The Deuteron D State Revisited

Determining the deuteron D-state admixture is a classic problem in nuclear physics. In its usual form, the problem is easily stated. Since the deuteron has a quadrupole moment, it cannot be a pure S state, but rather a D-state admixture is required. What, then, is the probability of finding the neutron and proton in the deuteron in a relative D state? After more than 40 years of study, the best answer one can give is $P_D = 3.3\text{-}9\%$, an uncertainty of some 40%.[1] This large uncertainty does not arise from experimental inaccuracies, but rather reflects a conceptual problem. Motivated in part by that problem, attention has turned recently to another deuteron property for quantifying the D state, the ratio of the D-state to S-state asymptotic normalization. The latest experiments for it are quoting *uncertainties* of 1.5-3%. The purpose of this Comment is to explain the origin of this great improvement in conceptual precision, and the nature of the experiments that give rise to it.

Considered as a neutron-proton (n-p) nonrelativistic bound state, the deuteron has a wave function of the form

$$\Psi = \frac{u(r)}{r} Y_{00}\,\chi^{(1)} + \frac{w(r)}{r}\,(Y_{2M}\,\chi^{(1)}), \tag{1}$$

where r is the relative n-p coordinate, $\chi^{(1)}$ the triplet spin wave function and Y_{lm} the usual spherical harmonic. The bracket in the D-wave part represents the coupling of the spin and orbit parts to the total spin one of the deuteron. Since the D-state admixture is small, the quadrupole moment is determined almost entirely by the cross term, which is proportional to the integral $\int uwr^2 dr$. The D-state probability itself, P_D, is given by $P_D = \int w^2 dr$. Therefore, P_D is not determined directly by the quadrupole moment and depends more sensitively on the small-r behavior of w than does that moment.

To determine P_D, therefore, one must either "measure" the small-r behavior of w or find a quantity that depends on P_D directly. Within the framework of

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the n–p bound-state model of Eq. (1), this is easy to do. For example the deuteron magnetic moment obtained from this wave function is given by $\mu_D = \mu_N + \mu_p - (3/2)(\mu_N \mu_p - 1/2)P_D$ in terms of the neutron and proton moments μ_N and μ_p.[2] From the measured deuteron magnetic moment one gets $P_D = 4\%$. However, there are relativistic and meson-exchange current corrections to the magnetic moment that are not part of the nonrelativistic treatment embodied in Eq. (1). These are of the same order of magnitude as the P_D contribution, and are themselves rather uncertain. Their inclusion makes the P_D obtained from the magnetic moment uncertain by some 40%. Many other experiments involving electromagnetic probes have been suggested that seem at first glance to determine P_D directly. Within the model based solely on the nonrelativistic n–p wave function of Eq. (1), they do determine P_D; on closer inspection, however, these experiments also require corrections from meson-exchange effects, relativistic effects or final-state interactions for their interpretation. These corrections are again sufficiently uncertain or model-dependent to cast comparable imprecision into the determination of P_D.

Alternatively, one can try to measure $w(r)$ directly with particular attention to getting at the short-range part that is needed for the P_D integral. For example, this can be done in a break-up experiment. To extract the small-r part of $w(r)$ from such experiments, one must deal with the final-state interactions.[3] Either one must be able to calculate them or one must suppress them by using very-high-energy or short-wavelength probes to induce the break-up. In this latter case, meson-exchange currents again appear.

All these difficulties in determining P_D or $w(r)$ arise from a common problem. Determining $w(r)$ at short distances requires some knowledge of the dynamics at these distances. In the break-up reaction, for example, for r inside the range of nuclear forces, it is likely that the neutron and proton will rescatter after the break-up, and we cannot disentangle the short-range properties of $w(r)$ without removing the effects of these rescatterings. But to do that requires knowledge of the short-range forces that determine both the rescatterings and $w(r)$. Meson-exchange currents are a manifestation of these same forces. Inside the range of nuclear forces, quantum mechanics does not permit a separation of the properties of $w(r)$, final-state interactions and meson-exchange effects. Thus, determining the short-range part of $w(r)$ requires a full dynamical theory. That is why $w(r)$, or alternatively P_D, *can* be determined given a dynamical framework and appropriate data. (Recall the case of the magnetic moment with the assumption of no meson degrees of freedom.) The difficulty is that there is no universally accepted dynamics, and different dynamical assumptions lead to very different values of P_D. The use of a full relativistic quantum field theory with meson degrees of freedom makes matters even more arbitrary. Friar[4] has shown that, starting with a field theory and systematically removing meson degrees of freedom by unitary transformation, it is possible to shuffle effects of the mesons between the wave function and the current operators in such a way as to make P_D essentially *arbitrary*.

