DUERR : I want to make a short comment on this l-parity, i.e., the parity which is connected with the reversal of l. I want to give a " nonmystical " interpretation of this. In order to incorporate parity in a strict fashion, you are forced, in a way, to double the number of components of the field operator. If you double the number of components of the field operator, however, you immediately run into the difficulty that you can write down five different invariant fourth order expressions, i.e., you have five Fermi interaction terms which will involve five different coupling constants. So you look for a method of doubling the components which does not increase the number of coupling constants. Now, in this special case which we have investigated, we introduce the doubling of the components in a very special fashion in the following sense : in the original equation l enters only quadratically. We now could slightly modify this theory in stating that the theory may also depend on the absolute value of l, i.e., it may depend on the sign of the square root of l^2. If you use this degree of freedom in that way, then of course, this is equivalent to doubling the number of components, and parity can be introduced, but you do not introduce new kinds of interactions.

DYNAMICAL THEORY OF ELEMENTARY PARTICLES SUGGESTED BY SUPERCONDUCTIVITY

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My talk is based on some work done in collaboration with Dr. Jona-Lasinio.

We would like to propose here a theory of elementary particles which is based on a mathematical analogy between the dynamics of relativistic particles and that of superconductors in the theory of Bardeen, Cooper and Schrieffer [1]. That there can exist such an analogy is not surprising. Both relativistic quantum field theory and solid state physics deal with many body problems of large media, and in fact we already know many instances where field theoretical techniques have been successfully applied to problems of solid state physics. We shall see presently that this interaction of the two branches of physics can be reciprocal, and that solid state physics can provide us with useful models which help us understand the dynamics of elementary particles.

I. We start with the comparison of the Dirac equation for a nucleon, say, and the Bogolubov-Valatin relation [2, 3] for an elementary excitation (quasi-particle) in a superconducting medium. They are given respectively by

$$\begin{aligned} E\psi_1 &= \boldsymbol{\sigma}\cdot\mathbf{p}\psi_1 + m\psi_2 \\ E\psi_2 &= -\boldsymbol{\sigma}\cdot\mathbf{p}\psi_2 + m\psi_1 \\ E &= \pm\sqrt{p^2+m^2} \end{aligned} \qquad (1)$$

and

$$\begin{aligned} E\psi_{p+} &= \varepsilon_p\psi_{p+} + \phi\psi^{\dagger}_{-p-} \\ E\psi^{+}_{-p-} &= -\varepsilon_p\psi^{\dagger}_{-p-} + \phi\psi_{p+} \\ E &= \sqrt{\varepsilon_p{}^2+\phi^2} \end{aligned} \qquad (2)$$

Here the Weyl representation is used for the Dirac equation : ψ_1, ψ_2 correspond to the eigenstates of chirality $\gamma_5 = \mp 1$. In Eq. (2), $\psi_{p\pm}$ is the wave function of an electron with momentum p and spin $\pm$ (up or down), so that $\psi^{\dagger}_{-p-}$ effectively represents a hole of momentum p and spin $+$; ε_p is the kinetic energy measured from the Fermi surface; ϕ is a constant characteristic of the theory of superconductivity initiated by Bardeen, Cooper and Schrieffer. In the ground state of superconductor, all the electrons are in the lower eigenstates of Eq. (2). To excite the system, it will take a finite amount of energy $\geq 2|\phi|$. In a similar way, it takes energy $\geq 2m$ to excite the ground state (vacuum) of the world by creating a nucleon antinucleon pair.

In the BCS-Bogolubov theory, the energy gap ϕ is obtained as a Hartree-Fock type of self-consistent potential (self-energy) arising from the phonon-mediated attractive interaction between electrons. It comes about because the attractive interaction produces correlated pairs of electrons with opposite momenta and spin near the Fermi surface, which have finite binding energy.

