Chapter 32

Phenomenology in the Higgs triplet model with A_4 symmetry

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Abstract

I will discuss phenomenology of doubly charged scalars of $SU(2)_L$-triplet fields in the simplest extension of the Higgs Triplet Model with the A_4 symmetry. It is shown that their decays into a pair of leptons have unique flavor structures which can be tested at the LHC if some of their masses are below the TeV scale. Sizable decay rates for $\tau \to \overline{\mu} ee$ and $\tau \to \overline{e}\mu\mu$ can be obtained naturally while other lepton flavor violating decays of charged leptons are almost forbidden in this model, which can be tested at the MEG experiment and future B factories. This talk is based on ref. [1].

32.1. Introduction

Neutrino oscillation measurements declared that neutrinos have masses although they are regarded as massless particles in the standard model of particle physics (SM). The experiments also uncovered the structure of the lepton flavor mixing matrix, the Maki-Nakagawa-Sakata (MNS) matrix U_{MNS}, which can be parametrized as

$$U_{\text{MNS}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad (32.1)$$

where c_{ij} and s_{ij} mean $\cos\theta_{ij}$ and $\sin\theta_{ij}$, respectively. Experimental results[1] for mixing angles are $\sin^2\theta_{23} \simeq 0.5$, $\sin^2\theta_{13} \simeq 0$, and $\sin^2\theta_{12} \simeq 0.3$.

[1] If all experimental data is used in 1 degree of freedom (d.o.f.) analysis where $\Delta\chi^2 = 9$ corresponds to 99.73% C.L. contour, we can have the strongest constraint on a single parameter. However, as the cost for the strong constraint, other parameters are not constrained at all. Therefore, even if there are constraints on several parameters (each of which is obtained in 1 d.o.f. analysis with all data), we must use only one of them in order to avoid multiple use of experimental data.

The most naive extension of the SM to accommodate the neutrino mass is the introduction of the right-handed neutrino ν_R which is a singlet under the SM gauge group. Then the Dirac mass of the neutrino can be obtained from $y_\nu \overline{L}\, i\sigma_2\, \Phi^* \nu_R$, where $\sigma_i (i = 1\text{-}3)$ are the Pauli matrices, $L = (\nu_L, \ell)^T$ is a lepton doublet of $SU(2)_L$, and $\Phi = (\phi^+, \phi^0)^T$ is the SM Higgs doublet. If the neutrino mass is given solely by the term in the same way as the generation of other fermion masses, it seems unnatural because the Yukawa coupling constant y_ν must be extremely small. We may expect that the neutrino mass is produced in a different way. If we accept the lepton number non-conservation, one possibility is the Majorana mass term $1/2\, m_\nu \overline{(\nu_L)^c}\, \nu_L$, where the superscript c denotes the charge conjugation. The mass term is allowed only for the neutrino among the SM fermions in order to keep $U(1)_{EM}$ gauge symmetry. Therefore the neutrino mass can naturally be very different from other fermion masses.

Before the breaking of $SU(2)_L \times U(1)_Y$ gauge symmetry to $U(1)_{EM}$ gauge symmetry, the weak-isospin I_3 and hypercharge Y of the Majorana mass term ($I_3 = 1, Y = -2$) should be compensated by those of scalar fields. If we do not introduce new scalar fields which have their vacuum expectation values (vev), the compensation is achieved by the SM Higgs doublet Φ as a dimension-5 operator $(\overline{L^c}\, i\sigma_2\, \Phi)(\Phi^T\, i\sigma_2\, L)$ or higher-dimensional ones. If we accept new scalar fields, the simplest way of the compensation is given by the Higgs Triplet Model (HTM) [2] where the Majorana mass is provided by a dimension-4 operator $h_{\ell\ell'}\, \overline{L_\ell^c}\, i\sigma_2\, \Delta\, L_{\ell'}$ with an $SU(2)_L$-triplet scalar field Δ of $Y = 2$. The new Yukawa coupling constants $h_{\ell\ell'} (\ell, \ell' = e, \mu, \tau)$ satisfy $h_{\ell\ell'} = h_{\ell'\ell}$. The triplet scalar field can be expressed as

$$\Delta = \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix}. \tag{32.2}$$

