$\bar{K}N$ scattering lengths from the experiments on kaonic hydrogen and kaonic deuterium

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Abstract. Within the framework of a low-energy effective field theory we consider the procedure of extraction of the S-wave kaon–nucleon scattering lengths a_0 and a_1 from a combined fit to the kaonic hydrogen and kaonic deuterium data. It is demonstrated that, if the present DEAR central values for the kaonic hydrogen ground-state energy and width are used in the analysis of the data, a solution for a_0 and a_1 exists only in a restricted domain of input values for the kaon-deuteron scattering length. We therefore conclude that forthcoming measurement of this scattering length imposes stringent constraints on the theoretical description of the kaon-deuteron interactions at low energies. Most of the results of this talk are contained in the recent paper [1].

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1 Introduction

Recently, the DEAR collaboration at LNF-INFN has performed a measurement of the energy level shift and width of the kaonic hydrogen ground state [2] with a considerably better accuracy than the earlier KpX experiment at KEK [3]. The preliminary result of DEAR is

$$\epsilon_{1s} = 193 \pm 37\ (\text{stat}) \pm 6\ (\text{syst})\ \text{eV}\,,$$

$$\Gamma_{1s} = 249 \pm 111\ (\text{stat}) \pm 30\ (\text{syst})\ \text{eV}\,. \qquad (1)$$

Now DEAR is being replaced by the SIDDHARTA experiment, which by 2007 plans measurements of both the energy shift and the width of kaonic hydrogen with a precision of several eV, i.e. at the few percent level. Moreover, SIDDHARTA will attempt the first ever measurement of the energy shift of the kaonic deuterium with a comparable accuracy and possibly, of other atomic complexes (kaonic helium and sigmonic atoms).

The necessity to perform measurements of the kaonic deuterium ground-state observables is evident from the following simple argument. Namely, the measurement of only the kaonic hydrogen spectrum does not allow one – even in principle – to extract independently both S-wave $\bar{K}N$ scattering lengths a_0 and a_1. The reason for this is that there are open inelastic channels below the $\bar{K}N$ threshold, rendering these scattering lengths complex. Consequently, one has to determine four independent quantities (real and imaginary parts of a_0 and a_1) that requires performing four independent measurements – e.g., the energy level shifts and widths of kaonic hydrogen *and* kaonic deuterium. However, even though it is clear that a_0 and a_1 can not be determined separately without measuring kaonic deuteron, it is still not *a priori* evident, whether it is possible to do so if one performs such a measurement. Note that to this end one needs theoretical input from three-body calculations, which relate a_0 and a_1 to the observables of the kaonic deuterium and one has to check, whether the uncertainties in these calculations are small enough not to hinder a determination of a_0 and a_1 from the outcome of the experiment.

A time-honored phenomenological approach to the kaon-deuteron interactions at low energies is based of Faddeev equations in the potential scattering theory, see, e.g. [4–9]. For the recent status of the problem, see e.g. [10] and references therein. In this approach, one predicts the numerical value of the K^-d scattering length, using "realistic" input two-body potentials in the calculations. On the other hand, one may derive an expression for the K^-d scattering length in a form of the (partially re-summed) multiple-scattering series, which algebraically relates this scattering length to a_0 and a_1 and which thus can be used to solve the problem of extracting a_0 and a_1 from the experiment (inverse problem). We stress that within the potential picture one may always refer to the exact solution of Faddeev equations for a numerical check of the validity of the multiple-scattering expansion which, in most cases, works reasonably well [10].

In recent years the investigations of the same problem within the effective field theory (EFT) framework have started to appear (see e.g. [1, 11]). An ultimate goal of

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these investigations is to perform a model-independent calculation of the kaon-deuteron scattering observables without referring to the exact form of phenomenological hadronic interactions. Moreover, this type of the approach has the potential to go beyond the approximations that have been used to derive the multiple-scattering series within the potential model. Namely, one may expect that using the EFT methods could allow one to systematically improve the accuracy of the calculations by including e.g. the effects, coming from higher-order terms in the effective-range expansion of the $\bar{K}N$ amplitudes. Other effects like the nucleon recoil, or the short-range three-body forces, should also be taken into account, if one aims at a high accuracy in final results, which should be compared to the precise data coming from future experiments.