The conclusion of all this is that, without a fully-fledged, calculable, universally accepted hadrodynamics, the short-range part of $w(r)$, and correspondingly P_D, cannot be unambiguously extracted from any experiment. The large uncertainty usually quoted for P_D represents, then, the latitude introduced by our imprecise control over the short-range dynamics. But if we need a full-scale dynamics to extract P_D, it is not an observable in the usual sense. In fact, if we had the full-scale dynamical understanding needed to extract it, we could calculate it from that dynamical scheme in the first place.

If P_D is not an observable, because the short-range part of $w(r)$ is not directly accessible, how can we quantify the D state? The existence of a deuteron quadrupole moment requires some D state. In other words, $w(r)$ may not be fully observable, but $w(r) = 0$ is certainly not consistent with the quadrupole moment. The answer is that only the short-range part of $w(r)$ is "obscured" by dynamics. The long-range part is not. Since the deuteron is so weakly bound, a good fraction of $w(r)$ – and of $u(r)$ – extends beyond the range of dynamics. From a knowledge of the deuteron binding energy and the angular momentum of the states, we know the functional form in the asymptotic region. Since this asymptotic wave function is, by definition, outside the range of forces, this magnitude – traditionally, but somewhat misleadingly, called the *asymptotic normalization* – is *directly accessible* to experiment *without* intervening dynamical assumption. It is this asymptotic normalization of the deuteron D state which is the observable, model-independent, measure of the deuteron D state.

How is such a quantity measured? Asymptotically (for large r), $u(r)$ or $w(r)$ go like $e^{-\alpha r}$, where α is given in terms of the deuteron binding energy B and the nuclear mass by $(MB)^{1/2}/\hbar$. (In fact, for $w(r)$ there is a polynomial in r associated with the D-wave nature of the state that multiplies the exponential factor.) The asymptotic normalization is the coefficient of this exponential factor in the normalized wave function. In momentum space, the exponential part of the wave function translates into a pole in the momentum-space wave function of the deuteron. Up to kinematic factors, the asymptotic normalization is the residue at that pole. The pole comes at unphysical values of the momentum, q ($q^2 = -\alpha^2$). The interior, dynamical or nonasymptotic parts of the wave function are associated with branch cuts in momentum that are deeper into the physical region. Because of the large size of the deuteron compared with the range of nuclear forces, the pole residue can be clearly separated from the dynamical cut and obtained from the data by extrapolation.

To measure the asymptotic normalization of w one must either find an experiment sensitive only to the asymptotic tail of $w(r)$ or find an experiment in which one can extrapolate the data to the wave-function pole. Since u has the same asymptotics as w, and since u is very much larger than w, the asymptotic signal or pole residue will be dominated by u unless the contribution of u is suppressed. Thus an experiment to measure the asymptotic part of w must involve a "filter" sensitive only to the D-wave part of the wave function. Such a filter is a second-rank polarization $T_{2\nu}$ in a scattering or reaction process involving the deuteron. Also, since it is the relative strength of the D state that we really want, it is convenient to quote not the asymptotic normalization of the D state, but the ratio of the D-state to S-state normalization. This dimensionless ratio, which is a measure of the asymptotic D to S strength, is called ρ_D.