One important aspect of the BCS theory is that the energy gap ϕ depends on the phonon-electron interaction constant in a non-analytic way; in other words such a solution cannot be obtained by perturbation. Another unusual aspect is that the approximation violates gauge invariance, as can be seen from the fact that the quasi-particle described by Eq. (2) is not an eigenstate of charge. This, however, is not a defect of the theory, but is rooted in the physical reality of superconductors. It has been shown [2, 4-7] that a quasi-particle is actually accompanied by polarization of the surrounding medium, and the overall charge conservation is established when both are taken together. The quantum of such polarization manifests itself as collective excitations of the medium formed by moving quasi-particle pairs. The existence of such collective excitations are a necessary consequence of the seeming contradiction between gauge invariance and a finite energy gap.

Considering these properties of superconductivity we are led to the following propositions about elementary particles. It is an old and attractive idea that the mass of a particle is a self-energy due to interaction. According to the present analogy, it will come about because of some attractive correlation between massless bare particles, and will be determined in a self-consistent way rather than by simple perturbation. Since a free massless fermion conserves chirality, let us further assume that the interaction also preserves chirality invariance, just as the electron-phonon system preserves gauge invariance. Then if an observed fermion (quasi-particle) can have a finite mass, there should also exist collective excitations of fermion pairs. Such excitations will behave like bosons, of zero fermion number, so that they may be called mesons. They will play the role of preserving the overall conservation of chirality, and from this we will be able to infer that they are pseudoscalar mesons, like the pions found in nature.

It is because of these interesting features that we would like to regard our theory as primarily dealing with nucleons and mesons. A unified understanding of all the baryons and leptons will be of course our ultimate goal, but for the moment we will content ourselves with a modest task.

II. Having explained our motivation, we have now to determine the precise nature of the dynamics, namely, the interaction of the bare nucleon field. Clearly our theory is a kind of compound particle model, of which there are many existing varieties. Among them, however, only the theory of Heisenberg [8] has a definite mathematical approach to handle the dynamical problems. We will also adopt in this work a nonlinear theory of the Heisenberg type which admits the γ_5-gauge group. In Heisenberg's theory an attempt has been made to derive, from a single fundamental spinor field, isospin and other internal degrees of freedom which characterize different particles. As we shall see in the following, we do not find such a possibility; we will have to start from a multi-component field in order to account for these particles.

As a simplified model, we adopt here a single four-component spinor field with the Lagrangian

$$\begin{aligned} L &= -\bar{\psi}\gamma_\mu\partial_\mu\psi + g_0[\bar{\psi}\psi\bar{\psi}\psi - \bar{\psi}\gamma_5\psi\bar{\psi}\gamma_5\psi] \\ &= -\bar{\psi}\gamma_\mu\partial_\mu\psi - \tfrac{1}{2}g_0[\bar{\psi}\gamma_\mu\psi\bar{\psi}\gamma_\mu\psi - \bar{\psi}\gamma_\mu\gamma_5\psi\bar{\psi}\gamma_\mu\gamma_5\psi] \end{aligned} \tag{3}$$

The second form follows from the first by the Fierz transformation. This Lagrangian admits both the ordinary gauge and the γ_5 gauge transformations:

$$\begin{aligned} &a)\quad \psi\to\exp[i\alpha]\psi \qquad \bar{\psi}\to\bar{\psi}\exp[-i\alpha] \\ &b)\quad \psi\to\exp[i\alpha\gamma_5]\psi \qquad \bar{\psi}\to\bar{\psi}\exp[i\alpha\gamma_5] \end{aligned} \tag{4}$$

where α is a constant phase.

Eq. (3) is not the only possible form of the γ_5 invariant interaction. It was chosen because of a certain advantage in incorporating isospin later on. We also remark that beside the non-linear spinor self-interaction, there are other interesting possibilities. For example, intermediate vector bosons have been considered in connection with the various conservation laws of strong interaction [9-12]. Our theory will work equally for these interactions, and the main characteristics of the results will not be much different; they are essentially determined by symmetry properties and simple dynamical conditions such as attraction or repulsion.

III. We will set up a Hartree-Fock procedure for calculating the self-energy of a nucleon. In the following we assume that all the quantities we calculate are somehow convergent. We will not indulge in speculations about the real mechanism behind this, and in practice introduce an *ad hoc* relativistic cut-off.

