

$$V \longrightarrow V + i(\Lambda - \Lambda^\dagger) \tag{4.64}$$

under $U(1)$ local group, and write instead

$$\int d^4\theta \Phi^\dagger e^{2gV} \Phi \tag{4.65}$$

which is gauge invariant. Furthermore, the product $\Phi^\dagger e^{2gV}\Phi$ in (4.65) is real making it a vector superfield. Then (4.65) is also supersymmetric.

We note that a superpotential of the form (4.33) cannot be introduced because it is not gauge invariant. However, if we consider more than one superfield Φ_i transforming as

$$\Phi_i \longrightarrow e^{-2iq_i g\Lambda}\Phi_i, \tag{4.66}$$

where q_i is the $U(1)$ charge corresponding to the superfield Φ_i, it is possible to have a superpotential of the form

$$W(\Phi_i) = \frac{1}{2}m_{ij}\Phi_i\Phi_j + \frac{1}{3}\lambda_{ijk}\Phi_i\Phi_j\Phi_k, \tag{4.67}$$

where m_{ij} is different from zero only if $q_i + q_j = 0$ and y_{ijk} is different from zero only when $q_i + q_j + q_k = 0$.

Finally, the total Lagrangian for a supersymmetric theory with $U(1)$ local invariance is written as

$$\mathcal{L} = \int d^2\theta \frac{1}{4}(W^a W_a + \bar{W}_{\dot{a}}\bar{W}^{\dot{a}}) + \int d^4\theta \Phi_i^\dagger e^{2q_i gV}\Phi_i + \left[\int d^2 W(\Phi_i) + \text{h.c}\right]. \tag{4.68}$$

In field components and considering the WZ gauge, (4.68) becomes

$$\begin{aligned}\mathcal{L} =& (D_\mu\phi_i)^\dagger(D^\mu\phi_i) + i\bar{\chi}_i\bar{\sigma}^\mu D_\mu\chi_i + F_i^\dagger F_i - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\lambda}_i\bar{\sigma}^\mu\partial_\mu\lambda_i + \frac{1}{2}D^2 \\ &+ (m_{ij}\phi_i F_j + \lambda_{ijk}\phi_i\phi_j F_k - \frac{1}{2}m_{ij}\chi_i\chi_j - \lambda_{ijk}\chi_i\chi_j\phi_k + \text{h.c.}) \\ &+ i\sqrt{2}gq_i(\phi_i^\dagger\chi_i\lambda - \phi\bar{\chi}_i\bar{\lambda}_i) - q_i\phi_i^\dagger\phi_i D,\end{aligned} \tag{4.69}$$

where

$$D_\mu\phi_i = \partial_\mu\phi_i + igq_iA_\mu\phi_i, \tag{4.70}$$

$$D_\mu \chi_i = \partial_\mu \chi_i + igq_i A_\mu \chi_i. \tag{4.71}$$

After eliminating the auxiliary fields F_i and D replacing its EOM in (4.69), we obtain

$$\begin{aligned}\mathcal{L} =&(D_\mu\phi_i)^\dagger(D^\mu\phi_i) + i\bar{\chi}_i\bar{\sigma}^\mu D_\mu\chi_i - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\lambda}_i\bar{\sigma}^\mu\partial_\mu\lambda_i \\ &- |m_{ij}\phi_j + \lambda_{ijk}\phi_j\phi_k|^2 - \left(\frac{1}{2}m_{ij}\chi_i\chi_j + \lambda_{ijk}\chi_i\chi_j\phi_k + \text{h.c}\right) \\ &+ i\sqrt{2}gq_i(\phi_i^\dagger\chi_i\lambda - \phi\bar{\chi}_i\bar{\lambda}_i) - \frac{1}{2}q_i^2\left(\phi_i^\dagger\phi_i\right)^2.\end{aligned} \tag{4.72}$$

4.4.2 Non-abelian case

The generalization of the previous discussion to the non-abelian case is relatively straightforward. Let us consider a theory which is invariant under a non-abelian local group G and suppose that it contains a chiral superfield Φ that transforms non-trivially under G:

$$\Phi \longrightarrow e^{-i\Lambda}\Phi, \tag{4.73}$$

$$\Phi^\dagger \longrightarrow \Phi^\dagger e^{i\Lambda^\dagger}, \tag{4.74}$$

where now Λ is the matrix

$$\Lambda \equiv 2gT^a\Lambda^a, \tag{4.75}$$

with T^a being the hermitian generators of G in the representation at which Φ transforms, and g is the gauge coupling constant corresponding to the group G.

The generalization of (4.65)

$$\int d^4\theta\Phi^\dagger e^V\Phi, \tag{4.76}$$

with V given by

$$V \equiv 2T^aV^a \tag{4.77}$$

will be SUSY invariant if the transformation for V is extended to

$$e^V \longrightarrow e^{-i\Lambda^\dagger}e^Ve^{i\Lambda}. \tag{4.78}$$

Also, the field strength W_a in (4.55) is generalized to

$$W_a = -\frac{1}{4}\bar{D}\bar{D}e^{-V}D_ae^V, \tag{4.79}$$

so that it transforms as

$$W_a \longrightarrow e^{-i\Lambda}W_ae^{i\Lambda}. \tag{4.80}$$

Finally, the full non-abelian superysmmetric Lagrangian is written as

$$\mathcal{L} = \int d^4\theta \frac{1}{4} \mathrm{Tr} \left(W^a W_a + \bar{W}_{\dot{a}} \bar{W}^{\dot{a}} \right) + \int d^4\theta \Phi^\dagger e^V \Phi, \tag{4.81}$$

where the superpotential was not considered. After writing it in terms of field components and eliminating auxiliary fields, it takes the form

$$\begin{aligned} \mathcal{L} =& (D_\mu \phi_i)^\dagger (D^\mu \phi_i) + i\bar{\chi}_i \bar{\sigma}^\mu D_\mu \chi_i - \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + i\bar{\lambda}^a_i \bar{\sigma}^\mu \partial_\mu \lambda^a_i \\ &+ i\sqrt{2} g \left[(\phi_i^\dagger T^a \chi_i) \lambda^a - \bar{\lambda}^a (\chi_i^\dagger T^a \phi_i) \right] - \frac{1}{2} g^2 (\phi_i^\dagger T^a \phi_i)(\phi_j^\dagger T^a \phi_j), \end{aligned} \tag{4.82}$$

where the covariant derivatives are

$$D_\mu \phi_i = \partial_\mu \phi_i + igT^a A^a_\mu \phi_i, \tag{4.83}$$

$$D_\mu \chi_i = \partial_\mu \chi_i + igT^a A^a_\mu \chi_i. \tag{4.84}$$

4.5 The Minimal Supersymmetric SM (MSSM)

In this section, we will study the minimal supersymmetric extension of the SM, also known as the MSSM. The purpose of this section is to get used to build supersymmetric theories. We will describe the field content of the theory as well as the Lagrangian. We will also introduce terminology and notation that will be useful in future chapters.

4.5.1 Field content

The first thing to note in order to construct a supersymmmetric version of the SM, is that every field within the SM must be part of a supermultiplet (either chiral or gauge). So, one adds a supersymmetric partner (superpartner) with opposite statistic but with the same gauge transformations for every field in the SM. In the MSSM, this is done in the following way: leptons and quarks belong to chiral supermultiplets, and the corresponding spin-0 superpartners are named by adding the letter 's' (for 'scalar') at the beginning of the SM field name; thereby they are called *sleptons* and *squarks*. Also, for the electron we have its superpartner, the selectron; for the neutrinos, the sneutrinos; for the muon, the smuon, and so on. Gauge bosons belong to vector multiplets and the corresponding spin-1/2 superpartners are named by adding the suffix '-ino' to the regular SM field names. Thus, they are called *gauginos*. The superpartner of the photon is called photino; that of the gluon is called gluino and so on. Finally, in the MSSM, it is necessary to introduce two Higgs doublets unlike the SM where there is just one (this will be explained

Names		**Spin-0**	**Spin-1/2**	$SU(3)_C$,$SU(2)_L$,$U(1)_Y$
sleptons, leptons	L_I	$\tilde{l}_I = \begin{pmatrix} \tilde{\nu}_{IL} \\ \tilde{l}_{IL} \end{pmatrix}$	$l_I = \begin{pmatrix} \nu_{IL} \\ l_{IL} \end{pmatrix}$	**1**, **2**, -1/2
	E_I	$\tilde{e}_I$	$e_I \equiv (e_{IR})^c$	**1**, **1**, 1
squarks, quarks	Q_I	$\tilde{q}_I = \begin{pmatrix} \tilde{u}_{IL} \\ \tilde{d}_{IL} \end{pmatrix}$	$q_I = \begin{pmatrix} u_{IL} \\ d_{IL} \end{pmatrix}$	**3**, **2**, 1/6
	U_I	$\tilde{u}_I$	$u_I \equiv (u_{IR})^c$	$\bar{\mathbf{3}}$, **1**, -2/3
	D_I	$\tilde{d}_I$	$d_I \equiv (d_{IR})^c$	$\bar{\mathbf{3}}$, **1**, 1/3
Higgs, higgsinos	H_U	$h_U = \begin{pmatrix} h_U^+ \\ h_U^0 \end{pmatrix}$	$\tilde{h}_U = \begin{pmatrix} \tilde{h}_U^+ \\ \tilde{h}_U^0 \end{pmatrix}$	**1**, **2**, 1/2
	H_D	$h_D = \begin{pmatrix} h_D^0 \\ h_D^- \end{pmatrix}$	$\tilde{h}_D = \begin{pmatrix} \tilde{h}_D^0 \\ \tilde{h}_D^- \end{pmatrix}$	**1**, **2**, -1/2

Table 4.1: Chiral supermultiplet fields in the MSSM with corresponding transformations under the gauge group. The index I runs from 1 to 3 and denotes each of the three families of particles. Adapted from [36].

afterwards). These Higgs doublets belong also to a chiral supermultiplet and the corresponding superpartners are called *Higgsinos*. All superpartners are denoted by putting a tilde above the normal symbol for the SM field. The superfields contained in the MSSM and their components are listed in Tables 4.1 and 4.2.

Names	**Spin-0**	**Spin-1/2**	$SU(3)_C$,$SU(2)_L$,$U(1)_Y$
gluino, gluon	$\tilde{G}$	G_μ	**8**, **1**, 0
winos, W bosons	$\tilde{W}$	W_μ	**1**, **3**, 0
bino, B boson	$\tilde{B}$	B_μ	**1**, **1**, 0

Table 4.2: Gauge supermultiplet fields in the MSSM with corresponding transformations under the gauge group. Adapted from [36].

4.5.2 The MSSM Lagrangian

To specify the Lagrangian of a supersymmetric theory we need to define its superpotential. The MSSM superpotential is given by

$$W_{MSSM} = y_u^{IJ} U_I Q_J \cdot H_U - y_d^{IJ} D_I Q_J \cdot H_D - y_e^{IJ} E_I L_J \cdot H_D + \mu H_U \cdot H_D. \quad (4.85)$$

Here, y_u, y_d and y_e are 3×3 matrices in family space. They are the same Yukawa couplings of the SM (eq. 3.1), now for three families. These trilinear terms constitute the supersymetric version of the Yukawa terms of the Standard model in the sense that they are responsible for giving mass to leptons and quarks after the Higss fields h_U^0 and h_D^0 acquire vacuum expectation values.

Now, we can justify the presence of two Higgs doublets in the MSSM rather than the single h field in the SM. If we refer to the Yukawa terms of the SM Lagrangian (3.36), we notice that the Higgs field h gives mass to the down quark and the electron when it acquires a vev while the responsible for giving mass to the up quark is the field $\tilde{h}$ defined as

$$\tilde{h} \equiv i\sigma_2 h^*.$$

As we argued in section 4.2, the superpotential must be holomorphic, that is, it cannot contain a field and its hermitian conjugate at the same time because it would spoil supersymmetry. Since a term such as $\tilde{h}$ would imply the hermitian conjugate of H_U, we must add a new Higgs field with the same quantum numbers as $\tilde{h}$, namely H_D.

The Lagrangian in eq. (4.85) is invariant under the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetry of the SM. $SU(3)$ indices for Q, U and D were suppressed. The '$\cdot$' notation means the $SU(2)$ invariant product of two $SU(2)$ doublets. That is, for two $SU(2)$ doublets A_i and B_i, with i denoting one of the two $SU(2)$ components of A or B, we define the $SU(2)$ invariant product

$$A \cdot B \equiv \epsilon^{ij} A_i B_j, \quad (4.86)$$

with $\epsilon^{11} = \epsilon^{22} = 0$, $\epsilon^{12} = 1$ and $\epsilon^{21} = -1$.

However W_{MSSM} is not the most general superpotential. There exist additional renormalizable terms that can be included in it, such as

$$W_{\Delta L=1} = \lambda_e^{IJK} L_I \cdot L_J E_K + \lambda_L^{IJK} L_I \cdot Q_J D_k + \mu_L^I L_I \cdot H_U \quad (4.87)$$

and

$$W_{\Delta B=1} = \lambda_B^{IJK} U_I D_J D_K. \tag{4.88}$$

It happens that these terms violates lepton ($\hat{L}$) and baryon number ($\hat{B}$) conservation respectively. These quantum numbers are assigned such that Q_I carry $\hat{B} = 1/3$ and $\hat{L} = 0$; D_I and U_I carry $\hat{B} = -1/3$ and $\hat{L} = 0$; L_I carry $\hat{B} = 0$ and $\hat{L} = 1$; E carries $\hat{B} = 0$ and $\hat{L} = -1$ and H_U carries $\hat{B} = \hat{L} = 0$. Therefore, the terms in $W_{\Delta L=1}$ violate lepton number by one unit of $\hat{L}$ and $W_{\Delta B=1}$ violates baryon number by one unit of $\hat{B}$. Violation of this quantum numbers have not been detected experimentally. Moreover, the presence of λ_L and λ_B would imply decay channels for the proton. Since we have never seen such decays, the strength of those coupling must be extremely and unnaturally small. One possibility is to postulate $\hat{B}$ and $\hat{L}$ conservation as a principle in the MSSM, but, actually, lepton and baryon number are violated by non-pertubative electroweak effects in the SM. Then a new symmetry called *R-parity* is postulated in order to avoid the terms in (4.87) and (4.88), maintaining the interaction terms in (4.85). R-parity is multiplicatively conserved and is defined by

$$R = (-1)^{3\hat{B}+\hat{L}+2s}, \tag{4.89}$$

where s denotes the spin of the particle.

The MSSM, as was formulated until now would predict that, for every particle in the SM, must exist a superpartner with the same mass. Experiments should have found such particles since those range of energies have already been explored. Not having discovered those *sparticles* suggests that supersymmetry must be broken spontaneously at some high energy in such a way that, at the energies already explored, supersymmetry is hidden. We do not know how such a spontaneous symmetry breaking mechanism works but we can postpone the answer to that question and study instead the physics after the spontaneous SUSY breaking. In order to do that we add terms to the Lagrangian that break SUSY explicitly. It is mandatory for these terms to be *soft*, that is, they must have coefficients with positive mass dimension in order to avoid introducing new divergences. Also they have to respect the gauge symmetry of the theory, $SU(3)_C \times SU(2)_L \times U(1)_Y$.

Completing the MSSM Lagrangian, we introduce all the possible soft SUSY-breaking terms [36]:

(i) Mass terms for gauginos:

$$-\frac{1}{2}\left(M_3 \tilde{G}^A \tilde{G}^A + M_2 \tilde{W}^a \tilde{W}^a + M_1 \tilde{B}\tilde{B} + \text{h.c.}\right). \tag{4.90}$$

(ii) Non-holomorphic mass terms for sfermions:

$$- m^2_{Q,IJ}\tilde{q}^\dagger_I\tilde{q}_J - m^2_{L,IJ}\tilde{l}^\dagger_I\tilde{l}_J - m^2_{U,IJ}\tilde{u}^\dagger_I\tilde{u}_J - m^2_{D,IJ}\tilde{d}^\dagger_I\tilde{d}_J - m^2_{E,IJ}\tilde{e}^\dagger_I\tilde{e}_J. \tag{4.91}$$

(iii) Mass terms for Higgs fields:

$$- m^2_{H_U} h^\dagger_U h_U - m^2_{H_D} h^\dagger_D h_D - (b\, h_U \cdot h_D + \text{h.c}). \tag{4.92}$$

(iv) Trilinear scalar couplings:

$$- a^{IJ}_U \tilde{u}_I\tilde{q}_J \cdot h_U + a^{IJ}_D \tilde{d}_I\tilde{q}_J \cdot h_D + a^{IJ}_E \tilde{e}_I\tilde{l}_J \cdot h_D + \text{h.c.}. \tag{4.93}$$

Then, the complete Lagrangian of the MSSM is of the form

$$\mathcal{L}_{\text{MSSM}} = \mathcal{L}_{\text{SUSY}} + \mathcal{L}_{\text{soft}}, \tag{4.94}$$

where $\mathcal{L}_{\text{SUSY}}$ is the SUSY invariant part which contains the kinetic terms for all the fields listed in Tables 4.1 and 4.2 as well as the supersymmetric gauge interaction terms invariant under the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ and those arising from the superpotential in (4.85); and $\mathcal{L}_{\text{soft}}$ contains all the soft-SUSY breaking terms showed in eqns. (4.90) $-$ (4.93).

Chapter 5

Extra Dimensions

The unification of the interactions has guided the development of physics through its history. For instance, in the seventeenth century, Newton realized that the force that attract objects to the Earth is the same as the one which causes the Moon to orbit around it and explain both with his gravitational theory. Later on, Maxwell succeeded in formulating the theory of electromagnetism, unifying the phenomena of magnetism and electricity.

Following this path, the Finnish physicist Gunnar Nordström, in 1914, introduced an extra spatial dimension for the first time in an attempt to unify Maxwell's theory of electromagnetism and a scalar version of gravity [37]. It must be noticed that the Einstein's Gravitational Theory was not formulated until 1915. Nordstrm's Scalar gravity failed in predicting the bending of light during a solar eclipse and was forgotten. In 1921, Theodor Kaluza published a theory where he attempted to unify now the electromagnetism and Einstein's theory of gravity, also adding a fourth spatial dimension [38]. Later, in 1926, Oskar Klein proposed that the extra dimension in Kaluza theory is curled up forming a circle [39]. That is, unlike the common four dimensions, the fifth one must be finite in size. This is a key concept because an infinite fifth dimension could be easily detected by us. Moreover the finite size of the extra dimension must be very small so we would not have noticed it so far. We say that the extra dimension is compactified and the compactification scale is given by the inverse radius of the circle. For us to be able to detect experimentally the extra dimension it is necessary to reach energies comparable to the compactification scale.

The Kaluza-Klein theory fails in accommodating chiral spinor fields, a fundamental feature of the SM [40]. But in modern theories this issue is overcome with the introduction of compactifications with singularities.

This chapter will serve as a review of the basic concepts of Extra Dimensions. The content of this part is mainly based on Refs. [41, 42, 43]. We will work in the simplest scenario, that is, we will add just one spatial extra dimension described by the coordinate $y = x^5$. Then, a general $5D$ theory is specified by an action of the form

$$S_5 = \int d^4x \int dy \; \mathcal{L}_5[\Phi(x^\mu, y)], \tag{5.1}$$

where Φ stands for a general 5-dimensional field. Also, we will consider a flat $5D$ metric $\eta_{MN} = diag(1, -1, -1, -1, -1)$. Throughout this dissertation, capitalized latin indices M, N run over $0, 1, 2, 3, 5$ and lowercase Greek indices μ, ν run over $0, 1, 2, 3$.

5.1 Compactification

We say that the theory is compactified on $\mathcal{M}_4 \times C$ if C is a compact (closed and bounded) space described by the coordinate $y = x^5$. $\mathcal{M}_4$ is the Minkowski spacetime. Then, the four dimensional Lagrangian is obtained by integrating the fifth coordinate y as

$$\mathcal{L}_4 = \int dy \; \mathcal{L}_5[\Phi(x^\mu, y)]. \tag{5.2}$$

C can be written as $C = M/G$, where M is a non-compact manifold and G is a discrete group that acts *freely* on M through operators τ_g which constitute a representation of G, namely, $\tau_{g_1 g_2} = \tau_{g_1}\tau_{g_2}$. That G acts freely on M means that only the operator τ_i related with the identity element $i \in G$ has fixed points ($\tau_i(y) = y, \;\; \forall y \in M$). Then, C is constructed by the identification of points in M that belong to the same orbit:

$$y \equiv \tau(y). \tag{5.3}$$

Also, we state that physics should depend only on the points in C, that is

$$\mathcal{L}_5[\Phi(x, y)] = \mathcal{L}_5[\Phi(x, \tau(y))]. \tag{5.4}$$

A necessary and sufficient condition to satisfy this equation is

$$\Phi(x, \tau_g(y)) = T_g \Phi(x, y), \tag{5.5}$$

where T_g are a representation of G, namely $T_{g_1 g_2} = T_{g_1} T_{g_2}$ for $g_1, g_2 \in G$ and are also elements of a symmetry group of the theory.

