

NLO–QCD EVENT GENERATORS IN GRACE*

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Automatic Feynman-amplitude calculation system, GRACE, has been extended to treat next-to-leading order (NLO) QCD calculations. Matrix elements of loop diagrams as well as those of tree level ones can be generated using the GRACE system. A soft/collinear singularity is treated using a leading-log subtraction method. Higher order re-summation of the soft/collinear correction by the parton shower method is combined with the NLO matrix-element without any double-counting in this method. An example of the event generator for $W +$ jet and di-photon processes are given for demonstrating a validity of this method.

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1. Introduction

As a next generation energy frontier experiment, the Large Hadron Collider (LHC) experiment [1] will start at CERN in 2008. LHC is a proton–proton colliding machine with a beam energy of 7 TeV. One of the main purpose of the experiment is to search for the Higgs boson, which is the only missing particle included in the well-established standard model [2], and to search for new phenomena beyond the standard model such as SUSY model. Moreover a precision measurement of standard model parameters is also an important purpose of LHC.

For a large scale experiment such as LHC, event generators which can reproduce physics phenomena based on the theoretical prediction are necessary. Especially for LHC event generators play a very important role to extract physics signal from QCD backgrounds, since the hadron–hadron collider has a large background of QCD jets. One may need event generators based on the QCD calculation, however, the lowest order results of the QCD calculations have very limited prediction power due to large uncertainty from renormalization scale ambiguity. At least NLO QCD calculation is needed

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to predict absolute values of the cross sections. (Much effort to calculate NNLO corrections [3] is being made for LHC experiments.) Usual analysis method for experimental data using QCD event generators is to use LO event-generators for predictions of distribution, which are corrected by the NLO calculations as an over-all factor (so-called *k-factor*). This kind of method is obviously not enough for the precision measurement at LHC, because the NLO correction may deform various distributions. When the signal is overlapped by a tail of the background distribution, one must be careful since the amount of the background around a tail region may be affected by the NLO correction very strongly. The NLO-QCD event generator is highly desirable to extract the physics signal from huge QCD backgrounds as much as possible. A lot of work has already been done [4] for developing NLO-QCD event generators.

In order to perform NLO-QCD calculations for lots of processes appearing in the LHC physics systematically and exactly, automatic system is necessary. A status of the GRACE development along this line is explained in this report.

2. What is GRACE?

A calculation of the matrix elements of hard-scattering processes is not a simple task when multiple partons are produced. The GRACE system [5], an automatic system for generating Feynman diagrams and a FORTRAN source-code to evaluate the amplitude, is developed to perform this kind of tasks. The system can in principle treat any number of external particles, and has been used for up to six fermions [6] in the final state within a practical CPU time. In the GRACE system, matrix elements are calculated numerically using a CHANNEL [7] library based on a helicity amplitude. CHANNEL contains routines to evaluate wave-functions/spinors at external states, interaction vertices, and particle propagators. Latest version of GRACE can treat minimal SUSY standard model up to one-loop level. One can obtain the latest version of GRACE from <http://www-sc.kek.jp/> and easily install into your Linux machine.

Based on this method, we have already published an event generator of various processes in proton-(anti-)proton collisions at tree-level named GR@PPA [8], which includes, for instance, $W/Z + \text{jets}$ (up to 4 jets) with the subsequent W/Z decay to a fermion pair with an interface to PYTHIA [9].

A schematic view of GRACE is shown in Fig. 1. GRACE consists of a diagram generator, amplitude (code) generator, and numerical integration/event generation parts. The diagram generator can generate all possible Feynman diagrams according to a model file. The model file defines particles and interactions appearing in the theory. Some part of the particle definition in the model file is shown in Fig. 2. A particle name, spin,

