Archives

Ref.TH.1532-CERN

INCLUSIVE TWO-PARTICLE CORRELATIONS FROM THE THERMODYNAMIC MODEL

G. Ranft *)
CERN -- Geneva

and

J. Ranft
Sektion Physik, Karl-Marx-Universität
Leipzig

A B S T R A C T

Within the thermodynamic model two-particle correlations are due to the fact that the two particles can emerge from one or from two fireballs; these fireballs can behave as leading or non-leading fireballs.

To compute two-particle correlations from the thermodynamic model the decay chain multiplicity function - which depends on the fireball mass and gives the number of decay generations - has to be specified. Several choices are considered. They all have no effect on single-particle distributions, but lead to significantly different predictions for the fully integrated two-particle correlation $R^{(2)}$.

We compute the average number of fireballs produced.

The high energy behaviour of the one and two-fireball contributions to $R^{(2)}$ is obtained. Our model accounts for the change of sign of $R^{(2)}$ as function of energy as is shown by the data and asymptotically contains the multi-Regge model and the diffractive excitation model as two limiting cases.

---

*) On leave from Sektion Physik, Karl-Marx-Universität, Leipzig, D.D.R.

Ref.TH.1532-CERN
20 July 1972

1. INTRODUCTION

From the analysis of a large number of data, it is known that the thermodynamic model (TM) describes well [1)-5)] the shapes and energy dependence of single-particle spectra. Especially, this model gives very naturally the increase with increasing energy of single particle spectra

$$E\frac{d^3N}{d^3p} = \frac{E}{\sigma_{inel}}\frac{d^3\sigma}{d^3p}$$

in the pionization region [4)-6)], which has been detected experimentally in comparing data at accelerator and ISR energies [7)]. Also other models like, e.g., the multi-Regge model (MRM) and the diffractive excitation model (DEM) are successful in describing the main features of single particle distributions in the accelerator energy region. The investigation of inclusive multiparticle distributions and correlations is expected to eliminate some of these models and therefore lead to a better understanding of the high energy production mechanism. If many particle distributions and correlations are to be studied with the thermodynamic model, it will be necessary to specify some features of the model which do not lead to significant differences in single particle distributions, but which might change the two-particle distributions significantly. They thus lead to a better understanding of the model.

Due to the large number of variables, it is difficult to describe even two-particle correlations in full generality. Therefore it is instructive to study partially integrated correlation functions which give, e.g., the correlation only in terms of the rapidities. Such a correlation function $C^{(2)}(y_1^*,y_2^*)$ has been studied with the multiperipheral thermodynamic model (MPTM) [8)] and compared successfully with $K^+p \to \pi^-\pi^-$ + anything data at 12 GeV/c [9),10)]. In Ref. 9) only the distribution of two pions emerging from the same fireball were considered. In the reaction studied at 12 GeV/c this is justified because in most events only one fireball will be produced at this energy and furthermore no through-going particles contribute. At higher energy and different reactions, more contributions have to be taken into account.

Here we write down all possible terms to the inclusive two-particle distribution in the thermodynamic model. Mainly the fully integrated distribution function and correlation and their energy dependence are studied.

The thermodynamic single-particle spectra have scaling behaviour and a flat central plateau at high s. This leads in the asymptotic limit to logarithmically increasing average multiplicities, $\langle n\rangle \sim \ln s$.

For the two-particle distribution according to the TM, several possible subdivisions of the rapidity function $G(\eta, \eta_\pm)$ [4] into the velocity weight function and the decay chain multiplicity function lead to widely different behaviour of the average value $\langle n(n-1)\rangle$ which might grow asymptotically like $\sim \ln s$, $\sim \ln^2 s$ or $\sim s^q$ with $0 < q \leq \frac{1}{2}$.

In Section 2 we discuss the single particle spectra which show scaling behaviour and a flat central plateau. This behaviour is not changed by several possible choices for the subdivision of the rapidity function $G(\eta)$ into the velocity weight function $F(\lambda, \gamma_\pm)$ of the fireballs and the decay chain multiplicity function $q(M_F)$.

In Section 3 the two-particle distribution is defined and all contributing mechanisms according to the TM, such as, one and two-fireball terms, are given as distributions and in fully integrated form. The general formalism includes correlations between newly produced particles, between leading and non-leading particles and between two leading particles.

In Section 4 the asymptotic behaviour of all contributions to the two-particle distribution and the integrated two-particle function is obtained.

In Section 5 the integrated two-particle correlations are evaluated at finite energies and compared with data for $\pi^-\pi^-$ production in collisions of two positively charged hadrons. Our model reproduces the change of sign of the two-particle correlation as a function of the energy.

2. INCLUSIVE SINGLE-PARTICLE SPECTRA IN THE THERMODYNAMIC MODEL

2.1 Inclusive single-particle spectra defined in terms of rapidity variables

Fireballs are excited hadrons which decay within the statistical thermodynamic bootstrap model [11),12),2)] in their rest frame isotropically according to

$$\frac{d^3N'}{d^3p'} = q(M_F)\, f_i(E_i', T(M_F)) \tag{2.1}$$

Here $f_i(E_i', T(M_F))$ is the Planck distribution [11]

$$f_i(E_i', T(M_F)) = \frac{V z_i}{(2\pi)^3}\left[\exp\left(\frac{E_i'}{T(M_F)}\right) \pm 1\right]^{-1} \quad \begin{cases} + \text{ Fermions} \\ - \text{ Bosons} \end{cases} \tag{2.2}$$

E'_i is the energy of particle i in the rest frame of the fireball from which it emerges. z_i is a statistical weight factor and V is the hadron volume. The temperature T of the fireball is connected with its mass M_F. $q(M_F)$ is called decay chain multiplicity function [2]. It accounts for the fact that the fireball does not decay in one step into the final stable hadrons. $q(M_F)$ gives the average number of generations in the decay of a fireball with mass M_F.

The single-particle spectrum in the centre-of-mass system is obtained by a superposition of decaying fireballs moving with different velocities in the c.m.s.,

$$\lambda = (\mathrm{sign}\,\beta)\,\frac{\gamma - 1}{\gamma_\pm - 1} \qquad -1 \leq \lambda \leq +1 \tag{2.3}$$

The Lorentz parameters γ and $\gamma_\pm$ are those of the fireball and the incoming particle a in forward (γ_+) and b in backward (γ_-) direction, respectively. For a newly produced particle 1 the single-particle distribution is

$$\rho^{(1)}(\vec{p}_1^{\,*}) = f^{(1)}(\vec{p}_1^{\,*}) = \frac{E_1^*}{\sigma_{inel}}\,\frac{d^3\sigma^*}{d^3p_1^*} = E_1^*\,\frac{d^3N^*}{d^3p_1^*} = \frac{d^3N^*}{dy_1^*\,d^2p_{\perp 1}} \tag{2.4}$$

$$= \int_{-1}^{1} d\lambda\, F(\lambda,\gamma_\pm)\, q(M_F(\lambda,\gamma_\pm))\left[E_1' f(E_1', T(M_F))\right] \tag{2.5}$$

c.m.s. variables are denoted by a star. E'_i has to be expressed in terms of c.m.s. quantities. The fireball mass M_F might depend on the velocity λ of the fireball and on $\gamma_\pm$.

