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Unveiling the evolution of rotating black holes in loop quantum cosmology

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In this work, we aim to discuss about the evolution of rotating black holes (RBHs) within the context of loop quantum cosmology. Here, the main part of our research work focuses on the impacts of angular momentum based rotating parameter and accretion efficiency on the lifetime of RBHs. Our study reveals that accretion of dark energy would not significantly affect the evolution of RBHs, however higher value of rotating parameter could slightly delay the evaporation times of RBHs. Our analysis also depicts that the maximum value of rotating parameter for evolution of any RBH is $10^{-8}M_i^2$, where M_i is the formation mass of RBH. Moreover, from our calculation we found that the maximum mass of a presently existing supermassive black hole would be 10^{48}g, if it undergoes rotation. Also from astrophysical constraint analysis, we found that there is a greater tendency for formation of black holes in loop quantum cosmology than standard model of cosmology.

Keywords Dark energy, Rotating black holes, Loop quantum cosmology

Black Hole is a large compact fascinating object, where the existence of gravity in the region of space-time is so strong that nothing can escape out of it. The most general solution for black hole was first successfully given by Karl Schwarzschild in 1916[1,2]. In this solution, Schwarzschild had focused on the space-time of black holes which were spherically symmetric and electrically neutral in nature, and could be explained only in terms of their masses. Later, Hans Reissner (1916) and Gunnar Nordstr*ö*m (1918) had provided a solution for those black holes, which can be explained in terms of spherically symmetric property of black hole having some mass and finite amount of electric charge[3–5]. In general, most astronomical bodies are rotating in nature. During the process of rotating body collapse, the rate of rotation would increase by maintaining its constant angular momentum. In nature, the existence of rotating black hole[6,7] played a prominent role in different astrophysical processes. Actually, the rotating black hole is an exceptional astronomical object which provides various features and properties that differs from a static black hole. In 1963, Roy Kerr (A famous New Zealand Mathematician) had discovered a solution for black holes whose space-times are rotating and electrically uncharged in nature[8,9]. On the basis of classical features, the resulting space-times can be explained by Kerr solution. For the rotating and charged black holes, Ezra Newman in 1965 provided a solution that explained the behavior of such class of black holes and commonly known as Kerr-Newman solution. According to the no-hair theorem[10], black holes are completely explained by the three physical quantities i.e. mass (M), charge (q) and angular momentum (J). Basing on this no-hair theorem, the black holes having only two physical parameters, mass and angular momentum, are called Kerr black holes. From a geometrical point of view, when we deviate from the spherical symmetry, a new metric comes into picture known as Kerr metric, that can describe the black hole in terms of their angular momentum and mass. This metric was developed 48 years after the Einstein field equations came into existence[8]. The equation for Kerr metric[8,11,12] is generally defined in terms of Boyer-Lindquist co-ordinates. The Kerr metric is only useful, when you take uncharged spinning black hole into consideration. A Kerr black hole is constituted of several structural terms such as spin axis, horizon, ergo sphere, static limit etc. The existence of angular momentum not only shows the space-time around Kerr black hole[13] is non-static but also matter can move very close to the orbit of the respective black holes. The study of such kind of black holes which contain an asymptotic notion of angular momentum in the dark energy submerged environment can provide an interesting result.

On the other hand, the quantization of gravity[14] is yet an unsettled issue and it requires a wide range of techniques to provide any kind of relevant solution. As we know, one of the greatest success of general relativity[15–19] is its geometrical techniques and it will be more suitable when we combine with the quantum mechanical techniques[20]. General theory of relativity and quantum theory cannot individually explain all the questions relate

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with the origin of the universe. Quantum gravity is the theory whose main objective is to combine both these theory into a unified theory. Loop Quantum Gravity (LQG)[21–25] is one of the characteristic features of quantum gravity theories which are completely non-perturbative and explicit background independent in nature[26]. The implication of LQG framework on cosmology to examine our whole universe is generally known as loop quantum cosmology (LQC)[27–30]. LQC is based on the theory which replaces the classical big bang singularity[31] by quantum big bounce[32]. This remarkable theory can resolve the theory of singularity and explain various unknown character that our universe would possess, by analyzing the evolution of various astronomical bodies. Furthermore, our observable universe is mainly filled with dark energy, dark matter and ordinary matter in different proportion[33,34] and recent observation shows that the present universe is largely dominated by dark energy[35–37]. By considering this dark energy domination[16,38–41] in LQC, the study of the RBHs dynamics may provide a captivating result than in the case of standard cosmology[42]and other theories of gravity[43,44].