Each of the general approaches we outlined have been tried for obtaining ρ_D. Knutson and Haeberli[5] suggested that the asymptotic part of $w(r)$ could be determined from $T_{2\nu}$ in sub-Coulomb deuteron stripping. Since the deuteron does not enter the nucleus in this case, but its trajectory is presumed to be known from distorted-wave calculations, the major contribution to the stripping comes from the asymptotic part of the wave function, and hence ρ_D can be determined. Using this method, Stephenson and Haeberli[6] recently reported $\rho_D = 0.02649 \pm 0.00043$. Alternatively, ρ_D can be obtained by studying the neutron-exchange pole contribution to T_{22} in deuteron–proton elastic scattering.[7] This exchange pole is the same as the pole in the wave function. In fact, as was first pointed out by Gruebler *et al.*,[8] extrapolation is unstable in T_{22}, while it is quite stable in $\sigma_{22} = T_{22}\,\mathrm{d}\sigma/\mathrm{d}\Omega$.[9] The exchange-pole signal is quite strong in σ_{22}, which indicates the dominance of the simple neutron-exchange process. The latest result of Gruebler *et al.*[10] from an average of a number of experiments at different energies is $\rho_D = 0.0259 \pm 0.0007$. These two remarkably precise values for ρ_D overlap and, though each method has some ambiguity in the extraction of ρ_D, one can see that ρ_D is far more precisely known than P_D. In the sub-Coulomb stripping method the problems are associated with the model dependence of the distorted-wave treatment. In the pole extrapolation, the major remaining difficulty is a better treatment of distance singularities, and particularly the effect of the Coulomb force. Pole-extrapolation methods have a long and sophisticated history in modern hadron physics, and these corrections should be relatively easily made. The major point is that these refinements in methods for extracting ρ_D should not obscure the fact that ρ_D is a well-defined observable; it is known to within ±1.5-3%. There are, in principle, no dynamical or meson-exchange corrections to it. (In particle physics, this pole residue is called the coupling constant or width.) By contrast, P_D, the fraction of the state that is n-p D state, is a model-dependent quantity that can only be extracted in the context of a complete and unambiguous dynamical description of n-p interactions including meson degrees of freedom.

It is true that ρ_D may not have the immediate intuitive appeal of P_D. In particular, if one looks at various modern nuclear-force models and the deuteron wave function that they yield, one finds that they all yield a value for ρ_D very close to the recently measured number, while these models have very different values (±40%) for P_D.[1] It should be clear from our discussion above that one should not conclude from this fact that P_D is a far more sensitive measure of the dynamics than ρ_D. Rather, one should conclude that P_D has considerable inherent arbitrariness (as we have discussed), while ρ_D is rather well fixed by nearly any sensible theory. In fact, Wong[11] showed long ago that the general constraints of analyticity and unitarity nearly completely determine ρ_D. It has been known for even longer that the asymptotic normalization of the S state is fixed by the triplet scattering length and effective range. Thus, ρ_D is not only an observable, but one deeply enmeshed in a consistent nucleon-nucleon dynamics.

The deuteron is only one of many multicomponent hadron bound states. For each of these the question of quantifying the components arises. It is clear from our discussion of the deuteron that the percentage of a component of any bound state is a model-dependent measure but that asymptotic normalization is not. Because of the relatively weak binding, the deuteron is a particularly easy place to determine asymptotics. For other hadronic bound states, the extraction of asymptotic normalization ratios is more challenging.[†]

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References

1. For a review of the deuteron, see D.W.L. Sprung, in *Few Body Problems and Particle Physics,* edited by R.J. Slobodrian, B. Cujac and K. Ramavataram (Les Presses de l'Université Laval, Quebec, 1975), pp. 475–493.
2. Cf. J.M. Blatt and V.F. Weisskopf, *Theoretical Nuclear Physics* (Wiley, New York, 1952), p. 109.
3. R.D. Amado, Phys. Rev. C **19**, 1473 (1979).
4. J.L. Friar, Phys. Rev. C **20**, 325 (1979).
5. L.D. Knutson and W. Haeberli, Phys. Rev. Lett. **35**, 558 (1975).
6. K. Stephenson and W. Haeberli, Phys. Rev. Lett. **45**, 520 (1980).
7. R.D. Amado, M.P. Locher and M. Simonius, Phys. Rev. C **17**, 403 (1978).
8. Gruebler, Burgi, Konig, Schmelzbach and Jenny, in *Lecture Notes in Physics 82,* edited by Zingl, Haftel and Zankel (Springer, Berlin, 1978), p. 248.

[†] For quark bound states that have no asymptotics, the model-dependent components are the only way to address the issue.

9. Amado, Locher, Martorell, Konig, White, Schmelzbach, Gruebler, Burgi and Jenny, Physics Lett. **79B,** 368 (1978); see also Conzett, Hinterberger, von Rossen, Seiler and Stephenson, Phys. Rev. Lett. **43,** 572 (1979).
10. Gruebler, Konig, Schmelzbach, Jenney and Sperisen, *Proceedings of the Fifth International Symposium on Polarization Phenomena in Nuclear Physics, Santa Fe 1980* (to be published).
11. D.Y. Wong, Phys. Rev. Lett. **2,** 409 (1959).