Nucleon-Pion Spectroscopy from Sparsened Correlators

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Image credit: Fermilab
Deep Underground Neutrino Experiment

- Beam from Fermilab to South Dakota to study $\nu$ oscillations
- Oscillation parameters depend on $P(\nu_\mu \rightarrow \nu_e)$ as function of $L/E$
- Experimental $\nu$ beams inherently broadband
- Will require reconstruction of $E_\nu$
- Need energy-dependent cross-sections for $\nu$-nucleus interactions

Image credit: B. Abi et al. (DUNE Collaboration), 2006.16043
**ν-N Cross-Sections**

![Graph showing ν cross section vs. Eν (GeV)](image_url)

- DUNE beam peaked at a couple GeV – in resonant region

Image credit: Adapted from J. A. Formaggio, G. P. Zeller (1305.7513)
Motivation

Nuclear many body methods relate neutrino-nucleus cross-sections to nucleon form factors (N. Rocco, 11:00; R. Gupta, 11:30)

Quasi-elastic regime – based on nucleon elastic form factor
  - Great work in previous talks toward estimating this

DIS regime – perturbative
  - Factorization theorems write this in terms of nucleon PDFs

Resonant regime – largest uncertainty is $N \rightarrow \Delta$ transition
  - Current estimates from electroproduction have $>10\%$ uncertainties (E. Hernández et al., 1001.4416)
  - Need 3% uncertainty for DUNE target precision (D. al., 2210.02455)

Results

Figure credit: 2210.02455
Neutrinoproduction

$$\nu_\mu N \rightarrow \mu \Delta$$

- Mediated through electroweak current
  $$\tilde{N}(\gamma_\mu - \gamma_\mu \gamma_5)\Delta$$
- Vector component known from $$eN \rightarrow e\Delta$$
- Axial component difficult to measure experimentally
- $$\Delta$$ resonance highly unstable
  $$\Delta \rightarrow N\pi, N\pi\pi$$
- Goal: Understand $$N\pi$$, $$N\pi\pi$$ spectrum up to $$m_\Delta$$
Related Work

- Long history of studying $N \rightarrow \Delta$ on lattice (C. Alexandrou et al., hep-lat/0607030)
- Much exciting recent work in $N\pi$ spectroscopy and scattering
  - L. Barca et al. (2211.12278), Bulava et al. (2208.03867), Silvi et al. (2101.00689, source of figures), Andersen et al. (1710.01557)
- Physical point calculations now possible (Alexandrou et al., 2307.12846)
- Can probe $\Delta$ resonance via $N\pi$ phase shifts
- $N\pi\pi$ systems still unexplored
Methodology

- Want to compute

\[ \langle N(\tau) \pi(\tau) \pi(\tau) \tilde{N}(0) \pi(0) \pi(0) \rangle \]

- Naïvely requires all-to-all propagators (timeslice-to-self \( \pi \) loops)

- Cost: \( O(V^2) \) for inversions, \( O(V^6) \) for contractions

- Contraction cost reduced to \( O(V^3) \) by computing sequential propagators through \( \pi \)

- Contraction cost further reduced by eightfold by parity projecting all quarks
Propagator Sparsening

- Nearby sites on lattices highly correlated
- Can compute propagators on coarse grid without much loss of information (Detmold et al., 1908.07050; Yuan Li, 2009.01029; Amarasinghe et al., 2108.10835)
  - In momentum space, corresponds to incomplete Fourier projection
- Loss of information further reduced by Gaussian smearing
- Sparsening by factor of $f$ in each direction reduces inversion costs by $f^3$ and seqprop construction cost in contractions by $f^9$
Ensemble Details

- $a = 0.15$ fm, $L = 4.8$ fm, $m_{\pi, P} = 135$ MeV HISQ ensemble from FNAL/MILC
- Clover fermions used for valence quarks ($m_{\pi, \text{val}} \approx 170$ MeV)
- Gradient flow smearing used to reduce mixed-action artifacts
- Propagators computed using QUDA multigrid inverter (M. Clark et al., 0911.3191, 1612.07873) on $8^3$ grid on each timeslice
- Gaussian smearing applied at source and sink
Contraction Code

- Standalone code to read in propagators from QUDA and compute $N_\pi$, $N_{\pi\pi}$ contractions
- Designed to support CPU and GPU targets
- Leverages MKL BLAS or cuBLAS for sequential propagator construction
- Performs all Wick contractions from these sequential propagators
$N\pi$, $p = 0$
$N\pi\pi, \ p = 0$

**Diagram Description:**

- The graph plots $a_{\text{eff}}$ versus $t/a$ for different states of the system.
- The states are indicated by different symbols and colors:
  - Blue circles: $N$
  - Red triangles with diagonal lines: $N\pi\pi, l=1/2$
  - Black diamonds: $N\pi\pi, l=3/2$
  - Green squares: $N\pi\pi, l=5/2$

- The horizontal lines represent the central values of the $a_{\text{eff}}$ for each state group.
- Error bars are shown for each point, indicating the statistical uncertainty.

**Equations:**

- $N\pi\pi$,
- $N\pi\pi, I = 1/2$
- $N\pi\pi, I = 3/2$
- $N\pi\pi, I = 5/2$
$N_\pi, |p| = 1$
Conclusions

- $N \rightarrow \Delta$ and therefore $N \rightarrow N\pi, N\pi\pi$ axial transitions needed for DUNE
- Spectroscopy calculations – first step in producing good $N\pi(\pi)$ interpolators
- Future plans:
  - Increased statistics
  - GEVP to study states in same parity/isospin sectors
  - Finite-volume phase shifts to study $\Delta$ resonance
  - 3-point functions for axial/vector form factors
Isospin Splitting in $N\pi$

![Graph showing isospin splitting in $N\pi$.](image-url)