In 1974, Hawking found that black holes emit thermal radiation as a result of quantum phenomena[45]. Generally the black hole evaporation depends upon its initial mass. This signifies that lesser is the initial mass, more quickly the black hole will evaporate. Again, the evolution of evaporating black holes was successfully examined by Page in his work by taking Schwarzschild and Kerr metric into consideration[46,47]. Especially, the evolution of rotating Kerr black hole was studied numerically by Page where he had considered Hawking evaporation process only[47]. But during the early evolution of black hole, the background environment was very hot and highly dense. So a very substantial amount of absorption of energy-matter, called accretion, from the surrounding goes into the black holes. In literature many works have been studied on the effect of accretion that is responsible for the enhancement of life time of black holes[48]. The effect of dark energy accretion on black holes has been studied by several researchers[49,50]. Similarly, the impact of radiation accretion on evolution of black holes was explained in the context of different theories like Brans-Dicke theory[51], standard cosmology[17] and brane world cosmology[52].

In this work, we study the evolution of rotating black holes (RBHs) within the context of loop quantum cosmology (LQC) by taking interacting dark energy into account. Here, actually we have examined the impact of dark energy accretion on evolution of RBHs and investigated the impact of rotating parameter on RBHs dynamics. In this article, we have also tried to impose constraints on the formation of RBHs from present astrophysical observations.

Basic framework

For a spatially flat FRW universe ($k = 0$) having scale factor $a(t)$ and energy density (ρ), the Friedmann equation, Raychaudhuri's equation and energy conservation equation in loop quantum cosmology[53–55] take the form

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho\left(1 - \frac{\rho}{\rho_c}\right), \tag{1}$$

$$\dot{H} = -4\pi G(\rho + p)\left(1 - \frac{2\rho}{\rho_c}\right), \tag{2}$$

and

$$\dot{\rho} + 3H(\rho + p) = 0 \tag{3}$$

respectively. Here $H = \frac{\dot{a}(t)}{a(t)}$ is the Hubble parameter, p is the pressure of the perfect fluid filling the universe and ρ_c symbolizes the critical value of energy density of the universe given by $\rho_c = \frac{\sqrt{3}}{16\pi^2\gamma^3}\rho_{pl}$ with $\gamma = \frac{\ln 2}{\pi\sqrt{3}}$ is the dimensionless Barbero-Immirzi parameter[56–58] and ρ_{pl} is the energy density of the universe in Planck time. We can also construct braneworld cosmology independently from Eq. (1) like Loop Quantum Gravity. Basically when the general relativity reaches its high curvature limits, we expect that Freidmann dynamics have to change. This modification is naturally possible by taking quantum gravity into consideration. We want to explain that how braneworld cosmology[59–61] can be formulated from the standard Freidmann equation and how can be it different from loop quantum gravity. Freidmann equation in standard cosmology gives $H^2 \propto \rho$. But the modified Friedmann equation in loop quantum cosmology is represented as $H^2 \propto \rho(1 - \frac{\rho}{\rho_c})$. This change in the Friedmann equation represents the discrete quantum nature of space time as predicted by loop quantum cosmology. The way the Friedmann equation is modified corresponds to a bouncing cosmology without singularities. But for braneworld cosmology case, the modified Friedmann equation in the Randall-Sundrum braneworld scenario (most widely studied) can be expressed as $H^2 \propto \rho(1 + \frac{\rho}{2\sigma})$, where σ is the brane tension and rests have their usual meaning. This variation in this effective Friedmann equation shows the existence of extra dimensions to this model. Also these theories are different from each other in different aspects, for example superinflation in the early universe and a nonsingular bounce in a contracting universe is the main features of LQC but it is absent in case of R-S braneworld scenario.

From the energy conservation equation, we can find the energy density of the universe for both radiation-dominated era and matter-dominated era as;

$$\rho \propto \begin{cases} a^{-4}, t < t_e \\ a^{-3}, t > t_e \end{cases} \tag{4}$$

where t_e is the radiation-matter equality.

By using the solution of Friedmann equation, one can get the scale factor $a(t)$ for different cosmic eras. For the radiation-dominated era ($t < t_e$), the scale factor varies as[26]

$$a(t)_{t<t_e} = \left[\frac{\rho_0 a_0^3 a_e}{\rho_c} + \left\{2\rho_0^{\frac{1}{2}} a_0^{\frac{3}{2}} a_e^{\frac{1}{2}} \sqrt{\frac{8\pi G}{3}}(t-t_e) + \left(a_e^4 - \frac{\rho_0 a_0^3 a_e}{\rho_c}\right)^{\frac{1}{2}}\right\}^2\right]^{\frac{1}{4}} \tag{5}$$

where subscript '0' represents the present value of the corresponding parameter and $a_e = a(t_e)$.