When $T_g = I, \;\; \forall g \in G$ (trivial representation of G), eq.(5.5) reduces to

$$\Phi(x, \tau_g(y)) = \Phi(x, y), \tag{5.6}$$

which defines an *ordinary compactification.*

When $T_g \neq I$ for some $g \in G$, eq. (5.5) defines a *Scherk-Schwarz compactification.* T_g is known as *Scherk-Schwarz twist* and it is said that the field $\Phi(x, y)$ satisfies *twisted boundary conditions.*

Let us describe the *compactification in a circle.* In this case, $M = \mathbb{R}$ (the set of real numbers), $G = \mathbb{Z}$ (the set of integer numbers) and $C = S^1$ (the circle). The elements of the group $\mathbb{Z}$ that act on $y \in \mathbb{R}$ are represented by:

$$\tau_n(y) = y + 2\pi n R, \tag{5.7}$$

where R is the radius of the circle. Then, if we identify $\tau_n(y) \equiv y$, we are left with an interval of the form $\langle y, y + 2\pi R]$. We can conveniently choose $y = \pi R$ to obtain the fundamental domain:

$$\langle -\pi R, \pi R] \, . \tag{5.8}$$

If we define a Scherk-Schwarz boundary condition for the field:

$$\Phi(x, y + 2\pi R) = T\Phi(x, y), \tag{5.9}$$

where T could be the element of a $\mathbb{Z}_2$ symmetry of the theory, we can complete the interval in (5.8) to $[-\pi R, \pi R]$. For fields with untwisted boundary conditions $T = +1$ (bosons) we make

$$\Phi(x, \pi R) = \Phi(x, -\pi R), \tag{5.10}$$

and the field is a single-valued function on the circle S^1.

For fields with twisted b.c.'s $T = -1$ (fermions) we make

$$\Phi(x, \phi R) = -\Phi(x, -\phi R), \tag{5.11}$$

and the field is a non single-valued function on S^1.

5.1.1 Orbifold Compactification

Just as we described ordinary and Scherk-Schwarz compactifications, we can define an *orbifold compactification* in a similar way. This type of compactification is used to obtain chiral fermions in a $4D$ low energy description as needed in the Standard Model.

Let us consider a compact manifold C and a discrete group H that acts *non freely* on C through the operators ζ_h which constitutes a representation of H. That H acts non freely on C means that there is more than one fixed point. We construct the orbifold $O = C/H$ by identifying points in C which differ by ζ_h for some $h \in H$. Also, we define that fields evaluated at these two points differ by some operator Z_h which is an element of a global or local symmetry of the theory, that is:

$$y \equiv \zeta(y), \tag{5.12}$$

$$\Phi(x, \zeta(y)) = Z_h \Phi(x, y). \tag{5.13}$$

O has singularities at the fixed points. Eqs.(5.12) and (5.13) specifies the orbifold compactification and are known as *orbifold boundary conditions*.

A useful orbifold construction is $O = S^1/\mathbb{Z}_2$ where S^1 is the circle and $\mathbb{Z}_2$ is the cyclic group of order 2, which relates two opposite points in S^1:

$$\zeta(y) = -y. \tag{5.14}$$

Also, we have for fields

$$\Phi(x, -y) = Z\Phi(x, y), \tag{5.15}$$

where $Z^2 = 1$, that is, in field spacetime, Z is a matrix with eigenvalues ± 1. The orbifold $O = S^1/\mathbb{Z}_2$ has two fixed points $y = 0$ and $y = \pi R$. Each of them defines a 4-dimensional spacetime slice also known as a *brane*.

5.2 Scalars

Let us consider a free massless real scalar field Φ in a $5D$ space with radius R compactified on the orbifold $O = S^1/\mathbb{Z}_2$ with fixed points $y = 0$ and $y = \pi R$. Its action is given by

$$S_5[\Phi] = \int d^5x \, \frac{1}{2} \partial_M \Phi \partial^M \Phi. \tag{5.16}$$

In order to derive the equation of motion (EOM) and to find the boundary conditions at the fixed points, we will follow the minimum action principle, $\delta S = 0$. The variation of the action in eq.(5.16) is:

$$\delta S_5[\Phi] = \int d^4x \;\; \partial^5 \Phi \delta \Phi \Big|_{y=0}^{y=\pi R} - \int d^5x \left[\partial_M \partial^M \Phi \right] \delta \Phi, \tag{5.17}$$

where we used the usual boundary conditions for fields at infinity: $\Phi|_{x^\mu=\inf} = 0$. By making $\delta S_5 = 0$ we obtain:

$$\partial_M \partial^M \Phi = 0, \tag{5.18}$$

$$\partial^5 \Phi \delta \Phi \Big|_{y=\pi R} - \partial^5 \Phi \delta \Phi \Big|_{y=0} = 0. \tag{5.19}$$

Eq. (5.18) is the EOM for the field Φ and Eq.(5.19) defines the allowed b.c.'s. In order to satisfy eq. (5.19) we need to impose Neumann or Dirichlet b.c.' s:

$$\text{Neumann(N):} \quad \partial_5 \Phi| = 0,$$

$$\text{Dirichlet(D):} \quad \Phi| = 0$$

that can appear in four combinations:

$$\begin{aligned}
(++) &= (\text{N at } y = 0,\ \text{N at } y = \pi R), \\
(+-) &= (\text{N at } y = 0,\ \text{D at } y = \pi R), \\
(-+) &= (\text{D at } y = 0,\ \text{N at } y = \pi R), \\
(--) &= (\text{D at } y = 0,\ \text{D at } y = \pi R).
\end{aligned} \tag{5.20}$$

Since the spacetime is compact, Φ can be Fourier expanded in the y coordinate as

$$\Phi(x, y) = \frac{1}{\sqrt{2\pi R}} \sum_{n=-\infty}^{+\infty} \phi^{(n)}(x) \xi_n(y), \tag{5.21}$$

where $\xi(y)$ form a complete set of orthogonal functions on the interval. The $\phi^{(n)}(x)$ fields are called *Kaluza-Klein modes*. Complete sets of functions $\xi_n(y)$ appropriate to the conditions in eq.(5.20) are:

$$\begin{aligned}
\xi_n^{(++)}(y) &= \cos\left[\frac{ny}{R}\right], & \xi_n^{(+-)}(y) &= \cos\left[\frac{(2n+1)y}{2R}\right], \\
\xi_n^{(--)}(y) &= \sin\left[\frac{ny}{R}\right], & \xi_n^{(-+)}(y) &= \sin\left[\frac{(2n+1)y}{2R}\right],
\end{aligned} \tag{5.22}$$

where $\xi_n^{(++)}$ satisfies a boundary condition of the type $(++)$ and so on, and n takes the values $0, 1, 2, ...$ Replacing the fields with different boundary conditions in the action in eq. (5.16), we obtain

$$S_5^{(++)} = \int d^4x \left[\frac{1}{2} \partial_\mu \phi^{(0)} \partial^\mu \phi^{(0)} + \sum_{n=1}^{\infty} \left(\partial_\mu \phi^{(n)} \partial^\mu \phi^{(n)} - \frac{n^2}{R^2} \phi^{(n)} \phi^{(n)} \right) \right], \tag{5.23}$$

$$S_5^{(--)} = \int d^4x \sum_{n=1}^{\infty} \left(\partial_\mu \phi^{(n)} \partial^\mu \phi^{(n)} - \frac{n^2}{R^2} \phi^{(n)} \phi^{(n)} \right), \tag{5.24}$$

$$S_5^{(\pm\mp)} = \int d^4x \sum_{n=-\infty}^{\infty} \frac{1}{2} \left(\partial_\mu \phi^{(n)} \partial^\mu \phi^{(n)} - \left(\frac{2n+1}{2R} \right)^2 \phi^{(n)} \phi^{(n)} \right). \tag{5.25}$$

Then, $S_5^{(++)}$ is equivalent to a $4D$ action describing a real scalar massless mode corresponding to $n = 0$ (*zero mode*) and infinite real scalar fields with mass $m_n = n/R$. For $S_5^{(--)}$, the Kaluza Klein (KK) modes form a tower of real fields with masses n/R. In the case of $S_5^{(\pm\mp)}$ the theory is equivalent to a $4D$ action of an infinite tower of real scalar fields with masses $m_n = (2n+1)/2R$.

If extra dimensions really exist, the Kaluza-Klein modes could be detected experimentally in the form of a bunch of particles with increasing masses. The lightest particle would correspond to the zero mode KK field. We note that the only fields that posses zero modes are those with (++) boundary conditions. Then, particles with these b.c.'s are expected to be detected at low energies.

We can see that the four combinations in eq.(5.20) can be obtained by different combinations of values of T and Z in eqs.(5.9) and (5.15). More specifically, we have:

$$(T = +1) \wedge (Z = +1) \longrightarrow (++), \tag{5.26}$$

$$(T = +1) \wedge (Z = -1) \longrightarrow (--), \tag{5.27}$$

$$(T = -1) \wedge (Z = +1) \longrightarrow (+-), \tag{5.28}$$

$$(T = -1) \wedge (Z = -1) \longrightarrow (-+). \tag{5.29}$$

So, in order to fully define the boundary conditions of a field we need to specify both transformations of the field under reflections about $y = 0$ denoted by Z and under translations by $2\pi R$ denoted by T. Or equivalently, we can specify Z transformation and its transformation under reflections about $y = \pi R$, which we denote by Z'. We must note that T and Z are related by $Z' = TZ$. Then, in terms of Z and Z' we have:

$$(Z' = +1) \wedge (Z = +1) \longrightarrow (++), \tag{5.30}$$

$$(Z' = -1) \wedge (Z = -1) \longrightarrow (--), \tag{5.31}$$

$$(Z' = -1) \wedge (Z = +1) \longrightarrow (+-), \tag{5.32}$$

$$(Z' = +1) \wedge (Z = -1) \longrightarrow (-+). \tag{5.33}$$

5.3 5D fermions

In $5D$ we need 5 Dirac matrices in order to satisfy the Clifford algebra. We already have a matrix in $4D$ that satisfy this condition: γ^5. So we will take as a basis of

Dirac matrices in $5D$ the following set:

$$\Gamma^M = (\gamma^\mu, -i\gamma_5) = \left(\begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}, \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} \right), \tag{5.34}$$

where $\sigma^\mu = (1, \overrightarrow{\sigma})$, $\bar{\sigma}^\mu = (1, -\overrightarrow{\sigma})$ and $\overrightarrow{\sigma}$ are the Pauli matrices.

We will work, as before, with a fifth extra dimension of radius R compactified on the orbifold S^1/Z_2. The Lagrangian for the $5D$ Dirac field is written as

$$S_5[\psi] = \int d^5x \bar{\Psi}(i\Gamma^M \partial_M - m_\Psi)\Psi. \tag{5.35}$$

We will follow the principle of minimum action as we did for the scalar case. The variation of (5.35) is:

$$\delta S_5[\Psi] = \int d^4x \bar{\Psi}\gamma_5 \delta\Psi|_{y=0}^{y=L} + \int d^5x \delta\bar{\Psi} \left(i\Gamma^M \partial_M - m_\Psi\right) \Psi - \int d^5x (i\partial_M \bar{\Psi}\Gamma^M + m_\psi \bar{\Psi})\delta\Psi. \tag{5.36}$$

Then, by making $\delta S_5[\Psi] = 0$, from the second and third term of (5.36), we obtain the EOM:

$$\left(i\Gamma^M \partial_M - m_\Psi\right) \Psi = 0 \tag{5.37}$$

and

$$\bar{\Psi} \left(i\overleftarrow{\partial}_M \Gamma^M + m_\Psi\right) = 0. \tag{5.38}$$

Also, from the first term in eq.(5.36), boundary conditions must be defined such that:

$$\bar{\Psi}\gamma_5\delta\Psi|_{y=L} - \bar{\Psi}\gamma_5\delta\Psi|_{y=0} = 0. \tag{5.39}$$

Expressing it in terms of $\Psi_{L,R} \equiv P_{L,R}\Psi$, with $P_L = (1-\gamma_5)/2$ and $P_R = (1+\gamma_5)/2$, we have

$$\left(\delta\bar{\Psi}_L \Psi_R - \delta\bar{\Psi}_R \Psi_L\right)|_{y=0}^{y=\pi R} = 0 \tag{5.40}$$

and the EOM for these fields are:

$$i\gamma^\mu \partial_\mu \Psi_R = (\partial_5 + m_\Psi)\Psi_L, \tag{5.41}$$

$$i\gamma^\mu \partial_\mu \Psi_L = (-\partial_5 + m_\Psi)\Psi_R. \tag{5.42}$$

Then, we see that if we were to impose b.c.'s to the fields Ψ_L and Ψ_R, these will not be independent but will be related by the EOM's (5.41) and (5.42). So, at each boundary $y = 0,\ y = \pi R$, there are two possible choices of b.c.'s:

$$\Psi_L = 0 \text{ which implies } \partial_5 \Psi_R(y) = m_\Psi \Psi_R(y) \tag{5.43}$$

or

$$\Psi_R = 0 \text{ which implies } \partial_5 \Psi_L(y) = -m_\Psi \Psi_L(y). \tag{5.44}$$

For the particular case in which $m_\Psi = 0$, these conditions simplify to

$$\Psi_L| = 0 \,(\text{Dirichlet}, -) \wedge \partial_5 \Psi_R| = 0 \,(\text{Neumann}, +) \tag{5.45}$$

or

$$\partial_5 \Psi_L| = 0 \,(\text{Neumann}, +) \wedge \Psi_R| = 0 \,(\text{Dirichlet}, -). \tag{5.46}$$

Then, we note that Ψ_R and Ψ_L obey opposite boundary conditions.

Similarly to the scalar case, each chiral component can be decomposed in terms of Kaluza-Klein modes as

$$\Psi_R(x, y) = \frac{1}{\sqrt{2\pi R}} \sum_{n=0}^{\infty} \psi_R^{(n)}(x) \xi_n^R(y) \tag{5.47}$$

and

$$\Psi_L(x, y) = \frac{1}{\sqrt{2\pi R}} \sum_{n=0}^{\infty} \psi_L^{(n)}(x) \xi_n^L(y). \tag{5.48}$$

For $m_\Psi = 0$ we can use the functions defined in eq.(5.22) as the orthogonal set of functions ξ_n^R or ξ_n^L taking into account that if ξ_n^R has $(++)$ boundary conditions, then ξ_n^L must obey the opposite boundary conditions, that is, $(--)$. The same happens with the other three possible b.c.'s.

From the Kaluza-Klein decomposition, we can see that for the $(\pm, \mp)$ fields, there is a tower of four dimensional Dirac fields with masses $m_n = (2n+1)\pi/2L$. For the $(\pm, \pm)$ fields the massive levels are at $m_n = n\pi/L$ and there is also a massless chiral field for $n = 0$ (zero mode) corresponding to $\xi_0^{(++)}$ since this is the only field that contains a non trivial solution ($\xi_0^{(--)}$ doesn't admit a Kaluza-Klein mode). The existence of chiral fields is fundamental when applied to the Standard Model and this is one of the main reasons why orbifold compactifications are preferred over other ones.

5.4 Spin-1 fields

The $5D$ action for a massless spin-1 field is

$$\begin{aligned}
S_5[A^\mu] &= \int d^5x \left[-\frac{1}{4} F_{MN} F^{MN} \right] \\
&= \int d^5x \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} F_{\mu 5} F^{\mu 5} \right] \\
&= \int d^5x \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} \partial_\mu A_5 \partial^\mu A_5 + \frac{1}{2} \partial_5 A_\mu \partial_5 A^\mu - \partial_5 A_\mu \partial^\mu A_5 \right] \\
&= \int d^5x \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} \partial_\mu A_5 \partial^\mu A_5 + \frac{1}{2} \partial_5 A_\mu \partial_5 A^\mu - \partial_5 A_5 \partial_\mu A^\mu \right] \\
&\quad - \int d^4x (A_\mu \partial^\mu A_5)|_{y=0}^{y=\pi R},
\end{aligned} \tag{5.49}$$

where in the last equality, it was used integration by parts and the usual vanishing of fields at infinity.

We note that there is a bulk term mixing A_μ and A_5. So, we will need to add a gauge fixing term in order to cancel this. This gauge fixing term is chosen to be

$$S_{GF} = \int d^5x \left[-\frac{1}{2\xi} \left(\partial_\mu A^\mu - \xi \partial_5 A_5 \right)^2 \right], \tag{5.50}$$

where ξ is an arbitrary gauge fixing parameter. Then, the Lagrangian becomes

$$\begin{aligned}
S_5'[A] =& S_5[A] + S_{GF} \\
=& \int d^5x \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2\xi} (\partial_\mu A^\mu)^2 + \frac{1}{2} \partial_5 A_\mu \partial_5 A^\mu + \frac{1}{2} \partial_\mu A_5 \partial^\mu A_5 - \frac{1}{2} \xi (\partial_5 A_5)^2 \right].
\end{aligned} \tag{5.51}$$

Now, we can proceed with the variational principle as the previous cases. Varying under δA_μ we obtain

$$\begin{aligned}
\delta S_5'[A] = \int d^5x \delta A_\mu & \left[\eta^{\mu\nu} \partial_\rho \partial^\rho - \left(1 - \frac{1}{\xi} \right) \partial^\mu \partial^\nu - \eta^{\mu\nu} \partial_5 \partial_5 \right] A_\nu \\
&+ \int d^4x \delta A_\mu [\partial_5 A^\mu - \partial^\mu A_5]|_{y=0}^{y=L}
\end{aligned} \tag{5.52}$$

and varying under δA_5, we have

$$\delta S_5'[A] = \int d^5x \delta A_5 [-\partial_\mu \partial^\mu + \xi \partial_5^2] A_5 - \int d^4x \delta A_5 \left[\xi \partial_5 A_5 - \partial_\mu A^\mu \right] |_{y=0}^{y=\pi R}. \tag{5.53}$$

Thus, making $\delta S' = 0$, we obtain the EOM

$$\left[\eta^{\mu\nu} \partial_\rho \partial^\rho - \left(1 - \frac{1}{\xi} \right) \partial^\mu \partial^\nu \right] A_\nu - \partial_5 \partial_5 A_\mu = 0, \tag{5.54}$$

$$\partial_\mu \partial^\mu A_5 - \xi \partial_5^2 A_5 = 0. \tag{5.55}$$

Also, boundary conditions must satisfy the equations

$$(\partial_5 A^\mu - \partial^\mu A_5)|_{y=0}^{y=\pi R} = 0, \tag{5.56}$$

$$(\xi \partial_5 A_5 - \partial_\mu A^\mu)|_{y=0}^{y=\pi R} = 0. \tag{5.57}$$

Then, we can identify two possibilities

$$(+) \equiv \quad A_5| = 0 \quad \Rightarrow \quad \partial_5 A_\mu| = 0 \tag{5.58}$$

or

$$(-) \equiv \quad A_\mu| = 0 \quad \Rightarrow \quad \partial_5 A_5| = 0. \tag{5.59}$$

Then, we have four cases labeled by

$$(+,+)\ (+,-)\ (-,+)\ (-,-), \tag{5.60}$$

where the first entry indicates the boundary condition at $y = 0$ and the second one the boundary condition at $y = \pi R$.

5.5 5D SUSY

In this part we will explore how supersymmetry manifests in a 5-dimensional theory. This will be useful in the next chapter when we construct supersymmetric UV completions in 5 dimensions for theories with folded supersymetry.

In $4D$ supersymmetry the smallest spinor is a Weyl spinor with 4 real components. Then, the minimum number of anticommuting generators of $4D$ supersymmetry amounts to four. On the other hand, in $5D$, the smallest spinor is the Dirac spinor with 8 real degrees of freedom meaning that there are, at least, 8 anticommuting generators, that is, $\mathcal{N} = 2$ supersymmetry from the $4D$ viewpoint. Therefore, $\mathcal{N} = 1$ supersymmetric theories in $5D$ have the same field content as a $\mathcal{N} = 2$ supersymmetric theory in $4D$. We will treat here two types of $5D$ supermultiplets: the hypermultiplet and the vector multiplet.

5.5.1 Hypermultiplet

Off-shell $5D$, $\mathcal{N} = 1$ hypermultiplets are formed by a Dirac spinor Ψ, two complex scalars ϕ and ϕ^c, and two complex auxiliary fields F and F^c. We can construct a 5-dimensional Lagrangian containing the components of the hypermultiplet but in order for us to be easier to work with, we will express the $5D$ supersymmetric Lagrangians in the language of the already studied 4-dimensional $\mathcal{N}=1$ supermultiplets. This was done in the articles of Ref. [63] and [64].