We introduce the rapidity of particle i

$$y_i^* = \cosh^{-1}\frac{E_i^*}{\mu_i}\;, \qquad \mu_i = \sqrt{m_i^2 + p_{\perp i}^2} \tag{2.6}$$

and the rapidity η of the fireball in the c.m.s.

$$\eta = \cosh^{-1}\gamma$$
$$= (\mathrm{sign}\,\lambda)\cosh^{-1}\left[1 + |\lambda|(\gamma_\pm - 1)\right] \qquad (2.7)$$

The energy E_i' of particle i is

$$E_i' = \mu_i \cosh(y_i^* - \eta) \qquad (2.8)$$

The velocity weight function has a slight energy dependence if single-particle distributions do not show a dip at $y^* \approx 0$ [2]. It is, in this case, for an asymmetric initial state [4]

$$F(\lambda, \gamma_\pm) = \frac{\tilde{F}(\lambda)}{|\tanh\eta|}$$
$$= \tilde{F}(\lambda)\cdot\frac{\lambda(\gamma_\pm - 1) + 1}{\sqrt{\lambda^2(\gamma_\pm - 1)^2 + 2\lambda(\gamma_\pm - 1)}} \qquad (2.9)$$

A good parametrization of $\tilde{F}(\lambda)$ is [2].

$$\tilde{F}(\lambda) = N\exp(-a|\lambda|) \quad , \qquad a = 0.76 \qquad (2.10)$$

N normalizes this function to 1 in the ranges $-1 \leq \lambda \leq 0$ and $0 \leq \lambda \leq +1$, respectively. We neglect the term ± 1 in $f(E_1', T(M_F))$, change the integration variable in (2.5) to η,

$$\lambda = (\mathrm{sign}\,\eta)\cdot\frac{\cosh\eta - 1}{\cosh\eta_\pm - 1} \quad , \qquad \eta_\pm = \cosh^{-1}\gamma_\pm$$
$$d\lambda = \frac{\sinh\eta}{\cosh\eta_\pm - 1}\,d\eta \qquad (2.11)$$

and have

$$\frac{d^3N^*}{dy_1^*\,d^2p_{\perp 1}} = \frac{V z_1}{(2\pi)^3}\int_{-\eta_-}^{\eta_+} d\eta\, G(\eta, \eta_\pm)\,\mu_1\cosh(y_1^* - \eta)\exp\left(-\frac{\mu_1}{T(M_F)}\cosh(y_1^* - \eta)\right) \qquad (2.12)$$

The function

$$G(\eta,\eta_\pm) = F(\lambda,\gamma_\pm(\eta_\pm))\, q(\lambda,\gamma_\pm(\eta_\pm))\,\frac{d\lambda}{d\eta} \tag{2.13}$$

is the only function of the TM which has to be determined from data on single particle distributions. Single-particle spectra - subject to experimental errors - do practically not allow to determine the functions $F(\lambda, \gamma_\pm(\eta_\pm))$ and $q(\lambda, \gamma_\pm(\eta_\pm))$ separately.

2.2 Different possible choices for the decay chain multiplicity function and velocity weight function

In Ref. 2) the relation

$$q = q_0\,\frac{\varepsilon(\lambda,\gamma_0)}{\varepsilon_0}\,, \qquad \varepsilon(\lambda,\gamma_0) = \varepsilon_0\gamma_0/\gamma$$

has been used for pp collisions. This has been generalized in Ref. 4) for asymmetric initial particles to

$$\begin{aligned} q(M_F) = q(\lambda;\gamma_\pm) &= q_0\,\frac{M_F^{max} - m_0}{m_p} \\ q_0 &\approx 1.65 \\ m_0 = m_a &\quad \text{for} \quad \lambda \geq 0 \\ m_0 = m_b &\quad \text{for} \quad \lambda < 0 \end{aligned} \tag{2.14}$$

for the dependence of the decay chain multiplicity on the mass M_F of the fireball. In (2.14) M_F^{max} is the maximal kinematically allowed mass of a fireball moving with λ ,

$$\begin{aligned} M_F^{max}(\lambda) &= \sqrt{s}\left(\gamma - \sqrt{(\gamma^2-1) + \frac{m_p^2}{s}}\right) \\ &\approx \sqrt{s}\left(\gamma - \sqrt{\gamma^2-1}\right) = \sqrt{s}\, e^{-|\eta|} = M_F^{max}(s,\eta) \end{aligned} \tag{2.15}$$

It has been shown [4] that the rapidity function (2.13) can be written in the form

$$\begin{aligned} G(\eta,\eta_\pm) &= \frac{\sinh\eta}{\cosh\eta_\pm - 1}\, F(\lambda,\gamma_\pm(\eta_\pm))\, q(\lambda,\gamma_\pm(\eta_\pm)) \\ &\approx C\, \exp\left[-a\cdot\exp(-|\eta-\eta_\pm|)\right]\left(1 - \frac{2m_0}{\sqrt{s}}\sinh\eta\right) \end{aligned} \tag{2.16}$$

$+$ for $\eta \geq 0$ and $m_o = m_a$

$-$ for $\eta < 0$ and $m_o = m_b$

and

$$C = q_o \cdot N \tag{2.17}$$

Calculating the decay chain multiplicity function $q(\lambda, \gamma_\pm)$, (2.14), with the maximal allowed fireball mass M_F^{max} means that we only consider the production of <u>one</u> fireball. If the production of more than one fireball is to be allowed, an average fireball mass M_F smaller than M_F^{max} has to be used in calculating $q(\lambda, \gamma_\pm) = q(\eta, \eta_\pm)$. The average fireball mass M_F might depend on the total energy s and on λ or η. $M_F(s, \eta)$ might be chosen in one of the following forms *)

i)

$$M_F^{(i)}(s,\eta) = \begin{cases} M_c \left(\frac{\sqrt{s}}{M_c} e^{-|\eta|} \right)^w & 0 \leq w \leq 1 \quad \text{for } |\eta| \leq \eta_{L\pm}(s, M_c) \text{ where } M_F^{(i)}(s,\eta) < M_F^{max} & (2.18a) \\ M_F^{max} & \text{otherwise} & (2.18b) \end{cases}$$

ii)

$$M_F^{(ii)}(s,\eta) = \begin{cases} M_c \ln \left(\frac{\sqrt{s}}{M_c} e^{-|\eta|} \right) & \text{for } \frac{\sqrt{s}}{M_c} \gg 1 \text{ if } M_F^{(ii)}(s,\eta) < M_F^{max} & (2.19a) \\ M_F^{max} & \text{otherwise} . & (2.19b) \end{cases}$$

In case i) for $w = 0$ the average mass of all fireballs produced is constant, $M_F = M_C$. For $w = 1$ the fireball mass is the maximal one, $M_F = M_F^{max}$, and only one fireball can be produced. Our choice of $0 \leq w \leq 1$ interpolates between these two limiting situations. Case ii) gives the slowest possible increase of the fireball mass M_F with s.

---

*) Instead of (2.18a) a more general parametrization would be

$$M_F^{(i)}(s,\eta) = M_c \left(\frac{\sqrt{s}}{M_c} \right)^u e^{-|\eta| v}$$

$0 \leq u \leq 1$, $0 \leq v \leq 1$. For simplicity we use (2.18a).

Because the condition $\sqrt{s}/M_C \gg 1$ can only be met at higher energies in Section 5 we shall not give numerical results for the case ii), although we discuss the asymptotic behaviour analytically.

Generally, we expect for fixed λ and s a distribution of fireball masses. For simplicity we perform here all calculations only for the average mass of a fireball moving with λ.