Similarly, for the matter-dominated era ($t > t_e$), the scale factor varies as

$$a(t)_{t>t_e} = \left[\frac{\rho_0 a_0^3}{\rho_c} + \left\{\frac{3}{2}\rho_0^{\frac{1}{2}} a_0^{\frac{3}{2}} \sqrt{\frac{8\pi G}{3}}(t-t_0) + \left(a_0^3 - \frac{\rho_0 a_0^3}{\rho_c}\right)^{\frac{1}{2}}\right\}^2\right]^{\frac{1}{3}}. \tag{6}$$

By using the Eqs. (4), (5) and (6), the expression for energy density ρ in radiation-dominated era and matter-dominated era can be written as[26]

$$\rho(t)_{t<t_e} = \rho_0\left[\frac{\rho_0}{\rho_c} + \left\{2\sqrt{\frac{8\pi G}{3}}\rho_0^{\frac{1}{2}}(t-t_e) + \frac{3}{2}\sqrt{\frac{8\pi G}{3}}\rho_0^{\frac{1}{2}}(t_e-t_0) + \left(1-\frac{\rho_0}{\rho_c}\right)^{\frac{1}{2}}\right\}^2\right]^{-1}, \tag{7}$$

and

$$\rho(t)_{t>t_e} = \rho_0\left[\frac{\rho_0}{\rho_c} + \left\{\frac{3}{2}\sqrt{\frac{8\pi G}{3}}\rho_0^{\frac{1}{2}}(t-t_0) + \left(1-\frac{\rho_0}{\rho_c}\right)^{\frac{1}{2}}\right\}^2\right]^{-1}. \tag{8}$$

Therefore, redshift (z) can be calculated for radiation-dominated era and matter-dominated era as

$$z_{t<t_e} = y\left[\frac{\rho_0}{\rho_c}y^3 + \left\{2\rho_0^{\frac{1}{2}} y^{\frac{3}{2}}\sqrt{\frac{8\pi G}{3}}(t-t_e) + \left(1-\frac{\rho_0}{\rho_c}y^3\right)^{\frac{1}{2}}\right\}^2\right]^{-\frac{1}{4}} - 1 \tag{9}$$

where

$$y = \left[\frac{\rho_0}{\rho_c} + \left\{\frac{3}{2}\rho_0^{\frac{1}{2}}\sqrt{\frac{8\pi G}{3}}(t_e-t_0) + \left(1-\frac{\rho_0}{\rho_c}\right)^{\frac{1}{2}}\right\}^2\right]^{-\frac{1}{3}} \tag{10}$$

and

$$z_{t>t_e} = \left[\frac{\rho_0}{\rho_c} + \left\{\frac{3}{2}\rho_0^{\frac{1}{2}}\sqrt{\frac{8\pi G}{3}}t_0\left(\frac{t}{t_0}-1\right) + \left(1-\frac{\rho_0}{\rho_c}\right)^{\frac{1}{2}}\right\}^2\right]^{-\frac{1}{3}} - 1 \tag{11}$$

respectively.

Assuming that the early universe is filled with radiation & matter and latter dark energy appeared due to their decay and using the present observational data[36] that 68.3% of the universe is filled with dark energy and present age of the universe is 13.82×10^9years. one can find the decay rate as $\Gamma = 1.5665 \times 10^{-18} s^{-1}$, i.e. we can write $\rho = \rho_x + \rho_m$ with $\rho_m = (1-\Gamma t)\rho$ and $\rho_x = \Gamma t \rho$. Here ρ_m is the radiation-matter density and ρ_x is the dark energy density having pressure $p_x = \omega\rho_x$. Again, the equation of state parameter (ω) for an interacting dark energy in loop quantum cosmology was found to be[62]

$$\omega = 0.01971\left(\frac{t}{t_0}\right)^2 - 1.0442\left(\frac{t}{t_0}\right). \tag{12}$$

But as per principle, the equation of state parameter of interacting dark energy should be always negative: This demands that interaction will continue upto $t \approx (\frac{1.0442}{0.01971})t_0 = 52.978 t_0 = 2.30985 \times 10^{19} s$.

The spacetime around a rotating black hole with mass M and angular momentum J can be explained by the line element as[11,63]

$$ds^2 = \left(1-\frac{2GMr}{c^2b^2}\right)c^2dt^2 + \frac{4GMra^* sin^2\theta}{cb^2}d\phi dt - \frac{b^2}{\Delta}dr^2 - b^2d\theta^2 - \left(r^2 + (a^*)^2 + \frac{2GMr(a^*)^2 sin^2\theta}{c^2b^2}\right)sin^2\theta d\phi^2. \tag{13}$$

Where $a^* = \frac{J}{M}$, $b^2 = r^2 + (a^*)^2 cos^2\theta$, $\Delta = r^2 - \frac{2GMr}{c^2} + (a^*)^2$. The (t,r,$\theta$,$\phi$) co-ordinates used here are called Boyer-Lindquist co-ordinates and are analogous to the Schwarzschild coordinates for a non rotating black hole.