In $4D$ language, hypermultiplets can be described as containing two $\mathcal{N}=1$ chiral superfields $\Phi(y)$ and $\Phi^c(y)$, where y is the fifth coordinate which is acting as a label. The $4D$ chiral supermultiplets are decomposed as

$$\Phi(y) = \phi(y) + \sqrt{2}\theta\chi(y) + \theta^2 F(y), \tag{5.61}$$

$$\Phi^c(y) = \phi^c(y) + \sqrt{2}\theta\chi^c(y) + \theta^2 F^c(y). \tag{5.62}$$

The $5D$ action for a free hypermultiplet can be written as

$$S_5^{\text{Hyp.}} = \int d^4x dy \left\{ \int d^4\theta \left(\Phi^\dagger\Phi + \Phi^{c\dagger}\Phi^c\right) + \left(\int d^2\theta \Phi^c(m+\partial_y)\Phi + \text{h.c.} \right) \right\}. \tag{5.63}$$

We see that this form of the Lagrangian is manifestly supersymmetric from the $4D$ viewpoint. Let us show that it is equivalent to the $5D$ Lagrangian that describes the components of the hypermultiplet. Expanding 5.63 in component fields, we have

$$\begin{aligned} S_5^{\text{Hyp.}} = \int d^5x \, \big[& (\partial^\mu\phi)^\dagger(\partial_\mu\phi) + i\bar{\chi}\bar{\sigma}^\mu\partial_\mu\chi + F^\dagger F + (\partial^\mu\phi^c)^\dagger(\partial_\mu\phi^c) + i\bar{\chi^c}\bar{\sigma}^\mu\partial_\mu\chi^c + F^{c\dagger}F^c \\ & + \phi^c\partial_5 F + \phi^{c\dagger}\partial_5 F^\dagger - \chi^c\partial_5\chi - \bar{\chi^c}\partial_5\bar{\chi} + F^c\partial_5\phi + F^{c\dagger}\partial_5\phi^\dagger \\ & + m(\phi^c F - \chi^c\chi + F^c\phi + \phi^{c\dagger}F^\dagger - \bar{\chi^c}\bar{\chi} + F^{c\dagger}\phi^\dagger) \big] . \end{aligned} \tag{5.64}$$

Now, the EOM for the auxiliary fields F and F^c are:

$$F = \partial_5\phi^{c\dagger} - m\phi^{c\dagger}, \tag{5.65}$$

$$F^c = -\partial_5\phi^\dagger - m\phi^\dagger. \tag{5.66}$$

Replacing in eq.(5.64), in order to eliminate F and F^c from the Lagrangian, we obtain

$$\begin{aligned} S_5^{\text{Hyp}} &= \int d^5x \, \big[(\partial^M\phi)^\dagger(\partial_M\phi) + (\partial^M\phi^c)^\dagger(\partial_M\phi^c) + i\bar{\chi}\bar{\sigma}^\mu\partial_\mu\chi + i\bar{\chi^c}\bar{\sigma}^\mu\partial_\mu\chi^c \\ & \qquad -\chi^c\partial_5\chi - \bar{\chi}^c\partial_5\bar{\chi} - m^2(|\phi^c|^2 + |\phi|^2) - m(\chi^c\chi + \bar{\chi}^c\bar{\chi}) \big] \\ S_5^{\text{Hyp}} &= \int d^5x \, \big[(\partial^M\phi)^\dagger(\partial_M\phi) + (\partial^M\phi^c)^\dagger(\partial_M\phi^c) - m^2(|\phi^c|^2 + |\phi|^2) + \bar{\Psi}(i\Gamma^M\partial_M - m)\Psi \big] . \end{aligned} \tag{5.67}$$

where $\Psi = (\chi, \bar{\chi}^c)$. The last action clearly describes a $5D$ theory with a Dirac spinor Ψ and the two complex scalars ϕ and ϕ^c.

5.5.2 Vector multiplet

A $D = 5$ vector multiplet is composed by a 5-vector gauge field A_M, a four component Dirac gaugino field Ξ and a scalar σ. In terms of $D = 4$, $\mathcal{N} = 1$ supersymmetry it can be described as being composed by a gauge supermultiplet V and a chiral supermultiplet Σ in such a way that their component decompositions are

$$V = \theta\sigma^\mu\bar{\theta}A_\mu + i\bar{\theta}^2\theta\lambda - i\theta^2\bar{\theta}\bar{\lambda} + \frac{1}{2}\bar{\theta}^2\theta^2 D, \tag{5.68}$$

$$\Sigma = \frac{1}{\sqrt{2}}(\sigma + iA_5) + \sqrt{2}\theta\lambda' + \theta^2 F, \tag{5.69}$$

where V is in the Wess-Zumino gauge.

The transformation of fields under a gauge transformation is given by

$$V \longrightarrow V + \Lambda + \bar{\Lambda}, \tag{5.70}$$

$$\Sigma \longrightarrow \Sigma + \sqrt{2}\partial_5\Lambda \tag{5.71}$$

and the gauge invariant action is given by

$$S_5^A = \int d^5x \left[\frac{1}{4g^2}\int d^2\theta W^a W_a + \text{h.c.} + \int d^4\theta \frac{1}{g^2}(\partial_5 V - \frac{1}{\sqrt{2}}(\Sigma + \Sigma^\dagger))^2\right]. \tag{5.72}$$

Chapter 6

Folded Supersymmetry

This chapter is devoted to the introduction of a new way to stabilize the weak scale against radiative corrections, namely, *Folded Supersymmetry*. It was introduced by Burdman, Chacko, Goh and Harnik in 2006, and the content of this part is completely based on their paper [1]. I just expanded some calculations and tried to give the reader a more detailed explanation.

As we said in section 3.4, the hierarchy problem arises due to quadratically divergent one loop corrections to the mass parameter of the Higgs field. The largest contribution is given by the diagram that involves the top quark running in the loop. One way to address the problem is to add new degrees of freedom and interactions that generate new one loop diagrams that make possible the cancellation of divergences. Examples of models that follow this path are: supersymmetry, little Higgs theories and twin Higgs theories. How they managed to remove divergences are roughly described in the following.

Supersymmetry accomplishes the cancellation of the top loop divergence by the introduction of new diagrams that involve stop fields running in the loop. These stops are scalar fields that have the same quantum numbers as the top quark apart from spin. In particular, they are charged under the $SU(3)_C$ color group of the SM. Little Higgs theories [44, 45, 46, 47, 48, 49] realize the Higgs as a pseudo Nambu Goldstone boson making necessary the introduction of 'top partners', which are fermions carrying SM color, that run in the loop and cancel out the divergence. The coupling parameters related to the top partners−Higgs interaction are related to the Yukawa couplings by a global symmetry. Twin Higgs theories also realize the Higgs as a pseudo Nambu Goldstone boson but the theory also respects a Z_2 discrete symmetry. The diagram that cancels out the top quark one loop divergence has the same form as in little Higgs theories but the fermionic field that runs in the loop is not necessarily charged under $SU(3)_C$.

Folded Supersymmetry is a tool to construct theories beyond the Standard Model that solve the hierarchy problem at least up to energies parametrically higher than the weak scale. The new one loop diagrams that cancel divergences in these models are the same as in supersymmetric theories but, as in twin Higgs models, the fields running in the loop are not charged under $SU(3)_C$. In this chapter we describe how these kind of models are constructed and how the can cancellations occur.

The main idea of Folded supersymmetry arises from the fact that, in the limit of large N, and for a certain class of theories, the correlation functions of orbifold non-supersymmetric *daughter theories* are identical to the corresponding correlation functions of the supersymmetric *parent theory*, modulo the rescaling of the gauge coupling constant. This statement was proved by using string theory perturbative techniques in [59, 60] and in the context of field theory in [61]. These results, in turn, follow from those in [56, 57, 58]. The concepts of parent and daughter theory as well as the orbifolding procedure will be explained in section 6.1.

As we already know, supersymmetry ensures that the masses of scalars are protected against quadratic divergences. So, the previous statement implies that in the orbifold daughter theory (that follows from a supersymmetric parent Lagrangian), the mass of the scalar fields must also be protected from radiative corrections, despite not being supersymmetric. The understanding of the mechanism that allows the cancellation of divergences can be used to construct extensions of the SM where the mass of the Higgs field is protected against radiative corrections.

6.1 Orbifolding a theory: Examples

In this part we will explain the procedure to *orbifold* a parent theory. First, we have to identify a discrete symmetry of the theory. "Orbifolding" the theory consists on eliminating all the fields that are charged (not invariant) under that symmetry. The remaining fields will be called *daughter fields*. The final daughter theory will be described by a Lagrangian that contains the terms of the parent theory Lagrangian that involves only daughter fields.

Following Ref. [1], we will give two examples of this procedure in order to clarify it, one for a supersymmetric gauge theory and one for a supersymmetric theory with Yukawa interactions. Then, we will identify the mechanism that guarantees the cancellation of divergences at one loop and give a prescription to construct models where such a cancellation is realized.

6.1.1 Example 1: Supersymmetric gauge theory

Let us start by orbifolding a supersymmetric gauge theory with a $U_c(2N)$ gauge symmetry and a global $U_f(2N)$ flavour symmetry down to a non-supersymmetric daughter theory with a $U(N) \times U(N)$ gauge symmetry.

The field content of the parent theory is specified by:

(i) A chiral supermultiplet, Q that transforms in the fundamental representation of $U_c(2N)$ and in the antifundamental representation of $U_f(2N)$:

$$Q \to e^{-2ig\Lambda_c} Q e^{i\alpha^G T_f^G}, \tag{6.1}$$

where $\Lambda_c \equiv T_c^G \Lambda_c^G$, and T_c^G and T_f^G are the generators of the $U_c(2N)$ and $U_f(2N)$ groups respectively (G runs from 1 to $(2N)^2$). The component fields of Q will be represented by $2N \times 2N$ matrices in the form:

$$Q = \left\{ \tilde{q} = \begin{pmatrix} \tilde{q}_{Aa} & \tilde{q}_{Ab} \\ \tilde{q}_{Ba} & \tilde{q}_{Bb} \end{pmatrix}, \qquad q = \begin{pmatrix} q_{Aa} & q_{Ab} \\ q_{Ba} & q_{Bb} \end{pmatrix} \right\}, \tag{6.2}$$

where A and B indices distinguish between the two $U(N)$ gauge groups inside $U_c(2N)$ and a and b indices distinguish between the two $U(N)$ global groups inside $U_f(2N)$. $\tilde{q}$ represents the scalar component field and q, the fermionic one.

(ii) A chiral supermultiplet, $\bar{Q}$ transforming in the antifundamental representation of $U_c(2N)$ and in the fundamental of $U_f(2N)$:

$$\bar{Q} \to e^{2ig\Lambda_c^*} \bar{Q} e^{-i\alpha^a T_f^{aT}}. \tag{6.3}$$

The component fields of $\bar{Q}$ will be represented as matrices in the form:

$$\bar{Q} = \left\{ \tilde{r} = \begin{pmatrix} \tilde{r}_{Aa} & \tilde{r}_{Ab} \\ \tilde{r}_{Ba} & \tilde{r}_{Bb} \end{pmatrix}, \qquad r = \begin{pmatrix} r_{Aa} & r_{Ab} \\ r_{Ba} & r_{Bb} \end{pmatrix} \right\}. \tag{6.4}$$

Indices A, B, a and b represent the same distinction as for the components of the superfield Q. $\tilde{r}$ is the scalar component field and r is the spin$-1/2$ spinor field.

(iii) A vector superfield $V \equiv 2gV^G T_c^G$ transforming as

$$e^V \to e^{-2ig\Lambda_c^\dagger} e^V e^{2ig\Lambda_c}, \tag{6.5}$$

whose component fields will be represented as:

$$V = \left\{ A_\mu = \begin{pmatrix} A_{\mu,AA} & A_{\mu,AB} \\ A_{\mu,BA} & A_{\mu,BB} \end{pmatrix}, \qquad \lambda = \begin{pmatrix} \lambda_{AA} & \lambda_{AB} \\ \lambda_{BA} & \lambda_{BB} \end{pmatrix}. \right\} \tag{6.6}$$

Here A and B, as before, distinguish between the two $U(N)$ subgroups inside the $U_c(2N)$ gauge group. A_μ is the spin-1 gauge field component and λ is the gaugino field component.

With these, we construct the supersymmetric Lagrangian:

$$\mathcal{L} = \int d^2\theta \frac{1}{2} \mathrm{Tr}(W^a W_a + \bar{W}^a \bar{W}_a) + \int d^4\theta \mathrm{Tr}(\Phi^\dagger e^V \Phi) + \int d^4\theta \mathrm{Tr}(\bar{\Phi}^\dagger e^{V^*} \bar{\Phi}), \tag{6.7}$$

which, expanded in component fields, takes the form:

$$\begin{aligned} \mathcal{L} =& \mathrm{Tr}\left\{ (D_\mu \tilde{q})^\dagger (D^\mu \tilde{q}) + i\bar{q}\bar{\sigma}^\mu D_\mu q + (D_\mu \tilde{r})^\dagger (D^\mu \tilde{r}) + i\bar{r}\bar{\sigma}^\mu D_\mu r \right\} \\ &+ \mathrm{Tr}\left\{ -\frac{1}{2}[W_{\mu\nu} W^{\mu\nu}] + 2i\bar{\lambda}\bar{\sigma}^\mu D_\mu \lambda \right\} \\ &+ ig\sqrt{2}\mathrm{Tr}\left\{ \tilde{q}^\dagger \lambda q - \bar{q}\bar{\lambda}\tilde{q} - \tilde{r}^\dagger \lambda^T r + \bar{r}\bar{\lambda}^* \tilde{r} \right\} \\ &- \frac{g^2}{2}\mathrm{Tr}(\tilde{q}^\dagger T_c^a \tilde{q})\mathrm{Tr}(\tilde{q}^\dagger T_c^a \tilde{q}) - \frac{g^2}{2}\mathrm{Tr}(\tilde{r}^\dagger T_c^{a*} \tilde{r})\mathrm{Tr}(\tilde{r}^\dagger T_c^{a*} \tilde{r}), \end{aligned} \tag{6.8}$$

where

$$W_{\mu\nu} = W_{\mu\nu}^G T^G; \qquad \text{for } G = 1, 2, ..., (2N)^2, \tag{6.9}$$

and

$$W_{\mu\nu}^G = \partial_\mu A_\nu^G - \partial_\nu A_\mu^G - g f^{GHI} A_\mu^H A_\nu^I. \tag{6.10}$$

Covariant derivatives are defined by

$$D^\mu \tilde{q} = \partial^\mu \tilde{q} + igT_c^G A^{G\mu} \tilde{q}, \tag{6.11}$$

$$D^\mu q = \partial^\mu q + igT_c^G A^{G\mu} q, \tag{6.12}$$

$$D^\mu \tilde{r} = \partial^\mu \tilde{r} - igT_c^{G*} A^{G\mu} \tilde{r}, \tag{6.13}$$

$$D^\mu r = \partial^\mu r - igT_c^{G*} A^{G\mu} r, \tag{6.14}$$

$$D^\mu \lambda = \partial^\mu \lambda + ig[A^\mu, \lambda]. \tag{6.15}$$

Now, in order to obtain the orbifold daughter theory we identify the discrete symmetries of the theory. These are:

1. A Z_2 symmetry labeled by $Z_{2\Gamma}$ which is an element of the $U_c(2N)$ gauge group generated by the $2N \times 2N$ diagonal matrix with N '+1' entries and N '−1' entries:

$$\Gamma = \begin{pmatrix} +1 & & & & & \\ & \ddots & & & & \\ & & +1 & & & \\ & & & -1 & & \\ & & & & \ddots & \\ & & & & & -1 \end{pmatrix} \tag{6.16}$$

under which the superfields transform as:

$$Q \to \Gamma Q, \quad \bar{Q} \to \Gamma^* \bar{Q}, \quad V \to \Gamma V \Gamma^\dagger. \tag{6.17}$$

2. A Z_2 symmetry labeled by Z_{2F} which is an element of the $U_f(2N)$ flavour group generated by the $2N \times 2N$ diagonal matrix with N '+1' entries and N '−1' entries:

$$F = \begin{pmatrix} +1 & & & & & \\ & \ddots & & & & \\ & & +1 & & & \\ & & & -1 & & \\ & & & & \ddots & \\ & & & & & -1 \end{pmatrix} \tag{6.18}$$

under which the superfields transform as:

$$Q \to Q F^\dagger, \quad \bar{Q} \to \bar{Q} F^T, \quad V \to V. \tag{6.19}$$

3. A Z_{2R} symmetry under which all the bosonic fields are even and all the fermionic fields are odd, that is they transform as:

$$\tilde{q} \to \tilde{q}, \quad q \to -q, \quad \tilde{\bar{q}} \to \tilde{\bar{q}}, \quad \bar{q} \to -\bar{q}, \quad A_\mu \to A_\mu, \quad \lambda \to -\lambda. \tag{6.20}$$

So, the combined discrete symmetry is $Z_{2\Gamma} \times Z_{2R} \times Z_{2R}$. We show now how each field behave under this symmetry. They will be either even (+) or odd (−) under it:

$$\tilde{q} = \begin{pmatrix} \tilde{q}_{Aa}(+) & \tilde{q}_{Ab}(-) \\ \tilde{q}_{Ba}(-) & \tilde{q}_{Bb}(+) \end{pmatrix}, \qquad q = \begin{pmatrix} q_{Aa}(-) & q_{Ab}(+) \\ q_{Ba}(+) & q_{Bb}(-) \end{pmatrix}, \tag{6.21}$$

$$\tilde{r} = \begin{pmatrix} \tilde{r}_{Aa}(+) & \tilde{r}_{Ab}(-) \\ \tilde{r}_{Ba}(-) & \tilde{r}_{Bb}(+) \end{pmatrix}, \qquad r = \begin{pmatrix} r_{Aa}(-) & r_{Ab}(+) \\ r_{Ba}(+) & r_{Bb}(-) \end{pmatrix}, \tag{6.22}$$

$$A_\mu = \begin{pmatrix} A_{\mu,AA}(+) & A_{\mu,AB}(-) \\ A_{\mu,BA}(-) & A_{\mu,BB}(+) \end{pmatrix}, \qquad \lambda = \begin{pmatrix} \lambda_{AA}(-) & \lambda_{AB}(+) \\ \lambda_{BA}(+) & \lambda_{BB}(-) \end{pmatrix}. \tag{6.23}$$

The next step to obtain the orbifolded daughter theory is to eliminate all the fields that are odd under $Z_{2\Gamma} \times Z_{2R} \times Z_{2R}$ and keep the terms in the Lagrangian that involve only daughter fields. The orbifolded Lagrangian (eq.A.17) and how it was obtained is shown in Appendix A. The remaining daughter theory is a non supersymmetric $U(N) \times U(N)$ gauge theory.

It was proved in [62] that this orbifold daughter Lagrangian gives us the same correlation functions as the parent one, up to the rescaling of the gauge coupling constant $g \to \sqrt{2}g$. The correspondence implies that the mass of $\tilde{q}_{Aa}$ is protected against quadratic divergences at any loop order due to the supersymmetry of the parent theory. The one loop quadratically divergent diagrams that contribute to the mass of $\tilde{q}_{Aa}$ are shown in Figure 6.1 and their contributions are, respectively:

$$6.1a: \ \frac{3g^2\Lambda^2 N}{32\pi^2}, \qquad 6.1b: \ -\frac{g^2\Lambda^2 N}{8\pi^2}, \qquad 6.1c: \ \frac{g^2\Lambda^2 N}{32\pi^2}. \tag{6.24}$$

The calculations are performed in detail in Appendix A. From 6.24 we check that, at one loop, divergences cancel out as was expected from the correspondence.

Let us note that if we look at the terms of the orbifolded Lagrangian that generate the one loop diagrams, we note that they have related coupling parameters. That is, the gauge coupling constant in A.18, the scalar-gaugino-fermion coupling constant in A.19 and the quartic scalar self-coupling in A.20 must be related in order for the cancellation to take place. However, there is not any symmetry of the theory that guarantees these connections. Then, it is important to build a theory that is valid at higher energies and explains the values of these coupling parameters. It is said that such a theory is an ultraviolet completion of the low energy theory. Such a theory will be postponed until section 6.3 where we set up a UV completion in 5 dimensions for the Yukawa interaction in the SM.

6.1.2 Example 2: Yukawa coupling

In this example (also discussed in Ref. [1]) we will explore how the procedure of orbifolding can be applied to theories with Yukawa couplings where matter fields are in bifundamental representations.

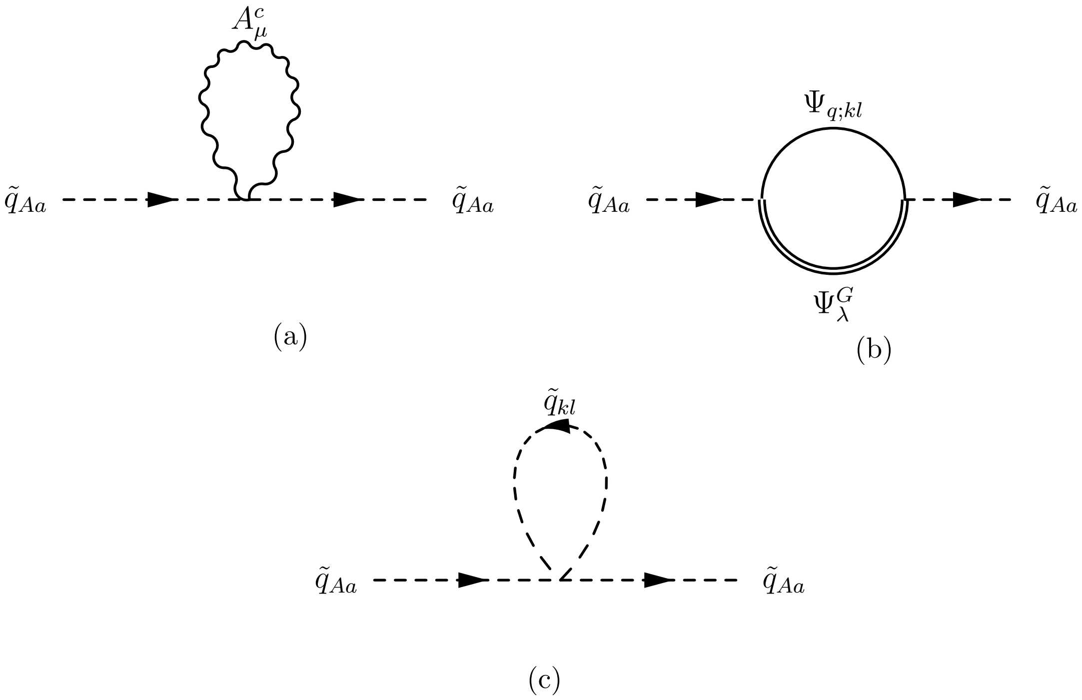


Figure 6.1: One loop contributions to the mass of $\tilde{q}_{Aa}$ due to (a) gauge interaction, (b) gaugino interaction and (c) autointeractions.