Certainly, these choices i) and ii) should have no effect on single-particle distributions. Therefore the function $G(\eta, \eta_\pm)$, Eq. (2.13) should not be affected by our choice for $q(M_F)$. This leads us to a redefinition of the function $F(\lambda, \gamma_\pm(\eta_\pm))$. Denoting by $\hat{q}(M_F) = \hat{q}(\lambda, \gamma_\pm(\eta_\pm))$ the decay chain multiplicity function

$$\hat{q}(M_F) = \hat{q}(\lambda, \gamma_\pm) = \hat{q}(s,\eta) = q_0 \frac{M_F(s,\eta) - m_0}{m_p} \tag{2.20}$$

in case i) $$\hat{q}(s,\eta) = q^{(i)}(s,\eta) = \frac{q_0}{m_p}\left[M_c\left(\frac{\sqrt{s}}{M_c} e^{-|\eta|}\right)^w - m_0\right] \tag{2.21}$$

in case ii) $$\hat{q}(s,\eta) = q^{(ii)}(s,\eta) = \frac{q_0}{m_p}\left[M_c\left(\ln\left(\frac{\sqrt{s}}{M_c} e^{-|\eta|}\right)\right) - m_0\right] \tag{2.22}$$

the new velocity weight function becomes

$$\hat{F}(\lambda, \gamma_\pm(\eta_\pm)) = F(\lambda, \gamma_\pm) \frac{q(\lambda, \gamma_\pm)}{\hat{q}(\lambda, \gamma_\pm)} \tag{2.23}$$

In case i) it is

$$\hat{F}(\lambda, \gamma_\pm(\eta_\pm)) = F(\lambda, \gamma_\pm) \frac{\left[M_c \cdot \frac{\sqrt{s}}{M_c}(\gamma - \sqrt{\gamma^2 - 1}) - m_0\right]}{\left[M_c\left(\frac{\sqrt{s}}{M_c}(\gamma - \sqrt{\gamma^2 - 1})\right)^w - m_0\right]} \tag{2.24}$$

which for high enough s is

$$\hat{F}(\lambda, \gamma_\pm(\eta_\pm)) \approx F(\lambda, \gamma_\pm)\left[\frac{\sqrt{s}}{M_c}(\gamma - \sqrt{\gamma^2 - 1})\right]^{1-w} \tag{2.25}$$

In case ii) it is

$$\hat{F}(\lambda,\gamma_\pm) = F(\lambda,\gamma_\pm)\,\frac{\left[M_c \frac{\sqrt{s}}{M_c}\left(\gamma-\sqrt{\gamma^2-1}\right)-m_0\right]}{\left[M_c \ln\left\{\frac{\sqrt{s}}{M_c}\left(\gamma-\sqrt{\gamma^2-1}\right)\right\}-m_0\right]} \tag{2.26}$$

2.3 Single-particle spectra for through-going particles

Diffractively produced fireballs of low mass (resonances or particles produced via diffraction dissociation) can lead to leading (through-going) particles.

In the TM the spectrum of a through-going particle 1 is given by several contributions [13], the most important of which is

$$E_1^* \frac{d^3N^*}{d^3p_1^*} = \frac{d^3N^*}{dy_1^* d^2p_{\perp 1}} =$$

$$= \frac{Vz_1}{(2\pi)^3}\int_{-1}^{1} d\lambda\, F_0(\lambda,\gamma_\pm)\left[\frac{E_1' f(E_1', T(M_F))}{N_I(T(M_F))}\right] \tag{2.27}$$

with γ_+ for $\lambda \geq 0$

γ_- for $\lambda < 0$

where $N_I(T(M_F))$ is given in Ref. 13). The decay chain multiplicity function is $q(\lambda,\gamma_\pm) \equiv 1$ in this case. The function $F_0(\lambda,\gamma_\pm)$ is parametrized as [2]

$$F_0(\lambda,\gamma_\pm) = N \exp(b|\lambda|) \quad , \quad b = 4.78 \tag{2.28}$$

As in Section 2.1, Eq. (2.12), the spectrum can be obtained in rapidity formulation with a function $G_0(\eta,\eta_\pm)$ defined as

$$G_0(\eta,\eta_\pm) = \frac{F_0(\lambda,\gamma_\pm)}{N_I(M_F(\lambda,\gamma_\pm))}\cdot\frac{d\lambda}{d\eta} \tag{2.29}$$

$G_0(\eta,\eta_\pm)$ has two maxima in the fragmentation region near η_+ and η_-.

2.4 The asymptotic behaviour of the thermodynamic single-particle spectra

The asymptotic behaviour of single-particle spectra for newly produced particles, as well as for through-going particles, has been discussed in Ref. 2), 4) and 5). It has been shown that the TM leads to

- scaling behaviour and limiting fragmentation at high s;
- a non-vanishing central plateau in the rapidity; and
- that the asymptotic value of the spectra at small x is reached from below [5].

The model gives a fair understanding [4)-6)] of the deviations from scaling at presently available energy which have been detected between 3.7 and 1500 GeV/c [7]. Furthermore, it has been shown [2)-4),6)] that $G(\eta, \eta_\pm)$ is a universal function with respect to

- the energy dependence between 3.7 and 1500 GeV/c; and
- the kind of primary and secondary hadrons in the collision.

It follows from this behaviour that the multiplicity grows asymptotically like $\ln s$ for newly produced particles and remains constant for through-going particles giving the asymptotic result

$$\langle n \rangle \sim \ln s + 2 \tag{2.30}$$

3. INCLUSIVE TWO-PARTICLE DISTRIBUTIONS AND CORRELATIONS ACCORDING TO THE THERMODYNAMIC MODEL

3.1 Definitions

The invariant distribution for single-particle production is

$$f^{(1)}(\vec{p}^*) = E^* \frac{d^3 N^*}{d^3 p^*} = \frac{E^*}{\sigma_{inel}} \frac{d^3\sigma^*}{d^3 p^*} \tag{3.1}$$

The stars denote the quantities in the c.m.s. The two-particle distribution is

$$f^{(2)}(\vec{p}_1^*, \vec{p}_2^*) = E_1^* E_2^* \frac{d^6 N^*}{d^3 p_1^* d^3 p_2^*} = \frac{1}{\sigma_{inel}} E_1^* E_2^* \frac{d^6 \sigma^*}{d^3 p_1^* d^3 p_2^*} \tag{3.2}$$

For convenience, we define furthermore the correlation functions [14)]

$$\rho^{(1)}(\vec{p}^*) = f^{(1)}(\vec{p}^*) \qquad (3.3)$$

$$\rho^{(2)}(\vec{p}_1^*, \vec{p}_2^*) = f^{(2)}(\vec{p}_1^*, \vec{p}_2^*) - \rho^{(1)}(\vec{p}_1^*)\,\rho^{(1)}(\vec{p}_2^*) \qquad (3.4)$$

$\rho^{(2)}(\vec{p}_1^*, \vec{p}_2^*)$ is the two-particle correlation function. The fully integrated single-particle distribution is

$$R^{(1)} \equiv F^{(1)} \equiv \int \frac{d^3p^*}{E^*} f^{(1)}(\vec{p}^*) \equiv \int \frac{d^3p^*}{E^*} \rho^{(1)}(\vec{p}^*) \equiv \langle n \rangle \qquad (3.5)$$

which is by definition equal to the average multiplicity $\langle n \rangle$. The fully integrated two-particle distribution is

$$F^{(2)} = \iint \frac{d^3p_1^*}{E_1^*} \frac{d^3p_2^*}{E_2^*} f^{(2)}(\vec{p}_1^*, \vec{p}_2^*) \qquad (3.6)$$

$$= \begin{cases} \langle n_i(n_i-1)\rangle & \text{for identical particles 1 and 2} \quad (3.7a) \\ \langle n_i n_j \rangle & \text{for non-identical particles 1 and 2} \quad (3.7b) \end{cases}$$

The fully integrated two-particle correlation is

$$R^{(2)} = \iint \frac{d^3p_1^*}{E_1^*} \frac{d^3p_2^*}{E_2^*} \rho^{(2)}(\vec{p}_1^*, \vec{p}_2^*) \qquad (3.8)$$

$$= \begin{cases} \langle n_i(n_i-1)\rangle - \langle n_i \rangle^2 & \text{for identical particles 1 and 2} \quad (3.9a) \\ \langle n_i n_j \rangle - \langle n_i \rangle \langle n_j \rangle & \text{for non-identical particles 1 and 2.} \quad (3.9b) \end{cases}$$

For identical particles $R^{(2)}$ is related to the dispersion of the multiplicity distribution

$$R^{(2)} = D^2 - \langle n \rangle \tag{3.10}$$

Due to

$$\frac{d^3p^*}{E^*} = dy^* d^2p_\perp = \pi\, dy^* dp_\perp^2 \tag{3.11}$$

it is convenient to study correlations starting with the rapidity formulation. We call

$$C^{(2)}(y_1^*, y_2^*) = \iint d^2p_{\perp 1}\, d^2p_{\perp 2}\, \rho^{(2)}(\vec{p}_1^{\,*}, \vec{p}_2^{\,*})$$

the two-particle rapidity correlation function.