The self-consistent method is not an entirely new concept in field theory. In its simplest form, it actually turns out to be identical with the renormalization procedure. When a Hamiltonian is given as a sum of free and interaction parts: $H = H_0 + H_1$, we introduce a self-energy Hamiltonian Σ, and split H as $(H_0+\Sigma)+(H_1-\Sigma) = H_0'+H_1'$. Σ will be assumed in a general form which yields linear equations for the fields so that we can define one-particle eigenstates. The vacuum is then defined with respect to these new eigenstates. Taking H_0' as the basis for perturbation calculation, we finally determine Σ from the requirement that the residual interaction H_1' no longer contains a self-energy part [7].

Applying this procedure to Eq. (3), we may assume a constant self-energy part $\Sigma = m$. If we calculate Σ explicitly in the lowest order, we get the following self-consistency equation:

$$m = g_0[2\mathrm{Tr}\{S_F^{(m)}(0)\} - 2\gamma_5\mathrm{Tr}\{\gamma_5 S_F^{(m)}(0)\} - \gamma_\mu\mathrm{Tr}\{\gamma_\mu S_F^{(m)}(0)\} + \gamma_\mu\gamma_5\mathrm{Tr}\{\gamma_\mu\gamma_5 S_F^{(m)}(0)\}] \quad (5)$$

Here $S_F^{(m)}(x)$ is the Feynman propagator for a Dirac particle with mass m. Only the first term of Eq. (5) survives after taking the trace. Introducing a sharp relativistic cut-off in momentum space, Eq. (5) reduces to

$$m = m\frac{g_0\Lambda^2}{2\pi^2}\left[1-\frac{m^2}{\Lambda^2}\log\left(1+\frac{\Lambda^2}{m^2}\right)\right] \quad (6)$$

which has two solutions: $m = 0$ and

$$\frac{4\pi^2}{g_0\Lambda^2} = 1-\frac{m^2}{\Lambda^2}\log\left(1+\frac{\Lambda^2}{m^2}\right) \quad (7)$$

The latter has a real solution $m \neq 0$ only if

$$0 < \frac{2\pi}{g_0\Lambda^2} < 1 .$$

The trivial solution $m = 0$ clearly corresponds to the perturbation result. On the other hand, the non-trivial solution cannot be obtained by straight perturbation theory from a γ_5 invariant Lagrangian. This solution is the analogue of the superconducting solution. The non-analytic dependence of m on g_0 is evident from Eq. (7).

In the following we shall assume that the condition for the existence of a non-trivial solution is met. Keep in mind, however, that the trivial solution also exists. We are therefore naturally interested in whether they are equally legitimate solutions, and if so, whether two different particles can coexist.

The first question cannot be answered in a definite way, though from the analogy with superconductivity we may expect the trivial solution to be unstable, and in fact we shall see some indication of it later. The answer to the second question turns out to be negative. This is because the two solutions represent two different time developments of the field: $\psi^{(0)}(\mathbf{r}, t)$ and $\psi^{(m)}(\mathbf{r}, t)$, so that their respective eigenmodes are related by a canonical transformation. If we define the corresponding vacuum states $\Omega^{(0)}$ and $\Omega^{(m)}$, the fields $\psi(\mathbf{r}, 0)$, $\bar{\psi}(r, 0)$ applied to $\Omega^{(0)}(\Omega^{(m)})$ will create only particles of mass zero (mass m). In this way we obtain two different Hilbert spaces H_1 and H_2 based on $\Omega^{(0)}$ and $\Omega^{(m)}$. It is easy to show that these two spaces are orthogonal to each other. That is, for any two states $\Phi^{(0)}$ and $\Phi^{(m)}$ taken out of H_1 and H_2 respectively [13], we have $(\Phi^{(0)}, \Phi^{(m)}) = 0$. Similarly, any operator built up of finite products of the field ψ and $\bar{\psi}$ will have no matrix elements between the two spaces. This means that there is a superselection rule operating between the two possible " worlds ".