Let us consider a supersymmetric theory with a $SU(2N)_1 \times SU(2N)_2 \times SU(2N)_3$ global symmetry and the superfields $Q_{12}(2N, \overline{2N}, 1)$, $Q_{23}(1, 2N, \overline{2N})$, $Q_{31}(\overline{2N}, 1, 2N)$ as its matter content. That is, superfields transform as:

$$Q_{12} \to U_1 Q_{12} U_2^\dagger, \tag{6.25}$$

$$Q_{23} \to U_2 Q_{23} U_3^\dagger, \tag{6.26}$$

$$Q_{31} \to U_3 Q_{31} U_1^\dagger. \tag{6.27}$$

The bosonic and fermionic field components of Q_{12} will be denoted as matrices of the form

$$Q_{12} = \left\{ \tilde{q}_{12} = \begin{pmatrix} \tilde{q}_{1A,2A} & \tilde{q}_{1A,2B} \\ \tilde{q}_{1B,2A} & \tilde{q}_{1B,2B} \end{pmatrix}, \qquad q_{12} = \begin{pmatrix} q_{1A,2A} & q_{1A,2B} \\ q_{1B,2A} & q_{1B,2B} \end{pmatrix} \right\} \tag{6.28}$$

and those components of Q_{23} and Q_{31} will have the same form. In this notation A and B distinguish between the two $SU(N)$ groups inside each $SU(2N)$ group. The symmetry of the theory allows the Yukawa term

$$\lambda \, \mathrm{Tr}(Q_{12} Q_{23} Q_{31}) \tag{6.29}$$

in the superpotential. Then, the Lagrangian for this theory is:

$$\mathcal{L} = \int d^4\theta \left[\mathrm{Tr}(Q_{12}^\dagger Q_{12}) + \mathrm{Tr}(Q_{23}^\dagger Q_{23}) + \mathrm{Tr}(Q_{31}^\dagger Q_{31})\right] + \int d^2\theta \left[\lambda \mathrm{Tr}\left(Q_{12}Q_{23}Q_{31}\right) + \text{h.c.}\right] \tag{6.30}$$

or, in field components,

$$\begin{aligned}\mathcal{L} =& \mathrm{Tr}\left(\partial_\mu \tilde{q}_{12}^\dagger \partial^\mu \tilde{q}_{12} + i\bar{q}_{12}\bar{\sigma}^\mu \partial_\mu q_{12}\right) + \mathrm{Tr}\left(\partial_\mu \tilde{q}_{23}^\dagger \partial^\mu \tilde{q}_{23} + i\bar{q}_{23}\bar{\sigma}^\mu \partial_\mu q_{23}\right) \\ &+ \mathrm{Tr}\left(\partial_\mu \tilde{q}_{31}^\dagger \partial^\mu \tilde{q}_{31} + i\bar{q}_{31}\bar{\sigma}^\mu \partial_\mu q_{31}\right) \\ &- \lambda \mathrm{Tr}\left(\tilde{q}_{31}q_{12}q_{23} + \tilde{q}_{12}q_{23}q_{31} + \tilde{q}_{23}q_{12}q_{31}\right) + \text{h.c.} \\ &- \lambda^2\left[\mathrm{Tr}\left(\tilde{q}_{23}^\dagger \tilde{q}_{12}^\dagger \tilde{q}_{12}\tilde{q}_{23}\right) + \mathrm{Tr}\left(\tilde{q}_{31}^\dagger \tilde{q}_{23}^\dagger \tilde{q}_{23}\tilde{q}_{31}\right) + \mathrm{Tr}\left(\tilde{q}_{12}^\dagger \tilde{q}_{31}^\dagger \tilde{q}_{31}\tilde{q}_{12}\right)\right].\end{aligned} \tag{6.31}$$

Now, the theory is invariant under the discrete symmetry $Z_{2\Gamma} \times Z_{2R}$. Here $Z_{2\Gamma}$ denote the discrete symmetry under which the superfields transform as:

$$Q_{12} \to \Gamma Q_{12}\Gamma^\dagger, \quad Q_{23} \to \Gamma Q_{23}\Gamma^\dagger, \quad Q_{31} \to \Gamma Q_{31}\Gamma^\dagger \tag{6.32}$$

with Γ defined in eq.(6.16) and Z_{2R} refers to the discrete symmetry under which the bosonic fields are even and the fermionic ones are odd. The field components are either even (+) or odd (−) under the action of $Z_{2\Gamma} \times Z_{2R}$. Explicitly, we have for Q_{12}:

$$\tilde{q}_{12} = \begin{pmatrix} \tilde{q}_{1A,2A}(+) & \tilde{q}_{1A,2B}(-) \\ \tilde{q}_{1B,2A}(-) & \tilde{q}_{1B,2B}(+) \end{pmatrix}, \qquad q_{12} = \begin{pmatrix} q_{1A,2A}(-) & q_{1A,2B}(+) \\ q_{1B,2A}(+) & q_{1B,2B}(-) \end{pmatrix} \tag{6.33}$$

and the transformation of the component fields of Q_{23} and Q_{31} are similar.

The orbifolded daughter theory is obtained by projecting out the fields that are odd under $Z_{2\Gamma} \times Z_{2R}$ and keeping the terms in the Lagrangian that contain only daughter fields. The daughter Lagrangian is showed in eq. (A.36) of Appendix A. This daughter theory is non-supersymmetric and invariant under an $SU(N)_{1A} \times SU(N)_{1B} \times SU(N)_{2A} \times SU(N)_{2B} \times SU(N)_{3A} \times SU(N)_{3B}$ global symmetry.

The one loop diagrams that contribute to the mass of $\tilde{q}_{1A,2A}$ with quadratic divergences are given by the diagrams in Figure 6.2. Their contributions are:

$$6.2a: \ -\frac{\lambda^2 N}{8\pi^2}, \qquad 6.2b: \ \frac{\lambda^2 N}{16\pi^2} \qquad 6.2c: \ \frac{\lambda^2 N}{16\pi^2} \tag{6.34}$$

They are worked out in section A.2 and, as we see, they cancel out exactly as expected by the correspondence with the parent theory. Since the other scalar fields have similar interaction as those of $\tilde{q}_{1A,2A}$, their masses will also be protected from quadratic divergences. As in the previous subsection, a UV completion is needed in order to understand why the coupling parameters take the particular values at low energies which allow the exact cancellation.

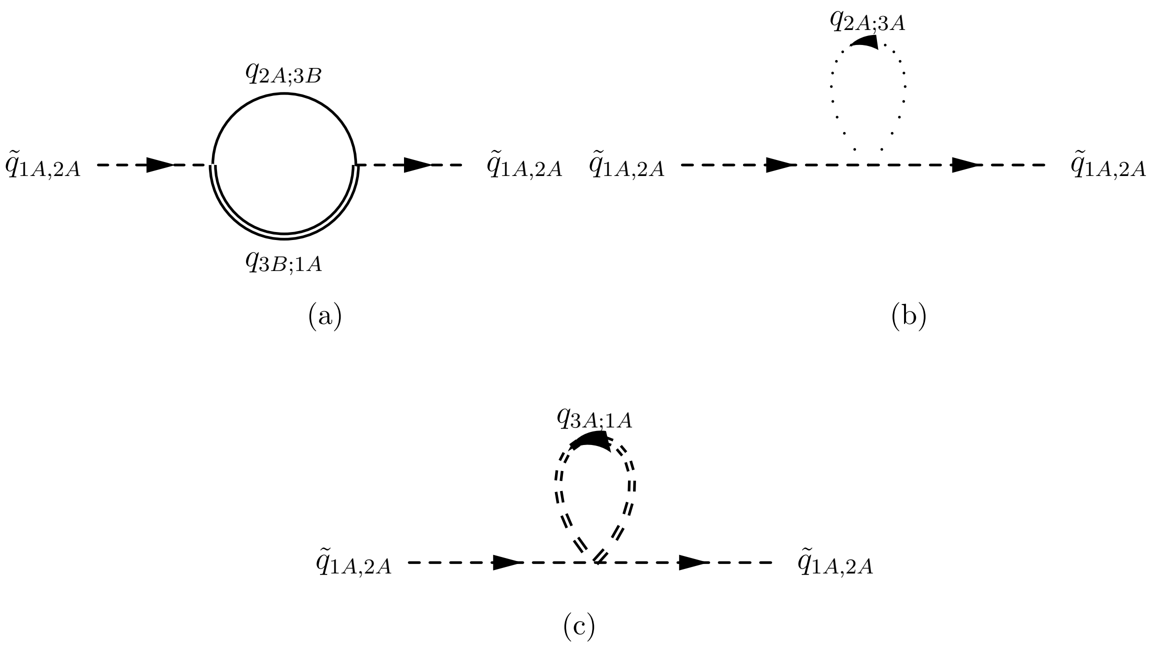


Figure 6.2: One loop contributions to the mass of $\tilde{q}_{1A,2A}$ due to (a) Yukawa interaction, and (b), (c) quartic scalar interactions.

6.2 Bifold protection and a prescription to construct an orbifolded theory

So far, we treated two examples in which the one loop divergences cancel out in a non-supersymmetric daughter theory. It would be useful to understand how this cancellation is happening in order to be able to construct much larger theories and eventually address the hierarchy problem in the SM.

Let us analyze how the cancellations operate. For that we will consider the theory with Yukawa interaction studied in subsection 6.1.2. We saw that the diagrams in Figure 6.2 generated from the orbifolded theory posses quadratic divergences that cancel out. Let us analyze how cancellations occur in the supersymmetric parent theory. In the parent theory there are three more diagrams that contribute with the mass of $\tilde{q}_{1A,2A}$. They are shown in Figure 6.3. We note that the only difference with the diagrams in Figure 6.2 are the different fields running in the loops. Actually, they have also the same value, that is, their contributions are:

$$6.3a: \ -\frac{\lambda^2 N}{8\pi^2}, \qquad 6.3b: \ \frac{\lambda^2 N}{16\pi^2} \qquad 6.3c: \ \frac{\lambda^2 N}{16\pi^2} \tag{6.35}$$

So, as it is expected for a supersymmetric theory, all quadratic divergences cancel out. However, we must notice that the fermionic loop diagram 6.2a could be canceled either by the bosonic loop diagrams 6.2b and 6.2c or by the bosonic loop diagrams 6.3b and 6.3c, and the same for the fermionic loop diagram 6.3a. Then, we say that the mass of $\tilde{q}_{1A,2A}$ enjoys *bifold protection* in the parent theory.

Let explain this in other way. The difference between the fields running in the loops of diagrams in Figure 6.2 and 6.3 relies on the $SU(2N)_3$ group index, that we will denote by 'i'. We assigned $3A$ for fields with index i running from 1 to N and $3B$ for fields with i running from $N+1$ to $2N$. In that way, we see that in the supersymmetric case, both fermionic loops combine to form one fermionic loop diagram with i running from 1 to $2N$ and the same for the bosonic loop diagrams. Then, the fermionic loop diagram with $i = 1, ...2N$ cancel out with bosonic loop diagrams with $i = 1, ...2N$ as expected in supersymmetry. However, we note that it is possible to cancel the fermionic loop diagram with $i = N+1, ..., 2N$ either with the bosonic loop diagrams with $i = 1, ..., N$ or the bosonic loop diagrams with $i = N+1, ...N$, and the same occurs for the fermionic loop diagrams with $i = 1, ..., N$.

The bifold protection of the mass of a scalar in the parent theory allows us to project out bosonic fields with index i running from $N+1$ to $2N$ and fermionic field with index i running from 1 to N and still have the cancellation.

With this information in mind, we are able to write down a prescription in order to construct a one loop radiatively stable theory starting from a theory with quadratically divergent contributions to the mass of a scalar arising from a specific interaction. The steps are the following:

1) Supersymmetrize the theory by adding the correspondent degrees of freedom (fermions or bosons).

2) Identify the index i that is summing from 1 to N in the relevant one loop diagrams and extend the theory by adding new fields and enlarging the gauge, global or discrete symmetries to have the index i now summed from 1 to $2N$ but with the same vertices in each graph. In the cases of $SU(N)$ gauge or global theories and Yukawa interactions, this can always be done in such a way that the mass of the scalar enjoys bifold protection. Also, the parent theory is invariant under the $Z_{2\Gamma}$ and Z_{2R} discrete symmetries.

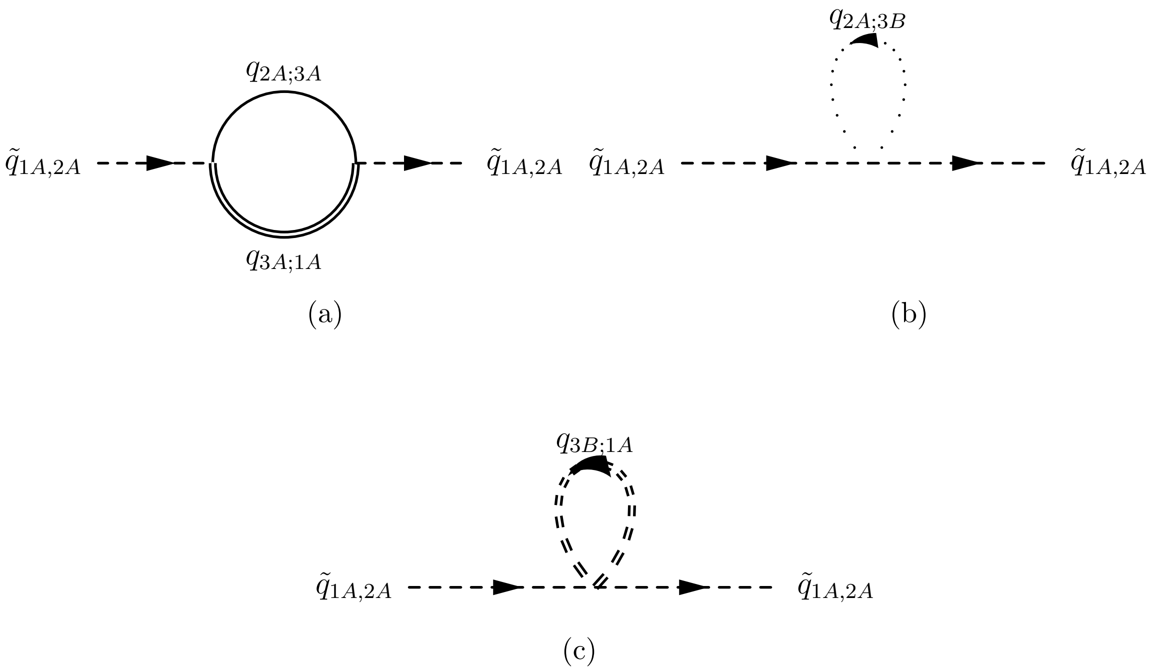


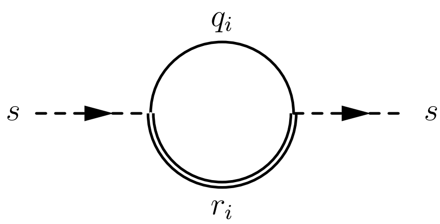
Figure 6.3: One loop diagrams that, together with those in Figure 6.2, contribute to the mass of $\tilde{q}_{1A,2A}$ in the parent supersymmetric theory described by the Lagrangian in A.32.

3) Project out the states odd under the combined $Z_{2\Gamma} \times Z_{2R}$ discrete symmetry.

Let us apply this prescription to a theory with a Yukawa coupling between a massless scalar singlet s and massless chiral Weyl-spinor fields q_i and r_i transforming in the fundamental and antifundamental representation of $U(N)$ respectively. Index i runs from 1 to N. The Lagrangian of this theory is written as:

$$\mathcal{L} = (\partial^\mu s)^\dagger (\partial_\mu s) + i\bar{q}_i \bar{\sigma}^\mu \partial_\mu q_i + i\bar{r}_i \bar{\sigma}^\mu \partial_\mu r_i - \lambda s q_i r_i - \lambda s^\dagger \bar{q}_i \bar{r}_i. \tag{6.36}$$

The only one loop contribution to the mass of s is given by the following diagram



which corresponds to a quadratic divergence:

$$-2\lambda^2 N \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2}. \tag{6.37}$$

We will extend this theory in order to have a cancellation of this quadratic divergence.

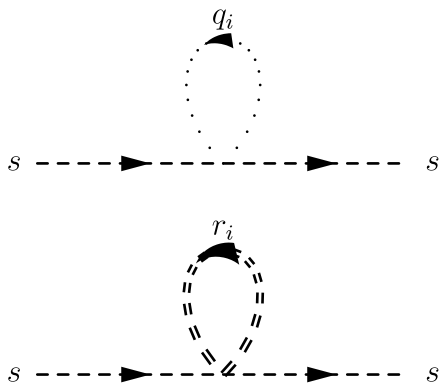
Following the rules established above, we first supersymmetrize the theory. We make this by adding fields with opposite spin than s, q_i and r_i, that is, we add the fields $\tilde{s}$ (spin 1/2), $\tilde{q}_i$ (spin 0) and $\tilde{r}_i$ (spin 0). Then we treat each couple of fields as components of a respective chiral superfield. We will denote these superfields as S (with component fields s and $\tilde{s}$), Q_i (with component fields q_i and $\tilde{q}_i$) and $\bar{Q}_i$ (with component fields r_i and $\tilde{r}_i$). The supersymmetric generalization of the Lagrangian in eq. (6.36) is

$$\mathcal{L} = \int d^4\theta \left(S^\dagger S + Q_i^\dagger Q_i + \bar{Q}_i^\dagger \bar{Q}_i \right) + \int d^2\theta \left(\lambda S Q_i \bar{Q}_i + \text{h.c.} \right), \tag{6.38}$$

which, written in terms of field components and integrating out the auxiliary fields, takes the form

$$\begin{aligned} \mathcal{L} =& (\partial_\mu s)^\dagger (\partial^\mu s) + i\bar{\tilde{s}}\bar{\sigma}^\mu \partial_\mu \tilde{s} + (\partial_\mu \tilde{q}_i)^\dagger (\partial^\mu \tilde{q}_i) + i\bar{q}_i \bar{\sigma}^\mu \partial_\mu q_i + (\partial_\mu \tilde{r}_i)^\dagger (\partial^\mu \tilde{r}_i) + i\bar{r}_i \bar{\sigma}^\mu \partial_\mu r_i \\ & - \lambda \left(s q_i r_i + \tilde{q}_i \tilde{s} r_i + \tilde{r}_i \tilde{s} q_i + \text{h.c.} \right) - \lambda^2 \left(\tilde{q}_i^\dagger \tilde{r}_i^\dagger \tilde{q}_j \tilde{r}_j + s^\dagger s \tilde{q}_i^\dagger \tilde{q}_i + s^\dagger s \tilde{r}_i^\dagger \tilde{r}_i \right). \end{aligned} \tag{6.39}$$

Now, as expected from supersymmetry, the previous fermion loop is canceled by the boson loops



with quadratically divergent contributions

$$\lambda^2 N \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2}, \tag{6.40}$$

each one. Indeed, if we sum the contributions in eqns.(6.40) and (6.37), the total quadratic divergence vanish out. We realise that the cancellation of the one loop divergent contributions to the mass of s is possible due to the presence of the following terms in the Lagrangian:

$$\mathcal{L} \supset (-\lambda s q_i r_i + \text{h.c}) - \lambda^2 (s^\dagger s \tilde{q}_i^\dagger \tilde{q}_i + s^\dagger s \tilde{r}_i^\dagger \tilde{r}_i). \tag{6.41}$$

The same pattern is present for the other scalars $\tilde{q}_i$ and $\tilde{r}_i$. Then, we can infer that the quadratic divergent contributions to mass of $\tilde{q}_i$ and $\tilde{r}_i$ also vanish out. Again, this is a consequence of supersymmetry.

Following step 2) of our prescription, we recognize the summed index i in the one loop corrections to the mass of s as the relevant one. Then, we promote the global $U(N)$ symmetry to a global $U(2N)$ symmetry in order to now have the index i running from 1 to $2N$. For this to happen, we have to add N Q's and N $\bar{Q}$'s. The new Lagrangian will have the same form as in eq.(6.39) just that now index i runs from 1 to $2N$. As before, quadratically divergent contributions to the mass of the scalars s, $\tilde{q}_i$ and $\tilde{r}_i$ vanish out. Also, now, the mass of s enjoys bifold protection.

This new enlarged theory is invariant under the discrete symmetry $Z_{2\Gamma} \times Z_{2R}$, where $Z_{2\Gamma}$ is the symmetry under which superfields transform as:

$$S \to S, \qquad Q \to \Gamma Q, \qquad \bar{Q} \to \Gamma \bar{Q}, \tag{6.42}$$

with Γ defined in 6.16, and Z_{2R} is the symmetry under which bosonic fields are even and fermionic fields are odd. The transformation properties of fields under this $Z_{2\Gamma} \times Z_{2R}$ are

$$s(+), \qquad \tilde{s}(-), \tag{6.43}$$

$$\tilde{q} = \begin{pmatrix} \tilde{q}_A(+) \\ \tilde{q}_B(-) \end{pmatrix}, \qquad q = \begin{pmatrix} q_A(-) \\ q_B(+) \end{pmatrix}, \tag{6.44}$$

$$\tilde{r} = \begin{pmatrix} \tilde{r}_A(+) \\ \tilde{r}_B(-) \end{pmatrix}, \qquad r = \begin{pmatrix} r_A(-) \\ r_B(+) \end{pmatrix}, \tag{6.45}$$

where indices A and B distinguish between the two $U(N)$ groups inside $U(2N)$.