3.2 Mechanisms for two-particle production in the thermodynamic model

We consider a multiperipheral thermodynamic model [8] in which particle clusters are produced along a multiperipheral chain via Pomeron and/or Regge exchange. We do not specify the internal cluster variables but consider the clusters as fireballs. Their decay properties are described by statistical thermodynamics [11),13]. The average number $\langle n_F \rangle$ of the produced fireballs is a function of s and depends furthermore on the parameters M_C and w in (2.21) and (2.22). Our parametrizations (2.20) to (2.25) allow to compute from kinematics and the normalization of $F(\lambda, \gamma_\pm)$, Eq. (2.9), the average number of fireballs $\langle n_F \rangle$ as function of s *).

At sufficiently high s, such that two or more fireballs are produced, the following mechanisms contribute to the two-particle distribution $f^{(2)}(\vec{p}_1^{\,*}, \vec{p}_2^{\,*})$

i) particles 1 and 2 emerge from the same fireball;

ii) particles 1 and 2 emerge from different fireballs.

We denote by D a light fireball (or resonance) with such quantum numbers that it can be produced diffractively. Furthermore we use the index F for a heavy fireball which can be produced diffractively or by other mechanisms. Then we have the following contributions to $f^{(2)}(\vec{p}_1^{\,*}, \vec{p}_2^{\,*})$

---

*) An approach to determine $\langle n_F \rangle$ from the DEM was described by K. Gottfried and O. Kofoed-Hansen, CERN preprint TH.1514 (1972).

- according to i) $$f_F^{(2)}(\vec{p}_1^*, \vec{p}_2^*) \tag{3.12}$$

- according to ii) $$f_{FF}^{(2)}(\vec{p}_1^*, \vec{p}_2^*) + f_{FD}^{(2)}(\vec{p}_1^*, \vec{p}_2^*) + f_{DD}^{(2)}(\vec{p}_1^*, \vec{p}_2^*) \tag{3.13}$$

From this we derive the fully integrated two-particle distribution

$$F^{(2)} = F_F^{(2)} + F_{FF}^{(2)} + F_{FD}^{(2)} + F_{DD}^{(2)} \tag{3.14}$$

The two-particle correlation is

$$\begin{aligned} \varrho^{(2)}(\vec{p}_1^*, \vec{p}_2^*) &= f_F^{(2)}(\vec{p}_1^*, \vec{p}_2^*) \\ &+ f_{FF}^{(2)}(\vec{p}_1^*, \vec{p}_2^*) + f_{FD}^{(2)}(\vec{p}_1^*, \vec{p}_2^*) + f_{DD}^{(2)}(\vec{p}_1^*, \vec{p}_2^*) \\ &- \varrho^{(1)}(\vec{p}_1^*) \cdot \varrho^{(1)}(\vec{p}_2^*) \end{aligned} \tag{3.15}$$

The fully integrated two-particle correlation for identical particles is

$$\begin{aligned} R^{(2)} &= \langle n(n-1) \rangle_F \\ &+ \langle n(n-1) \rangle_{FF} + \langle n(n-1) \rangle_{FD} + \langle n(n-1) \rangle_{DD} \\ &- \langle n \rangle^2 \end{aligned} \tag{3.16}$$

For non-identical particles a similar expression with (3.9b) results.

In the following we write down the various contributions to $f^{(2)}(\vec{p}_1^*, \vec{p}_2^*)$, we obtain $F^{(2)}$ and discuss the high energy behaviour of $F^{(2)}$ and $R^{(2)}$.

3.3 One fireball contribution

The two particles 1 and 2 emerge from a fireball statistically unconditional (uncorrelated) as long as the fireball is heavy enough such that energy, momentum, charge, strangeness conservation, etc., always remain satisfied without taking the kinematic limits imposed by the conservation laws into account explicitly.

In the rest frame of the decaying fireball with temperature $T(M_F) = T(\lambda)$ the two-particle spectrum is

$$\frac{d^6 N'_{12,F}}{d^3p'_1\, d^3p'_2} = q^2(\lambda,\gamma_\pm)\, f_1(E'_1, T(M_F))\, f_2(E'_2, T(M_F)) \tag{3.17}$$

In the c.m.s. the two-particle spectrum due to the superposition of fireballs moving with velocity λ is

$$E_1^* E_2^* \frac{d^6 N^*_{12,F}}{d^3p_1^*\, d^3p_2^*} = \frac{d^6 N^*_{12,F}}{dy_1^*\, dy_2^*\, d^2p_{1\perp}\, d^2p_{2\perp}} =$$

$$= \frac{V^2 z_1 z_2}{(2\pi)^6} \int_{-1}^{1} d\lambda\, F(\lambda,\gamma_\pm)\, q^2(\lambda,\gamma_\pm) \left[E'_1 E'_2\, f_1(E'_1, T(\lambda))\, f_2(E'_2, T(\lambda))\right] \tag{3.18}$$

All symbols have been explained already in Section 2. As in Section 2.1, the rapidities y_1^* and y_2^* of particles 1 and 2 in the c.m.s. as well as the fireball rapidity η [Eqs. (2.6) and (2.7)] can be introduced. As before, we neglect the ± 1 in $f_1(E'_1, T(\lambda))$ and $f_2(E'_2, T(\lambda))$. We introduce the rapidity weight function $G(\eta, \eta_\pm)$ of Eq. (2.13) and the function $\hat{q}(\eta, \eta_\pm)$ of Eqs. (2.21) and (2.22). Then we have

$$\frac{d^6 N^*_{12,F}}{dy_1^*\, dy_2^*\, d^2p_{1\perp}\, d^2p_{2\perp}} = \frac{V^2 z_1 z_2}{(2\pi)^6} \int_{-\eta_-}^{\eta_+} d\eta\, G(\eta,\eta_\pm)\cdot \hat{q}(\eta,\eta_\pm)\cdot$$

$$\cdot \prod_{i=1,2} \mu_i \cosh(y_i^* - \eta)\, \exp\left(-\frac{\mu_i}{T(\eta,\eta_\pm)} \cosh(y_i^* - \eta)\right) \tag{3.19}$$

Extending the $p_{i\perp}$ integrations $(i = 1,2)$ to infinity, we obtain the rapidity distribution

$$C_F^{(2)}(y_1^*, y_2^*) = \frac{d^2 N^*_{12,F}}{dy_1^*\, dy_2^*} = \frac{V^2 z_1 z_2}{(8\pi^2)^2} \int_{-\eta_-}^{\eta_+} d\eta\, G(\eta,\eta_\pm)\cdot \hat{q}(\eta,\eta_\pm)$$

$$\cdot \prod_{i=1,2} \cosh(y_i^* - \eta)\; I\left(\frac{\cosh(y_i^* - \eta)}{T(\eta,\eta_\pm)}\right) \tag{3.20}$$

Here we have abbreviated

$$I(A) = \int_0^\infty dp_{i\perp}^2 \, \mu_i \, e^{-\mu_i A} =$$

$$= e^{-m_i A} \left(\frac{2 m_i^2}{A} + \frac{2 m_i}{A^2} + \frac{4}{A^3} \right) \tag{3.21}$$

where μ_i and m_i are related by (2.6).