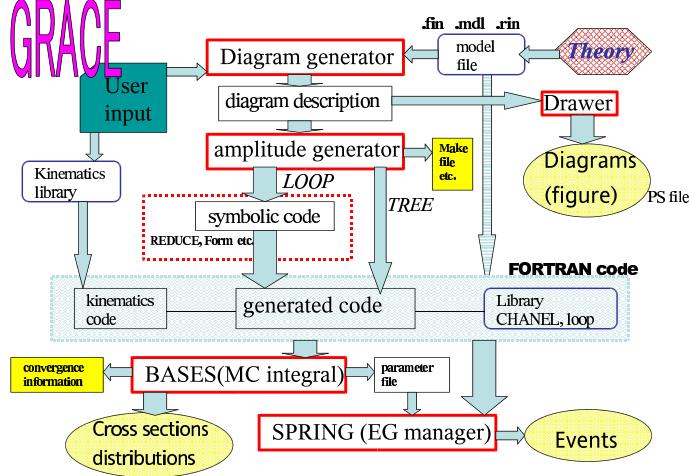


Fig. 1. A schematic view of a structure of GRACE.

```
%=====
% gauge bosons
%-----
Particle=W-plus["W+"]; Antiparticle=W-minus["W-"];
Gname={"W", "W^+", "W^-"};
PType=Vector; Charge=1; Color=1; Mass=amw; Width=agw;
PCode=2; KFCode=24; Gauge="wb";
Pend;
%
Particle=Z["Z0"]; Antiparticle=Particle;
Gname={"Z^0"};
PType=Vector; Charge=0; Color=1; Mass=amz; Width=agz;
PCode=4; KFCode=23; Gauge="zb";
Pend;
%
Particle=photon["A"]; Antiparticle=Particle;
Gname={"\gamma"};
PType=Vector; Charge=0; Color=1; Mass=ama; Width=0;
PCode=1; Massless; KFCode=22; Gauge="ab";
Pend;
%
Particle=gluon["g"]; Antiparticle=Particle;
Gname={"g"};
PType=Vector; Charge=0; Color=8; Mass=amg; Width=0;
PCode=8; Massless; KFCode=21;
Gauge="gl"; PSelect="gluon";
Pend;
```

Fig. 2. A particle definition part of the GRACE model file. Here gauge bosons are defined.

electric charge, color charge and so on are described in the file. User can add new particles in the file. Another part of the model file (interaction part) is shown in Fig. 3. Interacting particle names, order of the coupling constants, and variable names are defined in the file. User can also introduce new interactions into GRACE through the model file. In this case user has to prepare a FORTRAN subroutine to calculate such new interaction.

```

Vertex={u-bar, u, Z}; ELWK=1; FName=czuu(2,1/3);
  Vend;
Vertex={c-bar, c, Z}; ELWK=1; FName=czuu(2,2/3);
  Vend;
Vertex={t-bar, t, Z}; ELWK=1; FName=czuu(2,3/3);
  Vend;
Vertex={d-bar, d, Z}; ELWK=1; FName=czdd(2,1/3);
  Vend;
Vertex={s-bar, s, Z}; ELWK=1; FName=czdd(2,2/3);
  Vend;
Vertex={b-bar, b, Z}; ELWK=1; FName=czdd(2,3/3);
  Vend;
-----
% FFV (FFg)
-----
Vertex={u-bar, u, gluon}; QCD=1; FName=cguu(2,1/3);
  FType="V"; Vend;
Vertex={d-bar, d, gluon}; QCD=1; FName=cgdd(2,1/3);
  FType="V"; Vend;
Vertex={c-bar, c, gluon}; QCD=1; FName=cguu(2,2/3);
  FType="V"; Vend;
Vertex={s-bar, s, gluon}; QCD=1; FName=cgdd(2,2/3);
  FType="V"; Vend;
Vertex={b-bar, b, gluon}; QCD=1; FName=cguu(2,3/3);
  FType="V"; Vend;
Vertex={t-bar, t, gluon}; QCD=1; FName=cgdd(2,3/3);
  FType="V"; Vend;

```

Fig. 3. An interaction definition part of the GRACE model file. Interactions among fermions and gauge boson are described.

