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# UNIVERSALITY OF HADRON FORM FACTORS ABSTRACTED FROM THE PARTON MODEL*

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## ABSTRACT

The role played by the orbital angular momentum $L_z$ in parton models at $P = \infty$ is delineated. By postulating similar behavior for the parton wave functions of the low-lying baryons of equal $L_z$, we are able to relate various of the transition form factors for $\gamma_V N \rightarrow N^*$ to the elastic form factors of the nucleon.

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## I. INTRODUCTION

Matrix elements of local charge densities $<\psi' | J_0 | \psi>$ find very intuitive expressions in the language of the infinite momentum parton model[1]: as in ordinary quantum mechanics, they can be written as overlap integrals between the parton wave functions of the hadron states $\psi$ and $\psi'$. Several authors[2-4] have considered such expressions in various versions of the parton model. Unfortunately, the lack of specific information about the parton wave functions prevents the extraction of much information beyond the Drell-Yan[2]-West[3] relation.

In the present work, we consider the angular momentum properties of the wave functions involved in the overlap integrals, and are led through these considerations to propose a specific statement of universality among hadron form factors (Section II). The hypothesis can be directly tested in the case of the transition $\gamma_V p \rightarrow \Delta^+(1236)$, and this is done in Section III of the paper. In this section we also present some statements made by the model about certain other transition form factors ($\gamma_V N \rightarrow S_{11}(1525)$, $\gamma_V N \rightarrow D_{13}(1525)$) of experimental interest. Section IV of the paper contains some discussion of our results.

## II. FORM FACTORS IN THE PARTON MODEL

The physics discussed in this paper is viewed exclusively from the infinite momentum frame (IMF) introduced by Drell and Yan[2], in which the initial hadron momentum $P^\mu$ is the four vector $(P + m^2/2P,\ 0\ ,\ 0\ ,\ P)$, $P \rightarrow \infty$, and the four momentum carried by the current is $q^\mu = (m\nu/P,\ Q,\ 0,\ 0)$. As

$P \to \infty$, we have $P^2 = m^2$, $q^2 = -Q^2$, $q \cdot P = m\nu$. Many of the kinematical aspects of momentum space wave functions in the IMF are reviewed in an article by Kogut and Susskind[5], to which we refer the reader. In constructing form factors in the parton model, it will suffice (as discussed in Refs. 2 and 5) for us to consider the matrix element $\langle h' \underset{\sim}{P}' \lambda' | J_0 | h \underset{\sim}{P} \lambda \rangle_\infty$, where $\underset{\sim}{P}\lambda(\underset{\sim}{P}'\lambda')$ denote the momentum and helicity of hadron $h(h')$, $\underset{\sim}{P}' = \underset{\sim}{P} + \underset{\sim}{Q} = P\,\vec{i}_z + Q\,\vec{i}_x$ and the subscript $\infty$ on the matrix element denotes that the limit $P \to \infty$ is to be taken. The parton model then is a statement that

$$\langle h' P' \lambda' | J_0 | h P \lambda \rangle_\infty = \sum_a (\psi', j_0^a, \psi)_\infty \tag{1}$$

where $j_0^a$ is the bare charge density operator of parton type a, and $\psi(\psi')$ is the initial (final) hadron state expanded onto a Hilbert space of many-parton wave functions at infinite momentum. In momentum space, $\psi$ and $\psi'$ carry as arguments the longitudinal fractions $\eta_i$, the transverse momenta $\underset{\sim}{K}_i$, and the z-components of spin $\lambda_i$ of the partons. (The $\lambda_i$ differ from parton helicities only by corrections of $0(Q/P)$).

We now observe a simple fact which is essential to our discussion: <u>at infinite momentum the bare charge density $j_0^a$ cannot flip the spin of the parton from which it scatters</u>. E.g., in the case of spin 1/2 partons,

$$\begin{aligned} \langle \eta \underset{\sim}{K}' \lambda' | j^0 | \eta \underset{\sim}{K} \lambda \rangle_\infty &= e \left[ \bar{u}(\underset{\sim}{P}' \lambda') \gamma_0 u(P\lambda) \right]_\infty \\ &= e\,(2\eta P)\,\delta_{\lambda'\lambda'} \end{aligned} \tag{2}$$

with a normalization $u^+u = 2E$ for the spinors. From Eq. (1) it then follows that none of the parton spins are flipped during the scattering. Therefore, if the physical matrix element $\langle h' \underset{\sim}{P}' \lambda' | J_0 | h \underset{\sim}{P} \lambda \rangle_\infty$ does not vanish for $\lambda' \neq \lambda$, there must, in order that $\Delta J_z \neq 0$, occur a flip in $L_z$, the total z-component of orbital angular momentum of the partons; this consideration then implies the presence of components with different $L_z$'s, including $L_z \neq 0$, in the $\infty$-momentum parton wave function of any hadron with spin. For example, we shall see (in Eq. 12) that for the nucleon $\langle N \underset{\sim}{P}' -1/2 | J_0 | N \underset{\sim}{P} 1/2 \rangle \propto F_2(Q^2)$, the Pauli form factor. The non-vanishing of $F_2(Q^2)$ necessitates a classification scheme at infinite momentum in which neither $S_z$ nor $L_z$ are separately diagonal. This is a parton model realization of the conclusion of Dashen and Gell-Mann[6]. We shall have more to say about this in the concluding section of the paper.

Let us expand the hadron ket in terms of eigenstates $|L\rangle$, $|S\rangle$ of $L_z$ and $S_z$ respectively

$$|h, \underset{\sim}{P}, J_z = \lambda\rangle = \sum_{L+S=\lambda} C_L^{h\lambda} |L\rangle |S\rangle \tag{3}$$

with $\sum_L |C_L^{h\lambda}|^2 = 1$. The kets $|L\rangle$ and $|S\rangle$ should display additional labelling to indicate the intermediate angular momenta which couple to give the final values of L and S, but we omit this for the sake of typographic clarity.

It will now prove convenient to project the state vectors $|L\rangle$ onto kets $|\underset{\sim}{X}_1 \ldots \underset{\sim}{X}_n; \eta_1 \ldots \eta_n\rangle$ which provide a basis set in transverse position space. It is then a straightforward exercise starting from momentum space and Fourier transforming, to write the form factor in terms of the transverse position space wave functions. We shall only state the result here, leaving the derivation to Appendix A:

$$\lim_{P\to\infty} P^{-1} \langle h_2 \underset{\sim}{P}' \lambda_2 | J_0 | h_1 \underset{\sim}{P} \lambda_1 \rangle$$

$$= \sum_{n=1}^{\infty} \sum_a e_a \int \prod_{i=1}^{n} (d\eta_i/\eta_i)\, d^2\underset{\sim}{X}_i{}' \delta(\sum \eta_i \underset{\sim}{X}_i{}')\, \delta(\sum \eta_i - 1)$$

$$\sum_L \left(C_{L+\Delta\lambda}^{h_2\lambda_2}\right)^* \left(C_L^{h_1\lambda_1}\right) \left(\psi_{L+\Delta\lambda}^{h'} (\eta_1 .. \eta_a .. \eta_n;\ \underset{\sim}{X}_1{}' .. \underset{\sim}{X}_a{}' .. \underset{\sim}{X}_n{}')\right)$$

$$e^{i\underset{\sim}{Q} \cdot \underset{\sim}{X}_a'} \left(\psi_L^{h_1} (\eta_1 .. \eta_a .. \eta_n;\ \underset{\sim}{X}_1{}' .. \underset{\sim}{X}_a{}' .. \underset{\sim}{X}_n{}')\right) \tag{4}$$

with $\Delta\lambda \equiv \lambda_2 - \lambda_1$.

In Eq. (4), "a" labels the struck parton, with charge $e_a$. The $\underset{\sim}{X}_i'$ are equal to $\underset{\sim}{X}_i - \underset{\sim}{X}_0$, the displacement of $\underset{\sim}{X}_i$ from the "center of mass" $\underset{\sim}{X}_0 = \sum \eta_i \underset{\sim}{X}_i$. All the spin parts have dotted out, conforming to the previous discussion; and the independence of the $\psi$'s of the momenta P and P' is a result of the Galilean invariance in the infinite momentum frame[5].

Needless to say, we know little or nothing about the wave functions $\psi$, except for their normalization (derived in Appendix A):

$$\sum_{n=1} \int \prod_{i=1}^{n} (d\eta_i/\eta_i)\, d^2\underset{\sim}{X}_i{}'\ \delta(\sum \eta_i \underset{\sim}{X}_i{}')\ \delta(\sum \eta_i - 1)$$

$$\sum_{L, L'} (C_{L'}^{h'\lambda'})^* (C_L^{h\lambda}) \psi_{L'}^{h'} (\eta_1 .. \eta_n; \underset{\sim}{X}_i{}' .. \underset{\sim}{X}_n{}')\ \psi_L^{h}\ (\eta\ \ .. \eta_n;\ \underset{\sim}{X}_i{}' .. \underset{\sim}{X}_n{}')$$

$$= \delta_{\lambda'\lambda}\, \delta_{h'h} \tag{5}$$

However, let us explore the following line of reasoning. It is clear from the previous discussion on anomalous moments and L-S mixing that the $P\to\infty$ parton Hamiltonian is a fairly complex object, and that the wave functions for the hadrons in the IMF have little apparent relation to the usual "constituent quark" wave functions of the ordinary quark models, except perhaps through a unitary transformation given by the hadron dynamics. Now, consider as an example the singly charged non-strange baryons $\left(P(938), \Delta^+(1236), D^+_{13}(1525), \text{etc.}\right)$. The non-relativistic quark model predicts similar spatial wave functions for the P(938) and $\Delta^+(1236)$, but quite a different one for the $D^+_{13}(1525)$, since it is an $l = 2$ excitation. In the IMF, however, it _may_ be that the wave functions for a given $L_z$ are not too different for _any_ of the non-strange low-lying baryons of a given charge (so that the minimal quark makeup is identical - ppn in the above example). Let us weaken this statement some, and postulate that _for equal values of L, L' there is enough similarity in the wave functions such that the $Q^2$ behavior of the integrals $\int \psi^{h'}_{L'}\, e^{i\underset{\sim}{Q}\cdot \underset{\sim}{X}'_a}\, \psi^h_L$ is not very different for any of the low-lying baryons of a given charge and hypercharge_. We must however immediately stipulate that in the case $h_1 \neq h_2$, Q must be large enough to deaccentuate the orthogonality properties of the wave functions. Note that, in contrast to the case of deep inelastic electroproduction, the validity of Eq. (4) does not hinge on Q being large[7].