The one-fireball contribution $F_F^{(2)}$ to the fully integrated two-particle distribution $F^{(2)}$ is obtained by threefold integration

$$F_F^{(2)} = \frac{V^2 z_1 z_2}{(8\pi^2)^2} \int_{-y_1^{*\,max}}^{y_1^{*\,max}} dy_1^* \int_{-y_2^{*\,max}}^{y_2^{*\,max}} dy_2^* \int_{-\eta_-}^{\eta_+} d\eta \, G(\eta,\eta_\pm) \, \hat{q}(\eta,\eta_\pm)$$

$$\cdot \prod_{i=1,2} \cosh(y_i^* - \eta) \, I\left(\frac{\cosh(y_i^* - \eta)}{T(\eta,\eta_\pm)} \right) \tag{3.22}$$

The kinematic limits are

$$y_i^{*\,max} = \cosh^{-1} \left(\frac{s + m_i^2 - (m_a + m_b + m_i)^2}{2 m_i \sqrt{s}} \right) \tag{3.23}$$

$$\sim \ln \frac{\sqrt{s}}{2 m_i} \tag{3.24}$$

3.4 The two-fireball contribution

Here we consider the case that the two particles 1 and 2 emerge from two different fireballs moving with λ_1 and λ_2, respectively. The two-particle distribution in the c.m.s. is

$$\frac{d^6 N^*_{12,FF}}{dy_1^* dy_2^* d^2p_{1\perp} d^2p_{2\perp}} = \frac{\langle n_F(n_F-1)\rangle}{\langle n_F\rangle^2} \frac{V^2 z_1 z_2}{(2\pi)^6} \int_{-1}^{1} d\lambda_1 \int_{-1}^{1} d\lambda_2 \cdot F(\lambda_1,\gamma_\pm) \cdot$$

$$\cdot F(\lambda_2,\gamma_\pm) \, \hat{q}(\lambda_1,\gamma_\pm) \left[E_1' f_1(E_1', T(\lambda_1,\gamma_\pm)) \right] \left[E_2' f_2(E_2', T(\lambda_2,\gamma_\pm)) \right] \tag{3.25}$$

The factor

$$H = \frac{\langle n_F (n_F - 1)\rangle}{\langle n_F \rangle^2} \tag{3.26}$$

gives the probability of simultaneous production of two fireballs. If only one fireball is produced, due to this factor, this contribution to the two-particle distribution vanishes, as it should do.

Introducing the rapidities η_1 and η_2 of the two fireballs and their rapidity functions $G(\eta_i, \eta_\pm)$ $(i = 1,2)$ given in Eq. (2.12) similarly as was done in Eq. (3.19) we have

$$\frac{d^6 N^*_{12,FF}}{dy_1^* dy_2^* d^2 p_{1\perp} d^2 p_{2\perp}} = \frac{\langle n_F (n_F - 1)\rangle}{\langle n_F \rangle^2} \frac{V^2 z_1 z_2}{(2\pi)^6} \int_{-\eta_-}^{\eta_+} d\eta_1 \int_{-\eta_-}^{\eta_+} d\eta_2 \, G(\eta_1, \eta_\pm) \cdot$$

$$G(\eta_2, \eta_\pm) \prod_{i=1,2} \mu_i \cosh(y_i^* - \eta_i) \exp\left(- \frac{\mu_i}{T_i(\eta_i, \eta_\pm)} \cosh(y_i^* - \eta_i)\right) \tag{3.27}$$

Multiplying the integrand by a factor $[1-\exp(-a|\eta_1 - \eta_2|^2)]$ would avoid the two fireballs being produced with equal rapidities and would therefore work the same way as the factor (3.26). Apart from the factor H the integral (3.27) is equal to the product of the two single-particle distributions of particles 1 and 2.

Integrating (3.27) over y_1^*, y_2^*, η_1 and η_2 gives the contribution $F^{(2)}_{FF}$ to the two-particle distribution.

The contribution $F^{(2)}_{FD}$ to the total two-particle distribution $F^{(2)}$ can be computed as in (3.27) with one of the functions $G(\eta_i, \eta_\pm)$ $(i = 1$ or $2)$ replaced by $G_o(\eta_i, \eta_\pm)$, Eq. (2.29), as stated in Section 2.3, and putting $H = 1$. This term gives the correlation between a leading and a non-leading particle.

The correlation between two leading particles is given by $F^{(2)}_{DD}$. It is obtained like in (3.27) with both functions $G(\eta_i, \eta_\pm)$, $(i = 1$ and $2)$, replaced by the functions $G_o(\eta_i, \eta_\pm)$, $(i = 1$ and $2)$ and with $H = 1$. In this case the y_1^* integration extends only from $-\eta_-$ to zero and the y_2^* integration extends from zero to η_+.

4. THE ASYMPTOTIC BEHAVIOUR OF THE TWO-PARTICLE DISTRIBUTIONS AND CORRELATION FUNCTIONS

We discuss the one fireball contribution and the two fireball contributions for the choices i) and ii) of $\hat{q}(s,\eta)$, Eqs. (2.21) and (2.22).

4.1 The one-fireball contribution

Case i) Eq. (2.21) :

$$q^{(i)}(s,\eta) \approx \frac{q_0}{m_p}\left[M_c\left(\frac{\sqrt{s}}{M_c}\,e^{-|\eta|}\right)^W\right]$$

With this function $\hat{q}^{(i)}(M_F)$ and with the expression (2.16) for $G(\eta,\eta_\pm)$ the two-particle rapidity distribution is

$$C_F^{(2)}(y_1^*,y_2^*) = \frac{d^2N^*_{12,F}}{dy_1^*\,dy_2^*} = D\int_{-\eta_-}^{\eta_+} d\eta\,\exp\left(-a\exp(-|\eta-\eta_\pm|)\right)\cdot \tag{4.1}$$

$$\cdot\left(1-\frac{2m_0}{\sqrt{s}}\sinh\eta\right)e^{-|\eta|W}\prod_{i=1,2}\cosh(y_i^*-\eta)\,I\left(\frac{\cosh(y_i^*-\eta)}{T(s,\eta)}\right)$$

$$D = B\cdot C\cdot\left(\frac{\sqrt{s}}{M_c}\right)^W$$

$$B = \frac{V^2 z_1 z_2}{(8\pi^2)^2}\,\frac{q_0}{m_p}\,M_c \tag{4.2}$$

with C given in (2.17). We substitute

$$y_1^* - \eta = p$$

and

$$y_2^* - \eta = y_2^* - y_1^* - p = \Delta y^* - p \tag{4.3}$$

For $|\eta| \le \eta_{L\pm}$, where $\eta_{L\pm}$ were defined in (2.18a), i.e., away from the fragmentation region, it can be shown that in good approximation the expression

$$\left(1-\frac{2m_0}{\sqrt{s}}\sinh\eta\right) \approx 1 \tag{4.4}$$

holds. Furthermore, for $|\eta| \le \eta_{L\pm}$ we have $|\eta-\eta_\pm|$ large enough such that

$$\exp\left(-a\exp\left(-|\eta-\eta_\pm|\right)\right) \approx 1 \tag{4.5}$$

We approximate further

$$T(s,\eta) \longrightarrow T_0$$

and neglect the contribution to the integrand from the fragmentation region,

$$\eta_{L+} < \eta < \eta_+$$

and

$$-\eta_- < \eta < -\eta_{L-}$$

We then have

$$C_F^{(2)}(y_1^*, y_2^*) = D\int_{-\eta_{L-}+y_1^*}^{\eta_{L+}-y_1^*} dp\; e^{-|y_1^*+p|w} \cdot \cosh(-p)\cosh(\Delta y^*-p) \tag{4.6}$$

$$\cdot I\left(\frac{\cosh p}{T_0}\right) I\left(\frac{\cosh(\Delta y^*-p)}{T_0}\right)$$

Introducing for $I(A)$ the expression (3.21) and using equal masses for particles 1 and 2, $m_1 = m_2 = m$ we obtain

$$C_F^{(2)}(y_1^*, y_2^*) = D\,4m^4T_0^2\int_{-\eta_{L-}+y_1^*}^{\eta_{L+}-y_1^*} dp\cdot\exp\left[-|y_1^*+p|w - \frac{m}{T_0}\left\{\cosh p + \cosh(\Delta y^*-p)\right\}\right] \tag{4.7}$$

$$\cdot\left(1+\frac{T_0}{m\cosh p}+\frac{2T_0^2}{m^2\cosh^2 p}\right)\left(1+\frac{T_0}{m\cosh(\Delta y^*-p)}+\frac{2T_0^2}{m^2\cosh^2(\Delta y^*-p)}\right)$$