In our opinion, at present stage of the studies potential models provide useful testing ground for the validity of various schemes in EFT, since the relative size of different contributions can be directly estimated there by using numerical methods. In particular, the non-relativistic EFT, which we shall be using below, has a structure very similar to the potential model and leads to the same multiple-scattering series at leading order. On the other hand, a straightforward generalization of this approach leads to the inclusion of the higher-derivative local interactions in the kaon-nucleon Lagrangian, corresponding to the effective-range expansion of the $\bar{K}N$ amplitudes, as well as of the various terms describing relativistic corrections and short-range three-body interactions. It will be challenging to carry out a systematic quantitative analysis of these (sub-leading) effects, including the analysis of the theoretical error, with an aim to observe a consistent improvement of the theoretical precision. Note that, in order to be able to compare with the future accurate data coming from SIDDHARTA experiment, such calculations should be necessarily performed.

The main difference of the present study of the kaonic deuterium to previous work consists in the following. The existing approaches were exclusively concentrated on the prediction of the K^-d scattering length from the input $\bar{K}N$ scattering lengths. We are not aware of the "reversed" analysis in the literature, where the $\bar{K}N$ scattering lengths are determined from the input data of kaonic hydrogen and deuterium ground-state shift and width. However, this is exactly the type of the analysis that will be required in the near future for the SIDDHARTA data. In the present work we provide such an analysis. Moreover, in the absence of any experimental input for the deuteron, we argue in favor of using "synthetic" data instead: just scanning the complex plane ($\mathrm{Re}\,A_{\bar{K}d}$, $\mathrm{Im}\,A_{\bar{K}d}$) in a reasonable interval and using these values of the kaon-deuteron scattering length together with the K^-p elastic scattering length, which is measured in the kaonic hydrogen experiment, for extracting the values of a_0 and a_1. In this way we demonstrate that the reversed calculations, owing to the non-linear dependence of the kaon-deuteron amplitude on the $\bar{K}N$ scattering lengths, turn out to be much more sensitive to the theoretical input on the deuteron structure and the kaon-deuteron interactions, than a straightforward evaluation of the K^-d scattering length through the multiple-scattering series. This fact could potentially render a combined analysis of the hydrogen and deuterium data a beautiful testing ground for different EFT descriptions of the low-energy kaon-deuteron interactions and, as a result, might enable one to accurately determine the values of the scattering lengths a_0 and a_1.

2 Kaonic hydrogen and kaonic deuterium

In the experiments on hadronic atoms one measures the energy levels and widths of this sort of bound states. At present, there exists a well established systematic procedure for extracting the values of the pertinent hadronic scattering amplitudes at threshold from these measurements, based on non-relativistic effective Lagrangians (see e.g. [1, 12–17]). Below we merely present the result for the kaonic hydrogen and kaonic deuterium without a derivation. The details can be found in Refs. [1, 16, 17].

From the measurement of the (complex) energy shift of the hadronic atoms the elastic threshold amplitudes can be extracted by using following relations

$$\epsilon_{1s} - i\,\frac{\Gamma_{1s}}{2} = -2\alpha^3\mu_c^2\,a_p \times \left\{1 - 2\alpha\mu_c\,a_p\,(\ln\alpha - 1) + \cdots\right\}, \tag{2}$$

$$\epsilon_{1s}^d - i\,\frac{\Gamma_{1s}^d}{2} = -2\alpha^3\mu_r^2\,A_{\bar{K}d} \times \left\{1 - 2\alpha\mu_r\,A_{\bar{K}d}\,(\ln\alpha - 1) + \cdots\right\}. \tag{3}$$

Here, ϵ_{1s}, Γ_{1s} and ϵ_{1s}^d, Γ_{1s}^d are the ground-state strong shift and the width of the kaonic hydrogen and kaonic deuterium, respectively. Further, a_p and $A_{\bar{K}d}$ denote threshold amplitudes for the processes $K^-p \to K^-p$ and $K^-d \to K^-d$, and μ_c, μ_r denote the reduced masses of the K^-p and K^-d bound systems, respectively. In the following, we shall also refer to $A_{\bar{K}d}$ as to the "kaon-deuteron scattering length."