The last step is to assume that representations at $P = \infty$ are sufficiently mixed so that each of the different $L_z$ values is reasonably represented in the wave functions of all the low-lying non-strange baryons - more concisely, the $C^{h\lambda}_L$'s for given $\lambda$ are non-zero over the whole spread of the L's for any of the low-lying non-strange baryons.

The end result of all these speculations is an _experimental prediction: namely, that the matrix element $\langle h'\underset{\sim}{P}'\lambda'|J_0|h\underset{\sim}{P}\lambda\rangle_\infty$ can be written as $P\cdot F_{\Delta\lambda}(Q^2)\cdot G(Q^2)$, where $G(Q^2)$ contains all the dependence on h, h', $\lambda$, $\lambda'$, but is a very slowly varying function of $Q^2$_. The factor P originates from Eq. (3) inserted in the expansion (1).

To compare any specific case with experiment, we set

$$\lim_{P\to\infty} \frac{\langle h_2\, \underset{\sim}{P}'\, \lambda_1 + \Delta\lambda\, | J_0 |\, h_1\, \underset{\sim}{P}\lambda_1 \rangle}{\langle h_4\, \underset{\sim}{P}'\lambda_3 + \Delta\lambda\, | J_0 |\, h_3\, \underset{\sim}{P}\lambda_3 \rangle} = r(Q^2) \tag{6}$$

and hypothesize that for $Q^2$ away from zero (say $Q^2 \gtrsim 0.5\ \mathrm{GeV}^2$):

(i) r (which depends on all the h's and $\lambda$'s) is a very slowly varying function of $Q^2$ (i.e., much more slowly varying than the form factors themselves) and

(ii) $r(Q^2) \sim 1$. This is a guess based on the normalization conditions (3) and (5). A gross failure of this condition (say $r(Q^2) \gtrsim 10$ or $\lesssim 0.10$) would indicate that something has gone awry in our reasoning, namely that the wave functions are not all that similar in the IMF. So hypothesis (ii) will play an important role in the initial evaluation of predictions of the model.

A final word about isospin. In applications we shall generally choose $h_1 = h_3 = h_4$ = nucleon, $h_4$ = resonance. In this case, we shall test separately the hypotheses

$$\begin{aligned} &\langle N^*,\, \underset{\sim}{P}'\, \lambda_1 + \Delta\lambda\, | J_0^{V,S} |\, N,\, \underset{\sim}{P},\, \lambda_1 \rangle_\infty \\ &\qquad \approx r^{V,S}(Q^2, \Delta\lambda) \langle N^*,\, \underset{\sim}{P}',\, \lambda_3 + \Delta\lambda\, | J_0^{V,S} |\, N\, \underset{\sim}{P},\, \lambda_3 \rangle_\infty \end{aligned} \tag{7}$$

for the isovector (v) and isoscalar (s) currents, <u>if an isoscalar transition can occur</u> (i.e., for the I = 1/2 resonances).

We now turn to some examples.

## III. APPLICATIONS

### 1. $\gamma_V P \to \Delta^+(1236)$

It will suffice to examine $\lambda_1 = \lambda_3 = 1/2$, $\Delta\lambda = 0, -1$. According to the preceding discussion, we shall also deal in this case exclusively with the iso-vector current. Eq. (6) takes the form

$$\lim_{P\to\infty} \frac{\langle \Delta\, P' \pm \frac{1}{2}\, | J_0^V |\, N\, \underset{\sim}{P}\, \frac{1}{2} \rangle}{\langle N\, \underset{\sim}{P}' \pm \frac{1}{2}\, | J_0^V |\, N\, \underset{\sim}{P}\, \frac{1}{2} \rangle} = r_\pm^V(Q^2) \tag{8}$$

and $r_\pm^V(Q^2)$ should conform to hypotheses (i) and (ii).

In terms of standard invariants, we have for the nucleon

$$\langle N\, \underset{\sim}{P}' \lambda' | J_\mu | N\, \underset{\sim}{P}\lambda \rangle = \bar{u}\,(P'\lambda') \left[ (\tau_3 F_1^V + F_1^S)\gamma_\mu + i\sigma_{\mu\nu}\, q^\nu\, (\tau_3 F_2^V + F_2^S)/2m \right] u(P\lambda) \tag{9}$$

while for the $N \to \Delta$ transition we simplify matters by keeping only the magnetic dipole transition (this being a very good approximation to the data[8]) :

$$\langle \Delta\, \underset{\sim}{P}' \lambda' | J_\mu^V | N\, \underset{\sim}{P}\lambda \rangle = i\, F^*(Q^2)\, \epsilon_{\mu\alpha\beta\gamma}\, P'^\alpha\, q^\beta\, \bar{\psi}^\gamma\,(P'\lambda')\, u\,(P\lambda) \tag{10}$$

The spinors are normalized to $\bar{\psi}^\alpha \psi_\alpha = 2M$, $\bar{u}u = 2m$, where M and m are the masses of $\Delta(1236)$ and N(939), respectively.

Working in the infinite momentum frame $p^\mu = (P + m^2/2P, 0, 0, P)$, $q^\mu = (m\nu/P, Q, 0, 0)$ defined in Section II, it is a kinematical exercise to show that

$$\lim_{P\to\infty} P^{-1} < N \underset{\sim}{P}' \tfrac{1}{2} | J_0 | N \underset{\sim}{P} \tfrac{1}{2} > = 2(\tau_3 F_1^V + F_1^S) \quad , \tag{11}$$

$$\lim_{P\to\infty} P^{-1} < N \underset{\sim}{P}' -\tfrac{1}{2} | J_0 | N \underset{\sim}{P} \tfrac{1}{2} > = (Q/m)(\tau_3 F_2^V + F_2^S) \quad , \tag{12}$$

$$\lim_{P\to\infty} P^{-1} < \Delta \underset{\sim}{P}' \tfrac{1}{2} | J_0^V | N \underset{\sim}{P} \tfrac{1}{2} > = (Q^2/\sqrt{6})\, F^* \quad , \tag{13}$$

$$\lim_{P\to\infty} P^{-1} < \Delta \underset{\sim}{P}' -\tfrac{1}{2} | J_0^V | N \underset{\sim}{P} \tfrac{1}{2} > = \left[ Q(M+m)/\sqrt{6} \right] F^* \quad . \tag{14}$$

Ignoring the isoscalar pieces, we can then process Eqs. (11)-(14) through Eq. (8) to obtain the relations

$$\tfrac{1}{2} Q^2 F^*(Q^2) / \left[ \sqrt{6}\, F_1^V(Q^2) \right] = r_+(Q^2) \quad , \tag{15}$$

$$m(M+m)\, F^*(Q^2) / \left[ \sqrt{6}\, F_2^V(Q^2) \right] = r_-(Q^2) \quad , \tag{16}$$

with $r_\pm(Q^2)$ satisfying our hypotheses (i) and (ii).

Next, we compare (16) quantitatively with experiment. (We choose (16) rather than (15) because the necessary vanishing of $r_+(Q^2)$ at $Q^2 = 0$ probably impedes $r_+(Q^2)$ from becoming slowly varying until considerably higher values of Q than those at which $r_-(Q^2)$ attains this behavior.) $F^*(Q^2)$ is related to the form factor $G_M^*(Q^2)$ measured by Bartel et. al.[9] through the relation

$$G_M^*(Q^2) = \sqrt{\tfrac{2}{3}}\, m \sqrt{(M+m)^2 + Q^2}\; F^*(Q^2) \quad , \tag{17}$$

so that the condition (16) becomes

$$G_M^*(Q^2) \Big/ \left( 2 \sqrt{1 + Q^2/(M+m)^2}\; F_2^V(Q^2) \right) = r_-(Q^2) \tag{18}$$

For $Q^2 > 0.5$, the experimental results[8] $G_E^p(Q^2) \simeq G_M^p(Q^2)/(1+\kappa_p) \simeq G_M^n(Q^2)/\kappa_n \simeq (1 + Q^2/0.71)^{-2}$, $G_E^n \simeq 0$ combine to give

$$F_2^V(Q^2) \simeq 1.85\,(1 + Q^2/4m^2)^{-1}(1 + Q^2/0.71)^{-2} \tag{19}$$

In Fig. 1 is plotted the LHS of Eq. (18) for the data in Ref. 9 with $0.5\ \mathrm{GeV}^2 < Q^2 \leq 2.34\ \mathrm{GeV}^2$. It is seen that $r_-(Q^2)$ is indeed very slowly varying over this range of $Q^2$. (Statistically, in fact, it is quite consistent with being constant ($\simeq 0.76$) with a $\chi^2 = 8$ for 11 degrees of freedom. If we fit to the data of $Q^2 > 1$, the constant is 0.73 with a $\chi^2$ of 0.55 for 3 degrees of freedom.) Both the average value of $r_-(Q^2)$ and its behavior as a function of $Q^2$ are in remarkable agreement with our hypotheses (i) and (ii). [For comparison, a plot of $G_M^*(Q^2)/G_E^P(Q^2)$ in Ref. 9 shows this ratio decreasing much more rapidly with $Q^2$ than $r_-(Q^2)$, with a $\chi^2$ for a constant fit being over 40 for 11 degrees of freedom.]

We can now also check the consistency, with our model, of the magnetic dipole approximation for the $N \rightarrow \Delta$ transition: for large $Q^2$, the condition for the compatibility of (15) and (11) is that

$$F_2^V(Q^2) \simeq \left(2m(M+m)/Q^2\right) F_1^V(Q^2)\left(r_+(Q^2)/r_-(Q^2)\right). \tag{20}$$

The previously listed form factors give

$$F_2^V \simeq 3.15\,(m^2/Q^2)F_1^V \tag{21}$$

at large $Q^2$. Numerically, Eq. (20) reads

$$F_2^V \simeq 4.62\,(m^2/Q^2)F_1^V\left(r_+(Q^2)/r_-(Q^2)\right) \quad . \tag{20'}$$

We have just found $r_-(Q^2) \simeq \mathrm{const} \simeq 0.76$, so that agreement between (21) and (20') is obtained for $r_+(Q^2) \simeq \mathrm{const} \simeq 0.52$, completely within the limitations of our hypotheses (i) and (ii).