Rotating black holes and accretion of dark energy

Basically, Einstein-Maxwell theory deals with the black hole solutions whose charge and angular momentum are non zero commonly called as Kerr-Newman space time. Here we consider an uncharged rotating black hole i.e. Kerr black hole within the context of loop quantum cosmology. In this work, we study the effect of accretion of dark energy on the lifetime of the black holes[17,64–66]. The accretion rate of a black hole in the presence of interacting dark energy is given by[16,67]

$$\begin{aligned} \dot{M}_{acc} &= 4\pi f R_{BH}^2 \rho_x (1+\omega) \\ \implies \dot{M}_{acc} &= 4\pi f R_{BH}^2 \Gamma t \rho (1+\omega). \end{aligned} \tag{14}$$

Here, for radiation-dominated era $\rho = \rho_R$ and for matter-dominated era $\rho = \rho_M$, f is the accretion efficiency, and R_{BH} is the radius of the outer horizon of the rotating black hole with the mass M. Mathematically R_{BH} can be represented by

$$R_{BH} = r_+ = G\left(M + \sqrt{\left(M^2 - (a^*)^2\right)}\right) \tag{15}$$

where rotation parameter $a^* = \frac{J}{M}$ (with angular momentum J). In order to avoid naked singularity, the rotating black hole solution must obeys the inequality $M^2 \geq (a^*)^2$. Again, the exact value of f is not fixed as it depends upon the mean free paths of the surroundings particles of RBHs. Moreover, the accretion of dark energy will continue as long as the interaction becomes effective i.e. upto $t = 52.978t_0$. Now by using the expression of Eqs. (7), (12) and (15) in Eq. (14), we find the modified accretion rate for radiation-dominated era as

$$\begin{aligned}\left(\dot{M}_{acc}\right)_{rad} =& 4\pi f G^2\left(M + \sqrt{\left(M^2 - (a^*)^2\right)}\right)^2 \Gamma t \rho_0 \left[\frac{\rho_0}{\rho_c} + \left\{2\sqrt{\frac{8\pi G}{3}}\rho_0^{\frac{1}{2}}(t - t_e) + \frac{3}{2}\sqrt{\frac{8\pi G}{3}}\rho_0^{\frac{1}{2}}(t_e - t_0)\right.\right.\\ &\left.\left. + \left(1 - \frac{\rho_0}{\rho_c}\right)^{\frac{1}{2}}\right\}^2\right]^{-1}\left(1 + 0.01971\left(\frac{t}{t_0}\right)^2 - 1.0442\left(\frac{t}{t_0}\right)\right).\end{aligned} \tag{16}$$

Solving the above Eq. (16), we get

$$\begin{aligned}&\left(-3(a^*)^2 M + 2M^3 - 2\left(-(a^*)^2 + M^2\right)^{\frac{3}{2}}\right)\left(\frac{1}{3(a^*)^4}\right)\\ &= 3.04537 \times 10^{-94}t^2 - 1.39161 \times 10^{-74}t\\ &\quad + \left(1.18835 \times 10^{-57}\right) ln\left(2.4586 \times 10^{97}t^2 - 6.1626 \times 10^{114}t + 3.8616 \times 10^{131}\right)\\ &\quad + \left(56252.9\right) tan^{-1}\left(1.08926 \times 10^{44}t - 1.36512 \times 10^{61}\right) + 1.73924 \times 10^{-57} + Constant.\end{aligned}$$

Above equation shows that when you vary the time in a certain range, the change in mass is quite negligible. So we cannot say that mass change is always giving a constant value throughout the whole evolution. We can only say that the mass variation is negligibly small, as we move in cosmic time, for different formation mass of the RBH in radiation-dominated era, which is depicted on Figure 1. Similarly, we can calculate the modified accretion rate for matter-dominated era by using Eqs. (8), (12) and (15) in Eq. (14) as

