Gravitational wave echoes from quark stars

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Abstract. It has recently been reported that gravitational wave echoes at a frequency of about 72 Hz have been detected with the significance of 4.2σ from two neutron star merger event GW170817. Alternatively, the echoes of a gravitational wave can be obtained from ultra-compact stars, i.e., stars with compactness between 1/3 and 4/9. The mass-radius relation of ultra-compact stars is crossing the photon sphere line, where the line obeys the relation compactness equal to 1/3. Furthermore, it has recently been reported that strange stars within a simplest MIT-Bag model with an ultra-stiff equation of state (EoS) where the speed of sound equal to speed of light in vacuum can produce gravitational wave echoes with frequencies on the order of tens of kilohertz. For more realistic models of strange star equation of state, the speed of sound in the matter is physically less than one, and it should be density-dependent. In this report, we will check whether the more realistic EoS of quark stars can emit gravitational wave echoes or not. Here we employ two models of EoSs such as MIT-Bag models with speed of sound =1/√3 and CIDDM models with the density-dependent speed of sound.

1. Introduction

The interpretation of gravitation wave GW170817 event detected by LIGO-VIRGO interferometers is a signal of two neutron stars merging with an estimated total mass M_{tot} about 2.4 solar mass. The final results of merging could be a massive compact star or black hole [1]. The existence of gravitation wave (GW) echoes in GW17087 event has been studied for the first time in Ref. [2]. They obtained from their analysis that a tentative signal with a frequency of about 72 Hz with 4.2σ significance level exists. The discussions of the origin, mechanism, and possible source of GW echoes can be found in Ref. [3] and the references therein, including there an interpretation of this echo signal as originating from ultra-compact objects. Note that the ultra-compact objects are objects with compactness larger than compactness of ordinary compact objects like neutron stars. Ultra-compact objects have compactness between 1/3 and 4/9. Therefore, the mass-radius of an ultra-compact object can cross the photon sphere line in the star mass-radius relation. The photon sphere is a surface located at R=3GM where the circular photon orbits around the object are possible. The known examples of ultra-compact objects are such as Gravstars, Wormholes, Fuzzballs [4].

Strange stars are stars composed by up, down, and strange quarks. The formation of strange stars is based on the consequence of the idea that the presence of strange quark can lower the binding energy of quark matter in weak equilibrium below the one of ^{56}Fe. It is known in the literature as Bodmer and Witten hypothesis for the absolute stability of quark matter. Theoretical studies show that neutron stars may be converted to strange stars. However, the possible existence of quarks stars is still one of the most intriguing aspects of astrophysics, which has important implications for understanding the physics of strongly interacting matter. There are many models of strange quark matter in literature. The MIT-Bag model provides the most straightforward description of strange quark matter, where quarks are free and the average impact of confinement is provided through Bag constant. Another attractive strange quark matter model is confined isospin and density-dependent mass (CIDDM) model where its sound of speed is less than one and density-dependent. (Please see detail discussions about strange stars, e.g., [5] and the references therein.) Because strange stars are known very compact, the authors of Ref. [3] examined the possibility that ultracompact object predicted by GW170817 event is a strange star by evaluating the GW echoes produced by the strange star. However, in their calculation, they used the simplest MIT-Bag model with an unrealistic ultra-stiff equation of state (EoS), i.e., with speed of sound equal to one. We need to note the authors of Ref. [6] has shown the conjecture that the speed of sound in any medium is smaller than the velocity of light in vacuum divided by √3 is compatible with two things, neutron stars with masses around two solar mass and proper equation of state of hadronic matter at low densities. Therefore, in this work, we re-examine the possibility that ultra-compact object predicted by GW170817 event is a strange star by evaluating the GW echoes produced by the strange star but by using more realistic models of strange quark matter namely MIT-Bag models with a speed of sound =1/√3 and CIDDM models. In this work we want to check whether strange stars are an ultra-compact object or not.

The present paper is organized as follows. In Sec 2, we discuss the used formalism briefly. In sec 3, we provide the results and discussion, while sec 6, is for the conclusion.

2. Formalism

Here, we briefly discuss the strange matter based on MIT-Bag and CIDDM models. The strange matter composed by up, down, and strange quarks with the presence of electrons to make the quark matter electrically neutral. We assume that quark stars are static and spherically symmetric. Therefore, we can use Tolman-Oppenheimer-Volkoff (TOV) equations to obtain the mass and radius of the quark stars. We assume also that the critical temperature of the strange matter is larger than the actual temperature in quark stars due to the fact that quark stars are very dense, then the impact of temperature in equation of state of quark matter can be neglected. Here we use natural unit where the speed of light in vacuum c=1 and Planck constant ħ = 1.