Comparison with "Relativistic" SU(6)

In a sample comparison with an $N \rightarrow \Delta$ transition form factor obtained via symmetry considerations, we note the U(6,6) prediction of Salam, Delbourgo and Strathdee[10] that

$$F^*(Q^2) \propto \left[(M - m)^2 + Q^2\right]\left(\frac{E^* + m}{q^{*2}}\right) G_M^V(Q^2) \tag{22}$$

where we have omitted all $Q^2$-independent factors. We can most easily compare this with the prediction in the present work by calculating the ratio

$$\frac{\left[F^*(Q^2)\right]_{U(6,6)}}{\left[F^*(Q^2)\right]_{\text{present work}}} \propto \left[(M - m)^2 + Q^2\right]\left(\frac{E^* + m}{q^{*2}}\right) \frac{G_M^V(Q^2)}{F_2^V(Q^2)}$$

$$\propto \frac{\left[(M - m)^2 + Q^2\right]\left[(M + m)^2 + Q^2\right]\left[4m^2 + Q^2\right]}{\left[Q^4 + 2(M^2 + m^2)Q^2 + (M^2 - m^2)^2\right]} \tag{23}$$

Over the range of data shown in Fig. 1, this would have the U(6,6) form factor grow larger than ours by 46%. This represents a 5-6 standard deviation departure from the data.

2. $\gamma_V p \rightarrow D_{13}^+(1525)$

In this case, we work with the representation of the vertex function given by Bjorken and Walecka[11], which incorporates all three amplitudes for the excitation:

$$\langle N^* \underset{\sim}{P}' \lambda' | J\cdot e | NP\lambda \rangle$$

$$= \bar{\psi}^{\mu_1}(P'\lambda') \Big[ q_{\mu_1}(e\cdot q\, P\cdot q - e\cdot P\, q^2)\,(\tau_3 g_1^V + g_1^S)$$

$$+ 2\,\epsilon_{\mu_1 \alpha\beta\gamma}\, P^\alpha q^\beta S^\gamma\, (\tau_3 g_2^V + g_2^S)$$

$$+ i M q_{\mu_1}\, \gamma\cdot S\, \gamma^5 \left( \tau_3 (g_2^V + g_3^V) + (g_2^S + g_3^S) \right) \Big]\; U(P,\lambda) \qquad (24)$$

where $S_\gamma = \epsilon_{\gamma\lambda\nu\sigma} P^\lambda q^\nu e^\sigma = \epsilon_{\gamma\lambda\nu\sigma} P'^\lambda q^\nu e^\sigma$ .

In Appendix B we sketch the kinematic manipulations involved in obtaining the infinite momentum limit of (24). The result is

$$\lim_{P\to\infty} P^{-1} \langle D_{13} \underset{\sim}{P}' \tfrac{1}{2} | J_0^V | \underset{\sim}{P}' \tfrac{1}{2} \rangle = \tau_3 Q^2 \Big[ (2\omega^*(M+m) - Q^2)\, g_1^V$$

$$- 3M(M+m)\, g_2^V$$

$$+ M(2\omega^* + M + m)\, g_3^V \Big] / \sqrt{6} \qquad (25)$$

$$\lim_{P\to\infty} P^{-1} \langle D_{13} \underset{\sim}{P}' -\tfrac{1}{2} | J_0^V | \underset{\sim}{P}' \tfrac{1}{2} \rangle = \tau_3 Q \Big[ - (2\omega^* + M + m) Q^2\, g_1^V$$

$$+ 3 M Q^2\, g_2^V$$

$$+ M\,(2\omega^*(M+m) - Q^2)\, g_3^V \Big] / \sqrt{6} \qquad (26)$$

with $\omega^* = (M^2 - m^2 - Q^2)/2M$.

There are similar equations for $g_i^S$ , i = 1, 2, 3.

The $g_i$'s are related to the Feynman helicity amplitudes

$$\mathscr{F}_L = K \cdot \left( 1/\sqrt{6}(Q/M) g_1 \right)$$

$$\mathscr{F}_{3/2} = K \cdot (-g_2)$$

$$\mathscr{F}_{1/2} = K \cdot (g_3/\sqrt{3}) \tag{27}$$

where $K = 2\sqrt{2}\, M^{5/2} (E^* + m)^{1/2} q^{*2}$ .

The discussion of Section II and Eqs. (11), (12), (25) and (26) then lead to the following version of Eq. (7):

$$\left(2\omega^*(M+m) - Q^2\right) g_1^{V,S} - 3M(M+m)\, g_2^{V,S} + M(2\omega^* + M + m)\, g_3^{V,S}$$
$$\approx \sqrt{6}\, r_+^{V,S}(Q^2) \cdot 2F_1^{V,S}(Q^2) / Q^2 \tag{28}$$

$$-Q^3 (2\omega^* + M + m) g_1^{V,S} + 3MQ^3 g_2^{V,S} + QM\left(2\omega^*(M+m) - Q^2\right) g_3^{V,S}$$
$$\approx \sqrt{6}\, Q\, r_-^{V,S}(Q^2)\, F_2^{V,S} / m \tag{29}$$

with $r_\pm^{V,S}(Q^2)$ satisfying hypotheses (i) and (ii). (They are also different from the r's in Eq. (8))

What can we learn from Eqs. (28) and (29) ?

a) Since $F_1^S$ is not particularly small, Eq. (28) tells us that it is inconsistent for all the isoscalar amplitudes to vanish for $Q^2 \gtrsim 1$. Eq. (29) gives the added information that some linear combination of these is suppressed, due to the small size of $F_2^S$. The measured height of the second resonance peak deduced from electroproduction total cross sections from hydrogen and deuterium seem to show[12] a D/H ratio of $1.5 \pm 0.3$, and decreasing, near $Q^2 = 1$; the systematic deviation from a value of 2 indicates the presence of both I = 0 and I = 1 excitations of the peak. To actually separate the I = 1 and I = 0 components of the $D_{13}$, the isospin structure of the $S_{11}$ excitation must first be elucidated through a study of the electroproduction of $\eta$'s from deuterium.

b) Given our ignorance of the separate isospin amplitudes, we form the linear combinations $g_i^V + g_i^S$ to obtain a single pair of equations for the electroproduction amplitudes from protons:

$$Q^2\left[\left(2\omega^*(M+m) - Q^2\right)g_1^p - 3M(M+m)g_2^p + M(2\omega^* + M + m)\, g_3^p\right]$$
$$\approx 2\sqrt{6}\left(r_+^V F_1^V + r_+^S F_1^S\right) \tag{30}$$

$$Q\left[-Q^2(2\omega^* + M + m)\, g_1^p + 3MQ^2 g_2^p + M\left(2\omega^*(M+m) - Q^2\right) g_3^p\right]$$
$$\approx Q\sqrt{6}\,(r_-^V F_2^V + r_-^S F_2^S)\,/\,m$$
$$\approx Q\sqrt{6}\; r_-^V\, F_2^V/m \tag{31}$$

At $Q^2 = 0$, it is well known[13] that $g_2$ (which corresponds to helicity 3/2 in the $\gamma_V$-proton C.M.) is considerably larger than $g_3$ ($h = 1/2$), and of course $g_1$ does not contribute. However, at $Q^2 = 0$ our equations are not helpful: Eq. (30) is not valid (because of orthogonality), and Eq. (31) is an identity.

For $Q^2 > 0$, the amplitude analysis of the data is at present in a very confused state. Recent measurements on $\gamma_V\, p \rightarrow p\eta^o$ [14,15] allow one to estimate with some confidence the peak cross-section for $\gamma_V p \rightarrow S_{11}(1525)$. Subtracting this from the cross section for $\gamma_V \rightarrow$ (1525 peak) [12,16], one obtains an estimate for $\sigma(\gamma_V p \rightarrow D_{13}(1525))$. Three data points obtained in this way are shown in Fig. 2.

To compare with experiment, we shall state here the relation between peak cross sections and the helicity amplitudes. As shown in Appendix B,

$$(\sigma_h)_{peak} = 4\ \pi\ \alpha\ |\mathscr{F}_h|^2 / \left[\Gamma\, M(M^2 - m^2)\right] \tag{32}$$

where $\Gamma$ is the full width of the resonance, and $h = 1/2,\ 3/2,\ L$.

The measured cross section is

$$\begin{aligned}\sigma &= \sigma_T + \epsilon\ \sigma_L \\ &= \frac{1}{2}\left(\sigma_{1/2} + \sigma_{3/2}\right) + \epsilon\ \sigma_L\end{aligned} \tag{33}$$

where $\epsilon$, the longitudinal content of the virtual photon, is generally close to 1 in the resonance region for moderate values of $Q^2$.

Eqs. (30) and (31) are not solvable in any ordinary sense (since the r's are unknown, and there are three unknown amplitudes appearing in two equations). We shall examine the implications of Eqs. (30) and (31) by testing in turn three extreme possibilities:

a) $\sigma_L = \sigma_{1/2} = 0\ ,\ \ \sigma_{3/2} \neq 0$

In this case, we can obtain $g_2^p$ from Eq. (31) (with $g_1 = g_3 = 0$)

$$g_2^p \approx \sqrt{\frac{2}{3}}\ F_2^V\, r_-^V\ / \left(MmQ^2\right) \tag{34}$$

and calculate the cross section using Eqs. (27), (32) and (33). The result is

$$\sigma_{peak} = \frac{1}{2}\ \sigma_{3/2} \approx \frac{16\ \pi\ \alpha\ M^2\ (E^* + m)}{\Gamma\,(M^2 - m^2)\ m^2}\ \frac{q^{*4}}{Q^4}\ \frac{2}{3}\ |F_2^V|^2\ |r_-^V|^2 \tag{36}$$

A best fit to our three data points with a constant $r_-^V$ is shown in Fig. 2 as a dashed curve. Not only is the fit poor ($\lambda^2 = 25$ for 2 degrees of freedom) but the required value of $r_-^V$ is on the low side ($r_-^V = 0.19$). To obtain a better fit, one could have to allow r to vary by more than a factor of 2 over the $Q^2$ range of the data ($0.5 \lesssim Q^2 \lesssim 1.5$). So this solution is definitely unsatisfactory.

b) $\sigma_L = \sigma_{3/2} = 0, \quad \sigma_{1/2} \neq 0$

In this case we have from Eq. (31)

$$g_3^p \simeq \frac{\sqrt{6}\, r_-^V F_2^V}{mM\left(2\omega^*(M+m) - Q^2\right)} \tag{36}$$

$$\sigma_{peak} = 1/2\, \sigma_{3/2} = \frac{16\,\pi\,\alpha}{\Gamma}\, \frac{E^* + m}{M^2 - m^2}\, \frac{M^2}{m^2}\, \frac{q^{*4}}{\left(2\omega^*(M+m) - Q^2\right)} \cdot 2\, |F_2^V|^2\, |r_-^V|^2 \tag{37}$$

This solution is unphysical in that it develops a pole in the cross section when $2\,\omega^*(M+m) = Q^2$, which happens at $Q^2 = 1.04\ \mathrm{GeV}^2$.

c) $\sigma_{1/2} = \sigma_{3/2} = 0\,, \quad \sigma_L \neq 0$

The relevant equations in this case are

$$g_1^p \simeq \frac{-\sqrt{6}\; r_-^V\, F_2^V}{Q^2 m(2\omega^* + M + m)} \tag{38}$$

and

$$\sigma_{peak} \simeq \sigma_L = \frac{32\pi\alpha}{\Gamma}\, \frac{E^* + m}{M^2 - m^2}\, \frac{M^2}{m^2}\, \frac{q^{*4}}{Q^2}\, \frac{1}{(2\omega^* + M + m)^2} \cdot |F_2^V|^2\, |r_-^V|^2 \tag{39}$$

An excellent fit with constant $r_-^V$ (shown as a solid line in Fig. 2) is obtained for

$$r_-^V = 0.47$$

with $\chi^2 = 0.37$ for 2 deg/f.