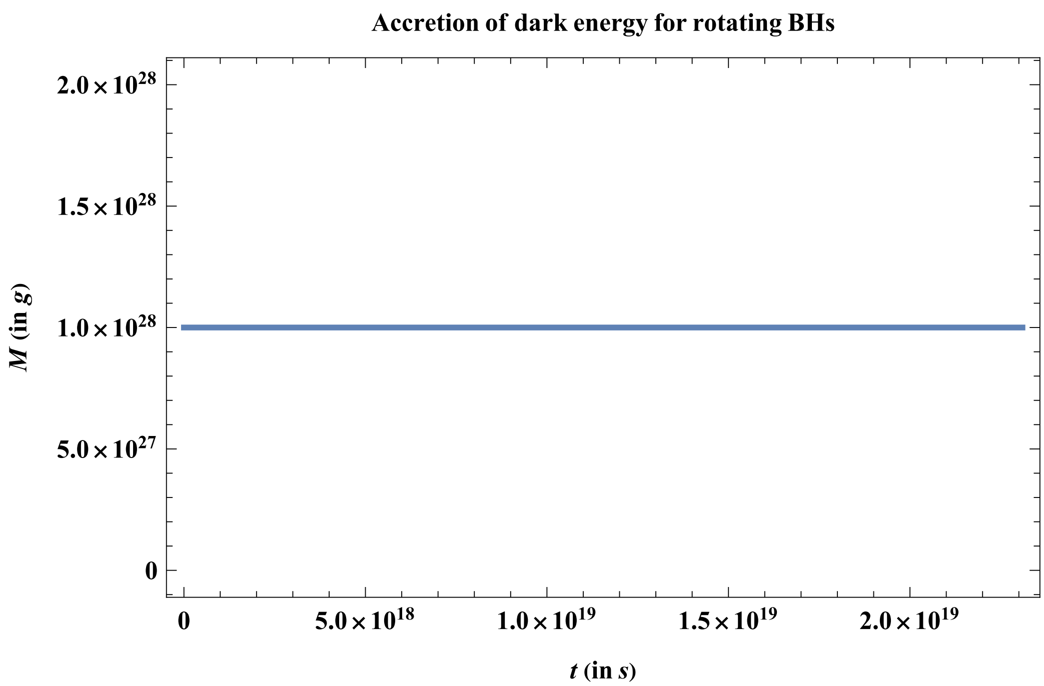

Figure 1. Variation of the RBH mass with time.

$$\left(\dot{M}_{acc}\right)_{mat} = 4\pi f G^2 \left(M + \sqrt{\left(M^2 - (a^*)^2\right)}\right)^2 \Gamma t \rho_0 \left[\frac{\rho_0}{\rho_c} + \left\{\frac{3}{2}\sqrt{\frac{8\pi G}{3}}\rho_0^{\frac{1}{2}}(t - t_0) + \left(1 - \frac{\rho_0}{\rho_c}\right)^{\frac{1}{2}}\right\}^2\right]^{-1} \left(1 + 0.01971\left(\frac{t}{t_0}\right)^2 - 1.0442\left(\frac{t}{t_0}\right)\right). \tag{17}$$

Solving the above Eq. (17), we get

$$\left(-3(a^*)^2 M + 2M^3 - 2\left(-(a^*)^2 + M^2\right)^{\frac{3}{2}}\right)\left(\frac{1}{3(a^*)^4}\right) = 5.41401 \times 10^{-94} t^2 - 2.46493 \times 10^{-74} t + \left(1.08763 \times 10^{-57}\right) ln\left(4.6099 \times 10^{96} t^2 - 1.5406 \times 10^{114} t + 1.2872 \times 10^{131}\right) + \left(85922.6\right) tan^{-1}\left(8.16946 \times 10^{43} t - 1.36512 \times 10^{61}\right) + 4.1038 \times 10^{-57} + Constant.$$

The solution of Eqs. (16) and (17) can be obtained by integrating the equations w.r.t time; which further exactly match with the results of non-rotating case at the limit $a^* = 0$. In Figure 1, we plot the variation of mass of a particular RBH, which is formed at $t = 10^{-10} s$ having rotating parameter $(a^*)^2 = 10^{-8} M_i^2$ for a single value of accretion efficiency $f = 1$. This graph shows that accretion of dark energy does not affect significantly the evolution of RBH in loop quantum cosmology. One of the reason behind this result is the absence of big bang in loop quantum cosmology. We know that energy formed during the big bang is the main cause for the rapid expansion of this universe. As the big bang is absent in LQC, the rate of expansion of this universe is slower than the standard cosmology and hence absorption of energy-matter from the surroundings become ineffective. The other factors like the symmetry of the black hole, the type of accretion rates and the type of the background fluids affect the accretion phenomena of Kerr black holes. Basically the existence of axial symmetry and stationarity of the Kerr black hole leads to slows down the accretion process[68]. In Kerr black hole, strong relativistic effects (mainly frame dragging effects) can influence the stability and dynamics of Kerr black hole environment which can reduce the steady accretion rate[69]. Several additional factors: angular momentum efficiency and radiative cooling process[70] affect significantly to the accretion of Kerr black hole. Higher radiation efficiency in Kerr black hole move matter-energy away, as it spirals inward that reduces the mass which goes into the black hole and hence reduces the growth of the black hole.