The complication actually does not end here. Since the finite mass in the Dirac equation violates γ_5 invariance, the vacuum $\Omega^{(m)}$ is clearly not an eigenstate of chirality. This will be possible only if the vacuum is degenerate with respect to the chirality quantum number. In fact we note that the non-trivial self-energy $\Sigma = m$ changes into

$$m(\cos 2\alpha + i\gamma_5 \sin 2\alpha)$$

under a γ_5 transformation, which equally satisfies Eq. (5). Corresponding to such a solution, a new vacuum $\Omega_\alpha^{(m)}$ has to be defined, so that eventually we get an infinite number of worlds distinguished by α running from 0 to 2π. Again we can show that a superselection rule operates between worlds having different α, which prevents such a degeneracy from becoming an observable effect. Indeed we can have only one kind of massive particle; and it makes no difference in physical predictions whatever the γ_5-gauge we work with.

IV. The observation just made explains some of the paradoxes connected with the γ_5 invariance. However, there still remains the question of chirality current conservation. Because of the γ_5 invariance of the Lagrangian (3), the field satisfies a continuity equation

$$\partial_\mu \bar{\psi} \gamma_\mu \gamma_5 \psi = 0 \tag{8}$$

On the other hand the non-trivial solution $\psi^{(m)}$, which describes an approximate one-particle state, does not satisfy Eq. (8) due to its finite mass. This implies that the free Dirac particle is not an adequate picture when interaction with other fields is involved. In general, the axial vector current vertex $\Gamma_{\mu 5}(p', p)$ between real particles should have the form $i\gamma_\mu\gamma_5 F_1(q^2) + \gamma_5 q_\mu F_2(q^2)$ where $q = p' - p$. The continuity equation restricts this further to

$$\Gamma_{\mu 5}(p', p) = F(q^2)[i\gamma_\mu\gamma_5 + 2m\gamma_5 q_\mu/q^2] \tag{9}$$

The second (" anomalous ") term will have a pole at $q^2 = 0$ if $F(0) \neq 0$. Using the launguage of the dispersion theory, it follows in this case that $F_2(q^2)$ has a contribution from an isolated zero-mass intermediate state which behaves like a pseudoscalar meson. The existence of such states was already anticipated in the beginning.

These general considerations can be explicitly corroborated by taking into account the effect of the residual interaction H_1' in an appropriate way.

Let us start with the collective states of the nucleon-antinucleon system. For this purpose we consider the scattering amplitude in the simple ladder approximation (Fig. 1). Corresponding to the four interaction types of Eq. (3), there will be four different modes. If we take the pseudoscalar interaction, and neglect for the moment the coupling with the pseudovector interaction, the sum of the graphs of Fig. 1 gives the scattering amplitude

$$M_p = -2g_0\gamma_5 \frac{1}{1-J_p(q^2)}\gamma_5$$

$$J_p(q^2) = \frac{ig_0}{2\pi^4}\int \mathrm{Tr}\{\gamma_5 S_F^{(m)}(p+\tfrac{1}{2}q)\gamma_5 S_F^{(m)}(p-\tfrac{1}{2}q)\}d^4p \tag{10}$$

The two γ_5's in M_p refer to the initial and final pairs.

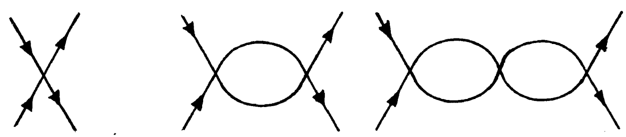

Fig. 1 Nucleon-antinucleon scattering graphs in the ladder approximation.

It is easy to see that if we put $q = 0$, J_p becomes equal to $2g_0 \mathrm{Tr} S_F^{(m)}(0)/m$, so that from the self-consistency condition (5) we have $J_p(0) = 1$. Since J_p is a function of q^2, this means that M_p has a pole at q^2, corresponding to a zero-mass pseudoscalar bound state.

We may make use of the dispersion relation to express $J_p(q^2)$ and the self-consistency condition as

$$\begin{aligned} J(q^2) &= 1 - \frac{g_0 q^2}{4\pi^2}\int_{4m^2}^{\Lambda^2} \frac{dk^2}{q^2+k^2}\sqrt{1-\frac{4m^2}{k^2}} \\ 1 = J(0) &= \int_{4m^2}^{\Lambda^2} dk^2 \sqrt{1-\frac{4m^2}{k^2}} \end{aligned} \tag{11}$$

Of course the cut-off Λ is different from the old one.