This is satisfactory on all counts.

As a result of this analysis, we predict that <u>if only one of the three excitation modes is present in the range $0.5 < Q^2 < 1.5\ \mathrm{GeV}^2$, it is most likely to</u>

be $\sigma_L$. It is extremely unlikely to be pure $\sigma_{3/2}$, and a helicity 1/2 dominated cross section is completely ruled out by the model. We note that the possibility of a purely longitudinal excitation is supported by a recent data analysis[17], but completely rejected in the quark model.[18]

One must note, however, that since the factor $2\omega^*(M+m) - Q^2$ multiplying $g_3$ in Eq. (31) is very small in the region of $Q^2 \sim 1$ GeV$^2$, a fair amount of $\sigma_{1/2}$ can be tolerated by our model if $\sigma_L$ is substantial in this region of $Q^2$.

We have not utilized Eq. (30) as a cross check on these results because of the ambiguity introduced by the presence of the two parameters $r_+^V$, $r_+^S$. It is interesting that even had we assumed equations such as

$$<N^{*+} \pm \tfrac{1}{2} \, |J_0| \, p\tfrac{1}{2}>_\infty \;\sim\; r_\pm^P <p \pm \tfrac{1}{2} \, |J_0| \, p\tfrac{1}{2}>_\infty$$

$$<N^{*o} \pm \tfrac{1}{2} \, |J_0| \, n\tfrac{1}{2}>_\infty \;\sim\; r_\pm^n <n \pm \tfrac{1}{2} \, |J_0| \, n>_\infty$$

instead of Eq. (7), the right hand side of Eq. (31) is replaced by $r_-^P\, F_2^P$. Since for $Q^2 < 4$ GeV$^2$ $F_2^V$ and $F_2^P$ are essentially equal, Eq. (31) is stable under this kind of change, with the unknown $r_-^V$ being replaced by $r_-^P$.

Let us examine the high $Q^2$ $\left(Q^2 >> (M+m)^2\right)$ behavior of Eqs. (30) and (31). If the scaling of $G_E$ and $G_M$ continued to hold at very large $Q^2$, then $F_1^V \sim F_1^S \approx Q^{-4}$, and $F_2^V \sim Q^{-6}$ ; Eqs. (30) and (31) take the asymptotic forms

$$- \left[(2M+m)/M\right] Q^2 g_1 - 3M(M+m)\, g_2 - Q^2 g_3 \sim Q^{-6} \tag{30'}$$

$$(Q^4/M)\, g_1 + 3MQ^2 g_2 - (2M+m)\, Q^2 g_3 \sim Q^{-6} \tag{31'}$$

These allow as a solution

$$g_2(Q^2) \leq \text{const} \cdot g_3(Q^2) \tag{40}$$

$$g_1(Q^2) \leq \text{const} \cdot g_3(Q^2)/Q^2 \tag{41}$$

From Eqs. (27) and (32), we have the predictions that for large $Q^2$

$$\sigma_L / \sigma_T \lesssim 0(Q^{-2}) \tag{42}$$

$$\sigma_{3/2} \Big/ \sigma_{1/2} \lesssim \text{const} \quad . \tag{43}$$

In fact, it is consistent for $\sigma_{3/2} \Big/ \sigma_{1/2} \to 0$ but not vice-versa. So the situation at large $Q^2$ is quite different from the one indicated near $Q^2 = 1$ in the preceding discussion. The predicted asymptotic vanishing of $\sigma_L / \sigma_T$ is again contrary to the result of the quark model.[18] It is a long kinematic excercise to show that as a result of equations such as (28) and (29), $\sigma_L / \sigma_T \to 0$ independent of the spin or normality of the resonance. The result is, however, not necessarily obtained in the Bjorken scaling limit $(Q^2/M^2 \to \text{const})$.

3. $\gamma_V P \to S^+_{11}(1525)$

We may repeat the whole preceding discussion in the present case, with the simplification of dropping the $g_2$ amplitude, given the lack of a helicity 3/2 state.

The analogues of Eqs. (30) and (31) are

$$Q^2 g_1^p (M - m) + M Q^2 g_3^p \approx 2 (r_+^V + F_1^V + r_+^S F_1^S) \tag{44}$$

$$-Q^2 g_1^p + M(M - m) g_3^p \quad \approx r_-^V F_2^V / m \tag{45}$$

with

$$\mathscr{F}_{1/2} = - \ 2 M^{5/2} (E^* + m)^{-1/2} q^{*2} \ \ 2 g_3 \tag{46}$$

$$\mathscr{F}_L = - \ 2 M^{5/2} (E^* + m)^{-1/2} q^{*2} (Q/M) g_1 \tag{47}$$

The formulae for the peak cross sections is this case are

$$(\sigma_L)_{peak} \approx \frac{8\pi\alpha}{\Gamma(M^2 - m^2)} \frac{M^2}{m^2} \frac{q^{*4}}{Q^2(E^* + m)} |r_-^V| \, |F_2^V|^2$$

$$\text{if} \quad \mathscr{F}_{1/2} = 0 \qquad (48)$$

$$(\sigma_T)_{peak} \approx \frac{4\pi\alpha}{\Gamma(M^2 - m^2)} \frac{M^2}{m^2} \frac{q^{*4}}{(E^* + m)} \frac{|r_-^V|^2}{(M - m)^2} \, 2\,|F_2^V|^2$$

$$\text{if} \quad \mathscr{F}_L = 0 \qquad (49)$$

a) $\sigma_{1/2} = 0$, $\sigma_L \neq 0$

The "best" fit, plotted as the dashed line in Fig. 3 against the data of Kummer, et. al.,[14] is obtained for

$$r_-^V = 0.48$$

with $\chi^2 = 25$ for 3 deg/f.

The fit is unsatisfacory, the variation required in $r_-^V$ in order to accommodate the data being a factor of 2.3 over the range of $Q^2$ considered.

b) $\sigma_L = 0$, $\sigma_{1/2} \neq 0$ (pure E1)

Shown as the solid line in Fig. 3, this fit is obtained for constant $r_-^V$

$$r_-^V = 0.39$$

with $\chi^2 = 1.5$ for 3 deg/f.

Thus, we may conclude that our model shows a definite preference for E1 dominance, this time in accord with the quark model.[18]

Finally, as in the case of the $D_{13}$, we find in the high-$Q^2$ limit

$$\sigma_L / \sigma_T \lesssim \text{const}/Q^2$$

for $Q^2 \gg (M + m)^2$.

## IV CONCLUSIONS

Our Hypotheses (i) and (ii) have allowed us to formulate a comprehensive scheme within which to correlate the $Q^2$-behavior (Hypothesis (i)) and (to some extent) the magnitude (Hypothesis (ii)) of the excitation form factors of the low-lying baryons to the elastic form factors of the nucleon. In the single case in which an experimental test is at present possible (the $N \to \Delta$ transition), our hypotheses (i) and (ii) are very well supported by the data. We have also sketched some implications of our model for the cross sections for $\gamma_V N \to D_{13}(1525)$ and $\gamma_V N \to S_{11}(1525)$. Within the realm of present experimental possibility, one prediction is of especial interest: <u>In the region of $Q^2 \simeq 1$, dominance of the cross section for $D_{13}$ excitation through a transverse helicity 1/2 initial state is ruled out, while excitation by a longitudinal photon is highly favored</u>. This is contrary to predictions of several versions of the non-relativistic quark model.[18] For much larger $Q^2$, ($Q^2 \gtrsim 6$ GeV$^2$), the prediction is that the ratio $\sigma_L / \sigma_T \lesssim \text{const}/Q^2$.

We close with some remarks about the parton structure of the hadrons at infinite momentum, based on the observations made in Section II: (1) The partons used here, if quarks, are to be viewed as "current quarks".[19] The wave functions of the low-lying baryon and meson states in terms of these is clearly different from the standard constituent quark qqq and $\bar{q}q$ wave functions such a conclusion, for example, casts great suspicion on the standard quark model predictions[20] for the spin dependent effects in deep-inelastic lepton scattering. (2) $J_0$ can induce transitions involving $\Delta J_z$ as high as $\pm 2J$, which implies that the $L_z$ content of the wave-function of a hadron of spin J extends at least over the range $-(J + 1/2)$ to $(J + 1/2)$ for the baryons, $-J$ to $J$ for the mesons. (3) The fact that neither $S_z$ nor $L_z$ are even approximated diagonal shows that the parton Hamiltonian at $P \rightarrow \infty$ is more complicated that simple string or spin-lattice models.[5] It is obviously of great theoretical interest to find the form and origin of the spin-orbit interaction piece.

## APPENDIX A: DERIVATION OF EQ. (4)

In this Appendix we shall stick to the notation of Ref. (5), with the exception of interchanging the role of the $\eta$'s and $\beta$'s.