Evolution of rotating black holes

The interplay between accretion and evaporation of rotating black holes represents the whole evolution of RBHs. The rate of change of mass due to Hawking evaporation is given by[42,48]

$$\dot{M}_{evp} = -4\pi R_{BH}^2 \sigma_H T_{BH}^4 \tag{18}$$

where σ_H is the Stefan's constant and T_{BH} is the Hawking temperature. The mathematical expression of T_{BH} for rotating uncharged black hole is[42,45,71]

$$T_{BH} = \frac{\sqrt{M^2 - (a^*)^2}}{4\pi G M\left(M + \sqrt{M^2 - (a^*)^2}\right)}. \tag{19}$$

Now by using the above expression, Eq. (18) modifies to

$$\dot{M}_{evp} = -\frac{\sigma_H}{G^2 64\pi^3}\frac{\left(M^2 - (a^*)^2\right)^2}{M^4\left(M + \sqrt{M^2 - (a^*)^2}\right)^2}. \tag{20}$$

In order to understand the whole evolution of RBH, one should know the total rate of change of mass of RBHs by considering both accretion and evaporation. Now the complete evolution equation becomes

$$\dot{M}_{tot} = -\frac{\sigma_H}{G^2 64\pi^3}\frac{\left(M^2 - (a^*)^2\right)^2}{M^4\left(M + \sqrt{M^2 - (a^*)^2}\right)^2} + 4\pi f R_{BH}^2 \rho(1 + \omega). \tag{21}$$

Since above Eq. (21) is not analytically solvable, we use numerical methods to solve it. Those RBHs are formed and evaporated in the radiation-dominated era, they all will follow this evolution Eq. (21). But for those RBHs which are lived in matter-dominated era, they are not affected by the accretion beyond time $t = 52.978 t_0$ during their evolution. Because during that era due to absence of interaction between dust and dark energy, the environment is not suitable enough for energy and matter absorption. So, during that time black hole evolution is only influenced by the evaporation term in the evolution equation. But, all the black holes obey evolution Eq. (21) during the time period $t = t_e$ to $t = 52.978 t_0$ in matter-dominated era.

Now we construct Table 1 by using the accretion and evaporation equations, which shows the variation of evaporation time (t_{evp}) for different values of formation time (t_i) and mass (M_i) of RBHs.

$f = 1$			
t_i (in s)	M_i (in g)	$(a^*)^2_{max}$	t_{evp} (in s)
10^{-23}	10^{15}	10^{22}	3.33×10^{16}
10^{-18}	10^{20}	10^{32}	3.33×10^{31}
10^{-13}	10^{25}	10^{42}	3.33×10^{46}
10^{-8}	10^{30}	10^{52}	3.33×10^{61}
10^{-3}	10^{35}	10^{62}	3.33×10^{76}
10^{2}	10^{40}	10^{72}	3.33×10^{91}
10^{7}	10^{45}	10^{82}	3.33×10^{106}
10^{9}	10^{47}	10^{86}	3.33×10^{112}
10^{10}	10^{48}	10^{88}	1.67×10^{17}
10^{11}	10^{49}	10^{90}	1.67×10^{17}
10^{13}	10^{51}	10^{94}	1.67×10^{17}

Table 1. Calculation of evaporation time (t_{evp}) and maximum value of rotating parameter (a^*_{max}) for different RBH mass and time at fixed accretion efficiency *f*.

Table 1 shows that how evaporation time changes, when we consider different formation masses of RBHs. We can see that when we increase the formation mass of RBHs, the evaporation time increases. It shows that early forming RBHs evaporate more quickly than the latter one. This fact is also verified in Figure 2. From Table 1, we also found that the maximum limit of rotating parameter restrict to a certain value which is $10^{-8}M_i^2$, where M_i is the formation mass. Moreover, from Table 1, we concluded that the rotating black holes having initial mass greater than 10^{48}g would be completely evaporated by present time. As we know the accretion efficiency *f* relies on some complex physical processes, however the precise value of *f* is not confirmed. But, here we found that *f* value becomes ineffective when RBH is investigated in the theory of LQC. Again we make Table 2 by using the accretion and evaporation equation, which shows the variation of evaporation time (t_{evp}) for different values of formation time (t_i) and mass (M_i) of RBHs at particular rotating parameter value.