2.1 *MIT-Bag Models*

The general form of the EOS of strange matter based on MIT-Bag model can be expressed as the relation between pressure P and energy density ε of strange matter such as

$$\varepsilon = 3p + 4B, \tag{1}$$

for realistic matter based on MIT-bag model with speed of sound $v_s = 1/\sqrt{3}$ and we should replace factor 3 in front pressure in Eq. (1) with factor 1 for ultra-stiff EoS. As suggested by Bodmer and Witten the bag constants B simulating the physical confinement with the acceptable value range is between 145 MeV $< B^{1/4} <$ 162 MeV. Detail discussion about the nature of MIT-Bag models can be found in Refs. [3-4].

2.2 *CIDDM Model*

For strange with EoS based on the confined-isospin-density-dependent-mass (CIDDM) model with additional scalar and vector Coulomb terms of strange quark matter with parameterκ_3=2500, κ_2^1=0.3, κ_2^2=0.8 and κ_1=0.46. Here κ_3 is isospin dependent parameter, κ_2^1 is scalar Coulomb parameter and $\kappa_2^1$is vector Coulomb parameter and κ_1 is harmonic oscillator parameter. The EoSs with scalar and vector Coulomb terms also pass the test for acceptable EoS to describe the quark matter (see the details of the EoS in Ref. [8] and the references therein). Note that the authors of Ref.[8] have found that within GR, if the Coulomb term is included, for the models where their parameters are consistent with strange quark matter absolute stability condition, the two sun-mass constraint prefers the maximum QS mass prediction of the model with the scalar Coulomb term to that of the model with the vector Coulomb term. With these models, we can obtain speed of sound less than 1 and density dependent. Detail discussion about the nature of CIDDM models can be found in Refs. [5,8].

2.3. TOV Equations

To obtain the mass-radius relation, we use the EOS strange matter calculated by using above models as input to solve TOV equations. The TOV equation is written in the form of differential equations as follows [3-4].

$$\frac{dp}{dr} = -\frac{G\varepsilon(r)m(r)}{r^2}\left[1+\frac{P(r)}{\varepsilon(r)}\right]\left[1+\frac{4\pi r^3 P(r)}{m(r)}\right]\left[1-\frac{2Gm(r)}{r}\right]^{-1}, \quad (2)$$

$$\frac{dm}{dr} = 4\pi r^3 \varepsilon(r), \quad (3)$$

$$\frac{d\Phi}{dr} = -\frac{1}{\rho + p}\frac{dp}{dr}, \quad (4)$$

with p is the pressure, and the speed of light in vacuum is equal to 1. Note that G is Newtonian gravitational constant and in the center m(0) ≈ 0, P(0) ≈ Pc, while in the edge of the star m(R) = M , P(R) = 0. Φ(r) is matric function, M is mass of the star and R is radius of the star. Note that the corresponding first order differential equations are solved numerically by using Runge-Kutta method.

2. 4. Frequency of GW Echoes

The gravitational waves emitted by stellar objects are partly reflected by the angular potential barrier on photon spheres. It is conceivable that photon is trapped for gravitational waves; whit the frequency is inversely proportional to the length of the trap. If the trap distance is small means that the star is getting closer to the photon sphere line or the signal GW frequency is high. The signal echo time is measured as the time of light from the center of the star to the photon spheres, according to [3,10]

$$\tau_{echo} = \int_0^{3M} \frac{dr}{\sqrt{e^{2\Phi(r)}\left(1-\frac{2m(r)}{r}\right)}}, \quad (5)$$

where m (r) and Φ (r) are determined by solving the TOV equation. The frequency of echoes of gravitational waves can be estimated by $\omega_{echo} = \pi / \tau_{echo}$. The estimated frequency is $1/2\tau_{echo}$.

3. Results and Discussion

In this section we will provide the results of numerical calculations based on the analysis. From the calculation results we obtain several plot results which are shown in Figs. 1-4.