The Majorana mass matrix $(m_\nu)_{\ell\ell'}$ for neutrinos is obtained as $(m_\nu)_{\ell\ell'} = \sqrt{2}\, v_\Delta h_{\ell\ell'}$, where the triplet vev $v_\Delta (= \sqrt{2}\, \langle \Delta^0 \rangle)$ breaks the lepton number by 2 units. Since the HTM does not introduce new fermions to the SM, neutrinos have no lepton number violating mixing (ex. mixing between ν_L and $(\nu_R)^c$) which is the key in the seesaw mechanism. Even if v_Δ is suppressed by a large mass scale, it is just a consequence of the soft-breaking (of the lepton number conservation) rather than the seesaw mechanism.

A doubly charged scalar H^{++} ($= \Delta^{++}$) is the characteristic particle in the HTM. Its decay into a pair of same-signed charged leptons ($H^{++} \to \bar{\ell}\, \bar{\ell'}$) will give a clear signal even in hadron colliders, and the flavor structure of the decay can give direct information on $(m_\nu)_{\ell\ell'}$ [3,4]. The doubly charged scalar can also contribute to flavor-violating decays of charged leptons (ex. $\tau \to \bar{\mu} ee$) at the tree level [5].

On the other hand, it seems interesting that the lepton flavor mixing has a nontrivial structure with two large mixings while the quark mixing structure is rather simple with only small mixings. There might be some underlying physics for the lepton flavor. A candidate for that is the A_4 symmetry which is a non-Abelian discrete group. The A_4 group is made from twelve elements of even permutations of four letters. The group has three 1-dimensional representations ($\underline{\mathbf{1}}$, $\underline{\mathbf{1}}'$, $\underline{\mathbf{1}}''$) and one 3-dimensional representation ($\underline{\mathbf{3}}$). Only $\underline{\mathbf{1}}$ is the A_4-invariant. The $\underline{\mathbf{3}}$ seems to be fit the tree flavors of leptons, and A_4 is the minimal one which has $\underline{\mathbf{3}}$. Some simple models based on the A_4 symmetry can be found in e.g., refs. [7,8,9]. Throughout this talk, I will use $\underline{\mathbf{3}}$ etc. for A_4-representations and "triplet" etc. for $SU(2)_L$-representations in order to avoid confusions.

The lepton mixing structure becomes the tribimaximal mixing form [6] ($\sin\theta_{23} = 1/\sqrt{2}$, $\sin\theta_{13} = 0$, and $\sin\theta_{12} = 1/\sqrt{3}$, which agree reasonably with neutrino oscillation data) without tuning Yukawa coupling constants if A_4 is broken to Z_3 and Z_2 in the charged lepton and neutrino sectors, respectively [9]. It seems attractive that the realization of the tribimaximal mixing can be expressed simply in terms of the symmetry breaking pattern. If the lepton flavor mixing structure is reproduced by a free fitting of parameters without such a guideline, there would be no worth to deal with symmetries (A_4 etc.) because such a fitting is also possible in the SM.

In this talk, I will present an extension of the HTM by using the A_4 group (we call the model as the A4HTM) and discuss phenomenology of doubly charged scalars in the model. We will see that the A4HTM has clear predictions which can be tested experimentally in near future.

	ψ^-_{1R}	ψ^-_{2R}	ψ^-_{3R}	$\Psi_{AL} = \begin{pmatrix} \psi^0_{AL} \\ \psi^-_{AL} \end{pmatrix}$
A_4	$\underline{\mathbf{1}}$	$\underline{\mathbf{1}}'$	$\underline{\mathbf{1}}''$	$\underline{\mathbf{3}}$
$SU(2)_L$	Singlet	Singlet	Singlet	Doublet
$U(1)_Y$	-2	-2	-2	-1

$\Phi_A = \begin{pmatrix} \phi^+_A \\ \phi^0_A \end{pmatrix}$	$\delta = \begin{pmatrix} \delta^+/\sqrt{2} & \delta^{++} \\ \delta^0 & -\delta^+/\sqrt{2} \end{pmatrix}$	$\Delta_A = \begin{pmatrix} \Delta^+_A/\sqrt{2} & \Delta^{++}_A \\ \Delta^0_A & -\Delta^+_A/\sqrt{2} \end{pmatrix}$
$\underline{\mathbf{3}}$	$\underline{\mathbf{1}}$	$\underline{\mathbf{3}}$
Doublet	Triplet	Triplet
1	2	2

Table 32.1
The leptons and the Higgs bosons in the A4HTM. The subscript $A = x, y, z$ denotes the index for $\underline{\mathbf{3}}$ of A_4; for example, $(\Psi_{xL}, \Psi_{yL}, \Psi_{zL})$ belongs to $\underline{\mathbf{3}}$ while each Ψ_{AL} is an $SU(2)_L$-doublet field.