One can see from Table 2 that the evaporation time is different for different initial mass for a particular rotating parameter value, and independent of accretion of dark energy. But the variation of rotating parameter slightly increases the life time of the RBHs in the theory of LQC. To optimize our results, we construct the Table 3 which shows the variation of the evaporation time with rotating parameter (a^*) for RBH having formation mass (10^{25}g). It explains that the evaporation time is slightly increases by increasing the value of rotating parameter (a^*). However the life span of RBHs significantly increases with increase in rotation parameter (a^*) in standard cosmology[42]. The main reason behind this discrepancy is due to the absence of big bang within the theory of LQC. The presence of insufficient energy in LQC directly indicates that RBHs, which are formed in radiation dominated era, could not accumulate more energy by its rotation. This may be the cause why evaporation time of RBHs comes earlier in LQC than in case of standard cosmology. The other possible factors behind this discrepancy are the symmetry of the black hole, the type of accretion rates and the type of the background fluids.

Here, we also shed some light on the evolution of supermassive rotating black holes in the framework of LQC. As we know supermassive rotating black holes (SMRBHs) are highly dense object and gigantic celestial structure existed at the center of galaxies. Recently large number of observations indicate the existence of early SMRBHs in different quasars. As per the recent data[72], a luminous quasar named J0313-1806 having luminosity $3.6 \times 10^{13} L_\odot$ existed at a red shift of $z = 7.642$ just after the big bang happens. After a long spectroscopic survey, researchers identify the presence of huge SMRBH having mass $(1.6 \pm 0.4) \times 10^9 M_\odot$ ($M_\odot = 1.98892 \times 10^{33}$g) at a distance of 670 million years that provides many puzzles in different theoretical models. In our work, we examine successfully that SMRBHs having mass greater than equal to $10^{48}\text{g} \approx 10^{15} M_\odot$, would have all been evaporated by present time. So in near future, if astrophysicists will observe RBHs having mass more than equal

$(a^*)^2 = 0.9 \times 10^{38}$			
t_i(in 10^{-14}s)	M_i (in 10^{24} g)	$(t_{evp})_{f=0}$ (in 10^{42} s)	$(t_{evp})_{f=1}$ (in 10^{42} s)
0.30	0.30	0.9000	0.9000
0.35	0.35	1.4292	1.4292
0.40	0.40	2.1333	2.1333
0.45	0.45	3.0375	3.0375
0.50	0.50	4.1667	4.1667

Table 2. Calculation of evaporation time (t_{evp}) for different value of RBH mass and time at fixed rotating parameter (a^*).

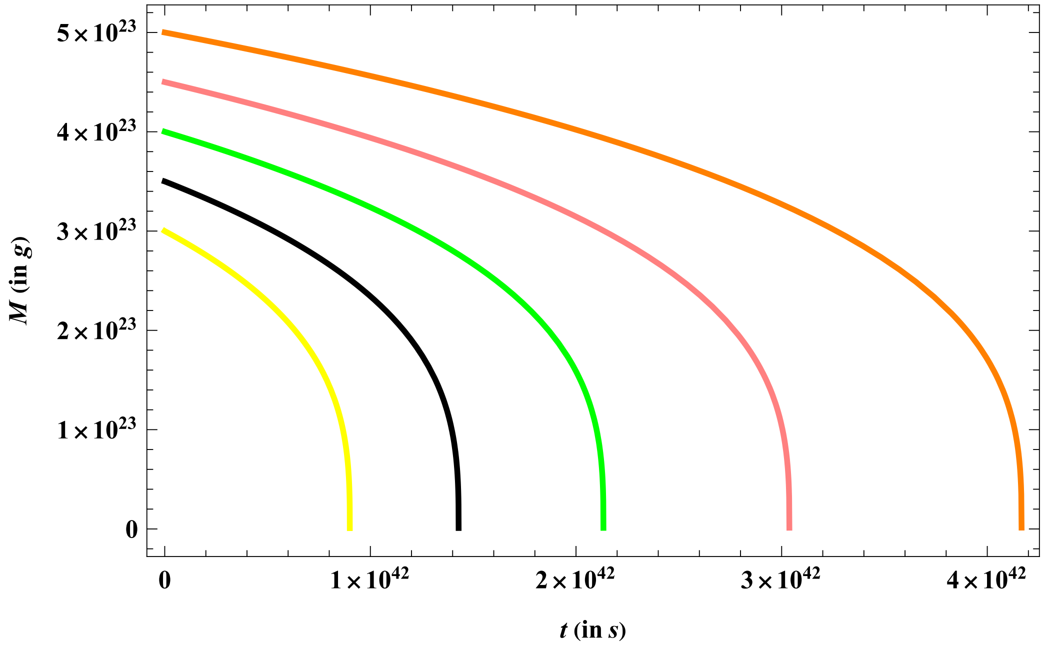

Figure 2. Variation of the RBH mass with evaporation time for a constant value of rotating parameter $((a^*)^2 = 0.9 \times 10^{38})$.