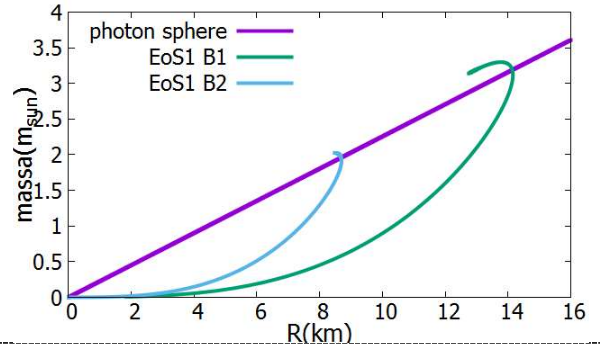

Figure 1. Mass-radius diagram for various stars, with equation of state EoS1

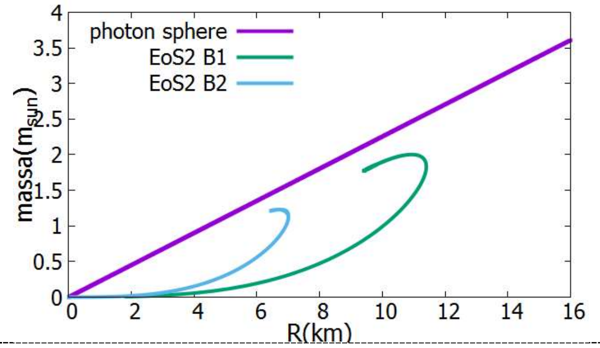

Figure 2. Mass-radius diagram for various stars, with equation of state EoS2

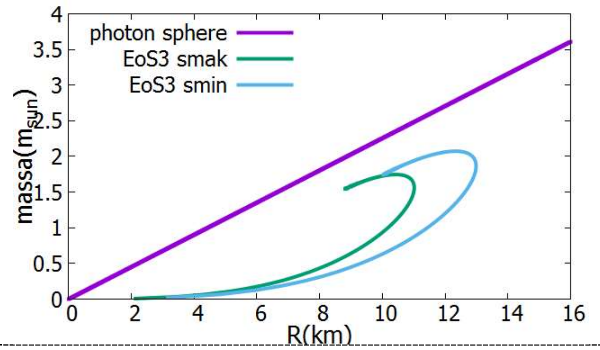

Figure 3. Mass-radius diagram for various stars, within CIDDM with scalar Coulomb model

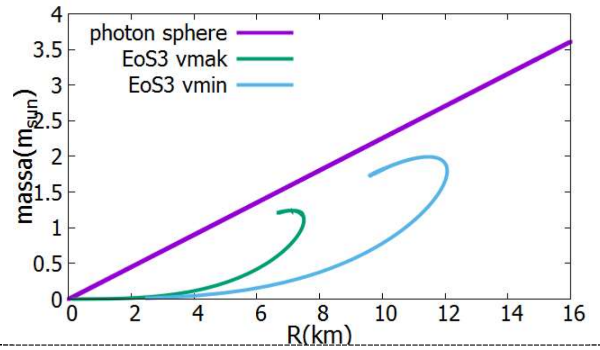

Figure 4. Mass-radius diagram for various stars within CIDDM with vector Coulomb model

Table 1. The echo results of gravitational waves of various stars

EoS	Model	MIT bag (MeV) (B1=145, B2=185)	ω_{echo} (kHz)
EoS1	$\varepsilon = p + 4B$	145	21.87
EoS1	$\varepsilon = p + 4B$	185	35.60
EoS2	$\varepsilon = 3p + 4B$	145	0
EoS2	$\varepsilon = 3p + 4B$	185	0
EoS3	CIDDM scalar	-	0
EoS3	CIDDM vector	-	0

It can be seen in Fig. 1, strange stars can cross the photon sphere line so that gravitational wave echos exist as the one obtained by Ref. [3] by using ultra-stiff EoS $\varepsilon = p + 4B$ and for the range of the acceptable B value. It is obvious in Fig. 2 that strange stars cannot cross the photon sphere line, where the EoS base on realistic MIT-Bag model and Figs. 3-4 for both kinds of CIDDM models. Therefore, it obvious for realistic strange stars, the stars do not emit the gravitational wave echoes. Note that in EoS 1 there is no interaction, the greater the constants for increasing values in both the mass and star radius, this indicates that the star is getting compressed. Moreover, at pressure p, the star's speed is the same as the speed of light ($v_s = 1$). Whereas in EoS 2, there is also no interaction with the same bag constant, and the prefactor in front the pressure become to 3, the star's speed is equal to $1/\sqrt{3}$ the speed of light ($v_s = 1/\sqrt{3}$). EoS 1 and EoS 2 both use MIT bag models with the same bag constants are B_1 = 145MeV and B_2= 185MeV. EoS 3 and EoS 4 have interactions. EoS 3 uses the CIDDM model with Coulomb scalar whereas in EoS 4 use the CIDDM model with Coulomb vector. It means that the compactness strange stars of depend significantly on the speed of sound. However, to emit GW echo, strange stars should have $v_s \geq 1$. This later fact is unrealistic.

4. Conclusions

Here we re-examine the possibility that ultra-compact object predicted by GW170817 event is a strange star by evaluating the GW echoes produced by the strange star but by using more realistic models of strange quark matter namely MIT-Bag models with a speed of sound $=1/\sqrt{3}$ and CIDDM models. We have shown that strange stars compactness depends on the speed of sound in strange quark matter. We confirm that strange stars with realistic EoS cannot be categorized as ultra-compact oobjects

5. References

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