32.2. Model

Table 32.1 shows particle contents in the A4HTM. No new fermion (ex. ν_R) is added to the SM, and only the scalar sector is extended. This model has three $SU(2)_L$-doublet and four $SU(2)_L$-triplet scalars. For realization of appropriate flavor structure of Yukawa coupling matrices, we do not rely on singlet scalars under the SM gauge group (the so-called flavons) in order to respect renormalizability which is preferred for predictability. For example, renormalizable Yukawa interactions of triplet scalars with the A_4 symmetry are expressed as

$$\left(\overline{(\Psi_{xL})^c},\ \overline{(\Psi_{yL})^c},\ \overline{(\Psi_{zL})^c} \right) \begin{pmatrix} h_\delta i\sigma_2\delta & h_\Delta i\sigma_2\Delta_z & h_\Delta i\sigma_2\Delta_y \\ h_\Delta i\sigma_2\Delta_z & h_\delta i\sigma_2\delta & h_\Delta i\sigma_2\Delta_x \\ h_\Delta i\sigma_2\Delta_y & h_\Delta i\sigma_2\Delta_x & h_\delta i\sigma_2\delta \end{pmatrix} \begin{pmatrix} \Psi_{xL} \\ \Psi_{yL} \\ \Psi_{zL} \end{pmatrix} + \text{h.c.}, \tag{32.3}$$

where h_δ and h_Δ are Yukawa coupling constants.

Let us just accept the following vev's without analyzing the scalar potential[2] (See Sec. III-A in ref. [1] for the detail):

$$\langle \phi^0_x \rangle = \langle \phi^0_y \rangle = \langle \phi^0_z \rangle = \frac{v}{\sqrt{6}}, \tag{32.4}$$

$$\langle \delta^0 \rangle = \frac{v_\delta}{\sqrt{2}}, \quad \langle \Delta^0_x \rangle = \frac{v_\Delta}{\sqrt{2}}, \quad \langle \Delta^0_y \rangle = \langle \Delta^0_z \rangle = 0. \tag{32.5}$$

Masses of charged leptons and neutrinos are given by the vev's in eqs. (32.4) and (32.4), respectively. In our convention of A_4-representations, vev's in eq. (32.4) break A_4 into Z_3 while ones in eq. (32.5) do into Z_2. Then, the tribimaximal mixing is obtained. However, note that this is just a mathematically beautiful reproduction of known values (lepton mixings). Here is the starting point of real physics although the mathematical beauty can be a motivation. In the next section, let us see predictions for phenomenology of doubly charged scalars which have not been measured yet. See ref. [1] for predictions on the mass of the lightest neutrino (or a sum rule of masses) and the Majorana phases which cannot be determined by oscillation measurements.

[2] In order to reduce the number of parameters in scalar potentials (not only in the A4HTM but also, for example, in extensions of two-Higgs-doublet-model with A_4), it is useful to notice relations of rearrangements of A_4-invariant combinations, which are similar to the Fierz transformation for the four-fermions. See Appendix B in ref. [1].

	e, ν_{eL}, H_3^{++}, H_4^{++}	μ, $\nu_{\mu L}$, H_2^{++}	τ, $\nu_{\tau L}$, H_1^{++}
Z_3-charges	1	ω	ω^2

Table 32.2
Z_3-charges of leptons and doubly charged scalars where $\omega \equiv \exp(2\pi i/3)$.