The integrand strongly decreases with increasing p and the value $p=0$ dominates the whole integral. We can estimate

$$C_F^{(2)}(y_1^*, y_2^*) = 4m^4T_0^2 D e^{-|y_1^*|w}\cdot e^{-\frac{m}{T_0}\cosh\Delta y^*}\cdot\Delta p \tag{4.8}$$

with some Δp. $F_F^{(2)}$ is obtained by integration over y_1^* and Δy^*. The integral over $d(\Delta y^*)$ is dominated by $\Delta y^* = 0$ and the y_1^* integral can be written as

$$2\int_0^{y_1^* \to \infty \text{ for } s\to\infty} dy_1^* \, e^{-|y_1^*| w} \xrightarrow[s\to\infty]{} \text{const.} \quad \text{for} \quad w>0 \tag{4.9}$$

The only s dependence of $F_F^{(2)}$ remains in D

$$F_F^{(2)} = B \cdot C \cdot \text{const.} \left(\frac{\sqrt{s}}{M_c}\right)^w + \text{const}'$$

and therefore

$$F_F^{(2)} \sim \left(\sqrt{s}\right)^w + \text{const.} \tag{4.10}$$

In particular, for $w=1$

$$F_F^{(2)} \sim \sqrt{s} + \text{const.} \tag{4.11}$$

holds, as is found in the DEM of Hwa and Lam [15].

For $w=0$ we find

$$F_F^{(2)} \sim \text{const.} \int_{-\eta_{L-}}^{\eta_{L+}} dy_1^* = \text{const.} (\eta_{L+} + \eta_{L-}) \sim \text{const.}\, 2\eta_{L+} \tag{4.12}$$

and because of $\eta_{L+} \sim \ln s$, it is

$$F_F^{(2)} = \langle n(n-1) \rangle_F \sim \text{const.} \ln s \tag{4.13}$$

Case ii) Eq. (2.22) : $q^{(ii)}(s,\eta) = \frac{q_0}{m_p}\left[M_c\left(\ln\frac{\sqrt{s}}{M_c} - |\eta|\right)\right]$

With this function $q^{(ii)}(s,\eta)$ and $G(\eta,\eta_\pm)$ of Eq. (2.16) the two-particle rapidity distribution is

$$C_F^{(2)}(y_1^*, y_2^*) = \frac{d^2 N^*_{12,F}}{dy_1^*\, dy_2^*} = B \cdot C \int_{-\eta_-}^{\eta_+} d\eta \, \exp\left(-a|\eta - \eta_\pm|\right) \cdot$$

$$\cdot \left(1 - \frac{2m_0}{\sqrt{s}} \sinh \eta\right)\left(\ln \frac{\sqrt{s}}{M_c} - |\eta|\right) \prod_{i=1,2} \cosh(y_i^* - \eta)\, I\left(\frac{\cosh(y_i^* - \eta)}{T(s,\eta)}\right) \tag{4.14}$$

Using the same method as in case i), we obtain from (4.14) the asymptotic behaviour of $F_F^{(2)}$ as

$$F_F^{(2)} = \langle n(n-1) \rangle_F \sim \ln^2 s \tag{4.15}$$

4.2 The two-fireball contribution

In Section 3.4, it was shown that the two-fireball contribution $f_{FF}^{(2)}(\vec{p}_1^*, \vec{p}_2^*)$ to the two-particle distribution $f^{(2)}(\vec{p}_1^*, \vec{p}_2^*)$ is equal to the product of the single-particle distributions for particles 1 and 2, besides the factor $H = \langle n_F(n_F-1)\rangle / \langle n_F \rangle^2$. Therefore, its asymptotic behaviour is

$$F_{FF}^{(2)} = \langle n(n-1) \rangle_{FF} \sim H \cdot \ln^2 s \tag{4.16}$$

Producing fireballs with a finite average mass independent of rapidity and primary energy [our case i) with $w = 0$] along a multiperipheral chain leads to the same prediction for the multiplicity $\langle n \rangle \sim \ln s$ and $\langle n(n-1)\rangle \sim$ $\sim \ln^2 s$ as the MPM *). So we find the MPM and DEM as two limiting cases of a thermodynamic model with the decay multiplicity function $q^{(i)}(s, \eta)$, Eq. (2.21), with $w = 0$ and $w = 1$.

Similarly to $f_{FF}^{(2)}$, as discussed in Section 3.4, the terms $f_{FD}^{(2)}(\vec{p}_1^*, \vec{p}_2^*)$ and $f_{DD}^{(2)}(\vec{p}_1^*, \vec{p}_2^*)$ factorize. They contain the contributions of one or two leading particles, respectively, to the two-particle spectrum. According to Section 4.1, the integration over the distribution of the produced particle leads to $\sim \ln s$. The integral of the distribution of the through-going particle is 1. Therefore we have

$$F_{DF}^{(2)} \sim \ln s \tag{4.17}$$

and

$$F_{DD}^{(2)} \sim \text{const.} \tag{4.18}$$

---

*) A MPM with factorized Pomeron exchange gives $R^{(2)} \sim \ln s$ due to cancellations in $\langle n(n-1)\rangle$ and $\langle n \rangle^2$. A general MPM would lead to $R^{(2)} \sim \ln^2 s$. We thank Dr. A. Bassetto for pointing this out to us.

In the Table, we collect our results on the asymptotic behaviour of the various contributions to the integrated two-particle distribution $F^{(2)}$ and the two-particle correlation $R^{(2)}$.

5. INTEGRATED TWO-PARTICLE DISTRIBUTION AND CORRELATION AT FINITE ENERGY

We have computed the fully integrated single-particle distribution $R^{(1)} \equiv \langle n \rangle$ and the one and two-fireball contributions $F_F^{(2)}$ and $F_{FF}^{(2)}$ to the integrated two-particle distribution $R^{(2)} \equiv \langle n(n-1) \rangle$ for negative pions produced in K^+p collisions. In this computation we have only used asymptotic kinematics for the $p_\perp$ integrations (extending the $p_\perp$ integral from 0 to ∞) and by the fact that we have treated the fireballs as rather heavy objects, for which energy and momentum conservation will not very stringently change the spectrum of one or two irradiated pions, in the sense discussed in Section 3.3. In this computation of $R^{(1)}$ and $R^{(2)}$ we have included the fragmentation region and we also have used the actual fireball temperature rather than to put $T \to T_0$.

In Fig. 1 we show the one-fireball contribution $F_F^{(2)}$ as a function of $\sqrt{s}$ for the decay chain multiplicity function $q^{(i)}(\eta, s)$ with $w = 0, \frac{1}{2}$ and 1, Eq. (2.21). Already at finite $\sqrt{s}$, at the beginning of the ISR energy region we find the asymptotic behaviour listed in the Table shown by our numerical computation. In this computation we have used $M_C = 4$ GeV.

From Eqs. (2.9), (2.20) and (2.25), we have computed the average number $\langle n_F \rangle$ of the fireballs produced as a function of $\sqrt{s}$ for our various choices of $\hat{q}(s, \eta)$ and for $M_C = 2, 3$ and 4 GeV. As can be seen from Fig. 2, even for constant fireball mass $M_F = M_C = 2$ GeV only at c.m.s. energies of 20 GeV there are in all collisions two or more fireballs produced. At $p_{lab} \sim 15$ to 20 GeV/c, we have even in this case ($M_C = 2$ GeV) less than 25% of two-fireball production. For larger M_C and for other choices of $\hat{q}(s, \eta)$, where the fireball mass M_F is growing with energy, the number of fireballs produced is increasing much slower.