An input file which includes particle names in initial and final state, interaction order and so on must be prepared before a graph generation. An example of the input file is shown in Fig. 4. A name of the input file is assumed to be `in.prc`. A command `grc` will generate an information file

```

%%%%%%%%%%%%%
Model="sm.mdl";           ← Name of model file
%%%%%%%%%%%%%
Process;                  ← loop order
  ELWK={2,2};             ← tree order
  QCD={3,1};              ← Order of  $\alpha$ 
  Initial={u u-bar};       ← Order of  $\alpha_s$ 
  Final ={gluon, w-plus, w-minus}; ← initial state particles
  Expand=Yes;
  Block=No;
  AnyCT=Yes;
  Kinem="2301";           ← final state particles
  Pend;                   ← kinematics number

```

Fig. 4. An example of the input file. The process of $u\bar{u} \rightarrow W^+W^-$ gluon is at 1-loop order of the QCD specified in this input file.

of all possible Feynman diagrams for the process defined in the input file. A command `gracefig` can visualize diagrams on the terminal as well as a post-script file. An example of the diagrams drawn by `gracefig` is shown in Fig. 5. To generate FORTRAN programs to evaluate the amplitude,

enter a command `grcfort`. All necessary program files to calculate cross sections of given process and to generate unit weight event are generated by this command. The numerical integration and event generation will be done using BASES/SPRING [10]. Appropriate kinematics routine can be chosen from program-library according to a singularity of the process. Experimental cuts can apply by using simple user routine.

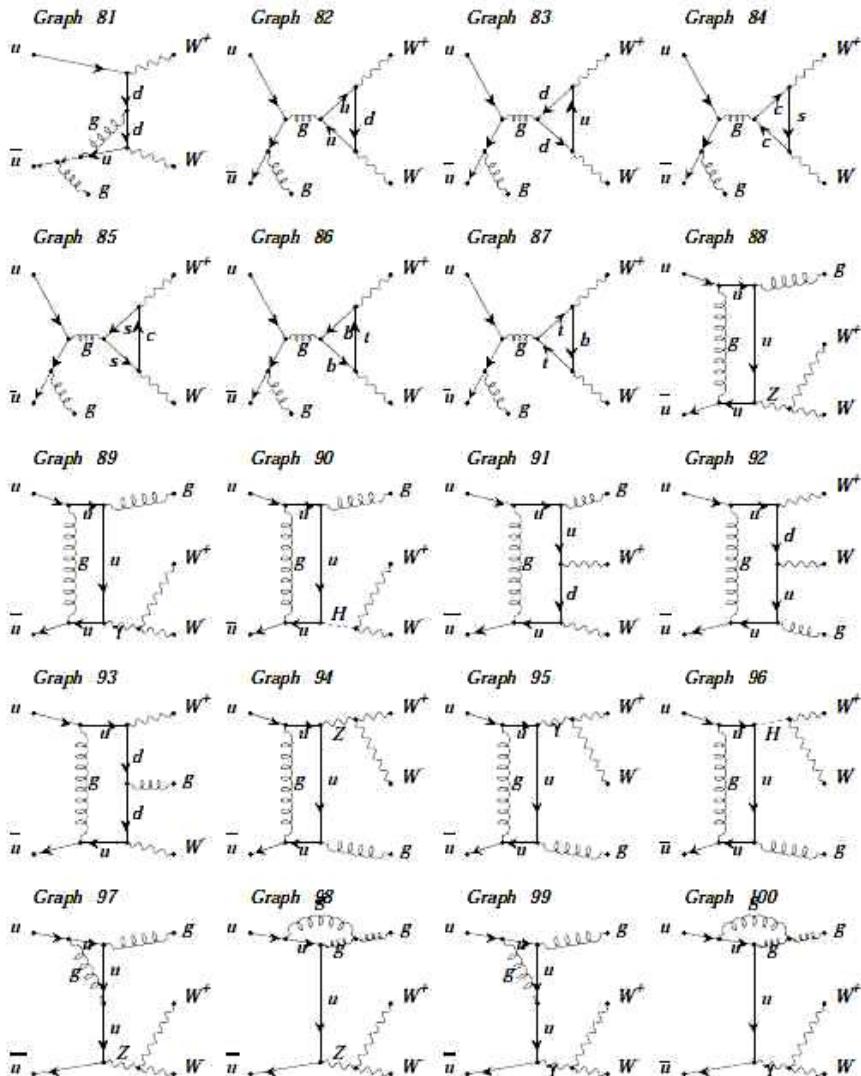


Fig. 5. A part of the diagrams of the process of $u\bar{u} \rightarrow W^+W^-$ gluon. There are 171 diagrams for this process in total.