Step 3) consists on projecting out all fields odd under the $Z_{2\Gamma} \times Z_{2R}$ discrete symmetry. By doing so, we obtain the final non-supersymmetric orbifolded daughter Lagrangian

$$\begin{aligned}\mathcal{L} =& (\partial_\mu s)^\dagger (\partial^\mu s) + (\partial_\mu \tilde{q}_A)^\dagger (\partial^\mu \tilde{q}_A) + i\bar{q}_B \bar{\sigma}^\mu \partial_\mu q_B + (\partial_\mu \tilde{r}_A)^\dagger (\partial^\mu \tilde{r}_A) + i\bar{r}_B \bar{\sigma}^\mu \partial_\mu r_B \\ & - \lambda \left(s q_B r_B + s^\dagger \bar{q}_B \bar{r}_B \right) - \lambda^2 \left(|\tilde{q}_A^T \tilde{r}_A|^2 + s^\dagger s \tilde{q}_A^\dagger \tilde{q}_A + s^\dagger s \tilde{r}_A^\dagger \tilde{r}_A \right),\end{aligned} \tag{6.46}$$

which is invariant under a $U(N)_A \times U(N)_B$ global symmetry. It is easy to see that the one loop quadratically divergent contributions to the mass of the scalar s are canceled out even though the theory is not supersymmetric (the terms in eq.(6.41)are still present). However, now, the quadratically divergent contributions to the mass of the scalar fields $\tilde{q}_A$ and $\tilde{r}_A$ do not cancel out. This means that at higher loop orders there will be divergences in the contributions to the mass of s that will not be canceled.

6.3 Application to the Standard Model

The most significant contribution of quadratic divergences to the mass of the Higgs field in Standard Model is given by the top loop diagram. In this section we address that sector of the SM by implementing the folded supersymmetry prescription and outline an ultraviolet completion for it.

The top Yukawa coupling in the Standard Model is given by

$$\mathcal{L}_{SM} \supset -y_t \left(\bar{q}_3 \tilde{H} u_3 + \bar{u}_3 \tilde{H}^\dagger q_3 \right). \tag{6.47}$$

We want to cancel the divergent contribution to the Higgs mass parameter. For that, we follow our prescription. Supersymmetrization implies having the term

$$y_t Q_3^\alpha \cdot H_U U_3^\alpha \tag{6.48}$$

in the superpotential, where Q_3, H_U and U_3 are the chiral superfields defined in Table 4.1 and α is the $SU(3)$ index which runs from 1 to 3, that is, it represents each of the three colors of quark fields. The term in eq.(6.48) corresponds to the first term in eq. (4.85) with the family indices I and J set to 3. As was stated in section 4.5.1, fields transform under $SU(3)$ and $SU(2)$ as

$$Q_3(\mathbf{3}, \mathbf{2}); \qquad H_U(\mathbf{1}, \mathbf{2}); \qquad U_3(\bar{\mathbf{3}}, \mathbf{1}). \tag{6.49}$$

The supersymmetric version of eq.(6.47), expressed in component fields, is

$$\begin{aligned} \mathcal{L} \supset &- y_t \left[\tilde{u}_3^\alpha q_3^\alpha \cdot \tilde{h}_u + \tilde{q}_3^\alpha \cdot \tilde{h}_u u_3^\alpha + h_u \cdot q_3^\alpha u_3^\alpha + \text{h.c.} \right] \\ &- y_t^2 \left[|\tilde{q}_3 \cdot h_u|^2 + |h_u|^2 |\tilde{u}_3|^2 + |\tilde{q}_3|^2 |\tilde{u}_3|^2 \right]. \end{aligned} \tag{6.50}$$

Then, we identify the index α in the loop contributions of the mass of h_u as the relevant one. In order to have a theory that enjoys bifold protection, we make this index to run from 1 to 6. This can be done in either of two ways: we can enlarge the symmetry group of the theory from $SU(3)$ to $SU(6)$, or we can extend it to an $SU(3) \times SU(3)$ and a discrete symmetry that interchange the two $SU(3)$. In this part we will follow the first approach.

After enlarging the $SU(3)$ gauge color group to an $SU(6)$ gauge group the Yukawa term in the superpotential have the same form as in (6.48), but now the fields transform as

$$Q_3(\mathbf{6}, \mathbf{2}); \quad H_U(\mathbf{1}, \mathbf{2}); \quad U_3(\bar{\mathbf{6}}, \mathbf{1}), \tag{6.51}$$

that is, Q_3 contains, in addition to the three color states of the SM, three new states charged under $SU(2)_L$ and $U(1)_Y$ but not under $SU(3)_C$ of the SM. Analogously U_3 contains, in addition to the 3 color states of the SM, three exotic states charged under $U(1)_Y$. These new fields will be called *Folded partners* (or *F-partners*, for short) of the corresponding MSSM fields.

The theory, now, is invariant under a discrete symmetry $Z_{2\Gamma}$ under which fields transform as

$$Q_3 \to -\Gamma Q_3, \quad U_3 \to -\Gamma^* U_3, \quad V_6 \to \Gamma V_6 \Gamma^\dagger, \tag{6.52}$$

where Γ is the 6×6 matrix defined in eq.(6.16) and V_6 is the vector superfield corresponding to the $SU(6)$ gauge group. Also, the theory is invariant under Z_{2R} under which bosonic fields are even and fermionic fields are odd. Under the combined symmetry $Z_{2\Gamma} \times Z_{2R}$, fields transform as

$$\tilde{q}_3 = \begin{pmatrix} \tilde{q}_A(-) \\ \tilde{q}_B(+) \end{pmatrix}, \quad q_3 = \begin{pmatrix} q_A(+) \\ q_B(-) \end{pmatrix}, \tag{6.53}$$

$$\tilde{u}_3 = \begin{pmatrix} \tilde{u}_A(-) \\ \tilde{u}_B(+) \end{pmatrix}, \quad u_3 = \begin{pmatrix} u_A(+) \\ u_B(-) \end{pmatrix}, \tag{6.54}$$

$$h_u(+), \quad \tilde{h}_u(-), \tag{6.55}$$

where indices A and B distinguish between the two $SU(3)$ groups contained in $SU(6)$.

Orbifolding out the odd states under the combined discrete group $Z_{2\Gamma} \times Z_{2R}$, the remaining Lagrangian contains only the terms

$$\mathcal{L} = -y_t \left[h_u \cdot q_A^\alpha u_A^\alpha + \text{h.c.}\right] - y_t^2 \left[|\tilde{q}_B \cdot h_u|^2 + |h_u|^2 |\tilde{u}_B|^2 + |\tilde{q}_B|^2 |\tilde{u}_B|^2\right] \tag{6.56}$$

from where the relevant couplings for the one loop corrections to the mass of h_u are

$$-\,y_t\left[h_u \cdot q_A^\alpha u_A^\alpha + \text{h.c.}\right] - y_t^2\left[|\tilde{q}_B \cdot h_u|^2 + |h_u|^2|\tilde{u}_B|^2\right]. \tag{6.57}$$

Here, we recognize the pattern in eq.(6.41) for h_u. That is, one loop divergent contributions to the mass of h_u vanish out. Furthermore, we note that the fermionic loop with fields charged under $SU(3)$ SM color is canceling out with bosonic loops not charged under $SU(3)$ SM color but under another $SU(3)$ color group.

Also, since the fields $\tilde{q}_B$ and $\tilde{u}_B$ do not couple to fermionic fields, there will be quadratically one loop contributions for their masses. Then, radiative stability of the mass of h_u is not guaranteed to orders higher than one.

6.3.1 Ultraviolet completion

In 6.57 are shown the relevant terms that allow the cancellation of divergences. As we see, it is vital for the cancellation to take place that the coupling parameter for the Yukawa term be y_t and that of the quartic interaction be y_t^2. However, there is not a symmetry that explain why they take those values. Then, we need an ultraviolet completion for the orbifolded theory. Here, we outline such UV completion.

We will consider a five-dimensional supersymmetric theory with an extra dimension of radius R compactified on S^1/Z_2. It is represented schematically in Figure 6.4 The fixed points are located in $y = 0$ and $y = \pi R$. The gauge symmetry of the theory is $SU(6)\times SU(2)_L \times U(1)_Y$. The $SU(6)$ gauge symmetry will be broken up to $SU(3)\times SU(3)\times U(1)$ by boundary conditions and at the same time supersymmetry will be broken by Scherk-Schwarz mechanism. Matter and gauge fields live in the bulk of $5D$ space while H_U and H_D are localized in the brane where $SU(6)$ is preserved which will happen to be at $y = 0$. As was explained in section (5.2), in order to fully specify the boundary conditions of the various fields we need to define the orbifold b.c.'s (transformations under reflections about $y = 0$) represented by Z and the Scherk-Schwarz b.c.'s (transformations under translations by $2\pi R$) represented by T. Or, analogously, we can define transformations under Z and transformations under reflections about $y = \pi R$ denoted by Z' ($Z' = TZ$).

A five-dimensional supersymmetric gauge multiplet $\hat{V}$ consists of a five-vector A_M, two Weyl spinors λ and λ', and a real scalar σ. From the viewpoint of $4D$ space the five dimensional theory posseses an $\mathcal{N} = 2$ supersymmetry. This $\mathcal{N} = 2$ supersymmetry is broken to a $\mathcal{N} = 1$ supersymmetry by the action of Z. $\hat{V}$ can be

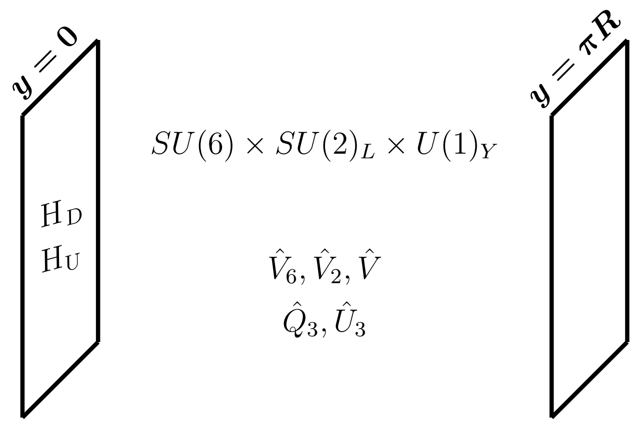


Figure 6.4: Sketch of the $5D$ theory that works as an ultraviolet completion for the orbifolded theory described by 6.56. Vertical planes represent the branes localized at $y = 0$ and $y = \pi R$ respectively. The gauge bulk symmetry is $SU(6) \times SU(2)_L \times U(1)_Y$. Gauge multiplets $(\hat{V}_6, \hat{V}_2, \hat{V})$ and matter hypermultiplets $(\hat{Q}_3, \hat{U}_3)$ are localized in the bulk, while H_U and H_D are localized in the $y = 0$ brane.

thought as composed by a $4D$ vector multiplet V with components (A_μ, λ), and a $4D$ chiral multiplet Σ with components $(\sigma + iA_5, \lambda')$. In order two break supersymmetry from $\mathcal{N} = 2$ to $\mathcal{N} = 1$, V and Σ must transform in a opposite way under Z. Since we want a low energy $4D$ effective non-supersymmetric theory, we cannot have this remaining $\mathcal{N} = 1$ supersymmetry. To correct this, we note that $\mathcal{N} = 2$ supersymmetry can be broken up to $\mathcal{N} = 1$ supersymmetry also by the action of Z'. But this $\mathcal{N} = 1$ supersymmetry must be different from that which survives the action of Z. For that, we break up $\hat{V}$ in an alternative way: a vector superfield V' with components (A_μ, λ'), and a chiral superfield Σ' with component fields $(\sigma + iA_5, -\lambda)$. Also, V' and Σ' must have different transformation properties under Z'. Then, although either Z or Z' separately breaks supersymmetry from $\mathcal{N} = 2$ to $\mathcal{N} = 1$, the combined action of Z and Z' breaks it completely.

A hypermultiplet $\hat{H}$ consists of two bosonic fields $\tilde{q}$ and $\tilde{q}^c$, and two fermionic fields q and q^c. From the four dimensional point of view, the theory possesses $\mathcal{N} = 2$ supersymmetry and can be decomposed in the two $4D$ chiral multiplets Q (with component fields $(\tilde{q}, q)$) and Q^c (with component fields $(\tilde{q}^c, q^c)$). We want Z to break up $\mathcal{N} = 2$ to $\mathcal{N} = 1$ supersymmetry, then Q and Q^c must transform differently under Z. We also want Z' to break up $\mathcal{N} = 2$ to $\mathcal{N} = 1$ supersymmetry (different from the previous $\mathcal{N} = 1$ SUSY). For that, we decompose $\hat{Q}$ alternatively into the chiral superfields Q' (with component fields $\tilde{q}^{*c}$ and q) and Q'^c (with component fields $(-\tilde{q}^*, q^c)$) and make them transform differently under Z'. Then, under the combined action of Z and Z', supersymmetry is completely broken.

In the present example we want to break $SU(6)$ to $SU(3) \times SU(3) \times U(1)$ gauge symmetry. We choose Z to preserve $SU(6)$ and Z' to breaks $SU(6)$. Then, denoting the five dimensional $SU(6)$ vector multiplet by $\hat{V}_6$, under the action of Z, V_6 is even while Σ_6 is odd. However, under the action of Z', superfields transform as

$$V_6' \to \Gamma V_6' \Gamma^\dagger, \qquad \Sigma_6' \to -\Gamma \Sigma_6' \Gamma^\dagger. \tag{6.58}$$

Explicitly, we have the field transformations:

$$\begin{aligned} \text{Under } Z: \qquad & V_6 \;\; [A_{\mu,6}(+),\; \lambda_6(+)] \\ & \Sigma_6 \;\; [\rho_6(-),\; \lambda_6'(-)]\,. \end{aligned} \tag{6.59}$$

$$\begin{aligned} \text{Under } Z': \qquad & V_6' \left[\begin{pmatrix} A_{\mu,6,AA}(+) & A_{\mu,6,AB}(-) \\ A_{\mu,6,BA}(-) & A_{\mu,6,BB}(+) \end{pmatrix}, \begin{pmatrix} \lambda'_{6,AA}(+) & \lambda'_{6,AB}(-) \\ \lambda'_{6,BA}(-) & \lambda'_{6,BB}(+) \end{pmatrix} \right] \\ & \Sigma_6' \left[\begin{pmatrix} \rho_{6,AA}(-) & \rho_{6,AB}(+) \\ \rho_{6,BA}(+) & \rho_{6,BB}(-) \end{pmatrix}, \begin{pmatrix} \lambda_{6,AA}(-) & \lambda_{6,AB}(+) \\ \lambda_{6,BA}(+) & \lambda_{6,BB}(-) \end{pmatrix} \right]. \end{aligned} \tag{6.60}$$

where $\rho_6 \equiv \sigma_6 + iA_{5,6}$.

As we saw in section 5.2, only the fields which are even under both Z and Z' will have zero modes at low energies when decomposed in Kaluza-Klein modes. Then, we notice that the only fields that will have a zero mode at low energies are the gauge bosons corresponding to the gauge group $SU(3) \times SU(3) \times U(1)$, $A_{\mu,6,AA}$ and $A_{\mu,6,BB}$, together with the fermionic fields $\lambda_{6,BA}$ and $\lambda_{6,AB}$. Furthermore, we also want to leave $SU(2)_L \times U(1)_Y$ unbroken; therefore, for the corresponding vector fields $\hat{V}$, we will choose that, under Z, V is even and Σ is odd; and under Z', V' is even and Σ' is odd. Making this leaves us only with the gauge boson fields corresponding to the groups $SU(2)_L$ and $U(1)_Y$ at low energies.

For matter fields, we introduce the hypermultiplet $\hat{Q}_3$ which transforms as $(6, 2)$ under $SU(6) \times SU(2)$ and has hypercharge $1/6$. The boundary conditions for fields are defined as

$$\text{Under } Z: \qquad Q_3 \to +Q_3, \quad Q_3^c \to -Q_3^c, \tag{6.61}$$

or, in terms of component fields

$$Z: \qquad \tilde{q}_3 = \begin{pmatrix} \tilde{q}_A(+) \\ \tilde{q}_B(+) \end{pmatrix}, \; q_3 = \begin{pmatrix} q_A(+) \\ q_B(+) \end{pmatrix}, \qquad \tilde{q}_3^c = \begin{pmatrix} \tilde{q}_A^c(-) \\ \tilde{q}_B^c(-) \end{pmatrix}, \; q_3^c = \begin{pmatrix} q_A^c(-) \\ q_B^c(-) \end{pmatrix}. \tag{6.62}$$

and

$$\text{Under } Z': \qquad Q_3' \to \Gamma Q_3', \quad Q_3'^c \to -\Gamma^* Q_3'^c, \tag{6.63}$$

or, in terms of component fields

$$Z': \qquad \tilde{q}_3^c = \begin{pmatrix} \tilde{q}_A^c(+) \\ \tilde{q}_B^c(-) \end{pmatrix}, \; q_3 = \begin{pmatrix} q_A(+) \\ q_B(-) \end{pmatrix} \qquad \tilde{q}_3 = \begin{pmatrix} \tilde{q}_A(-) \\ \tilde{q}_B(+) \end{pmatrix}, \; q_3^c = \begin{pmatrix} q_A^c(-) \\ q_B^c(+) \end{pmatrix}. \tag{6.64}$$

Then the only fields that have zero modes are q_A and $\tilde{q}_B$.

We also introduce the hypermultiplet $\hat{U}_3$ which transforms as $(\bar{6}, 1)$ under $SU(6) \times SU(2)$ and has hypercharge $-2/3$. Boundary conditions are defined as

$$\text{Under } Z: \qquad U_3 \to +U_3, \quad U_3^c \to -U_3^c, \tag{6.65}$$

or, in terms of component fields

$$Z: \qquad \tilde{u}_3 = \begin{pmatrix} \tilde{u}_A(+) \\ \tilde{u}_B(+) \end{pmatrix}, \; u_3 = \begin{pmatrix} u_A(+) \\ u_B(+) \end{pmatrix}, \qquad \tilde{u}_3^c = \begin{pmatrix} \tilde{u}_A^c(-) \\ \tilde{u}_B^c(-) \end{pmatrix}, \; u_3^c = \begin{pmatrix} u_A^c(-) \\ u_B^c(-) \end{pmatrix}. \tag{6.66}$$

and

$$\text{Under } Z': \qquad U_3' \to \Gamma^* U_3', \quad U_3'^c \to -\Gamma U_3'^c, \tag{6.67}$$

or, in terms of component fields

$$Z': \qquad \tilde{u}_3^c = \begin{pmatrix} \tilde{u}_A^c(+) \\ \tilde{u}_B^c(-) \end{pmatrix}, \; u_3 = \begin{pmatrix} u_A(+) \\ u_B(-) \end{pmatrix}, \qquad \tilde{u}_3 = \begin{pmatrix} \tilde{u}_A(-) \\ \tilde{u}_B(+) \end{pmatrix}, \; u_3^c = \begin{pmatrix} u_A^c(-) \\ u_B^c(+) \end{pmatrix}. \tag{6.68}$$

Then the only fields that have zero modes are u_A and $\tilde{u}_B$.

Fields that do not vanish at the $y = 0$ brane are those which are even under Z transformation. They are the vector superfield $V_6 = (A_{\mu,6}, \lambda_6)$ and the chiral superfields $Q_3 = (\tilde{q}_3, q_3)$ and $U_3 = (\tilde{u}_3, u_3)$. Then, in the brane located at $y = 0$, it is preserved an $\mathcal{N} = 1$ supersymmetry and the field content is given by the superfields V_6, Q_3, U_3 and, as we anticipated, the chiral supermultiplets H_U and H_D.