The residue of M_p at the pole determines the coupling constant of this " meson " to the nucleon:

$$\frac{G_1^2}{4\pi} = \left[\frac{1}{2\pi}\int_{4m^2}^{\Lambda^2} \frac{dk^2}{k^2}\sqrt{1-\frac{4m^2}{k^2}}\right]^{-1} \tag{12}$$

which is still logarithmically divergent with Λ.

We can extend this procedure to other types of interactions. We find that iteration of scalar and vector interactions gives rise to poles at finite mass, whereas the axial vector interaction produces only a resonance. The coupling of pseudoscalar and pseudovector interactions does not affect the pseudoscalar pole at zero-mass, but will give rise to an additional pseudovector (derivative) interaction with the nucleon.

In a similar way if we work with a two nucleon system in the ladder approximation, we find a scalar bound state. The results of these calculations are listed in Table I.

In view of the approximation involved, there is no reason to regard the results more than qualitatively correct except for the pseudoscalar meson mass. We want to point out, however, a few interesting features :

(a) The ordering of the meson levels is exactly what we can expect for the non-relativistic approximation to the Lagrangian (3).

(b) The coupling constants do not depend on g_0 explicitly.

(c) The scalar meson has zero binding energy, being independent of g_0 or Λ. We may therefore speculate that this has a more general basis.

Let us next come to the question of the axial vector vertex Eq. (9). The proof that the vertex really satisfies the continuity equations is based on a generalized Ward identity.

$$L(p')\gamma_5+\gamma_5 L(p) = -q_\mu \Gamma_{\mu 5}(p',p)$$

where (13)

$$L(p) \equiv -i\gamma\cdot p-\Sigma(p)$$

As in the case of ordinary gauge invariance, this relation also tells us how to make a selective summation of perturbation terms in order to maintain γ_5 invariance. In our present approximation to Σ, the necessary vertex graphs are shown in Fig. 2. Summing up these terms we get indeed Eq. (9) with $F(q^2)=1$.

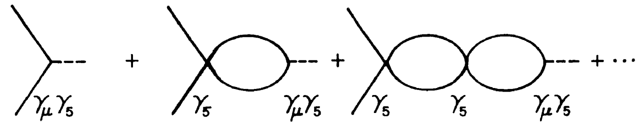

Fig. 2 The class of axial vector graphs which together lead to invariant results.

V. We have so far explored some interesting consequences of our theory based on the model Lagrangian (3). We shall now discuss briefly how to make the model more realistic by some generalizations. We need at least (1) inclusion of isotopic spin, and (2) making the " pion " mass finite.

(1) Instead of Eq. (3) consider

$$L= -\bar\psi\gamma_\mu\partial_\mu\psi + g_0[\bar\psi\psi\bar\psi\psi - \sum_{i=1}^{3}\bar\psi\gamma_5\tau_i\psi\bar\psi\gamma_5\tau_i\psi] \quad (14)$$

where ψ is now an 8-component spinor and the τ_i's

Table I. Spectrum of bound states

Type		Spectroscopic notation	Mass	Coupling : $[G^2/4\pi]^{-1}$
Mesons	pseudoscalar	$(^1S_0)$	0	$\frac{1}{2\pi}\int_{4m^2}^{\Lambda^2}\frac{dk^2}{k^2}\sqrt{1-\frac{4m^2}{k^2}}$
	scalar	$(^3P_0)$	$2m$	$\frac{1}{2\pi}\int\frac{dk^2}{k^2-4m^2}\sqrt{1-\frac{4m^2}{k^2}}$
	vector	$(^3S_1)$	$2m>\mu\gtrsim 1.6m$	$\frac{1}{3\pi}\int\frac{dk^2}{k^2-\mu^2}\sqrt{1-\frac{4m^2}{k^2}}\left(1+\frac{2m^2}{k^2}\right)$
" Deuteron " scalar		$(^1S_0)$	$2m>\mu\gtrsim 1.4m$	$\frac{2}{\pi}\int\frac{dk^2}{k^2-\mu^2}\sqrt{1-\frac{4m^2}{k^2}}$

are the usual isospin matrices. This Lagrangian admits the ordinary gauge transformation, isospin rotation, and isospin$\times\gamma_5$ rotation groups, of which the latter two together form a four-dimensional rotation group [14]).