32.3. Phenomenology of doubly charged scalars

At first, we must obtain mass eigenstates of relevant particles to our discussion. Although we take vev's in eqs. (32.4) and (32.5) motivated by the lepton flavor mixing, we can ignore triplet vev's because the tree-level constraint from the ρ parameter results in $(v_\delta^2 + v_\Delta^2)/v^2 \lesssim 0.01$. Thus Z_3 symmetry remains approximately in the A4HTM, and this makes everything simple. Physical particles (mass eigenstates) should be classified by their Z_3-charges. Since triplet vev's are ignored hereafter, we use flavor eigenstates for massless neutrinos. Table 32.2 shows Z_3-charges of charged leptons, neutrinos, and four doubly charged scalars H_i^{++} $(i = 1\text{-}4)$ made from four triplet fields. It is clear that the flavor symmetry is not the original A_4 but the remaining Z_3. In that sense, τ and $\overline{\mu}$ have the same flavor (the same Z_3-charge).

Next, let us investigate $H_i^{++} \to \overline{\ell}\,\overline{\ell'}$. Yukawa interactions in eq. (32.3) are rewritten by using mass eigenstates. The Yukawa interactions of H_i^{++} are $(h_{i\pm\pm})_{\ell\ell'}\,\overline{(\ell_L)^c}\,\ell'_L\,H_i^{++}$. Yukawa coupling constants $(h_{i\pm\pm})_{\ell\ell'}$ are given by

$$
\begin{aligned}
h_{1\pm\pm} &= \frac{1}{\sqrt{3}}\,h_\Delta \begin{pmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix}, \qquad h_{2\pm\pm} = \frac{1}{\sqrt{3}}\,h_\Delta \begin{pmatrix} 0 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \\
h_{3\pm\pm} &= \frac{1}{\sqrt{3}}\,h_\Delta \cos\theta_{\pm\pm} \begin{pmatrix} 2 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} + h_\delta e^{i\alpha_{\pm\pm}} \sin\theta_{\pm\pm} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}, \\
h_{4\pm\pm} &= -\frac{1}{\sqrt{3}}\,h_\Delta \sin\theta_{\pm\pm} \begin{pmatrix} 2 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} + h_\delta e^{i\alpha_{\pm\pm}} \cos\theta_{\pm\pm} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix},
\end{aligned}
\tag{32.6}
$$

where $\theta_{\pm\pm}$ and $\alpha_{\pm\pm}$ are mixing parameters of doubly charged scalars. These coupling constants result in unique flavor structures of H_i^{++} decays into same-sign charged leptons as listed in Table. 32.3. For example, H_1^{++} can decay only into $\overline{e}\,\overline{\mu}$ and $\overline{\tau}\,\overline{\tau}$. Many zeros for BR($H_i^{++} \to \overline{\ell}\,\overline{\ell'}$) is given by the conservation of Z_3-charges. Since ratios of nonzero parts (ex. BR($H_1^{++} \to \overline{\tau}\,\overline{\tau}$)/BR($H_1^{++} \to \overline{e}\,\overline{\mu}$) $= 2$) cannot be determined by Z_3 symmetry, these are consequences of original A_4 symmetry. Therefore both of A_4 and Z_3 can be tested by measuring leptonic decays of $H_i^{\pm\pm}$ at the LHC if they are right enough to be produced.

Doubly charged scalars contribute also to lepton flavor violating decays of charged leptons at the tree level. However, only $\tau \to \overline{e}\mu\mu$ and $\tau \to \overline{\mu}ee$ are allowed by the conservation of Z_3-charges as shown in Table 32.3. Thus, a stringent constraint BR($\mu \to \overline{e}ee$) $< 1.0 \times 10^{-12}$ [10] is satisfied without fine tuning of parameters. The Z_3 symmetry also forbids $\ell \to \ell'\gamma$ which look possible at the 1-loop level. Then it is easy to expect sizable effects on τ decays. By virtue of these predictions, the A4HTM can be tested in the MEG experiment and future B-factories even if H_i^{++} are too heavy to be produced at the LHC. Of course, the A4HTM is excluded easily if decays forbidden in the model are discovered. This is an excellent feature of the model due to its high predictability.