We now write the part of the $5D$ Lagrangian that is relevant for the cancellation of the one loop divergences in a $4D$ supersymmetric language following what was studied in subsection 5.5.1.

$$\begin{aligned}\mathcal{L}_5 = &\int d^4\theta \left[Q_3^\dagger Q_3 + Q_3^{c\dagger} Q_3^c + U_3^\dagger U_3 + U_3^{c\dagger} U_3^c\right] + \int d^4\theta \left[Q_3^c \partial_5 Q_3 + U_3^c \partial_5 U_3 + \text{h.c.}\right] \\ &+ \delta(y) \left\{ \int d^4\theta H_U^\dagger H_U + \int d^2\theta \left[y_t Q_3^\alpha \cdot H_U U_3^\alpha + \text{h.c.}\right] \right\}.\end{aligned} \tag{6.69}$$

It is convenient to write Q_3, Q_3^c, U_3 and U_3^c as

$$Q_3 = \begin{pmatrix} Q_A \\ Q_B \end{pmatrix}, \quad Q_3^c = \begin{pmatrix} Q_A^c \\ Q_B^c \end{pmatrix}, \quad U_3 = \begin{pmatrix} U_A \\ U_B \end{pmatrix}, \quad U_3^c = \begin{pmatrix} U_A^c \\ U_B^c \end{pmatrix}. \tag{6.70}$$

Then, (6.69) becomes

$$\begin{aligned}\mathcal{L}_5 = &\int d^4\theta \left[Q_A^\dagger Q_A + Q_A^{c\dagger} Q_A^c + U_A^\dagger U_A + U_A^{c\dagger} U_A^c + Q_B^\dagger Q_B + Q_B^{c\dagger} Q_B^c + U_B^\dagger U_B + U_B^{c\dagger} U_B^c\right] \\ &+ \int d^4\theta \left[Q_A^c \partial_5 Q_A + U_A^c \partial_5 U_A + Q_B^c \partial_5 Q_B + U_B^c \partial U_B + \text{h.c.}\right] \\ &+ \delta(y) \left\{ \int d^4\theta H_U^\dagger H_U + \int d^2\theta \left[y_t H_U \cdot Q_A U_A + y_t H_U \cdot Q_B U_B + \text{h.c.}\right] \right\}.\end{aligned} \tag{6.71}$$

By eliminating auxiliary fields and expanding $5D$ fields in Kaluza-Klein modes (the derivation is made in Appendix B) we obtain the Lagrangian

$$\mathcal{L} = \mathcal{L}_{(0)} + \mathcal{L}^{(KK)}, \tag{6.72}$$

where $\mathcal{L}_{(0)}$ is the part of the relevant Lagrangian that contains the zero modes:

$$\begin{aligned}\mathcal{L}_{(0)} = &[\text{kinetic terms}] \\ &- y_t^2 |h_u \tilde{q}_{B0}|^2 - y_t^2 |h_u|^2 |\tilde{u}_B|^2 - (y_t h_u q_{A0} u_{A0} + \text{h.c.})\end{aligned} \tag{6.73}$$

and $\mathcal{L}^{(KK)}$ is the part of the relevant Lagrangian that contains Kaluza-Klein modes:

$$\begin{aligned}\mathcal{L}^{(KK)} = &[\text{Kinetic terms}] \\ &+ \sum_{n,m} \big[y_t h_u q_{Bn} u_{Bm} + y_t^2 |h_u|^2 |\tilde{q}_{An}|^2 + y_t^2 |h_u|^2 |\tilde{u}_{Am}|^2 \\ &\qquad + y_t h_u q_{An} u_{Am} + y_t^2 |h_u|^2 |\tilde{q}_{Bm}|^2 + y_t^2 |h_u|^2 |\tilde{u}_{Bm}|^2\big] \\ &+ \sum_{n} \big[y_t h_u q_{An} u_{A0} + y_t^2 |h_u|^2 |\tilde{q}_{Bn}|^2 + y_t^2 |h_u|^2 |\tilde{u}_{B0}|^2\big] \\ &+ \sum_{m} \big[y_t h_u q_{A0} u_{Am} + y_t^2 |h_u|^2 |\tilde{q}_{B0}|^2 + y_t^2 |h_u|^2 |\tilde{u}_{Bm}|^2\big] .\end{aligned} \tag{6.74}$$

We note that 6.73 is equivalent to the Lagrangian in 6.57 which indicates that, indeed, the theory is a UV completion.

Chapter 7

Conclusions

The Standard Model of particle physics is a well tested theory that predicts experimental data with great accuracy up to the range of energies currently reached. Nevertheless, we saw in Chapter 3 that experimental and theoretical issues make evident that it is incomplete and there must be new physics waiting to be understood at higher energies. One of the main problems Standard Model faces is known as Hierarchy problem and arises when we consider that the theory is valid up to a certain high energy Λ usually took as the Planck scale. Within this assumption, the Higgs mass receives large quantum corrections that are quadratically sensitive to Λ, requiring a delicate fine tuning between the bare Higgs mass parameter and the quantum corrections in order to obtain a fix value of the physical Higgs mass (measured to be 125 GeV).

One of the possible solutions to hierarchy problem is based on supersymmetry which was studied in Chapter 4. It accomplishes to avoid the large quadratic contributions by adding a new field known as superpartner for every field in the SM. The superpartners have the same properties as ordinary fields except for the spin which differ by 1/2. They generate new quadratically sensitive contributions that have the same value as that of the SM but with opposite sign making the cancellation possible and solving the hierarchy problem. However, no experimental evidence of supersymmetry has been found so far.

In Chapter 6, we studied two examples of orbifolded theories where quadratic divergences due to one loop corrections to the mass of a scalar are canceled out. One for a theory with gauge interactions and the other for a theory with Yukawa coupling. We understood that cancellations take place because in the parent theory the scalar mass enjoys bifold protection. It served us to write down a prescription for constructing a folded supersymmetric theory. Then, we used that prescription to build a folded supersymmetric toy model out of the Yukawa interaction in Standard Model. We focused in the top Yukawa interaction because it is what contributes most to quadratic divergences in the SM. By doing so, we noticed that the orbifolded theory possesses an unexplained relation between coupling parameters of different interaction terms. So, we required to build a UV completion that account for that relation. The outlined UV completion was constructed in a 5-dimensional spacetime (whose basic concepts were reviewed in Chapter 5) with $\mathcal{N} = 1$ supersymmetry and made use of appropriate Scherk-Schwarz and orbifold boundary conditions to break supersymmetry and obtain the correct effective folded supersymmetric theory at low energies.

As future perspectives we intend to analyze realistic model where folded supersymmetry is realized not only for the Yukawa sector but also for the gauge sector.

Appendix A

Derivation of orbifolded Lagrangians

In this part, it is shown in detail the calculations that were made to obtain the orbifolded Lagrangians in the various examples worked in this dissertation as well as their corresponding one loop diagrams that contributes with quadratic divergences.

A.1 Supersymmetric gauge theory

In section 6.1.1 we applied the orbifold procedure to the supersymmmetric Lagrangian invariant under the $U(2N)_c$ gauge group and the global $U(2N)_f$ group:

$$\begin{aligned}\mathcal{L} =& \mathrm{Tr}\left\{(D_\mu\tilde{q})^\dagger(D^\mu\tilde{q}) + i\bar{q}\bar{\sigma}^\mu D_\mu q + (D_\mu\tilde{r})^\dagger(D^\mu\tilde{r}) + i\bar{r}\bar{\sigma}^\mu D_\mu r\right\}\\ &+ \mathrm{Tr}\left\{-\frac{1}{2}[W_{\mu\nu}W^{\mu\nu}] + 2i\bar{\lambda}\bar{\sigma}^\mu D_\mu\lambda\right\}\\ &+ ig\sqrt{2}\mathrm{Tr}\left\{\tilde{q}^\dagger\lambda q - \bar{q}\bar{\lambda}\tilde{q} - \tilde{r}^\dagger\lambda^T r + \bar{r}\bar{\lambda}^*\tilde{r}\right\}\\ &- \frac{g^2}{2}\mathrm{Tr}(\tilde{q}^\dagger T_c^a\tilde{q})\mathrm{Tr}(\tilde{q}^\dagger T_c^a\tilde{q}) - \frac{g^2}{2}\mathrm{Tr}(\tilde{r}^\dagger T_c^{a*}\tilde{r})\mathrm{Tr}(\tilde{r}^\dagger T_c^{a*}\tilde{r}).\end{aligned} \tag{A.1}$$

Fields that remain after orbifolding are:

$$\tilde{q} = \begin{pmatrix} \tilde{q}_{Aa} & 0 \\ 0 & \tilde{q}_{Bb} \end{pmatrix}, \qquad q = \begin{pmatrix} 0 & \tilde{q}_{Ab} \\ \tilde{q}_{Ba} & 0 \end{pmatrix}, \tag{A.2}$$

$$\tilde{r} = \begin{pmatrix} \tilde{r}_{Aa} & 0 \\ 0 & \tilde{r}_{Bb} \end{pmatrix}, \qquad r = \begin{pmatrix} 0 & \tilde{r}_{Ab} \\ \tilde{r}_{Ba} & 0 \end{pmatrix}, \tag{A.3}$$

$$A_\mu = \begin{pmatrix} A_{\mu,Aa} & 0 \\ 0 & A_{\mu,Bb} \end{pmatrix}, \qquad \lambda = \begin{pmatrix} 0 & \lambda_{AB} \\ \lambda_{BA} & 0 \end{pmatrix}, \tag{A.4}$$

Then, covariant derivatives reduce to

$$\begin{aligned} D_\mu \tilde{q} &= \begin{pmatrix} \partial_\mu \tilde{q}_{Aa} & 0 \\ 0 & \partial_\mu \tilde{q}_{Bb} \end{pmatrix} + ig \begin{pmatrix} A_{\mu,Aa} & 0 \\ o & A_{\mu,BB} \end{pmatrix} \begin{pmatrix} \tilde{q}_{Aa} & 0 \\ 0 & \tilde{q}_{Bb} \end{pmatrix} \\ &= \begin{pmatrix} \partial_\mu \tilde{q}_{Aa} + igA_{\mu,AA}\tilde{q}_{Aa} & 0 \\ 0 & \partial_\mu \tilde{q}_{Bb} + igA_{\mu,BB}\tilde{q}_{Bb} \end{pmatrix} \\ &\equiv \begin{pmatrix} D_\mu \tilde{q}_{Aa} & 0 \\ 0 & D_\mu \tilde{q}_{Bb} \end{pmatrix}, \end{aligned} \tag{A.5}$$

$$\begin{aligned} D_\mu \tilde{r} &= \begin{pmatrix} \partial_\mu \tilde{r}_{Aa} + igA_{\mu,AA}\tilde{r}_{Aa} & 0 \\ 0 & \partial_\mu \tilde{r}_{Bb} + igA_{\mu,BB}\tilde{r}_{Bb} \end{pmatrix} \\ &\equiv \begin{pmatrix} D_\mu \tilde{r}_{Aa} & 0 \\ 0 & D_\mu \tilde{r}_{Bb} \end{pmatrix}, \end{aligned} \tag{A.6}$$

$$\begin{aligned} D_\mu q &= \begin{pmatrix} 0 & \partial_\mu q_{Ab} \\ \partial_\mu q_{Ba} & 0 \end{pmatrix} + ig \begin{pmatrix} A_{\mu,AA} & 0 \\ 0 & A_{\mu,BB} \end{pmatrix} \begin{pmatrix} 0 & q_{Ab} \\ q_{Ba} & 0 \end{pmatrix} \\ &= \begin{pmatrix} 0 & \partial_\mu q_{Ab} + igA_{\mu,AA}q_{Ab} \\ \partial_\mu q_{Ba} + igA_{\mu,BB}q_{Ba} & 0 \end{pmatrix} \\ &\equiv \begin{pmatrix} 0 & D_\mu q_{Ab} \\ D_\mu q_{Ba} & 0 \end{pmatrix}, \end{aligned} \tag{A.7}$$

$$\begin{aligned} D_\mu r &= \begin{pmatrix} 0 & \partial_\mu r_{Ab} + igA_{\mu,AA}r_{Ab} \\ \partial_\mu r_{Ba} + igA_{\mu,BB}r_{Ba} & 0 \end{pmatrix} \\ &\equiv \begin{pmatrix} 0 & D_\mu r_{Ab} \\ D_\mu r_{Ba} & 0 \end{pmatrix}, \end{aligned} \tag{A.8}$$

$$\begin{aligned} D_\mu \lambda = &\begin{pmatrix} 0 & \partial_\mu \lambda_{AB} \\ \partial_\mu \lambda_{BA} & 0 \end{pmatrix} + ig \left[\begin{pmatrix} A_{\mu,AA} & 0 \\ 0 & A_{\mu,BB} \end{pmatrix} \begin{pmatrix} 0 & \lambda_{AB} \\ \lambda_{BA} & 0 \end{pmatrix} \right. \\ &\qquad\qquad \left. - \begin{pmatrix} 0 & \lambda_{AB} \\ \lambda_{BA} & 0 \end{pmatrix} \begin{pmatrix} A_{\mu,AA} & 0 \\ 0 & A_{\mu,BB} \end{pmatrix} \right] \\ = &\begin{pmatrix} 0 & \partial_\mu \lambda_{AB} + ig(A_{\mu,AB} - \lambda_{AB}A_{\mu,BB}) \\ \partial_\mu \lambda_{BA} + ig(A_{\mu,BB}\lambda_{BA} - \lambda_{BA}A_{\mu,AA}) & 0 \end{pmatrix}, \end{aligned} \tag{A.9}$$

Also, the field strength becomes

$$\begin{aligned}
W_{\mu\nu} &= \begin{pmatrix} \partial_\mu A_{\nu,AA} & 0 \\ 0 & \partial_\mu A_{\nu,BB} \end{pmatrix} \begin{pmatrix} \partial_\nu A_{\mu,AA} & 0 \\ 0 & \partial_\nu A_{\mu,BB} \end{pmatrix} \\
&+ ig \left[\begin{pmatrix} A_{\mu,AA} & 0 \\ 0 & A_{\mu,BB} \end{pmatrix} \begin{pmatrix} A_{\nu,AA} & 0 \\ 0 & A_{\nu,BB} \end{pmatrix} \right. \\
&- \left. \begin{pmatrix} A_{\nu,AA} & 0 \\ 0 & A_{\nu,BB} \end{pmatrix} \begin{pmatrix} A_{\mu,AA} & 0 \\ 0 & A_{\mu,BB} \end{pmatrix} \right] \\
&= \begin{pmatrix} \partial\mu A_{\nu,AA} - \partial_\nu A_{\mu,AA} + ig[A_{\mu,AA}; A_{\nu;AA}] & 0 \\ 0 & \partial_\mu A_{\nu,BB} + ig[A_{\mu,BB}; A_{\nu,BB}] \end{pmatrix} \\
&\equiv \begin{pmatrix} W_{\mu\nu,AA} & 0 \\ 0 & W_{\mu\nu,BB} \end{pmatrix}.
\end{aligned} \tag{A.10}$$

Applying the orbifold procedure will imply to decompose the generators T_c^G of the $U(2N)_c$ group into $N \times N$ block matrices of the form

$$T^G = \begin{pmatrix} T_1^G & \tilde{T}^G \\ \tilde{T}^{G\dagger} & T_2^G \end{pmatrix}. \tag{A.11}$$

The $4N^2$ generators will be chosen such that:

1. The first N^2 generators ($G = 1, ..., N^2$) will have the form

$$\begin{pmatrix} 0 & M^{i,j} \\ M^{i,j} & 0 \end{pmatrix}, \tag{A.12}$$

   where the element (m, n) of the matrices $M^{i,j}$ is given by

$$(M^{i,j})_{mn} = \frac{1}{2}\delta^i_m \delta^j_n, \tag{A.13}$$

   that is, the matrix $M^{i,j}$ has all its entries equal to zero except for the entry (i, j) which is equal to $1/2$. The indices i and j both run from 1 to N giving rise the N^2 matrices $M^{i,j}$.

2. The next N^2 generators ($G = N^2 + 1, ..., 2N^2$) will have the form

$$\begin{pmatrix} 0 & -iM^{i,j} \\ iM^{i,j} & 0 \end{pmatrix}. \tag{A.14}$$

For $G = 2N^2+1, ..., 3N^2$, the generators will have the form

$$\begin{pmatrix} T^c & 0 \\ 0 & 0 \end{pmatrix}, \tag{A.15}$$

where T^c are the N^2 generators of $U(N)$ and they satisfy $\mathrm{Tr}(T^cT^d) = \frac{1}{2}\delta^{cd}$. Indices c and d both run from 1 to N^2.

3. For $G = 3N^2+1, ..., 4N^2$, the generators will be of the form

$$\begin{pmatrix} 0 & 0 \\ 0 & T^c \end{pmatrix}. \tag{A.16}$$

Then, the matrices T_1^G will be zero except for the values $G = 2N^2+1, ..., 3N^2$ where they are equal to T^c. Then, the matrices T_2^G will be zero except for the values $G = 3N^2+1, ..., 4N^2$ where they are equal to T^c. And the matrices $\tilde{T}^G$ will be zero except for the values $G = 1, ..., N^2$ where they take the values of $M^{i,j}$ and for the values $G = N^2+1, ...2N^2$ where they take the values of $-iM^{i,j}$.

Then, the orbifold daughter Lagrangian is

$$\begin{aligned}
\mathcal{L} =&\mathrm{Tr}\left\{(D_\mu\tilde{q}_{Aa})^\dagger(D^\mu\tilde{q}_{Aa}) + (D_\mu\tilde{q}_{Bb})^\dagger(D^\mu\tilde{q}_{Bb}) + iq^\dagger_{Ba}\bar{\sigma}^\mu D_\mu q_{Ba} + iq^\dagger_{Ab}\bar{\sigma}^\mu D_\mu q_{Ab}\right\} \\
&+ \mathrm{Tr}\left\{(D_\mu\tilde{r}_{Aa})^\dagger(D^\mu\tilde{r}_{Aa}) + (D_\mu\tilde{r}_{Bb})^\dagger(D^\mu\tilde{r}_{Bb}) + ir^\dagger_{Ba}\bar{\sigma}^\mu D_\mu r_{Ba} + ir^\dagger_{Ab}\bar{\sigma}^\mu D_\mu r_{Ab}\right\} \\
&- \frac{1}{2}\mathrm{Tr}\left[W_{\mu\nu,AA}W^{\mu\nu}_{AA} + W_{\mu\nu,BB}W^{\mu\nu}_{BB}\right] \\
&+ \mathrm{Tr}\left\{2i\lambda^\dagger_{BA}\left[\partial_\mu\lambda_{BA} + ig(A_{\mu,BA}\lambda_{BA} - \lambda_{BA}A_{\mu,AA})\right] + \text{h.c.}\right\} \\
&+ ig\sqrt{2}\mathrm{Tr}\left(\tilde{q}^\dagger_{Aa}\lambda_{AB}q_{Ba} - q^\dagger_{Ba}\lambda^\dagger_{AB}\tilde{q}_{Aa} + \tilde{q}^\dagger_{Bb}\lambda_{BA}q_{Ab} - q^\dagger_{Ab}\lambda^\dagger_{BA}\tilde{q}_{Bb}\right) \\
&+ ig\sqrt{2}\mathrm{Tr}\left(-\tilde{r}^\dagger_{Aa}\lambda^T_{BA}r_{Ba} + r^\dagger_{Ba}\lambda^*_{BA}\tilde{r}_{Aa} - \tilde{r}^\dagger_{Bb}\lambda^T_{AB}r_{Ab} + r^\dagger_{Ab}\lambda^*_{BA}\tilde{r}_{Bb}\right) \\
&- \frac{g^2}{2}\left[\mathrm{Tr}(\tilde{q}^\dagger_{Aa}T_1^G\tilde{q}_{Aa})\mathrm{Tr}(\tilde{q}^\dagger_{Aa}T_1^G\tilde{q}_{Aa}) + \mathrm{Tr}(\tilde{q}^\dagger_{Bb}T_2^G\tilde{q}_{Bb})\mathrm{Tr}(\tilde{q}^\dagger_{Bb}T_2^G\tilde{q}_{Bb})\right] \\
&- \frac{g^2}{2}\left[\mathrm{Tr}(\tilde{r}^\dagger_{Aa}T_1^{G*}\tilde{r}_{Aa})\mathrm{Tr}(\tilde{r}^\dagger_{Aa}T_1^{G*}\tilde{r}_{Aa}) + \mathrm{Tr}(\tilde{r}^\dagger_{Bb}T_2^{G*}\tilde{r}_{Bb})\mathrm{Tr}(\tilde{r}^\dagger_{Bb}T_2^{G*}\tilde{r}_{Bb})\right].
\end{aligned} \tag{A.17}$$

We want to find the one loop corrections to the mass of $\tilde{q}_{Aa}$. Therefore, now we will focus just on the terms involving this field to write the corresponding Feynman rules. For that, let us introduce a bit of notation that will be valid only for the calculations that will be made from now on in this section:

1. The $N \times N$ matrix $\tilde{q}_{Aa}$ will be denoted just by $\tilde{q}$. Do not confuse it with the $2N \times 2N$ matrix with the same notation in eq. A.2.

2. The $N \times N$ matrix q_{Ba} will be denoted just by q. Do not confuse it with the $2N \times 2N$ matrix in A.2 which is denoted in the same way.

Using these conventions, let us list the Feynman rules that involve $\tilde{q}$:

(i) Expanding the term $\mathrm{Tr}\left[(D_\mu \tilde{q}_{Aa})^\dagger (D^\mu \tilde{q}_{Aa})\right] = \mathrm{Tr}\left[(D_\mu \tilde{q})^\dagger (D^\mu \tilde{q})\right]$ in the first line of A.17, we have:

$$\begin{aligned}\mathrm{Tr}\left[(D_\mu \tilde{q})^\dagger (D^\mu \tilde{q})\right] =& \mathrm{Tr}\left[\left(\partial_\mu \tilde{q} + igA^G_\mu T^G_1 \tilde{q}\right)^\dagger \left(\partial^\mu \tilde{q} + igA^{\mu G} T^G_1 \tilde{q}\right)\right] \\ =& \partial_\mu \tilde{q}^*_{ji} \partial^\mu \tilde{q}_{ji} - igA^c_\mu (T^c)_{kj} \tilde{q}^*_{ki} \partial^\mu \tilde{q}_{ji} + ig\partial_\mu \tilde{q}^*_{ji} A^{\mu c} (T^c)_{jk} \tilde{q}_{ki} \\ &+ g^2 A^c_\mu (T^c)_{kj} \tilde{q}^*_{ki} A^{\mu d} (T^d)_{jl} \tilde{q}_{li},\end{aligned} \tag{A.18}$$

where T'^c are the hermitian generators of the $U(N)$ group, with c running from 1 to N^2. The last three terms in the last line of A.18 generate the Feynman rules shown in Figures A.1 and A.2.