Following the same mathematical procedure as before, it is easy to see that Eq. (14) gives rise to zero-mass pseudoscalar mesons of isospin 1 (but not of isospin 0). In addition we find a scalar meson ($T=0$), two vector mesons ($T=0$ and 1), as well as two deuteron-like states ($J=1^+$, $T=0$ and $J=0^+$, $T=1$) with a mass spectrum similar to Table 1.

(2) Although it is not inconceivable that a γ_5 invariant theory can yield a finite "pion" mass, for the moment the simplest way to achieve this end will be to allow a small amount of violation of the invariance by introducing, for example, a finite bare mass m_0 in the Lagrangian.

As a result, the degeneracy of nucleon mass with respect to γ_5-rotation is lost, and we find in general three solutions: one trivial, and two non-trivial ones of opposite signs. This circumstance is illustrated in Fig. 3; note that for large enough m_0, the two smaller solutions disappear. It goes without saying that again a superselection rule holds between these solutions.

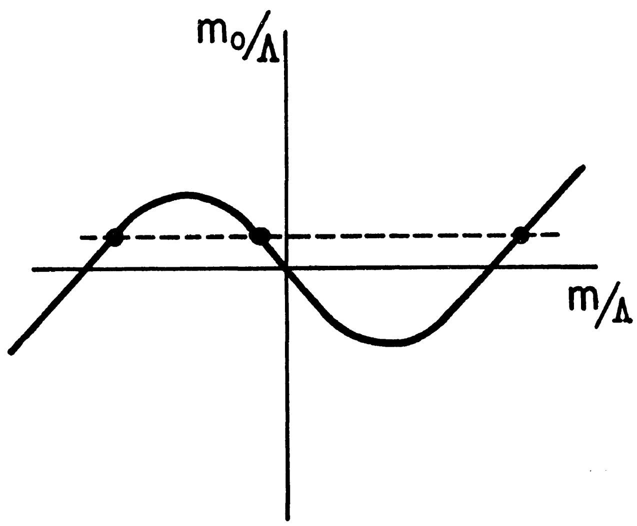

Fig. 3 Plot of observed mass m *vs.* bare mass m_0.

The meson masses corresponding to each solution will also be shifted from the previous case, but it turns out that only the largest mass solution gives a real pion mass, whereas the other two leads to imaginary values (ghost pions). This is an indication that only the largest m is the true solution of the Hamiltonian.

For the change of the nucleon and meson mass values we have a general relation $\delta m^2 \sim \delta\mu^2$, so that the biggest change occurs for the pion mass.

VI. We are now in a position to discuss the predictions of our theory on the basis of the generalized model of the previous section. We shall cover both strong and weak interactions.

1. Strong interaction

The theory contains three input parameters g_0, Λ and m_0, which we can fix by the three observable parameters m (nucleon mass), G_π (pion-nucleon coupling) and μ_π (pion mass). From them, we can in principle calculate all observable quantities. Among other things, we have obtained heavier meson states under a simple approximation. These results should be in general sensitive to the type of the interaction assumed. But it is interesting that already we get two vector mesons with the right properties that are anticipated from the nucleon electromagnetic structure, nuclear forces, and various other considerations [12, 15-17]). The rather high mass values obtained here could conceivably be due to the poor approximation. For instance, when we know that pions and other mesons exist, it would certainly not be legitimate to neglect their role in determining their own properties. However, to incorporate them in the self-consistent formalism would be a highly involved task.

A new prediction in our particular model is about the scalar meson of isospin 0. Actually it had zero binding energy, and if the γ_5 invariance violation is taken into account, it becomes a low energy (several MeV) nucleon-antinulceon resonance. We have speculated that this might even be quantitatively correct.