2.1. *NLO-QCD calculation in GRACE*

NLO-QCD event generator may consists of following elements:

- lowest order amplitude,
- loop oder amplitude,
- loop integrals,
- soft/collinear correction,
- real parton emission,
- parton distribution function,
- parton shower,
- rejection of double-countings,
- hadronization and peripheral routines,
- integration of multi initial/final processes into one package.

How to treat each item in the GRACE NLO-QCD system will be explained shortly as follows. For the detailed description, please refer previous report [11].

Lowest order and real parton emission amplitude are based on the tree level calculation, then GRACE system can treat them. Separation between real parton emission and soft/collinear correction is performed through a hybrid method combining a phase-space slicing and leading-log subtraction methods. That can supply cut-point independent results numerically stably. Loop amplitude with 2- and 3-point functions are embedded in the GRACE system as effective vertices. More than 4-point amplitudes are obtained using a purely numerical method. Four-point loop integrals including tensor integral are obtained using hyper-geometric functions analytically [12]. A fast and numerically stable FORTRAN routine is developed to evaluate loop integration based on that analytical formulae. Soft and collinear effects are resumed using parton shower method. We used a LO parton shower in forward evolution scheme. At low energy scale (at a b -quark mass) x -distribution is borrowed from reliable Parton Distribution Function (PDF). From this low energy scale to the hard-scattering energy scale, initial parton evolved using LO-QCD parton shower. In order to avoid a double-counting of leading-logarithmic terms between matrix elements and parton shower, these terms are subtracted from matrix elements. Moreover, when parton shower and the real parton emission is combined, it must be avoided that a parton from matrix elements overlaps that from parton shower in the same phase

volume. In order to avoid this kind of double-counting we require virtuality ordering on the matrix elements determined from maximum virtuality from the parton shower. This virtuality ordering restricts the phase space for the final state parton emission. This method works well on the simple Drell–Yan process as shown in Fig. 6. A transverse momentum distribution of

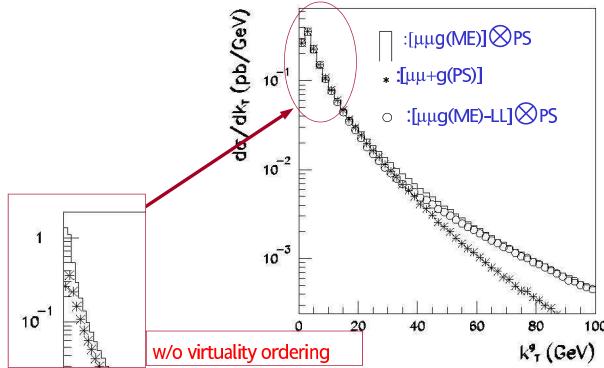


Fig. 6. Transverse momentum distributions of gluon in the Drell–Yan process. The gluon distribution emitted from matrix elements with virtuality ordering is shown in a solid histogram, and that from parton shower (leading gluon in the shower) is shown in dots. In a left lower histogram the result without virtuality ordering is also shown.

a gluon emitted from matrix elements with virtuality ordering is compared with distributions from parton shower (leading gluon jet). The distributions agree with each other very well. On the other hand distributions without virtuality ordering show disagreement as seen in the left-lower histogram in Fig. 6. This agreement shows that the virtuality ordering method works very well.

In hadron collider experiments, many processes with different initial and final state particles give the same event topology. Then it is desirable that the event generator can be used without any modifications to combine those different processes from user side. For this purpose we are developing NLO-QCD event generators in the framework of GR@PPA [8] which can combine many sub-processes into one package and gives an interface to PYTHIA [9] or HERWIG [13].

2.2. Examples of event generators made by GRACE

As examples of a prototype event generator made by GRACE, $W +$ jet and 2γ processes are explained in this section.