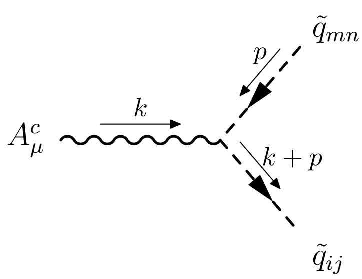


Figure A.1: Trilinear interaction between the scalar $\tilde{q}_{Aa}$ and the gauge fields, with Feynman rule: $-ig(2p_\mu + k_\mu)(T^c)_{im}\delta_{nj}$

(ii) Expanding the term $ig\sqrt{2}\mathrm{Tr}\left(\tilde{q}^\dagger_{Aa}\lambda_{AB} q_{Ba} - q^\dagger_{Ba}\lambda^\dagger_{AB}\tilde{q}_{Aa}\right)$ in the fifth line of A.17,

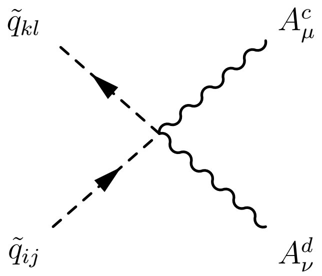


Figure A.2: Quartic interaction between the scalar field $\tilde{q}_{Aa}$ and the gauge fields, with Feynman rule: $ig^2(T^c)_{im}(T^d)_{mk}\delta_{jl}\eta_{\mu\nu}$

we obtain:

$$ig\sqrt{2}\text{Tr}\left(\tilde{q}^\dagger\lambda^G\tilde{T}^G q - q^\dagger\bar{\lambda}^G\tilde{T}^{G\dagger}\tilde{q}\right) = ig\sqrt{2}\left[\tilde{q}^*_{ji}(\tilde{T}^G)_{jk}\lambda^G q_{ki} - \bar{q}_{ji}\bar{\lambda}^G(\tilde{T}^{G*})_{kj}\tilde{q}_{ki}\right]$$
$$= ig\sqrt{2}\left[-\tilde{q}^*_{ji}(\tilde{T}^G)_{jk}(\Psi^G_\lambda)^T(\tilde{C}^{-1}P_L)\Psi_{q,ki} + \Psi^T_{q,ji}(\tilde{C}^{-1}P_R)(\Psi^G_\lambda)(\tilde{T}^{G*})_{kj}\tilde{q}_{ki}\right], \tag{A.19}$$

where Ψ^G_λ and Ψ_q are the Majorana fields constructed from λ and q respectively, as in 2.42. Also, $\tilde{C}^{-1} = -C\gamma^0$. $C = -i\gamma^2$ as it was defined in 2.40. The Feynman rules associated to the terms in A.19 are shown in Figures A.3 and A.4.

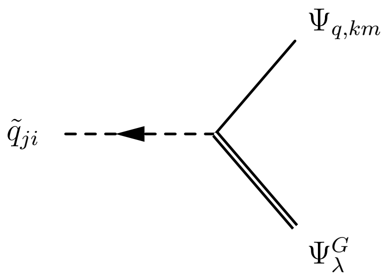


Figure A.3: Interaction between the scalar $\tilde{q}_{Aa}$, the gaugino λ and the fermionic field q_{Ba}, with Feynman rule: $g\sqrt{2}(\tilde{T}^G)_{jk}(\tilde{C}^{-1}P_L)\delta_{im}$.

(iii) From the term $-\frac{g^2}{2}\left[\text{Tr}(\tilde{q}^\dagger_{Aa}T^G_1\tilde{q}_{Aa})\text{Tr}(\tilde{q}^\dagger_{Aa}T^G_1\tilde{q}_{Aa})\right] = -\frac{g^2}{2}\left[\text{Tr}(\tilde{q}^\dagger T^G_1\tilde{q})\text{Tr}(\tilde{q}^\dagger T^G_1\tilde{q})\right]$ in A.17 we obtain

$$-\frac{g^2}{2}\left[\text{Tr}(\tilde{q}^\dagger T^G_1\tilde{q})\text{Tr}(\tilde{q}^\dagger T^G_1\tilde{q})\right] = -\frac{g^2}{2}\left[\text{Tr}(\tilde{q}^\dagger T^c\tilde{q})\text{Tr}(\tilde{q}^\dagger T^c\tilde{q})\right]$$
$$= -\frac{g^2}{2}\left[\tilde{q}^*_{ji}(T^c)_{jk}\tilde{q}_{ki}\tilde{q}^*_{nm}(T^c)_{np}\tilde{q}_{pm}\right]. \tag{A.20}$$

The corresponding Feynman rule is shown in Figure A.5.

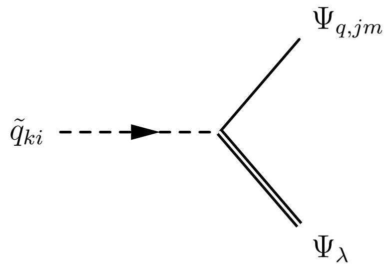


Figure A.4: Interaction between the scalar $\tilde{q}_{Aa}$, the gaugino λ and the fermionic field q_{Ba}, with Feynman rule: $-g\sqrt{2}(\tilde{T}^{G\dagger})_{jk}(\tilde{C}^{-1}P_R)\delta_{im}$.

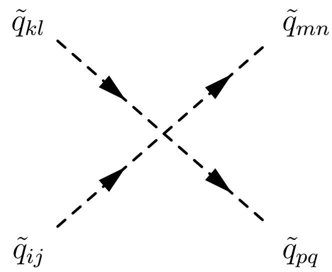


Figure A.5: Quartic scalar self-interaction with Feynman rule: $-ig^2\left(T^c_{mk}T^c_{pi}\delta_{nl}\delta_{qj} + T^c_{mi}T^c_{pk}\delta_{nj}\delta_{ql}\right)$.

Now that we know the Feynman rules that involves the scalar field $\tilde{q}_{Aa}$, we are able to find the one loop contributions to the mass of $\tilde{q}_{Aa}$. The relevant Feynman diagrams are shown in Figures A.6,

Let us find what is the quadratically divergent contribution of each o the diagrams. For the diagram in Figure A.6 we have:

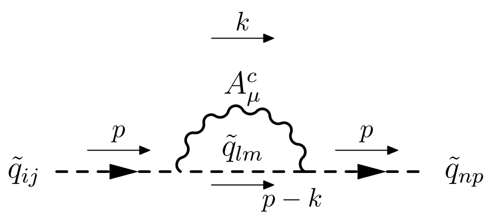


Figure A.6: One loop contribution to the mass of $\tilde{q}_{Aa}$ due to gauge fields.

$$
\begin{aligned}
-iM^{(1)} =& \sum_{l,m=1}^{N}\sum_{c=1}^{N^2}\int \frac{d^4k}{(2\pi)^4}\Bigg\{[-ig(2p_\mu - k_\mu)(T^c)_{li}\delta_{jm}]\left[\frac{i}{(p-k)^2}\right]\left[\frac{-i(\eta^{\mu\nu}-(1-\xi)\frac{k^\mu k^\nu}{k^2})}{k^2}\right] \\
&\times [-ig(2p_\nu - k_\nu)(T^c)_{nl}\delta_{pm}]\Bigg\} \\
=& \int \frac{d^4k}{(2\pi)^4}\left\{-g^2\frac{(2p-k)_\mu(2p-k)^\mu k^2 - (1-\xi)(2p_\mu-k_\mu)k^\mu k^\nu(2p_\nu-k_\nu)}{(p-k)^2(k^2)^2}\right\} \\
&\times \sum_{l,m=1}^{N}\sum_{c=1}^{N^2}(T^c)_{li}(T^c)_{nl}\delta_{jm}\delta_{pm} \\
=& -g^2\int \frac{d^4k}{(2\pi)^4}\left\{\frac{4p^2k^2 - 4(p\cdot k)^2 + \xi\,[4(p\cdot k)^2 - 4(p\cdot k)k^2 + k^4]}{(p-k)^2(k^2)^2}\right\} \\
&\times \sum_{l=1}^{N}\sum_{c=1}^{N^2}(T^c)_{li}(T^c)_{nl}\delta_{jp}
\end{aligned}
\tag{A.21}
$$

We are interested just in the quadratically divergent contribution. That is

$$
\begin{aligned}
M^{(1)}_{\text{quad}} =& -ig^2\xi\int\frac{d^4k}{(2\pi)^4}\frac{1}{(k-p)^2}\sum_{c=1}^{N^2}(T^cT^c)_{ni}\delta_{jp} \\
=& -\frac{\xi g^2 N\Lambda^2}{32\pi^2}\delta_{jp}\delta_{ni},
\end{aligned}
\tag{A.22}
$$

where we used the fact that, for the $U(N)$ generators, we have

$$
\sum_{c=1}^{N^2}(T^cT^c)_{ni} = \frac{N}{2}\delta_{ni}. \tag{A.23}
$$

For the diagram in Figure A.7, we have:

$$
\begin{aligned}
-iM^{(2)} =& \sum_{c=1}^{N^2}\sum_{m=1}^{N}\int\frac{d^4k}{(2\pi)^4}\left[ig^2(T^c)_{nm}(T^c)_{mi}\delta_{jp}\eta^{\mu\nu}\right]\left[\frac{-i\left(\eta_{\mu\nu}-(1-\xi)\frac{k_\mu k_\nu}{k^2}\right)}{k^2}\right] \\
=& g^2\sum_{c=1}^{N^2}\int\frac{d^4k}{(2\pi)^4}\left(\frac{3}{k^2}+\frac{\xi}{k^2}\right)(T^cT^c)_{ni}\,\delta_{jp} \\
=& (-i)\times\frac{(3+\xi)g^2\Lambda^2}{16\pi^2}\delta_{jp}\sum_{c=1}^{N^2}(T^cT^c)_{ni}.
\end{aligned}
\tag{A.24}
$$

Then,

$$
M^{(2)}_{\text{quad}} = \frac{(3+\xi)g^2\Lambda^2 N}{32\pi^2}\delta_{jp}\delta_{ni}. \tag{A.25}
$$

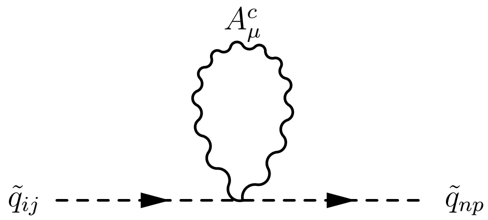


Figure A.7

For the diagram in Figure A.8, we obtain:

$$\begin{aligned}
-iM^{(3)} &= (-1)\sum_{G}^{4N^2}\sum_{k,l}^{N}\int\frac{d^4k}{(2\pi)^4}\left[\sqrt{2}g(\tilde{T}^G)_{nk}\delta_{pl}\right]\left[-g\sqrt{2}(\tilde{T}^{G\dagger})_{ki}\delta_{jl}\right] \\
&\qquad\times \mathrm{Tr}\left[\left(\tilde{C}^{-1}P_L\right)\left(-\frac{i}{\not{p}+\not{k}}\tilde{C}\right)\left(\tilde{C}^{-1}P_R\right)\left(-\frac{i}{\not{k}}\tilde{C}\right)\right] \\
&= -2g^2\int\frac{d^4k}{(2\pi)^4}\sum_{G=1}^{4N^2}\sum_{k=1}^{N}(\tilde{T}^{G\dagger})_{nk}(\tilde{T}^G)_{ki}\frac{\mathrm{Tr}\left[\left(\tilde{C}^{-1}P_L\right)\left((\not{k}+\not{p})\tilde{C}\right)\left(\tilde{C}^{-1}P_R\right)\left(\not{k}\tilde{C}\right)\right]}{k^2(k+p)^2}\delta_{jp} \\
&= -4g^2\int\frac{d^4k}{(2\pi)^4}\frac{(k+p)k}{k^2(k+p)^2}\sum_{G}^{4N^2}(\tilde{T}^{G\dagger}\tilde{T}^G)_{ni}\delta_{jp} \\
&= -4g^2\delta_{jp}\int\frac{d^4k}{(2\pi)^4}\frac{1}{k^2}\sum_{G}^{4N^2}(\tilde{T}^{G\dagger}\tilde{T}^G)_{ni} + \text{non-quadratically divergent terms.}
\end{aligned} \tag{A.26}$$

The sum in G actually reduces to

$$\sum_{G}^{4N^2}(\tilde{T}^{G\dagger}\tilde{T}^G)_{ni} = \sum_{p,q=1}^{N}\left(2M^{p,q\dagger}M^{p,q}\right)_{ni} = \frac{N}{2}\delta_{ni}. \tag{A.27}$$

With this, we obtain that the quadratic part of $-iM^{(3)}$ is

$$\begin{aligned}
-iM^{(3)}_{\text{quad}} &= -\,2g^2N\delta_{jp}\delta_{ni}\int\frac{d^4k}{(2\pi)^4}\frac{1}{k^2} \\
&= -\,2g^2\delta_{jp}\delta_{ni}\left(-i\frac{\Lambda^2}{16\pi^2}\right).
\end{aligned} \tag{A.28}$$

Then

$$M^{(3)}_{\text{quad}} = -\frac{g^2\Lambda^2N}{8\pi^2}. \tag{A.29}$$

Finally, from the diagram in Figure A.9, we have

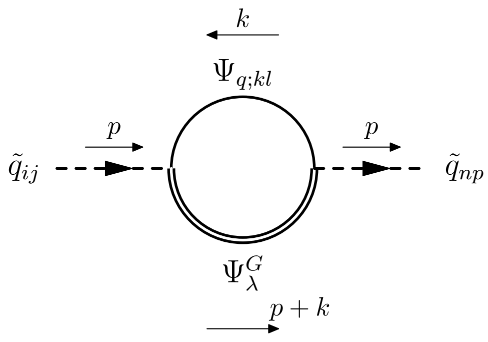


Figure A.8

$$
\begin{aligned}
-iM^{(4)} = &\frac{1}{2}\sum_{l,k=1}^{N}\sum_{c=1}^{N^2}\int\frac{d^4k}{(2\pi)^4}\left[-ig^2\left(T^c_{ni}T^c_{kk}\delta_{pj}\delta_{ll}+T^c_{nk}T^c_{ki}\delta_{pl}\delta_{lj}\right)\right]\left[\frac{i}{k^2}\right]\\
=&\frac{1}{2}g^2\left(\frac{\delta_{ni}}{\sqrt{2N}}\sqrt{\frac{N}{2}}\delta_{pj}N+\frac{N\delta_{ni}}{2}\delta_{pj}\right)\int\frac{d^4k}{(2\pi)^4}\frac{1}{k^2}\\
=&\frac{1}{2}g^2N\delta_{ni}\delta_{pj}\left(\frac{-i\Lambda^2}{16\pi^2}\right).
\end{aligned}
\tag{A.30}
$$

Then

$$
M^{(4)}_{\text{quad}} = \frac{g^2\Lambda^2 N}{32\pi^2}\delta_{ni}\delta_{pj}. \tag{A.31}
$$

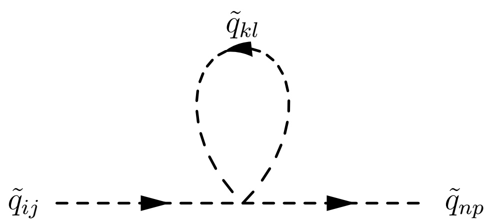


Figure A.9

A.2 Yukawa coupling

In section 6.1.2, we applied the orbifold procedure to the a supersymmetric Lagrangian containing the term

$$
\begin{aligned}
\mathcal{L} =& \mathrm{Tr}\left(\partial_\mu \tilde{q}_{12}^\dagger \partial^\mu \tilde{q}_{12} + i\bar{q}_{12}\bar{\sigma}^\mu \partial_\mu q_{12}\right) + \mathrm{Tr}\left(\partial_\mu \tilde{q}_{23}^\dagger \partial^\mu \tilde{q}_{23} + i\bar{q}_{23}\bar{\sigma}^\mu \partial_\mu q_{23}\right) \\
&+ \mathrm{Tr}\left(\partial_\mu \tilde{q}_{31}^\dagger \partial^\mu \tilde{q}_{31} + i\bar{q}_{31}\bar{\sigma}^\mu \partial_\mu q_{31}\right) \\
&- \lambda \mathrm{Tr}\left(\tilde{q}_{31} q_{12} q_{23} + \tilde{q}_{12} q_{23} q_{31} + \tilde{q}_{23} q_{12} q_{31}\right) + \mathrm{h.c.} \\
&- \lambda^2 \left[\mathrm{Tr}\left(\tilde{q}_{23}^\dagger \tilde{q}_{12}^\dagger \tilde{q}_{12} \tilde{q}_{23}\right) + \mathrm{Tr}\left(\tilde{q}_{31}^\dagger \tilde{q}_{23}^\dagger \tilde{q}_{23} \tilde{q}_{31}\right) + \mathrm{Tr}\left(\tilde{q}_{12}^\dagger \tilde{q}_{31}^\dagger \tilde{q}_{31} \tilde{q}_{12}\right)\right].
\end{aligned}
\tag{A.32}
$$

Fields that remain after orbifolding are:

$$
\tilde{q}_{12} = \begin{pmatrix} \tilde{q}_{1A,2A} & 0 \\ 0 & \tilde{q}_{1B,2B} \end{pmatrix}, \qquad q_{12} = \begin{pmatrix} 0 & q_{1A,2B} \\ q_{1B,2A} & 0 \end{pmatrix}
\tag{A.33}
$$

and analogous expressions for Q_{23} and Q_{31}. Let us calculate some relevant products before obtaining the daughter theory:

$$
\begin{aligned}
&\mathrm{Tr}\left(\tilde{q}_{23}^\dagger \tilde{q}_{12}^\dagger \tilde{q}_{12} \tilde{q}_{23}\right) \\
&= \mathrm{Tr}\left[\begin{pmatrix} \tilde{q}_{2A,3A}^\dagger \tilde{q}_{1A,2A}^\dagger & 0 \\ 0 & \tilde{q}_{2B,3B}^\dagger \tilde{q}_{1B,2B}^\dagger \end{pmatrix} \begin{pmatrix} \tilde{q}_{1A,2A} \tilde{q}_{2A,3A} & 0 \\ 0 & \tilde{q}_{1B,2B} \tilde{q}_{2B,3B} \end{pmatrix}\right] \\
&= \mathrm{Tr}\left(\tilde{q}_{2A,3A}^\dagger \tilde{q}_{1A,2A}^\dagger \tilde{q}_{1A,2A} \tilde{q}_{2A,3A}\right) + \mathrm{Tr}\left(\tilde{q}_{2B,3B}^\dagger \tilde{q}_{1B,2B}^\dagger \tilde{q}_{1B,2B} \tilde{q}_{2B,3B}\right),
\end{aligned}
\tag{A.34}
$$

$$
\begin{aligned}
\mathrm{Tr}\left(\tilde{q}_{12} q_{23} q_{31}\right) &= \mathrm{Tr}\left[\begin{pmatrix} \tilde{q}_{1A,2A} & 0 \\ 0 & \tilde{q}_{1B,2B} \end{pmatrix} \begin{pmatrix} 0 & q_{2A,3B} \\ q_{2B,3A} & 0 \end{pmatrix} \begin{pmatrix} 0 & q_{3A,1B} \\ q_{3B,1A} & 0 \end{pmatrix}\right] \\
&= \mathrm{Tr}\left(\tilde{q}_{1A,2A} q_{2A,3B} q_{3B,1A}\right) + \mathrm{Tr}\left(\tilde{q}_{1B,2B} q_{2B,3A} q_{3A,1B}\right).
\end{aligned}
\tag{A.35}
$$

And analogous expressions for the other quartic and trilinear terms in eq.(A.32). Using these results, the daughter Lagrangian is:

$$\begin{aligned}
\mathcal{L} = & \operatorname{Tr}\left(\partial_\mu \tilde{q}^\dagger_{1A,2A}\partial^\mu \tilde{q}_{1A,2A} + \partial_\mu \tilde{q}^\dagger_{1B,2B}\partial^\mu \tilde{q}_{1B,2B} + i\bar{q}_{1A,2B}\bar{\sigma}^\mu \partial_\mu q_{1A,2B} + i\bar{q}_{1A,2B}\bar{\sigma}^\mu \partial_\mu q_{1A,2B}\right) \\
& + \operatorname{Tr}\left(\partial_\mu \tilde{q}^\dagger_{2A,3A}\partial^\mu \tilde{q}_{2A,3A} + \partial_\mu \tilde{q}^\dagger_{2B,3B}\partial^\mu \tilde{q}_{2B,3B} + i\bar{q}_{2A,3B}\bar{\sigma}^\mu \partial_\mu q_{2A,3B} + i\bar{q}_{2A,3B}\bar{\sigma}^\mu \partial_\mu q_{2A,3B}\right) \\
& + \operatorname{Tr}\left(\partial_\mu \tilde{q}^\dagger_{3A,1A}\partial^\mu \tilde{q}_{3A,1A} + \partial_\mu \tilde{q}^\dagger_{3B,1B}\partial^\mu \tilde{q}_{3B,1B} + i\bar{q}_{3A,1B}\bar{\sigma}^\mu \partial_\mu q_{3A,1B} + i\bar{q}_{3A,1B}\bar{\sigma}^\mu \partial_\mu q_{3A,1B}\right) \\
& - \lambda \operatorname{Tr}\left(\tilde{q}_{1A,2A} q_{2A,3B} q_{3B,1A} + \tilde{q}_{1B,2B} q_{2B,3A} q_{3A,1B}\right) + \text{h.c.} \\
& - \lambda \operatorname{Tr}\left(\tilde{q}_{2A,3A} q_{3A,1B} q_{1B,2A} + \tilde{q}_{2B,3B} q_{3B,1A} q_{1A,2B}\right) + \text{h.c.} \\
& - \lambda \operatorname{Tr}\left(\tilde{q}_{3A,1A} q_{1A,2B} q_{2B,3A} + \tilde{q}_{3B,1B} q_{1B,2A} q_{2A,3B}\right) + \text{h.c.} \\
& - \lambda^2 \left[\operatorname{Tr}\left(\tilde{q}^\dagger_{2A,3A}\tilde{q}^\dagger_{1A,2A}\tilde{q}_{1A,2A}\tilde{q}_{2A,3A}\right) + \operatorname{Tr}\left(\tilde{q}^\dagger_{2B,3B}\tilde{q}^\dagger_{1B,2B}\tilde{q}_{1B,2B}\tilde{q}_{2B,3B}\right)\right] \\
& - \lambda^2 \left[\operatorname{Tr}\left(\tilde{q}^\dagger_{3A,1A}\tilde{q}^\dagger_{2A,3A}\tilde{q}_{2A,3A}\tilde{q}_{3A,1A}\right) + \operatorname{Tr}\left(\tilde{q}^\dagger_{3B,1B}\tilde{q}^\dagger_{2B,3B}\tilde{q}_{2B,3B}\tilde{q}_{3B,1B}\right)\right] \\
& - \lambda^2 \left[\operatorname{Tr}\left(\tilde{q}^\dagger_{1A,2A}\tilde{q}^\dagger_{3A,1A}\tilde{q}_{3A,1A}\tilde{q}_{1A,2A}\right) + \operatorname{Tr}\left(\tilde{q}^\dagger_{1B,2B}\tilde{q}^\dagger_{3B,1B}\tilde{q}_{3B,1B}\tilde{q}_{1B,2B}\right)\right].
\end{aligned} \tag{A.36}$$

We will focus on corrections to the mass of $\tilde{q}_{1A,2A}$. For that, we take the terms involving this field and write down their Feynman rules:

(i) From the Yukawa interaction terms in A.36, we have

$$\begin{aligned}
& -\lambda \operatorname{Tr}\left(\tilde{q}_{1A,2A} q_{2A,3B} q_{3B,1A}\right) + \text{h.c.} \\
& \quad = \lambda \operatorname{Tr}\left[\tilde{q}_{1A,2A}\Psi^T_{q,2A,3B}\tilde{C}^{-1}P_L\Psi_{q,3B,1A} + \tilde{q}^\dagger_{1A,2A}\Psi^T_{q,2A,3B}\tilde{C}^{-1}P_L\Psi_{q,3B,1A}\right].
\end{aligned} \tag{A.37}$$

The corresponding Feynman rules are shown in Figures A.10 and A.11.