In distinction to the Nova model [16] where the novas only are produced diffractively, we also allow the fireballs to be produced via quantum number exchange. Therefore we do not expect any problem with neutron production, which arises in the Nova model [17].

In Fig. 3 we plot the two-particle correlation $R^{(2)}$ for all negative pions produced in collisions of two positive initial particles versus the multiplicity of the negative pions, $<n_->$. We compare with data on π^+p and pp collisions, as collected by Białas et al.[18]. At low $<n_->$ or low s our calculation gives negative $R^{(2)}$, in agreement with the data. Two of our choices $q^{(i)}$ with $w > 0$ giving $F^{(2)} \sim s^{w/2} + \mathrm{const.}\, \ln^2 s$ show the change of sign of $R^{(2)}$, as seems to be required by the data, although the errors are very large. The solution with $q^{(i)}$ $w = 0$, that is constant fireball mass, seems to be unfavoured by the data. This is the solution which gives $R^{(2)}$ in agreement with the MPM.

Certainly, to draw more definite conclusions, better experimental data as well as a theoretical calculation taking into account all kinematic limits due to the finite mass of the fireballs are needed.

Similarly as in Fig. 3, the correlation $R^{(2)}$ of all charged particles can be computed. It shows again a negative behaviour for small $<n>$ and small $\sqrt{s}$ and later changes sign.

ACKNOWLEDGEMENTS

We should like to acknowledge discussions with Dr. R. Hagedorn. One of us (G.R.) wants to express her gratitude to the CERN Theoretical Study Division for its hospitality.

<u>TABLE</u> : The asymptotic behaviour of single-particle distributions $F^{(1)}$, two-particle distributions $F^{(2)}$ and two-particle correlations $R^{(2)}$ for the various choices of the decay chain multiplicity function $\hat{q}(s,\eta)$.

$\hat{q}(s,\eta)$	$F^{(1)}$	$F_F^{(2)}$	$F_{FF}^{(2)}$	$F_{DF}^{(2)}$	$F_{DD}^{(2)}$	$R^{(2)}$	Remarks
$\frac{q_o}{m_p}M_C$ $(w=0)$	$\sim \ln s$	$\sim \ln s$	$\sim \ln^2 s$	$\sim \ln s$	~ 1	$\sim \text{const.} \ln^2 s$	$R^{(2)}$ like in MPM (see *) p. 19)
$\frac{q_o}{m_p}M_C\left(\frac{\sqrt{s}}{M_C}e^{-\lvert\eta\rvert}\right)^w$ $0 < w < 1$	$\sim \ln s$	$\sim s^{w/2}$	$\sim \ln^2 s$	$\sim \ln s$	~ 1	$\sim s^{w/2} + \text{const.} \ln^2 s$	
$\frac{q_o}{m_p}M_C\left(\frac{\sqrt{s}}{M_C}e^{-\lvert\eta\rvert}\right)$ $w = 1$	$\sim \ln s$	$\sim \sqrt{s}$	$\sim \ln^2 s$	$\sim \ln s$	~ 1	$\sim s^{\frac{1}{2}} + \text{const.} \ln^2 s$	$F_F^{(2)}$ like in DEM
$\frac{q_o}{m_p}\left[M_C\left(\ln\frac{\sqrt{s}}{M_C} - \lvert\eta\rvert\right)\right]$	$\sim \ln s$	$\sim \ln^2 s$	$\sim \ln^2 s$	$\sim \ln s$	~ 1	$\sim \text{const.} \ln^2 s$	

R E F E R E N C E S

1) J. Ranft, Phys.Letters 36B, 225 (1971).

2) R. Hagedorn and J. Ranft, CERN preprint TH.1440 (1972);
J. Ranft, Karl-Marx-Universität preprint TUL.49 (1971).

3) E. Matthäus and J. Ranft, Phys.Letters 40B, 230 (1972).

4) Htun Than and J. Ranft, Karl-Max-Universität preprint KMU-HEP.7207 (1972).

5) Htun Than, J. Kripfganz, V. Müller, G. Ranft, J. Ranft and R. Blutner, Karl-Marx-Universität preprint KMU-HEP.7206 (1972).

6) G. Ranft and J. Ranft, "Single particle distributions and the thermodynamic model", 3rd International Colloquium on Multiparticle Dynamics, June 20-24, 1972, Zakopane.

7) For a review of the experimental situation, see :
J.C. Sens, Rapporteur Talk at the Oxford Conference, April 1972;
T. Ferbel, Rapporteur Talk at the 3rd International Colloquium on Multiparticle Dynamics, June 20-24, 1972, Zakopane.

8) G. Ranft and J. Ranft, Phys.Letters 32B, 207 (1970).

9) G. Ranft and J. Ranft, Phys.Letters 40B, 131 (1972).

10) W. Ko, Phys.Rev.Letters 28, 935 (1972).

11) R. Hagedorn, Suppl. Nuovo Cimento 3, 147 (1965).

12) S.C. Frautschi, Phys.Rev. D3, 2125 (1971);
S.C. Frautschi and C.J. Hamer, Phys.Rev. D4, 2125 (1971);
W. Nahm, preprint, Universität Bonn PI-2-102 (1971).

13) R. Hagedorn and J. Ranft, Suppl. Nuovo Cimento 6, 169 (1968).

14) K. Wilson, Cornell University preprint CLNS-131 (1971);
A. Bassetto, M. Toller and L. Sertorio, Nuclear Phys. B34, 1 (1971);
A.H. Mueller, Phys.Rev. D4, 150 (1971);
H.D.I. Abarbanel, Phys.Rev. D3, 2227 (1971).

15) R.C. Hwa, Phys.Rev.Letters 26, 1143 (1971);
R.C. Hwa and C.S. Lam, Phys.Rev.Letters 27, 1098 (1971).

16) M. Jacob and R. Slansky, Phys.Letters 37B, 408 (1971); Phys.Rev. D5, 1847 (1972).

17) D.R.O. Morrison, Rapporteur Talk at the 3rd International Colloquium on Multiparticle Dynamics, June 20-24, 1972, Zakopane.

18) A. Białas, K. Fiałkowski and R. Wit, Nuclear Phys. B43, 413 (1972);
A. Białas, K. Fiałkowski, R. Wit and K. Zalewski, Phys.Letters 39B, 211 (1972);
A. Białas and K. Zalewski, Nuclear Phys. B42, 325 (1972).

FIGURE CAPTIONS

Figure 1 : The contribution $F_F^{(2)}$ of two particles emerging from one fireball F to the integrated two-particle distribution plotted versus the c.m.s. energy $\sqrt{s}$ for the decay chain multiplicity function

$$q^{(i)}(s,\eta) = \frac{q_0}{m_p} M_c \left(\frac{\sqrt{s}}{M_c} e^{-|\eta|} \right)^w$$

for $w = 0, \frac{1}{2}$, and 1. The parameter M_C is 4 GeV.

Figure 2 : The average number $\langle n_F \rangle$ of fireballs produced as obtained from Eqs. (2.9) and (2.20) to (2.25) plotted as a function of the c.m.s. energy $\sqrt{s}$ and the parameter $M_C = 2$, 3 and 4 GeV. The decay chain multiplicity function used is

$$q^{(i)}(s,\eta) = \frac{q_0}{m_p} M_c \left(\frac{\sqrt{s}}{M_c} e^{-|\eta|} \right)^w$$

for $w = 0, \frac{1}{2}$ and 1.

Figure 3 : The correlation function $R^{(2)} = \langle n_-(n_- - 1) \rangle$ for $\pi^- \pi^-$ produced in collisions of two positively charged hadrons plotted versus $R^{(1)} = \langle n_- \rangle$ which is $\sim \ln s$. The computation was done for K^+p initial state with the decay chain multiplicity function

$$q^{(i)}(s,\eta) = \frac{q_0}{m_p} M_c \left(\frac{\sqrt{s}}{M_c} e^{-|\eta|} \right)^w$$

$w = 0, \frac{1}{2}$ and 1. We compare with the data of π^+p and pp into $\pi^- \pi^-$ compiled by Białas et al.[18].