### 1. Normalization in Momentum Space

As in Ref. (5), we define kets $|\beta_i \underset{\sim}{K}_i>$ which describe partons with longitudinal momentum $\beta_i P$, $P \rightarrow \infty$, and transverse momentum $\underset{\sim}{K}_i$. These are normalized in a manner invariant to finite boosts in the z directions

$$<\beta_i' \underset{\sim}{K}_i' | \beta_i \underset{\sim}{K}_i> = \beta_i \, \delta(\beta_i - \beta_i') \, \delta(\underset{\sim}{K}_i - \underset{\sim}{K}_i') \tag{A1}$$

We now construct the ket for a hadron with longitudinal momentum $\beta P$, transverse momentum $\underset{\sim}{K}$:

$$|\beta \underset{\sim}{K}> = \sum_N \int \prod_{i=1}^{N} \frac{d\beta_i}{\beta_i} \, d^2 \underset{\sim}{K}_i \; \beta \, \delta(\beta - \Sigma \beta_i) \; \delta^2(\underset{\sim}{K} - \Sigma \underset{\sim}{K}_i)$$

$$f_{\beta \underset{\sim}{K}} (\beta_1, .. \beta_n ; \underset{\sim}{K}_1, .. \underset{\sim}{K}_n) \, |\beta_i \underset{\sim}{K}_i> \tag{A2}$$

The normalization

$$<\beta K | \beta' \underset{\sim}{K}'> = \beta \, \delta(\beta - \beta') \, \delta(\underset{\sim}{K} - \underset{\sim}{K}') \tag{A3}$$

of these kets implies

$$\sum_n \prod_{i=1}^{n} \int (d\beta_i/\beta_i) \, d^2 \underset{\sim}{K}_i \;\; \delta(1 - \sum_i (\beta_i/\beta) \;\; \delta^2(\underset{\sim}{K} - \Sigma \underset{\sim}{K}_i)$$

$$|f_{\beta \underset{\sim}{K}}(\beta_1, .. \beta_n ; \underset{\sim}{K}_1, .. \underset{\sim}{K}_n)|^2 = 1 \tag{A4}$$

## 2. Invariances in $\beta$K Space

Lorentz and rotational invariance imply[5] that the infinite momentum wave function $f_{\beta \underset{\sim}{K}}(\beta_1 \ldots \beta_n ; \underset{\sim}{K}_1 \ldots \underset{\sim}{K}_n)$ is a function of the longitudinal fractions $\eta_i \equiv \beta_i/\beta$, and of the quantities $\underset{\sim}{P}_i \equiv \underset{\sim}{K}_i - \eta_i \underset{\sim}{K}$. There is no other $\beta$ or $\underset{\sim}{K}$ dependence. Note that

$$\Sigma \eta_i = 1$$

$$\Sigma \underset{\sim}{P}_i = 0$$

Mathematically,

$$f_{\beta \underset{\sim}{K}}(\beta_1 \ldots \beta_n ; \underset{\sim}{K}_1 \ldots \underset{\sim}{K}_n) = f(\eta_1 \ldots \eta_n ; \underset{\sim}{P}_1 \ldots \underset{\sim}{P}_n) \qquad \text{(A5)}$$

Since

$$\begin{aligned} \delta^2(\underset{\sim}{K} - \Sigma \underset{\sim}{K}_i) &= \delta^2(\Sigma \underset{\sim}{P}_i + \underset{\sim}{K} \Sigma \eta_i - \underset{\sim}{K}) \\ &= \delta^2(\Sigma \underset{\sim}{P}_i) \end{aligned} \qquad \text{(A6)}$$

we have the normalization

$$\sum_n \int \prod_{i=1}^{n} \frac{d\eta_i}{\eta_i} \; \delta(1 - \Sigma \eta_i)\, \delta(\Sigma \underset{\sim}{P}_i) \; | f(\eta_1 \ldots \eta_n ; \underset{\sim}{P}_1 \ldots \underset{\sim}{P}_n)|^2 = 1 \qquad \text{(A7)}$$

## 3. Transformations to $\underset{\sim}{X}$-space

We now procede to express the form factor in terms of wave functions in transverse position space.

Firstly, since $[\underset{\sim}{X}, P_z] = 0$, we can define a parton basis diagonal in the transverse position $\underset{\sim}{X}_i$ and longitudinal momentum $\beta_i \underset{\sim}{P}$, with normalization

$$\langle \beta_i' \underset{\sim}{X}_i' \,|\, \beta_i \underset{\sim}{X}_i \rangle = \beta_i\, \delta(\beta_i - \beta_i')\, \delta^2(\underset{\sim}{X}_i - \underset{\sim}{X}_i') \qquad \text{(A8)}$$

The ket for a hadron is then written

$$|\beta \underset{\sim}{K}> = \sum_n \int \Pi \frac{d\beta_i}{\beta_i} d^2 \underset{\sim}{X}_i \, \beta\delta(\beta - \Sigma\beta_i) \, \psi_{\beta \underset{\sim}{K}}(\beta_1 \ldots \beta_n ; \underset{\sim}{X}_1 \ldots \underset{\sim}{X}_n) \, |\beta_i \underset{\sim}{X}_i > \quad \text{(A9)}$$

Our normalizations (A1) and (A8) imply

$$<\beta_i \underset{\sim}{X}_i | \beta_i' \underset{\sim}{K}_i > = \frac{1}{2\pi} e^{i \underset{\sim}{K}_i \cdot \underset{\sim}{X}_i} \beta_i \, \delta(\beta_i - \beta_i') \quad \text{(A10)}$$

On inserting the expansion

$$|\beta_i \underset{\sim}{K}_i> = \int d^2 \underset{\sim}{X}_i <\beta_i \underset{\sim}{X}_i | \beta_i \underset{\sim}{K}_i > |\beta_i \underset{\sim}{X}_i >$$
$$= \frac{1}{2\pi} \int d^2 \underset{\sim}{X}_i \, e^{i \underset{\sim}{K}_i \cdot \underset{\sim}{X}_i} |\beta_i \underset{\sim}{X}_i > \quad \text{(A11)}$$

into Eq. (A2), we can compare Eqs. (A2) and (A9) to obtain the relation

$$\psi_{\beta \underset{\sim}{K}}(\beta_1 \ldots \beta_n ; \underset{\sim}{X}_1 \ldots \underset{\sim}{X}_n)$$
$$= (2\pi)^{-n} \int \prod_{i=1}^{n} d^2 \underset{\sim}{K}_i \, \delta(\underset{\sim}{K} - \Sigma \underset{\sim}{K}_i) \, e^{i \underset{\sim}{K}_i \cdot \underset{\sim}{X}_i} f(\eta_1 \ldots \eta_n ; \underset{\sim}{P}_1 \ldots \underset{\sim}{P}_n) \quad \text{(A12)}$$

$$\therefore \quad \psi_{\beta \underset{\sim}{K}}(\eta_1 \ldots \eta_n ; \underset{\sim}{X}_1 \ldots \underset{\sim}{X}_n)$$
$$= (2\pi)^{-n} \int \prod_{i=1}^{n} d^2 \underset{\sim}{P}_i \, \delta(\Sigma \underset{\sim}{P}_i) \, e^{i \underset{\sim}{P}_i \cdot \underset{\sim}{X}_i} e^{i (\sum_{i=1}^{n} \eta_i \underset{\sim}{X}_i) \cdot \underset{\sim}{K}} f(\eta_1 \ldots \eta_n ; \underset{\sim}{P}_1 \ldots \underset{\sim}{P}_n)$$

$$= e^{i \underset{\sim}{K} \cdot \underset{\sim}{X}} (2\pi)^{-n} \int \prod_{i=1}^{n} d^2 \underset{\sim}{P}_i \, \delta(\Sigma \underset{\sim}{P}_i) \, e^{i \underset{\sim}{P}_i \cdot \underset{\sim}{X}_i} f(\eta_1 \ldots \eta_n ; \underset{\sim}{P}_1 \ldots \underset{\sim}{P}_n) \quad \text{(A12)}$$

$$\equiv \frac{1}{2\pi} e^{i \underset{\sim}{K} \cdot \underset{\sim}{X}} \psi(\eta_1 \ldots \eta_n ; \underset{\sim}{X}_1 \ldots \underset{\sim}{X}_n) \quad \text{(A13)}$$

where

$$\underset{\sim}{X} \equiv \sum_{i=1}^{n} \eta_i \underset{\sim}{X}_i \quad \text{(A14)}$$

So Galilean invariance of $\not{f}$ has placed the entire $\underset{\sim}{K}$ dependence of $\psi_{\underset{\sim}{K}}$ into a phase factor. Note that $\underset{\sim}{X}$ is the "C.M." coordinate, if we make use of the $\beta_i \leftrightarrow m_i$ analogy. The $\beta$ dependence has disappeared.

We can also examine the consequence of translational invariance. This is simply a statement of the independence of $\not{f}$ on any external position coordinate. Hence we have for any $\underset{\sim}{a}$

$$\psi(\eta_1 \cdots \eta_n; \underset{\sim}{X}_1 \cdots \underset{\sim}{X}_n)$$

$$= \psi(\eta_1 \cdots \eta_n; \underset{\sim}{X}_1 - a \cdots \underset{\sim}{X}_n - \underset{\sim}{a}) \qquad \text{(A15)}$$

we shall often choose $\underset{\sim}{a} = \underset{\sim}{X} = \Sigma \beta_i \underset{\sim}{X}_i$.

The normalization of the $\psi_{\underset{\sim}{K}}$'s is

$$\int \frac{d\eta_i}{\eta_i} d^2\underset{\sim}{X}_i \ \delta(1 - \sum_i \eta_i) \psi^*_{\underset{\sim}{K}'} (\eta_1 \cdots \eta_n; \underset{\sim}{X}_1 \cdots \underset{\sim}{X}_n) \psi_{\underset{\sim}{K}}(\eta_1 \cdots \eta_n; \underset{\sim}{X}_1 \cdots \underset{\sim}{X}_n)$$

$$= \delta^2(\underset{\sim}{K} - \underset{\sim}{K}') \qquad \text{(A16)}$$

From (A13) and (A15) we have

$$\psi_K(\eta_1 \cdots \eta_n ; \underset{\sim}{X}_1 \cdots \underset{\sim}{X}_n) = \frac{1}{2\pi} e^{i\underset{\sim}{K}\cdot\underset{\sim}{X}} \psi(\eta_1 \cdots \eta_n ; \underset{\sim}{X}'_1 \cdots \underset{\sim}{X}'_n) \qquad \text{(A17)}$$

where $\underset{\sim}{X}'_i = \underset{\sim}{X}_i - \underset{\sim}{X}$. What is the normalization of the $\psi$'s ? To do this conveniently, we perform a transformation from the n variables $\underset{\sim}{X}_i$ to the n+1 variables $\underset{\sim}{X}'_i$, $\underset{\sim}{X}$, subject to the condition $\Sigma \eta_i \underset{\sim}{X}'_i = 0$.