$t_i = 10^{-13}$s, $M_i = 10^{25}$g	
$(a^*)^2$	$t_{evp}(in 10^{42}$ s)
10^{37}	3333.32123
10^{38}	3333.32123
10^{39}	3333.32124
10^{40}	3333.32125
10^{41}	3333.32138
10^{42}	3333.32273

Table 3. Variation of the evaporation time with rotating parameter (a^*) for fixed value of RBH mass and time.

to 10^{48}g in any galaxy, then it will put challenges to theoretical cosmologists to encounter the mystery behind such type of gigantic supermassive black holes.

Astrophysical constraints on the RBH mass fraction

Black holes formation in different cosmological eras must be supported by the matter density of the universe in that respective eras. Several cosmological observations can be taken as a suitable candidate to enforce constraints on the number density of the RBHs in different cosmic time. These constraints behave as the responsible candidates for the RBHs formation mass spectrum in different models of cosmology. Various constraints such as present matter density of the universe, present photon spectrum, Distortion of the cosmic microwave background spectrum, Nucleosynthesis constraints and Deuterium photodisintegration constraint[73] were studied in standard cosmology. Also one can study important constraints from various limits of γ-ray background[74,75] and observed galactic antiprotons and antideuterons[76,77]. In our work, we calculate the initial mass spectrum of RBHs by taking γ-ray background limit into consideration within the theory of loop quantum cosmology.

The mass fraction of the universe is going into RBHs at any time t is represented by[26,78,79]

$$\beta(t) = \left[\frac{\Omega_{RBH}(t)}{\Omega_R}\right](1+z)^{-1} \tag{22}$$

where $\Omega_{RBH}(t)$ shows the present density parameter interconnected with RBHs forming at time t having value $< 10^{-8}$, z is the redshift linked with time t and Ω_R represent the present microwave background density with value 10^{-4}. Here we consider that RBHs are formed in the radiation-dominated era. So in this environment, the redshift equation becomes

$$(1+z)^{-1} = \left(\frac{a(t)}{a(t_e)}\right)\left(\frac{a(t_e)}{a(t_0)}\right). \tag{23}$$

By inserting the scale factor expression in above Eq. (23) in terms of M_i, we can get the modified redshift equation as

$$(1+z)^{-1} = G^{\frac{1}{2}}\left[G^{-2}\frac{\rho_0}{\rho_c}\left(\frac{a_0}{a_e}\right)^3 + \left\{2\rho_0^{\frac{1}{2}}\left(\frac{a_0}{a_e}\right)^{\frac{3}{2}}\sqrt{\frac{8\pi G}{3}}(M_i - M_e) + G^{-1}\left(1-\frac{\rho_0}{\rho_c}\left(\frac{a_0}{a_e}\right)^3\right)^{\frac{1}{2}}\right\}^2\right]^{\frac{1}{4}} \times u \tag{24}$$

where $u = \frac{1}{y}$. By using Eq. (24) in Eq. (22) and taking the values of $\Omega_{RBH}(t)$ and Ω_R, one can find the bound on the mass fraction for presently evaporating RBHs as

$$\beta(M_i) < 7.2821 \times 10^{-5}. \tag{25}$$

Here our result is much greater than the values found in case of standard cosmology and scalar-tensor theory[42,80], which implies there is a greater tendency for formation of black holes in loop quantum cosmology than any other cosmological models.

Conclusion

In this work, we have successfully explained the evolution of rotating black holes (RBHs) by introducing the concept of interacting dark energy within the context of loop quantum cosmology. First, we have evaluated the accretion rate of RBHs by using the expressions of energy density (ρ), equation of state parameter (ω) and radius of RBHs. Subsequently, we have calculated the evaporation rate of RBHs by applying the Hawking evaporation mechanism. From our analysis, we found that the effect of accretion of dark energy would be insignificant in influencing the evolution of RBHs. Again, we have analyzed the impact of rotation on evolution of black holes. This study made us to put a constraint on the maximum value of rotating parameter. The allowed values of this parameter was found to be within the range of 0 and $10^{-8}M_i^2$, where M_i is the formation mass of RBH. Further, we have found that the life span of a black hole having rotation would be slightly greater than that of its non-rotating counterpart. By taking accretion of dark energy and Hawking evaporation into account, we have shown the complete evolution of RBHs in different cosmic eras. Finally, our study predicted that the supermassive RBHs having mass greater than equal to 10^{48}g, would have been all evaporated by present time. Also from astrophysical constraint analysis, we concluded that there is a greater tendency for formation of black holes in loop quantum cosmology than standard cosmology and scalar-tensor theory.

Ethics declaration

The research and analysis conducted by the authors are fully and accurately reflected in the paper. Nowhere is this work being considered for publication.

Data availibility

The datasets used and/or analysed during the current study available from the corresponding author upon reasonable request.

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