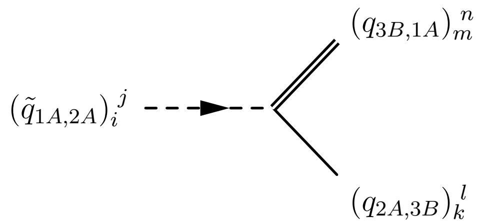


Figure A.10: Yukawa interaction with Feynman rule: $i\lambda\tilde{C}^{-1}P_L\delta_{jk}\delta_{lm}\delta_{ni}$

(ii) From the quartic scalar interaction terms in A.36 we have

$$-\lambda^2\left[\operatorname{Tr}\left(\tilde{q}^\dagger_{2A,3A}\tilde{q}^\dagger_{1A,2A}\tilde{q}_{1A,2A}\tilde{q}_{2A,3A}\right) + \operatorname{Tr}\left(\tilde{q}^\dagger_{1A,2A}\tilde{q}^\dagger_{3A,1A}\tilde{q}_{3A,1A}\tilde{q}_{1A,2A}\right)\right], \tag{A.38}$$

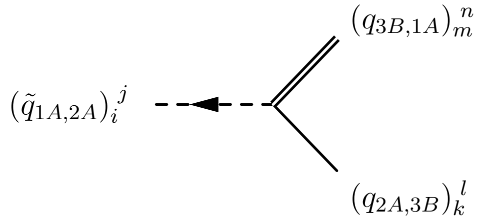


Figure A.11: Yukawa interaction with Feynman rule: $i\lambda\tilde{C}^{-1}P_R\delta_{jk}\delta_{lm}\delta_{ni}$.

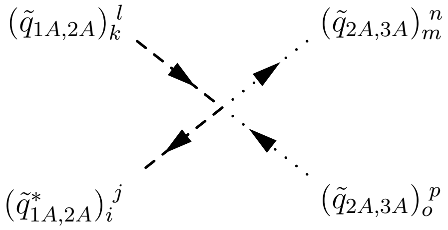


Figure A.12: Quartic scalar interaction with Feynman rule: $-i\lambda^2\delta_{mj}\delta_{ki}\delta_{to}\delta_{np}$.

which generates the Feynman rules shown in Figures A.12 and A.13.

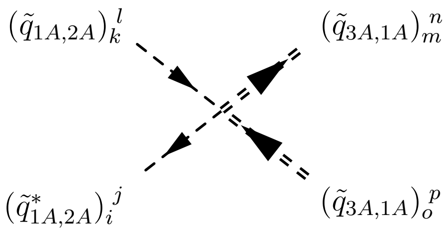


Figure A.13: Quartic scalar interaction with Feynman rule:$-i\lambda^2\delta_{mj}\delta_{ki}\delta_{to}\delta_{np}$.

Thus, with the listed Feynman rules, the one loop diagrams that contribute to the mass of $\tilde{q}_{1A,2A}$ are given by those shown in Figure A.14, A.15 and A.16.

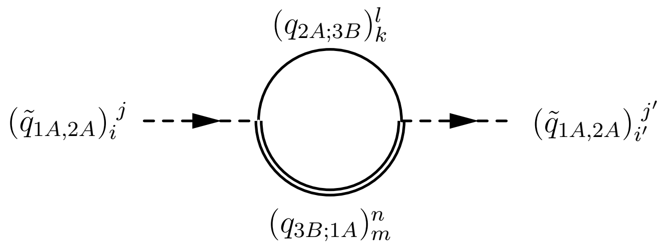


Figure A.14: One loop contribution to the mass of $1A\tilde{,}2A$ from the Yukawa interaction.

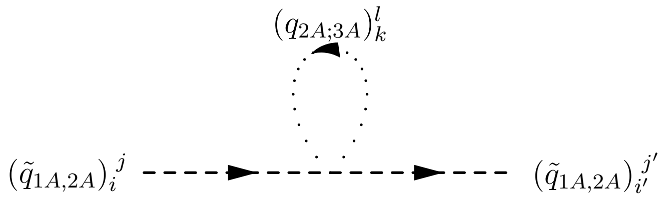


Figure A.15: One loop contribution to the mass of $\tilde{q}_{1A,2A}$ from the quartic scalar interaction $\tilde{q}_{1A,2A}$-$\tilde{q}_{2A,3A}$.

Let us calculate the contribution of each of them. For the diagram in Figure A.14 we have:

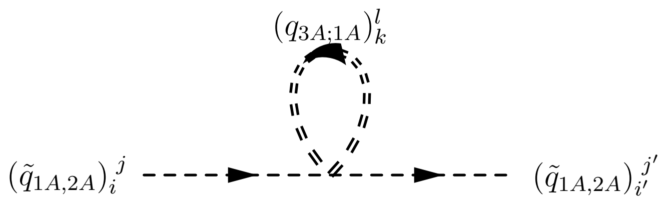


Figure A.16: One loop contribution to the mass of $\tilde{q}_{1A,2A}$ from the quartic scalar interaction $\tilde{q}_{1A,2A}$-$\tilde{q}_{3A,1A}$.

$$\begin{aligned}
-iM^{(1)} &= (-1)\sum_{k,l,m,n}^{N}\int\frac{d^4k}{(2\pi)^4}\left\{[i\lambda\delta_{jk}\delta_{lm}\delta_{ni}]\,[i\lambda\delta_{j'k}\delta_{lm}\delta_{ni'}]\right.\\
&\qquad\qquad\left.\times\ \mathrm{Tr}\left[\tilde{C}^{-1}P_L\left(\frac{-i}{\not{k}-\not{p}}\tilde{C}\right)(\tilde{C}^{-1}P_R)\left(\frac{-i}{\not{k}}\tilde{C}\right)\right]\right\}\\
&= -\lambda^2\delta_{ii'}\delta_{jj'}\int\frac{d^4k}{(2\pi)^4}\frac{\mathrm{Tr}[P_L(\not{k}-\not{p})P_R\not{k}]}{(k-p)^2k^2}\sum_{l,m}\delta_{lm}\delta_{lm}\\
&= -2\lambda^2N\int\frac{d^4k}{(2\pi)^4}\frac{1}{k^2}\delta_{ii'}\delta_{jj'}\\
&= -2\lambda^2\left(-i\frac{\Lambda^2}{16\pi^2}\right)\delta_{ii'}\delta_{jj'}.
\end{aligned} \tag{A.39}$$

Then

$$M^{(1)} = -\frac{\lambda^2N}{8\pi^2}. \tag{A.40}$$

For the diagram in Figure A.15 we obtain

$$\begin{aligned}
-iM^{(2)} &= \sum_{l,k=1}^{N}\int\frac{d^4k}{(2\pi)^4}\left[-i\delta_{kj'}\delta_{ii'}\delta_{kj}\delta_{ll}\right]\left[\frac{i\lambda^2}{k^2}\right]\\
&= \lambda^2\left(\sum_{l=1}^{N}\delta_{ll}\right)\int\frac{d^4k}{(2\pi)^4}\frac{1}{k^2}\delta_{ii'}\delta_{jj'}\\
&= -i\frac{\lambda N}{16\pi^2}.
\end{aligned} \tag{A.41}$$

Then

$$M^{(2)} = \frac{\lambda^2N}{16\pi^2}. \tag{A.42}$$

The diagram in Figure A.16 has the same form as that in Figure A.15. Then, its contribution is the same:

$$M^{(3)} = \frac{\lambda^2N}{16\pi^2}. \tag{A.43}$$

Appendix B

Radiative corrections in the 5-dimensional $SU(6)$ model

The $SU(6)$ ultraviolet theory described by the Lagrangian in 6.69, with the definitions in 6.70, takes the form

$$\begin{aligned}\mathcal{L}_5 = &\int d^4\theta \left[Q_A^\dagger Q_A + Q_A^{c\dagger} Q_A^c + U_A^\dagger U_A + U_A^{c\dagger} U_A^c + Q_B^\dagger Q_B + Q_B^{c\dagger} Q_B^c + U_B^\dagger U_B + U_B^{c\dagger} U_B^c\right] \\ &+ \int d^4\theta \left[Q_A^c \partial_5 Q_A + U_A^c \partial_5 U_A + Q_B^c \partial_5 Q_B + U_B^c \partial U_B + \text{h.c.}\right] \\ &+ \delta(y)\left\{\int d^4\theta H_U^\dagger H_U + \int d^2\theta \left[y_t H_U \cdot Q_A U_A + y_t H_U \cdot Q_B U_B + \text{h.c.}\right]\right\}.\end{aligned} \tag{B.1}$$

Written in terms of component fields, it takes the form

$$\begin{aligned}\mathcal{L}_5 = &\\ &\Big[(\partial^\mu \tilde{q}_A)^\dagger(\partial_\mu \tilde{q}_A) + i\bar{q}_A \bar{\sigma}^\mu \partial_\mu q_A + F_{Q,A}^\dagger F_{Q,A} + (\partial^\mu \tilde{q}_A^c)^\dagger(\partial_\mu \tilde{q}_A^c) + i\bar{q}_A^c \bar{\sigma}^\mu \partial_\mu q_A^c + F_{Q,A}^{c\dagger} F_{Q,A}^c \\ &+ (\partial^\mu \tilde{u}_A)^\dagger(\partial_\mu \tilde{u}_A) + i\bar{u}_A \bar{\sigma}^\mu \partial_\mu u_A + F_{U,A}^\dagger F_{U,A} + (\partial^\mu \tilde{u}_A^c)^\dagger(\partial_\mu \tilde{u}_A^c) + i\bar{u}_A^c \bar{\sigma}^\mu \partial_\mu u_A^c + F_{U,A}^{c\dagger} F_{U,A}^c \\ &+ (\partial^\mu \tilde{q}_B)^\dagger(\partial_\mu \tilde{q}_B) + i\bar{q}_B \bar{\sigma}^\mu \partial_\mu q_B + F_{Q,B}^\dagger F_{Q,B} + (\partial^\mu \tilde{q}_B^c)^\dagger(\partial_\mu \tilde{q}_B^c) + i\bar{q}_B^c \bar{\sigma}^\mu \partial_\mu q_B^c + F_{Q,B}^{c\dagger} F_{Q,B}^c \\ &+ (\partial^\mu \tilde{u}_B)^\dagger(\partial_\mu \tilde{u}_B) + i\bar{u}_B \bar{\sigma}^\mu \partial_\mu u_B + F_{U,B}^\dagger F_{U,B} + (\partial^\mu \tilde{u}_B^c)^\dagger(\partial_\mu \tilde{u}_B^c) + i\bar{u}_B^c \bar{\sigma}^\mu \partial_\mu u_B^c + F_{U,B}^{c\dagger} F_{U,B}^c\Big] \\ &+ \big[\tilde{q}_A^c \partial_5 F_{Q,A} - q_A^c \partial_5 q_A + F_{Q,A}^c \partial_5 \tilde{q}_A + \tilde{u}_A^c \partial_5 F_{U,A} - u_A^c \partial_5 u_A + F_{U,A}^c \partial_5 \tilde{u}_A \\ &\quad + \tilde{q}_B^c \partial_5 F_{Q,B} - q_A^c \partial_5 q_B + F_{Q,B}^c \partial_5 \tilde{q}_B + \tilde{u}_B^c \partial_5 F_{U,B} - u_B^c \partial_5 u_B + F_{U,B}^c \partial_5 \tilde{u}_B + \text{h.c.}\big] \\ &+ \delta(y)\Big[(\partial_\mu h_u)^\dagger(\partial^\mu h_u) + i\bar{\tilde{h}}_u \bar{\sigma}^\mu \partial_\mu \tilde{h}_u + F_H^\dagger F_H \\ &\quad + y_t \Big(h_u \cdot \tilde{q}_A F_{U,A} + h_u \cdot F_{Q,A} \tilde{u}_A + F_H \cdot \tilde{q}_A \tilde{u}_A - h_u \cdot q_A u_A - \tilde{h}_u \cdot \tilde{q}_A u_A - \tilde{h}_u \cdot q_A \tilde{u}_A + \text{h.c.}\Big) \\ &\quad + y_t \Big(h_u \cdot \tilde{q}_B F_{U,B} + h_u \cdot F_{Q,B} \tilde{u}_B + F_H \cdot \tilde{q}_B \tilde{u}_B - h_u \cdot q_B u_B - \tilde{h}_u \cdot \tilde{q}_B u_B - \tilde{h}_u \cdot q_B \tilde{u}_B + \text{h.c.}\Big)\Big].\end{aligned} \tag{B.2}$$

Here, the first bracket contains the usual $4D$ kinetic terms with the auxiliary fields $F_{Q,A}$, $F_{Q,A}^c$, $F_{U,A}$, $F_{U,A}^c$, $F_{Q,B}$, $F_{Q,B}^c$, $F_{U,B}$ and $F_{U,B}^c$ contained in the superfields Q_A,

Q^c_A, U_A, U^c_A, Q_B, Q^c_B, U_B and U^c_B respectively. Now, in order to eliminate these auxiliary fields, we replace their EOM

$$F_{Q,A} = \partial_5 \tilde{q}^{c*}_A - y_t h^*_u \tilde{u}^*_A \delta(y), \tag{B.3}$$

$$F^c_{Q,A} = -\partial_5 \tilde{q}^*_A, \tag{B.4}$$

$$F_{U,A} = \partial_5 \tilde{u}^{c*}_A - y_t h^*_u \tilde{q}^*_A \delta(y), \tag{B.5}$$

$$F^c_{U,A} = -\partial_5 \tilde{u}^*_A, \tag{B.6}$$

$$F_{Q,B} = \partial_5 \tilde{q}^{c*}_B - y_t h^*_u \tilde{u}^*_B \delta(y), \tag{B.7}$$

$$F^c_{Q,B} = -\partial_5 \tilde{q}^*_B, \tag{B.8}$$

$$F_{U,B} = \partial_5 \tilde{u}^{c*}_B - y_t h^*_u \tilde{q}^*_B \delta(y), \tag{B.9}$$

$$F^c_{U,B} = -\partial_5 \tilde{u}^*_B \tag{B.10}$$

into (B.2). The relevant terms for the one loop radiative corrections to the mass of H, in the action, are

$$\begin{aligned} S_5 \supset & \int d^5x \, [\text{5D kinetic terms}] \\ & - y_t \int d^5x \delta(y) \left[|h_u \cdot \tilde{q}_A|^2 + |h_u|^2 |\tilde{u}_A|^2 + h_u \cdot q_A u_A - \tilde{h}_u \cdot \tilde{q}_A u_A - \tilde{h}_u \cdot q_A \tilde{u}_A + \text{h.c.} \right] \\ & - y_t \int d^5x \delta(y) \left[|h_u \cdot \tilde{q}_B|^2 + |h_u|^2 |\tilde{u}_B|^2 + h_u \cdot q_B u_B - \tilde{h}_u \cdot \tilde{q}_B u_B - \tilde{h}_u \cdot q_B \tilde{u}_B + \text{h.c.} \right]. \end{aligned} \tag{B.11}$$

Expanding the fields in Kaluza-Klein modes according to their boundary conditions

$$\tilde{q}_A = \frac{1}{\sqrt{2\pi R}} \sum_{n=0}^{\infty} \tilde{q}_{An} \cos\left[\frac{(2n+1)y}{2R}\right], \tag{B.12}$$

$$\tilde{u}_A = \frac{1}{\sqrt{2\pi R}} \sum_{n=0}^{\infty} \tilde{u}_{An} \cos\left[\frac{(2n+1)y}{2R}\right], \tag{B.13}$$

$$q_A = \frac{1}{\sqrt{2\pi R}} \sum_{n=0}^{\infty} q_{An} \cos\left[\frac{ny}{R}\right], \tag{B.14}$$

$$u_A = \frac{1}{\sqrt{2\pi R}} \sum_{n=0}^{\infty} u_{An} \cos\left[\frac{ny}{R}\right], \tag{B.15}$$

$$\tilde{q}_B = \frac{1}{\sqrt{2\pi R}} \sum_{n=0}^{\infty} \tilde{q}_{Bn} \cos\left[\frac{ny}{R}\right], \tag{B.16}$$

$$\tilde{u}_B = \frac{1}{\sqrt{2\pi R}} \sum_{n=0}^{\infty} \tilde{u}_{Bn} \cos\left[\frac{ny}{R}\right], \tag{B.17}$$

$$q_B = \frac{1}{\sqrt{2\pi R}} \sum_{n=0}^{\infty} q_{Bn} \cos\left[\frac{(2n+1)y}{2R}\right], \tag{B.18}$$

$$u_B = \frac{1}{\sqrt{2\pi R}} \sum_{n=0}^{\infty} u_{Bn} \cos\left[\frac{(2n+1)y}{2R}\right] \tag{B.19}$$

gives us

$$\mathcal{L} = \mathcal{L}_{(0)} + \mathcal{L}^{(KK)}, \tag{B.20}$$

where $\mathcal{L}_{(0)}$ is the part of the relevant Lagrangian that contains the zero modes:

$$\begin{aligned} \mathcal{L}_{(0)} =& [\text{kinetic terms}] \\ & - y_t^2 |h_u \tilde{q}_{B0}|^2 - y_t^2 |h_u|^2 |\tilde{u}_B|^2 - (y_t h_u q_{A0} u_{A0} + \text{h.c.}) \end{aligned} \tag{B.21}$$

and $\mathcal{L}^{(KK)}$ is the part of the relevant Lagrangian that contains Kaluza-Klein modes:

$$\begin{aligned} \mathcal{L}^{(KK)} =& [\text{Kinetic terms}] \\ & + \sum_{n,m} \left[y_t h_u q_{Bn} u_{Bm} + y_t^2 |h_u|^2 |\tilde{q}_{An}|^2 + y_t^2 |h_u|^2 |\tilde{u}_{Am}|^2 \right. \\ & \qquad \left. + y_t h_u q_{An} u_{Am} + y_t^2 |h_u|^2 |\tilde{q}_{Bm}|^2 + y_t^2 |h_u|^2 |\tilde{u}_{Bm}|^2 \right] \\ & + \sum_{n} \left[y_t h_u q_{An} u_{A0} + y_t^2 |h_u|^2 |\tilde{q}_{Bn}|^2 + y_t^2 |h_u|^2 |\tilde{u}_{B0}|^2 \right] \\ & + \sum_{m} \left[y_t h_u q_{A0} u_{Am} + y_t^2 |h_u|^2 |\tilde{q}_{B0}|^2 + y_t^2 |h_u|^2 |\tilde{u}_{Bm}|^2 \right]. \end{aligned} \tag{B.22}$$

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