Define

$$\underset{\sim}{X}_{n+1} = \sum_{i=1}^{n} \eta_i \underset{\sim}{X}'_i$$

$$\underset{\sim}{X}_{n+1} = \underset{\sim}{X}$$

so that

$$\underset{\sim}{X}_i = \underset{\sim}{X}'_i + X'_{n+1} \qquad\qquad i = 1 \ldots n$$

So in any integral we have

$$\int \prod_{i=1}^{n} d^2 \underset{\sim}{X}_i \ldots = \int \prod_{i=1}^{n+1} d^2 \underset{\sim}{X}_i \; \delta\,(\underset{\sim}{X}_{n+1}) \ldots$$

$$= \int J \prod_{i=1}^{n} d^2 \underset{\sim}{X}'_i \; d^2 \underset{\sim}{X} \; \delta\,(\sum_{i=1}^{n} \eta_i \underset{\sim}{X}'_i) \ldots \tag{A18}$$

where J is the Jacobian

$$\begin{vmatrix}
\frac{\delta X_1}{\delta X'_1} \cdots & \frac{\delta X_1}{\delta X'_{n+1}} & \frac{\delta X_1}{\delta Y'_1} \cdots & \frac{\delta X_1}{\delta Y'_{N+1}} \\
\vdots & & & \vdots \\
\frac{\delta X_{n+1}}{\delta X'_1} \cdots & \frac{\delta X_{n+1}}{\delta X'_{n+1}} & \frac{\delta X_{n+1}}{\delta Y'_1} \cdots & \frac{\delta X_{n+1}}{\delta Y'_{n+1}} \\
\frac{\delta Y_1}{\delta X'_1} \cdots & \frac{\delta Y_1}{\delta X'_{n+1}} & \frac{\delta Y_1}{\delta Y'_1} \cdots & \frac{\delta Y_1}{\delta Y'_{n+1}} \\
\vdots & & & \vdots \\
\frac{\delta Y_{n+1}}{\delta X'_1} \cdots & \frac{\delta Y_{n+1}}{\delta X'_{n+1}} & \frac{\delta Y_{n+1}}{\delta Y'_1} \cdots & \frac{\delta Y_{n+1}}{\delta Y'_{n+1}}
\end{vmatrix}$$

The matrix elements are

$$\frac{\delta X_i}{\delta X'_j} = \delta_{ij} \qquad\qquad \text{for } i,\ j \neq n+1$$

$$\frac{\delta X_{n+1}}{\delta X'_j} = \eta_j \qquad\qquad \text{for } j \neq n+1$$

$$\frac{\delta X_j}{\delta X'_{n+1}} = 1 \qquad \text{for } j \neq n+1$$

$$\frac{\delta X_{n+1}}{\delta X'_{n+1}} = 0$$

and similarly for all $\frac{\delta Y_j}{\delta Y_k}$ . Of course, all $\frac{\delta X_j}{\delta Y_k}$ , $\frac{\delta Y_j}{\delta X_k}$ vanish.

The Jacobean matrix then looks like

$$\left|\begin{array}{c|c} \begin{matrix} 1 & 0 & 0 \ldots & 0 & 1 \\ 0 & 1 & 0 \ldots & 0 & 1 \\ 0 & 0 & 1 \ldots & 0 & 1 \\ \vdots & & & & \vdots \\ \beta_1 & \beta_2 & \beta_3 \cdot\cdot & \beta_n & 0 \end{matrix} & 0 \\ \hline 0 & \begin{matrix} 1 & 0 & 0 & \ldots & 0 & 1 \\ 0 & 1 & 0 & \ldots & 0 & 1 \\ 0 & 0 & 1 & \ldots & 0 & 1 \\ \vdots & & & & & \vdots \\ \beta_1 & \beta_2 & \beta_3 & \ldots & \beta_n & 0 \end{matrix} \end{array}\right|$$

Subtract (in each submatrix) the first n columns, respectively, from the last column. One obtains for the determinant

$$\begin{vmatrix} 1 & 0 & \ldots & 0 & 0 \\ 0 & 1 & \ldots & 0 & 0 \\ \vdots & & & & \vdots \\ 0 & 0 & \ldots & 1 & 0 \\ \eta_1 & \eta_2 & & \eta_n & (-\Sigma\,\eta_i) \end{vmatrix}$$

Expanding by the last column, we obtain $(-\Sigma\,\eta_i)1 = -\Sigma\,\eta_i = -1$.

For the two dim nsional case we get

$$J = (-\Sigma\,\eta_i)^2 = 1 \quad .$$

So now, using (A16), (A17) and (A18) the normalization condition becomes

$$\frac{1}{(2\pi)^2}\int \prod_{i=1}^{n} \frac{d\eta_i}{\eta_i}\, \delta(1-\Sigma\eta_i)\, d^2\underset{\sim}{X}'_i\; \delta(\Sigma\,\eta_i\,\underset{\sim}{X}'_i)\, d^2\underset{\sim}{X}\; e^{i(\underset{\sim}{K}-\underset{\sim}{K}')\cdot\underset{\sim}{X}}$$

$$\psi^*(\eta_1\ldots\eta_n;\underset{\sim}{X}'_1\ldots\underset{\sim}{X}'_n)\;\psi(\eta_1\ldots\eta_n;\underset{\sim}{X}'_1\ldots\underset{\sim}{X}'_n)$$

$$= \delta^2(\underset{\sim}{K}-\underset{\sim}{K}')$$

and hence

$$\prod_{i=1}^{n}\int\frac{d\eta_i}{\eta_i}\,\delta(1-\Sigma\eta_i)\,d^2\underset{\sim}{X}'_i\;\delta(\Sigma\eta_i\,\underset{\sim}{X}'_i)\,|\psi(\eta_1\ldots\eta_n;\underset{\sim}{X}'_1\ldots\underset{\sim}{X}'_n)|^2 = 1 \qquad \text{(A19)}$$

We now come to consider the vertex function $\langle\eta'\,K'\,|J_\mu|\,\eta\,\underset{\sim}{K}\rangle$, where $J_\mu$ is the electromagnetic current, coupling to the charges $e_q$ of the partons. As discussed in Drell and Yan, we shall specialize to the "good" currents $\mu = 0$ or 3.

In particular we shall choose $\mu = 0$.

In the parton model, we write

$$J_0 = \sum_a j_{0a} \qquad \text{(A21)}$$

where $j_{0a}$ is the charge density operator for parton a. By hypothesis, $j_{0a}$ has only a point-type coupling. In momentum space, this translates into

$$\langle\beta_a\,\underset{\sim}{K}'_a|\,j_{0a}\,|\beta_a\,\underset{\sim}{K}_a\rangle = e_a\,\beta_a\,P\,\delta_{\lambda_a\lambda'_a} \qquad \text{(A22)}$$

where $\lambda_a$ ($\lambda'_a$) is the initial (final) helicity of the parton a. If $\eta_a \neq \eta'_a$, a slightly more complicated form is obtained.

Introducing the expansion (A11) for the bet $|\beta \underset{\sim}{K}_a>$, Eq. (A22) in position space reads

$$<\beta_a \underset{\sim}{X}'_a | j_{0a} | \beta_a \underset{\sim}{X}_a> = (2\pi)^2 e_a \beta_a P \, \delta(\underset{\sim}{X}_a) \, \delta(\underset{\sim}{X}'_a)$$

$$= (2\pi)^2 e_a \beta_a P \, \delta(\underset{\sim}{X}_a) \, \delta(\underset{\sim}{X}'_a - \underset{\sim}{X}_a) \qquad \text{(A22)}$$

We now make use of the expansions (A2) for $|\beta \underset{\sim}{K}>$, (A21) for $J_0$, and the result (A23) to write

$$<\beta \underset{\sim}{K}' | J_0 | \beta \underset{\sim}{K}> = \beta P \sum_n \sum_a e_a \int \prod_{i=1}^{n} \left\{ \frac{d\eta_i}{\eta_i} \, \delta(1 - \Sigma \eta_i) \, d^2 \underset{\sim}{X}_i \right\} \delta(\underset{\sim}{X}_a)$$

$$e^{-i \underset{\sim}{Q} \cdot \underset{\sim}{X}} \, \psi_f^* (\eta_1 \cdots \eta_n ; \underset{\sim}{X}_1 \cdots \underset{\sim}{X}_n) \, \psi_i (\eta_1 \cdots \eta_n ; \underset{\sim}{X}_1 \cdots \underset{\sim}{X}_n) \qquad \text{(A29)}$$

where

$$\underset{\sim}{X} = \sum_{i=1}^{n} \eta_i \underset{\sim}{X}_i \, .$$

Equation A24 results after doing the $\beta'_i$, $\underset{\sim}{X}'_i$ integrations, and using the fact that in our IMF, $\beta = \beta'$, which forces $\beta_a = \beta'_a$ since all the other $\beta_i = \beta'_i$.

Making use of our previous result on transformation of variables, we can rewrite (A24) as

$$<\beta \underset{\sim}{K}' | J_0 | \beta K> = \beta P \sum_n \sum_a{}' e_a \int \prod_{i=1}^{n} \frac{d\eta_i}{\eta_i} \, \delta(1 - \Sigma \eta_i)$$

$$\Pi d^2 \underset{\sim}{X}'_i \, d\underset{\sim}{X} \;\; \delta(\Sigma \eta_i \underset{\sim}{X}'_i) \, \delta(\underset{\sim}{X}'_a + \underset{\sim}{X}) \, e^{-i \underset{\sim}{Q} \cdot \underset{\sim}{X}}$$

$$\psi_f^* (\eta_1 \cdots \eta_n ; \underset{\sim}{X}'_1 \cdots \underset{\sim}{X}'_n) \, \psi_i (\eta_1 \cdots \eta_n ; \underset{\sim}{X}'_1 \cdots \underset{\sim}{X}'_n) \qquad \text{(A25)}$$

The X integration is now trivial, and we obtain our final result

$$<\beta \underset{\sim}{\underline{K}}' | J_0 | \beta \underset{\sim}{\underline{K}}>_\infty = \beta P \sum_n \sum_a e_a \int \prod_{i=1}^{n} \frac{d\eta_i}{\eta_i} \, \delta(1 - \Sigma\eta_i)$$

$$\Pi \, d^2 \underset{\sim}{X}'_i \, \delta(\Sigma\eta'_i \underset{\sim}{X}'_i) \, e^{i Q \cdot X'_a} \quad \psi_f^*(\eta_1 \cdots \eta_n; \underset{\sim}{X}'_i \cdots X'_n) \quad \psi_i(\eta_1 \cdots \eta_n; \underset{\sim}{X}'_i \cdots \underset{\sim}{X}'_n) \tag{A.26}$$

## APPENDIX B: KINEMATICS

We begin with a statement of our conventions, and sketch the derivation of relevant formulae.

The matrix element of the current $J_\mu$ is taken between hadron states $|h\underset{\sim}{P}\lambda>$, $|h'\underset{\sim}{P}'\lambda'>$, normalized such that

$$<h'\underset{\sim}{P}'\lambda' \mid hP\lambda> = (2\pi)^3\, 2E_p\, \delta^3(\underset{\sim}{P}' - \underset{\sim}{P})\, \delta_{hh'} \,. \tag{B1}$$

The baryon isobars will be described by Rarita-Schwinger spinors normalized to

$$\bar{\psi}^{\mu_1 \cdots \mu_{J-1/2}}\, \psi_{\mu_1 \cdots \mu_{J-1/2}} = 2M \,. \tag{B2}$$

As usual, one constructs the tensor

$$W_{\mu\nu} \equiv (2\pi)^{-1} \int d^4x\, e^{iq\cdot x} <P\lambda \,|\, [J_\mu(x),\, J_\nu(0)] \,|\, P\lambda> \tag{B3}$$

$$= (2\pi)^3 \sum_n \delta^4(P + q - P_n) <n\,|J_\nu(0)|\,P\lambda> <n|J_\mu(0)|\,P\lambda>^* \tag{B4}$$

where (B4) follows from (B3) in the electroproduction region.