2. Weak interaction

We shall assume here the universal four-fermion weak interaction as far as bare nucleons are concerned. Our theory then enables us to predict the effect of the strong interactions on the weak interaction matrix elements. First of all, it is easy to see that there will be no renormalization on the vector β-decay current, which is conserved,[18] as is generally true for a model of this type. In addition we have an approximate axial vector current conservation. The breakdown of γ_5 symmetry will destroy the form of the vertex, Eq. (9), but for low energy processes this manifests itself most dramatically only through the change of the position of the pion pole.[19] In other words, we have for the neutron-proton axial vector vertex

$$\Gamma_{\mu 5}(p', p) = F_1(q^2) i\gamma_\mu \gamma_5 + F_2(q^2)\gamma_5 q_\mu/(q^2+\mu_\pi^2) \tag{15}$$

where still $F_1 \sim F_2/2m \sim F(q^2)$ is slowly varying for $q^2 \ll (3\mu_\pi)^2$. The second term is then negligible for β-decay. $F_1(0)$ gives the amount of axial vector renormalization g_A/g_V, which is $\sim$1.25 experimentally. Eq. (15) is very similar to the result of Goldberger and Treiman [20] obtained under special assumptions. One of its most interesting consequences is that it gives a relation between g_A, G_π and the pion decay (axial vector) coupling g_π [19-23].

$$2mg_A = 2mF_1(0) \sim F_2(-\mu_\pi^2) = \sqrt{2}G_\pi g_\pi \tag{16}$$

which follows by interpreting the residue at the pole according to the dispersion theory. Eq. (16) agrees well with the observed values of these constants.

As for the g_A/g_V ratio, the theory should predict a definite value. Unlike the vector current case, we have not found a guarantee that γ_5 invariance leads to no renormalization (although it holds within the present approximation). Nevertheless, it is somewhat tempting to postulate that the finite renormalization does appear only through the breakdown of the invariance, which will mostly arise from the pion mass. Under this assumption we obtain

$$g_A/g_V - 1 \sim \frac{3}{\pi}\frac{G_\pi^{\,2}}{4\pi}\left(\frac{\mu_\pi}{2m}\right)^2 \log\left(\frac{m}{\mu_\pi}\right)^2 \sim 0.3\,. \tag{17}$$

We have not yet attempted to incorporate the strange particles in our theory. But a formal generalization can be made on the form of the strangeness violating vector and axial vector currents, where the K particle will replace the pion. On this rather crude basis estimations have been made about the various decay modes of strange particles, which are in general accord with observation. However, we do not regard this as particularly significant until a more rigorous foundation is given.

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DISCUSSION

DUERR: I would like to question Nambu with respect to his last remarks on the g_A/g_V ratio. Do you have any indication in your theory that you get such a weak interaction besides your interactions from which you have calculated your masses, or do you consider this question only in the general case of the occurrence of any axial vector and vector interactions whatever the strength of this interaction might be?

NAMBU: The weak interaction is here considered as an additional process which has, at the present stage, no relation to the strong interaction.

BREIT: My understanding is that your theory predicts the mass of a vector meson that is reasonably large and that the coupling constant is of the order of the pion-nucleon coupling constant. I wonder whether it is possible to indicate what it is in the theory to limit the coupling constant to a reasonably narrow range. In connection with nucleon-nucleon scattering it would be rather interesting to know whether a factor of 3 or 4 is allowed in the coupling constant and what kind of limits there are on the mass on the vector meson, e.g., whether the correction by a factor of 2 or so is ruled out.

NAMBU: I cannot trust my results in any quantitative way. The diagrams I have drawn are very bad approximations from the point of view of dispersion theory because I am taking into account only the distant cut coming from nucleon-antinucleon states, and there will be a violent objection to this proceedure from dispersion theorists, of whom I used to be one (laughter)—but in this case I do not know how to do it in a more satisfactory way, so I just have a simple minded approximation. To satisfy the invariance requirements we must sum up a certain set of Feynman diagrams, and there is a certain definite restriction because of the invariance properties which must be preserved at each step of approximation. This is what I want to emphasize.

BLUDMAN: There is one qualitative feature; you have obtained g_A/g_V greater than one whereas at least in nonrelativistic field theories there is a tendency for this to come out less than one. Do you have more faith in this *qualitative* feature, and, if so, why did you get that result?