A D D E N D U M

Since this paper was written, the measurement of G. Carleton et al. [19)*)] at NAL at 205 GeV/c became known. Their results are $\langle n_- \rangle = 2.82 \pm 0.81$ and $R^{(2)}_{(-)} = \langle n_-(n_- - 1)\rangle - \langle n_- \rangle^2 = 0.95 \pm 0.21$. In our Figure 3 these data lie halfway between our curves with $w = 1$ and $w = \frac{1}{2}$, thus suggesting that the actual mass of the fireballs increases with energy like $M_F \sim M_C \left(\frac{\sqrt{s}}{M_C} e^{-|\eta|}\right)^w$, $\frac{1}{2} < w < 1$.

* * * * *

19) G. Carleton, Y. Cho, M. Derrick, R. Engelmann, T. Fields, L. Hyman, K. Jaeger, U. Mehtani, B. Musgrave, Y. Oren, D. Rhines, P. Schreiner, H. Yuta, L. Voyvodic, R. Walker, J. Whitmore, H.B. Crawley, Z. Ming Ma and R.G. Glasser, "Charged particle multiplicity distribution from 200 GeV pp interactions", NAL preprint (1972).

---

*) We thank Drs. D. Horn and M. Kugler for making this paper available to us.

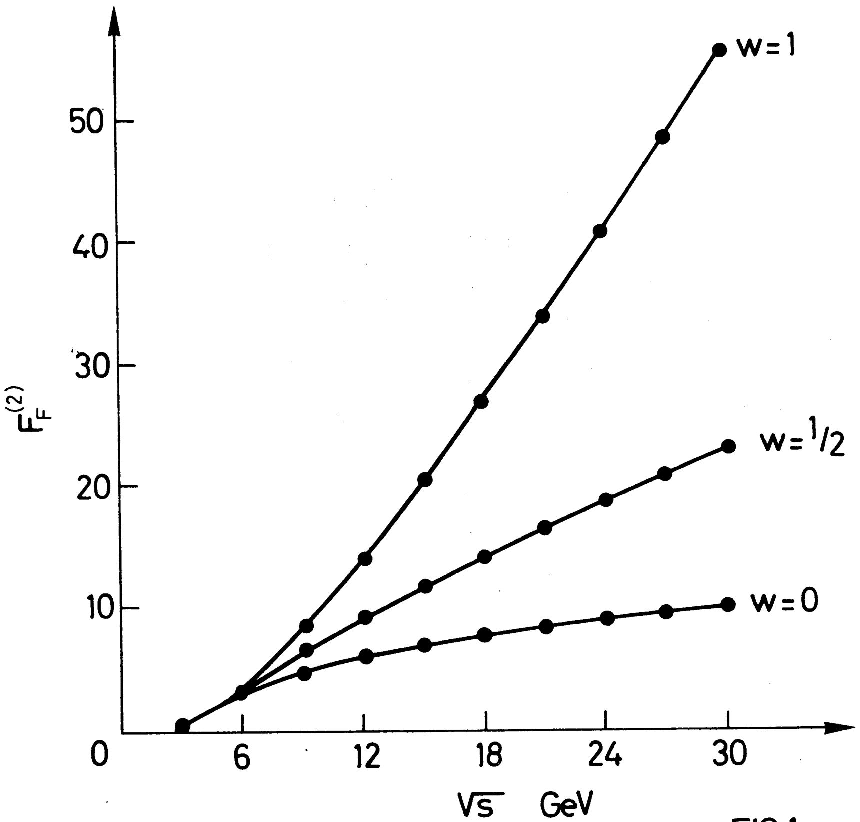
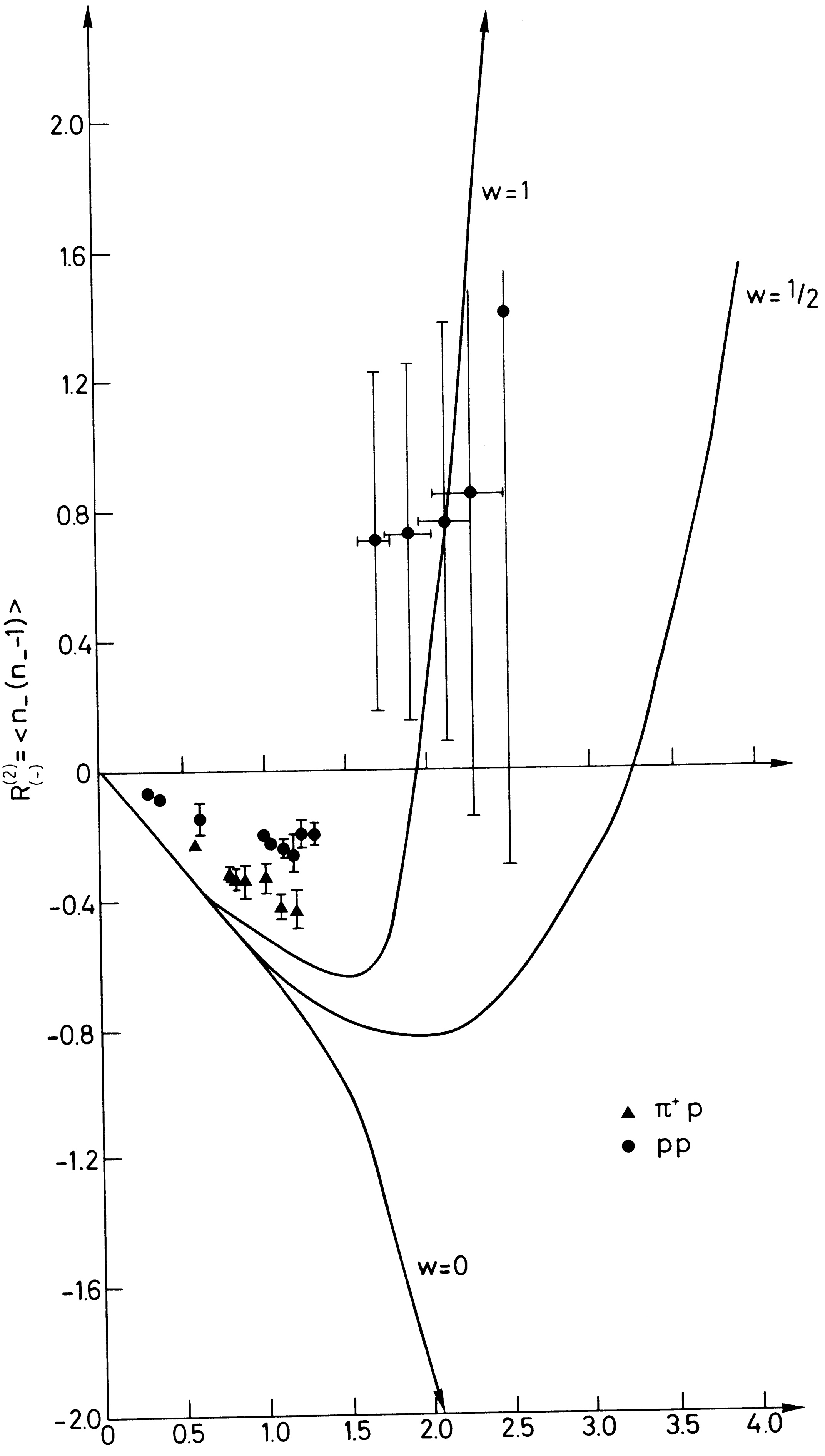


FIG.1



2.0
1.6
1.2
0.8
0.4
0
-0.4
-0.8
-1.2
-1.6
-2.0
$R^{(2)}_{(-)} = \langle n_-(n_- - 1)\rangle$
0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
<n->
w = 1
w = 1/2
w = 0
▲ $\pi^+ p$
● pp

FIG. 3