2.2.1. $W +$ jet process

As an example of a process with colored parton in the final state, we chose the $W +$ jet process. Very simple experimental cuts as transverse momentum of W and of jet greater than 20 GeV are applied. The PDF of CTEQ5L is used with factorization energy scale (and renormalization one) set to be W boson mass. Tree level cross section of 3.56×10^4 pb is obtained with the double-count rejection. If we do not require any double-count rejection, the cross section is 4.26×10^4 pb, which means double-counting effect is about 20%. Cancellation of the infra-red singularity is numerically checked at order of 10^{-10} . Full corrected cross sections are obtained to be 7.06×10^4 pb. Transverse momentum distributions of the process are shown in Fig. 7.

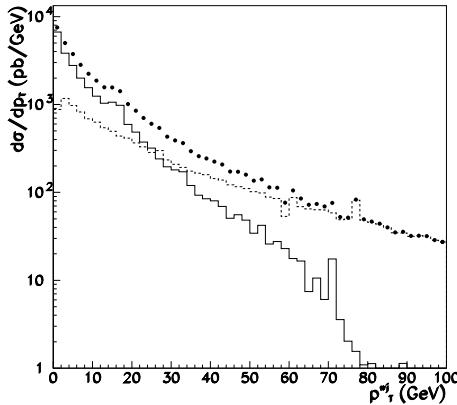


Fig. 7. Transverse momentum distributions of the $W +$ jet system. A solid histogram shows those of tree+virtual+soft/collinear correction, a dashed one shows those of real parton emission with leading-log subtraction. Filled circles show the result with full correction.

2.2.2. 2γ process

The 2γ process is an important process for the Higgs search for a mass range between 110 GeV to 150 GeV. To understand the detailed behavior of the QCD backgrounds, NLO-QCD event generator of this process is desirable. There already exists NLO event generator, DIPHOX [14], developed by Guillet *et al.*, however it is not a full exclusive event generator. Then we decided to create the event generator for this process.

Very simple cuts of the energy of γ greater than 10 GeV, polar angle greater than 10 degree, and opening angle between two γ greater than 10 degree are required for the test purpose. The PDF of CTEQ5L is used with

factorization energy scale (and renormalization one) set to be a center of mass energy of $\gamma\gamma$ system. Tree level cross section of 7.35×10^2 pb is obtained with the double-count rejection. Cancellation of the infra-red singularity is numerically checked at order of 10^{-10} . Full corrected cross sections are obtained to be 2.25×10^3 pb. Transverse momentum distributions of the process are shown in Fig. 8.

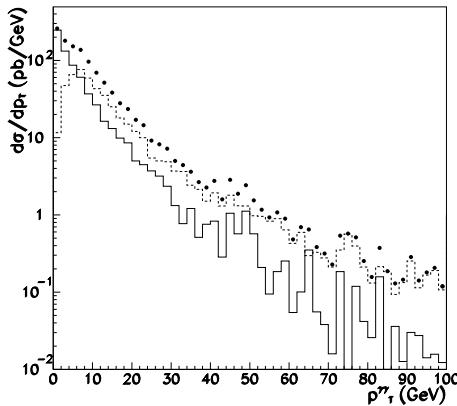


Fig. 8. Transverse momentum distributions of the 2γ system. A solid histogram shows those of tree+virtual+soft/collinear correction, a dashed one shows those of real parton emission with leading-log subtraction. Filled circles show the result with full correction.

2.3. Summary

Automatic Feynman amplitude generator GRACE is extended to treat NLO-QCD calculations. Tree-level calculations can be done up to 6 particle (and more) final state. For loop corrections, two- and three-point amplitudes are implemented in the system as an effective vertices. More than 4-point amplitudes are numerically evaluated using helicity amplitude method. Numerically stable loop integration package for box integral are developed up to tensor integral. The double-counting problems are treated by the leading-log subtraction method and virtuality ordering. As a prototype event generator, the NLO-QCD programs for $W +$ jet and 2γ processes are constructed. Those event generators (and more) will be implemented in the GR@PPA system and open to the LHC experimentalists in near future.

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