NAMBU: I calculate the renormalization effects due to various mesons. I just take the pion state. The pion state is the lowest mass state so the contribution from this state is the largest. The other mesons have about the same constants but are of larger mass. If you break invariance, the masses of the heavier mesons change very little; also the nucleon mass changes very little. So I just took the meson contribution to this renormalization effect. I took the difference between the zero mass case and the finite mass case; the contribution diminishes as the mass grows. The difference has the opposite sign to what one normally expects. The point is that the correction is negative but this becomes less negative as the mass increases. So the difference is positive.

BOGOLUBOV: I wish to make a small comment. In a more realistic approach everything largely depends on the cut-off. It is very interesting to consider this problem using a simple model which is of interest from the mathematical point of view. So one of my collaborators (Tavkelidze) has considered a Thirring-type one-dimensional model, in which massless fermions interact with massive bosons. His calculations are not based on the self-consistent principle but on the ordinary Feynman diagram approach. The result is that there is a degeneracy in such a simple case.

DUERR: I want to ask the following questions. In a theory which is γ_5 invariant, as you have assumed, you apparently get a spinor particle with finite mass. Now this γ_5 invariance still seems to imply in your case that you can only get a π-meson of zero mass. You remove this difficulty by employing a higher approximation and again making use of the degeneracy of the ground state.

NAMBU: I have a number of conjectures about this. Look at a similar problem in superconductivity: Because of the Coulomb interaction between the electrons the form factor $F(q^2)$ I wrote down is 0 for $q^2 = 0$, and in that case my argument fails and it turns out that there is no low lying collective excitation in the corresponding superconducting medium. This excitation becomes instead a massive plasma mode. But if you apply this to weak interactions, then the axial vector part vanishes in the low momentum limit so I do not want to take this alternative. If you take advantage of more degrees of freedom like baryons, and possibly leptons, then there may be some kind of asymmetry so you write down kind something like an effective bare mass. This is a sheer speculation.

FUKUDA: I wonder whether the solution leading to the energy gap has anything to do with Heisenberg's solution. In the theory of superconductivity the attractive forces near the Fermi sea play a very important role in producing dynamical correlations which are important for the energy gap. Here the nonlinear term is more important for the energy gap of the elementary particle.

NAMBU: In this case the situation is the same as in superconductivity. The interaction I assume is attractive between the nucleon and the antinucleon, as you see from the condition that g_0 has to be positive. If you take the opposite sign, you do not get a nontrivial solution.

SUDARSHAN: You say that you have left out the nearby singularities and have taken the far away ones. I would like to ask if you think it possible to consider a calculation in which, if the nearby singularities are also included, it would finally look like a dispersion calculation which is fashionable these days. And if the answer is affirmative what do you think about the concept of local fields which is often said to be essential in the derivation of dispersion relations?

NAMBU: If we want to improve our calculations it is best to use the dispersion relations. Then we will talk about nearby poles and branch cuts, and this may look eventually like the approach of Chew and Mandelstam. However, in my approach, I consider the bare coupling constant and the bare mass very seriously whereas in the dispersion approach we do not talk about them, and you know that to have a convergent result I have to introduce a cut-off parameter. Whether this will come about naturally in the dispersion theory in a self-consistent way I do not know. We might need a new concept such as the indefinite metric of Heisenberg.

HEISENBERG: It is certainly an excellent idea to compare the mathematical situation in superconductivity with that for the elementary particles, and some of the mathematical problems we have to deal with are so similar to the older problems that we should connect the different approaches to some kind of new mathematical method. For instance, you mentioned that in your case the Hilbert space is divided up into different sectors. If you compare this not with superconductivity but with ferromagnetism, we see that this is again the case. The lowest state of a ferromagnet is the one where all the magnets are oriented in one direction and of course you can turn the whole thing around, but you can never, by applying a finite number of ψ's on the ground state which has an infinite number of electrons, turn around the whole magnet. Therefore these states are entirely separated. I just mention this to say that here we have come to a very general mathematical problem, and we should now get accustomed to those problems in which the lowest state is degenerate. There are many different problems in physics that belong to this class. If one develops a new mathematical theory to deal with this class of problems, in the near future we may make great progress in the theory of elementary particles.