One can also expand

$$W_{\mu\nu} = -W_1(g_{\mu\nu} - q_\mu q_\nu/q^2) + W_2\, \tilde{P}_\mu\, \tilde{P}_\nu \tag{B5}$$

where $\tilde{P}_\mu = P_\mu - (P\cdot q)\, q_\mu/q^2$. The usual "$\nu W_2$" having a maximum value of approximately 0.3 in the scaling region is equal to $\frac{1}{2}(P\cdot q)W_2$ as defined in Eq. (B5).

The S-matrix for the scattering of a <u>real</u>, transverse photon helicity $\lambda_\gamma$ from a nucleon, helicity $\lambda$, into a final state n is

$$S_{fi} = \delta_{fi} + i(2\pi)^4 \delta^4(P_n - P - q)\;\; e<n|J\cdot \epsilon(\lambda_\gamma)\,|P\lambda> \tag{B6}$$

The total cross section is then given by

$$2E_N 2E_\gamma \quad \sigma_{\lambda\lambda_\gamma} = e^2(2\pi)^4 \sum_n \delta^4(P_n - P - q) \, | \langle n | J \cdot \epsilon(\lambda_\gamma) | P\lambda \rangle |^2$$

$$= \frac{1}{2} \alpha \, \epsilon_\mu W^{\mu\nu} \epsilon_\nu \tag{B7}$$

For a single particle final state (isobar), the sum over n is simply performed:

$$2E_N 2E_\gamma \quad \sigma_{\lambda\lambda_\gamma} = (2\pi)^4 \int \frac{d^4P_n}{(2\pi)^3} \delta^4(P_n - P - q) \, \delta(P_n^2 - M^2) \, e^2 \, | \langle n | J \cdot \epsilon | P\lambda \rangle |^2$$

$$= 8\pi^2 \alpha \, \delta((P+q)^2 - M^2) \, | \mathscr{F}_h |^2 \tag{B8}$$

where $\mathscr{F}_h$ is the C.M. Feynman amplitude $\langle N^* \underset{\sim}{Q} \, h | J \cdot \epsilon(q, \lambda_\gamma) | N \, \underset{\sim}{P} \, \lambda \rangle$, where $h = \lambda_\gamma - \lambda$. For real photons, the invariant $2E_N E_\gamma = M^2 - m^2$. One usually retains this flux normalization for virtual photons, with the result

$$\sigma_h = \frac{4\pi\alpha}{M^2 - m^2} \; \frac{\Gamma M}{(s - M^2)^2 + (\Gamma M)^2} \; | \mathscr{F}_h |^2 \tag{B9}$$

where we have gone from the $\delta$-function to the Breit-Wigner form.

The observed quantity $\sigma = \sigma_T + \epsilon \sigma_L$ is given by

$$\sigma = \sigma_T + \epsilon \sigma_L = \frac{1}{2} (\sigma_{1/2} + \sigma_{3/2}) + \epsilon \sigma_L$$

In the resonance region, $\epsilon \approx 1$ for small electron scattering angles and moderate values of $Q^2$.

<u>The Infinite Momentum Limit</u>

We shall need to evaluate $\langle N^* \, P' \lambda' | J_0 | N \, P \, \lambda \rangle$ in the IMF. As in the text we write (following Ref (11))

$< P + q,\ \lambda' \,|J\cdot \epsilon| P\ \lambda >$

$$= \bar{\psi}^{\mu_1 \cdot\cdot \mu_{J-1/2}} (P + q,\ \lambda')\ q_{\mu_2} \cdot\cdot\ q_{\mu_{J-1/2}}$$

$$\times \Big[ q_{\mu_1} (\epsilon \cdot q\, P \cdot q - \epsilon \cdot P\, q^2)\, g_1$$

$$+ 2\, \epsilon_{\mu_1 \alpha\beta\gamma}\, P^\alpha q^\beta S^\gamma\ g_2$$

$$+ i M q_{\mu_1}\ \gamma \cdot S \gamma^5 (g_2 + g_3) \Big]\ u(P,\ \lambda)$$

$$\equiv F_1 + F_2 + F_3 \tag{B10}$$

where $S_\gamma = \epsilon_{\gamma\lambda\nu\sigma} P^\lambda q^\nu \epsilon^\sigma = \epsilon_{\gamma\lambda\nu\sigma} (P + q)^\lambda q^\nu \epsilon^\sigma$. Our metric and $\gamma$-matrices those of Bjorken and Drell,[22] and the normalization of $\epsilon_{\mu\nu\lambda\sigma}$ is such that $\epsilon_{0123} = +1$. We normalize the spinor wave functions to $\bar{\psi}^{\mu_1 \cdot\cdot}\ \psi_{\mu_1 \cdot\cdot} = 2M$, $\bar{u}u = 2m$. Equation (B10) holds for the normal excitations $1/2^+ \to 1/2^+,\ 3/2^-,\ 5/2^+ \ldots$ A similar form holds for the abnormal excitations $1/2^+ \to 1/2^-,\ 3/2^+ \ldots$ with $u \to \gamma^5 u$.

In obtaining the infinite momentum limit (IML) of (B10), it will be convenient for us to reduce the second term in (B10) to an alternate form. Using the identity

$$\epsilon_{\mu\alpha\beta\gamma}\, \epsilon^{\gamma}_{\ \lambda\nu\sigma} = (g_{\mu\lambda} g_{\alpha\nu} g_{\beta\sigma} + g_{\mu\nu} g_{\alpha\sigma} g_{\beta\lambda} + g_{\mu\sigma} g_{\alpha\lambda} g_{\beta\nu}$$
$$- g_{\mu\lambda} g_{\alpha\sigma} g_{\beta\nu} - g_{\mu\nu} g_{\alpha\lambda} g_{\beta\sigma} - g_{\mu\sigma} g_{\alpha\nu} g_{\beta\lambda}) \tag{B11}$$

we can write

$$\vec{v}_{\mu_1} \equiv \epsilon_{\mu_1\alpha\beta\gamma} P^\alpha q^\beta S^\gamma$$

$$= P_{\mu_1}(P\cdot q \; q\cdot\epsilon - P\cdot\epsilon \; q^2)$$

$$+ q_{\mu_1}(P\cdot\epsilon \; P\cdot q - m^2 \; q\cdot\epsilon)$$

$$+ \epsilon_{\mu_1}(m^2 q^2 - (P\cdot q)^2) \tag{B12}$$

with the second term becoming

$$F_2 = 2g_2 \bar{\psi}^{\mu_1 \cdots \mu_{J-1/2}} v_{\mu_1} q_{\mu_2} \cdots q_{\mu_{J-1/2}} u$$

$$= 2g_2 \bar{\psi}^{\mu_1 \cdots \mu_{J-1/2}} (P+q, \lambda') \; q_{\mu_2} \cdots q_{\mu_{J-1/2}}$$

$$\Big[q_{\mu_1} (P\cdot\epsilon \; P\cdot q - m^2 \; q\cdot\epsilon - P\cdot q \; q\cdot\epsilon + P\cdot\epsilon \; q^2)$$

$$+ \epsilon_{\mu_1} (m^2 q^2 - (P\cdot q)^2)\Big] \; u(P,\lambda) \tag{B13}$$

In going from the first to the second line of the last equation we have made use of the fact that $\bar{\psi}^{\mu_1 \cdots \mu_{J-1/2}} P_{\mu_1} = -\bar{\psi}^{\mu_1 \cdots \mu_{J-1/2}} q_{\mu_1}$.

Now set $\epsilon_\mu = g_{\mu 0}$. With our choice $q = (m\nu/P, Q, 0, 0)$, we find the IML of $S_\gamma$ to be

$$\left[S_\gamma\right]_\infty = (0, 0, -PQ, 0) \tag{B14}$$

After taking the IML of all scalars such as $q \cdot \epsilon$, $P \cdot \epsilon$, etc., we find

$$
\begin{aligned}
<P'\lambda'|J_0|P\lambda>_\infty &= \left[Q^2 g_1 + 2(m\nu - Q^2)\, g_2\right] \\
&\times \left[\bar{\psi}^{\mu_1 \cdots \mu_{J-1/2}} q_{\mu_1} \cdots q_{\mu_{J-1/2}} u\right]_\infty - 2m^2(\nu^2 + Q^2)\, g_2 \\
&\times \left[\bar{\psi}^{\mu_1 \cdots \mu_{J-1/2}} \epsilon_{\mu_1} q_{\mu_2} \cdots q_{\mu_{J-1/2}}\right]_\infty \\
&- MPQ(g_2 + g_3) \left[\bar{\psi}^{\mu_1 \cdots \mu_{J-1/2}} q_{\mu_1} \cdots q_{\mu_{J-1/2}}\, i\gamma^2\gamma^5 u\right]_\infty
\end{aligned}
\tag{B15}
$$

The rest is straightforward. The Rarita-Schwinger wave function is expanded in the usual way

$$
\begin{aligned}
\psi^{\mu_1 \cdots \mu_{J-1/2}}(P'\lambda') &= \sum_{\lambda_1 \cdots \lambda_{J-1/2}, \sigma} C(J-1/2,\ 1/2,\ J; \sigma,\ \lambda' - \sigma) \\
&<\lambda_1 \cdots \lambda_{J-1/2}|J-1/2, \sigma> \ \epsilon_{\mu_1}(P'\lambda_1) \cdots \epsilon_{\mu_{J-1/2}}(P', \lambda_{J-1/2}) \\
&U(P',\ \lambda' - \sigma)
\end{aligned}
$$

where

$$
\begin{aligned}
&<\lambda_1 \cdots \lambda_{J-1/2}|J-1/2,\ \sigma> \\
&= 2^{J-1/2 - \Sigma|\lambda_i|} \left[\frac{(J-1/2+\sigma)!\ (J-1/2-\sigma)!}{(2J-1)!}\right]^{1/2} \cdot \delta_{\Sigma\lambda_i, \sigma}
\end{aligned}
\tag{B16}
$$

and the various dot products taken. Useful limits are the following:

$$\vec{\epsilon}(P + q,\ 0) \cdot q \ = \ (m\nu \ - Q^2)/M$$

$$= \omega^* \text{ (as defined after Eq. (26) in the text)}$$

$$\epsilon(P + q, \pm 1) \circ q \ = \ \pm \quad Q/\sqrt{2}$$

Useful spinor IML's are

$$\left[ \overline{U}(P',\ 1/2) \begin{Bmatrix} 1 \\ \gamma^5 \end{Bmatrix} u(P,\ 1/2) \right]_\infty = M \pm m$$

$$\left[ \overline{U}(P',\ -1/2) \begin{Bmatrix} 1 \\ \gamma^5 \end{Bmatrix} u\ (P,\ 1/2) \right]_\infty = -Q$$

$$\left[ \overline{U}(P',\ 1/2) \begin{Bmatrix} i\gamma^2\gamma^5 \\ i\gamma^2 \end{Bmatrix} u(P,\ 1/2) \right]_\infty = -Q$$

$$\left[ \overline{U}(P',\ -1/2) \begin{Bmatrix} i\gamma^2\gamma^5 \\ i\gamma^2 \end{Bmatrix} u(P, 1/2) \right]_\infty = -(M \pm m)$$

where $P' = P + q = \left(P + \frac{m^2+Q^2}{2P},\ Q,\ 0,\ P\right)$. Formula such as (30), (31) in the text follow from a detailed application of the procedure sketched in this appendix.