	BR($H_i^{++} \to \overline{\ell}\,\overline{\ell'}$)											LFV decays
	ee	:	$\mu\mu$	:	$\tau\tau$	:	$e\mu$	:	$e\tau$	:	$\mu\tau$	of charged leptons
$H_1^{\pm\pm}$	0	:	0	:	2	:	1	:	0	:	0	None
$H_2^{\pm\pm}$	0	:	2	:	0	:	0	:	1	:	0	$\tau_L \to \overline{e_L}\mu_L\mu_L$
$H_3^{\pm\pm}$	$R_3^{\pm\pm}$	:	0	:	0	:	0	:	0	:	1	$\tau_L \to \overline{\mu_L}e_Le_L$
$H_4^{\pm\pm}$	$R_4^{\pm\pm}$	:	0	:	0	:	0	:	0	:	1	$\tau_L \to \overline{\mu_L}e_Le_L$

Table 32.3
Ratios of decays of $H_i^{\pm\pm}$ into a pair of same-signed charged leptons in the A4HTM. Here $R_3^{\pm\pm}$ and $R_4^{\pm\pm}$ are combinations of model parameters. Contributions of $H_i^{\pm\pm}$ to $\tau \to \overline{\ell}\ell'\ell''$ at the tree level are also shown. Note that all of $H_i^{\pm\pm}$ does not contribute to $\mu \to \overline{e}ee$ and $\ell \to \ell'\gamma$ at the tree and 1-loop level, respectively.

32.4. Conclusions

I have presented a renormalizable model, the A4HTM, which is an extension of the HTM with A_4 symmetry. The A4HTM is compatible with the tribimaximal mixing. Phenomenology in the model is restricted by an approximately remaining Z_3 symmetry. Then sharp predictions have been obtained. It has been shown that leptonic decays of $H_i^{\pm\pm}$ have characteristic flavor structures which would be tested at the LHC if they are light enough to be produced. Even if $H_i^{\pm\pm}$ are too heavy to be produced, they can affect on flavor violating decays of charged leptons. The Z_3 symmetry allows only $\tau \to \overline{e}\mu\mu$ and $\tau \to \overline{\mu}ee$. The prediction would be tested at the MEG experiment and future B-factories.

REFERENCES

1. T. Fukuyama, H. Sugiyama and K. Tsumura, Phys. Rev. D **82**, 036004 (2010).
2. W. Konetschny and W. Kummer, Phys. Lett. B **70**, 433 (1977); J. Schechter and J. W. F. Valle, Phys. Rev. D **22**, 2227 (1980); T. P. Cheng and L. F. Li, Phys. Rev. D **22**, 2860 (1980); M. Magg and C. Wetterich, Phys. Lett. B **94**, 61 (1980).
3. J. Garayoa and T. Schwetz, JHEP **0803**, 009 (2008); M. Kadastik, M. Raidal and L. Rebane, Phys. Rev. D **77**, 115023 (2008); A. G. Akeroyd, M. Aoki and H. Sugiyama, Phys. Rev. D **77**, 075010 (2008); P. Fileviez Perez, T. Han, G. y. Huang, T. Li and K. Wang, Phys. Rev. D **78**, 015018 (2008).
4. S. T. Petcov, H. Sugiyama and Y. Takanishi, Phys. Rev. D **80**, 015005 (2009).
5. E. J. Chun, K. Y. Lee and S. C. Park, Phys. Lett. B **566**, 142 (2003); M. Kakizaki, Y. Ogura and F. Shima, Phys. Lett. B **566**, 210 (2003); A. G. Akeroyd, M. Aoki and H. Sugiyama, Phys. Rev. D **79**, 113010 (2009); T. Fukuyama, H. Sugiyama and K. Tsumura, JHEP **1003**, 044 (2010).
6. P. F. Harrison, D. H. Perkins and W. G. Scott, Phys. Lett. B **530**, 167 (2002).
7. E. Ma and G. Rajasekaran, Phys. Rev. D **64**, 113012 (2001); E. Ma, Mod. Phys. Lett. A **17**, 289 (2002).
8. Phys. Rev. D **70**, 031901 (2004); M. Hirsch, A. Villanova del Moral, J. W. F. Valle and E. Ma, Phys. Rev. D **72**, 091301 (2005) [Erratum-ibid. D **72**, 119904 (2005)]; M. Hirsch, A. S. Joshipura, S. Kaneko and J. W. F. Valle, Phys. Rev. Lett. **99**, 151802 (2007).
9. G. Altarelli and F. Feruglio, Nucl. Phys. B **720**, 64 (2005); Nucl. Phys. B **741**, 215 (2006).
10. U. Bellgardt *et al.* [SINDRUM Collaboration], Nucl. Phys. B **299**, 1 (1988).