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FIGURE CAPTIONS

1. Ratio $G_M^*(Q^2)/[2(1 + Q^2/(M + m)^2)^{1/2} F_2^V(Q^2)]$ vs $Q^2$. Data taken from ref. (9).

2. Theoretical fits to total $D_{13}$ excitation cross sections. Data extracted from Fig. 7 in A.B. Clegg, ref. (15).

3. Theoretical fits to total $S_{11}$ excitation cross section. Data taken from ref. (14), based on a branching ratio $(S_{11} \to \eta p)/(S_{11} \to \text{all}) = 0.55$.

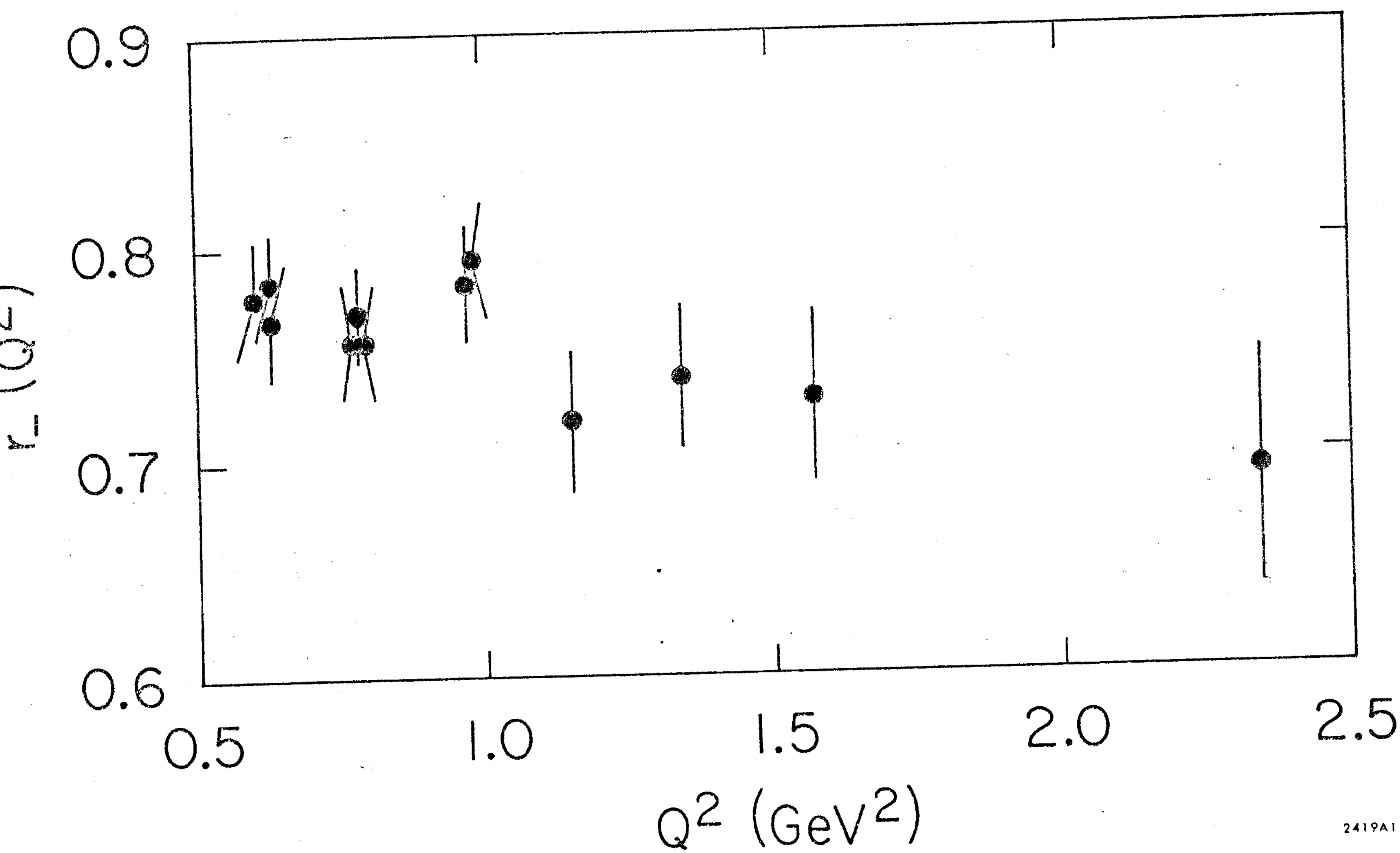


Fig. 1

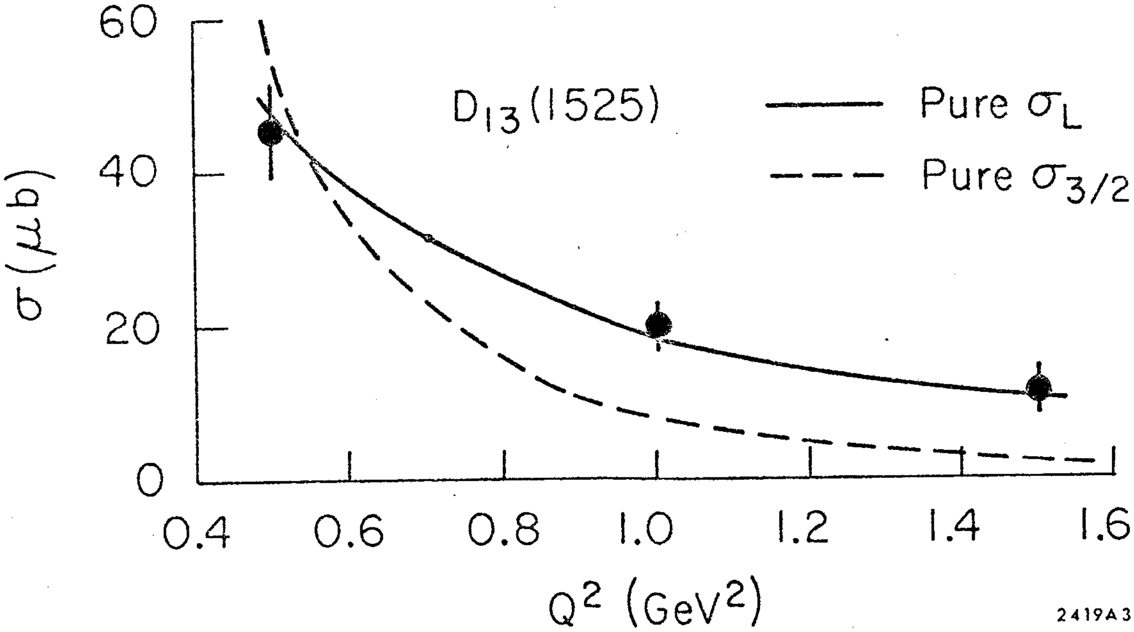
60
40
20
0
σ (μb)
D13(1525)
Pure σL
Pure σ3/2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
Q2 (GeV2)
2419A3

Fig. 2

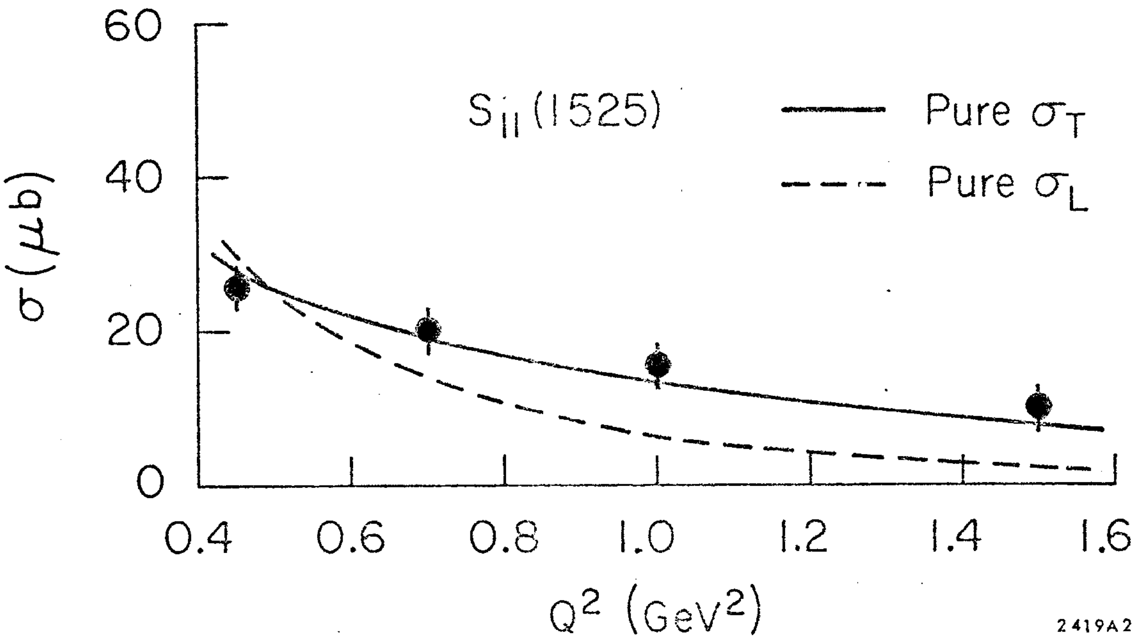
60
40
20
0
σ (μb)
S11(1525)
Pure σT
Pure σL
0.4
0.6
0.8
1.0
1.2
1.4
1.6
Q2 (GeV2)
2419A2

Fig. 3