The above universal relations are exact at next-to-leading order in the isospin-breaking parameters: the fine-structure constant α and the up- and down-quark mass difference $m_d - m_u$. The threshold amplitudes a_p and $A_{\bar{K}d}$ contain, by definition, isospin-breaking corrections up to and including $O(\alpha, (m_d - m_u))$.

The analysis of the data proceeds as follows. From the experiment, one extracts two complex quantities a_p and $A_{\bar{K}d}$. At the next step, one expresses a_p and $A_{\bar{K}d}$ in terms of a_0 and a_1 in order to get their values from the experiment. In the case of $A_{\bar{K}d}$ this implies addressing three-body problem in the kaon-two-nucleon sector (see below). Note also that from now on we omit virtual photons: they were primarily needed to create bound states of kaons with the proton and the deuteron. We believe that at the accuracy of presently available calculations, virtual photon corrections to hadronic observables can be safely neglected.

3 Multiple-scattering series for the kaon-deuteron amplitude

The effective field theory, which is used to describe K^-d scattering, is constructed in the following manner:

a) The $\bar{K}N$ sector is described by the non-relativistic effective Lagrangian of the type considered in Ref. [16] (without virtual photons)

$$\begin{aligned}\mathcal{L}_{\bar{K}N} &= \psi^\dagger \left\{ i\partial_t - m_p + \frac{\nabla^2}{2m_p} + \cdots \right\} \psi \\ &+ \chi^\dagger \left\{ i\partial_t - m_n + \frac{\nabla^2}{2m_n} + \cdots \right\} \chi \\ &+ (K^-)^\dagger \left\{ i\partial_t - M_K + \frac{\nabla^2}{2M_K} + \cdots \right\} K^- \\ &+ (\bar{K}^0)^\dagger \left\{ i\partial_t - M_{\bar{K}^0} + \frac{\nabla^2}{2M_{\bar{K}^0}} + \cdots \right\} \bar{K}^0 \\ &+ \tilde{d}_1\, \psi^\dagger\psi\, (K^-)^\dagger K^- + \tilde{d}_2(\psi^\dagger\chi\, (K^-)^\dagger \bar{K}^0 + h.c.) \\ &+ \tilde{d}_3\chi^\dagger\chi\, (\bar{K}^0)^\dagger \bar{K}^0 + \tilde{d}_4\chi^\dagger\chi\, (K^-)^\dagger K^- + \cdots\,, \end{aligned} \quad (4)$$

where ψ, χ, K^- and $\bar{K}^0$ stand for non-relativistic proton, neutron, K^- and $\bar{K}^0$ fields, respectively, m_p, m_n, $M_K = M_{K^-}$ and $M_{\bar{K}^0}$ denote the masses of these particles and $\tilde{d}_i$, $i = 1,2,3,4$ are expressed through the threshold scattering amplitudes in pertinent channels, reducing to certain combinations of a_0 and a_1 in the isospin limit. The inclusion of derivative interactions, corresponding to the higher-order terms in the effective-range expansion of the $\bar{K}N$ interactions, is straightforward.

b) The interactions between the nucleons are described in ChPT with non-perturbative pions (for a recent review, see e.g. [18]). In the effective theory, which is used here, the $\bar{K}N$ and NN sectors do not talk to each other, by construction. Therefore, the only input that one needs from the NN sector is that the nucleon-nucleon potential in the momentum space in CM frame is given by a known function $V_{NN}(\mathbf{p},\mathbf{q};\Lambda)$, where Λ denotes the cutoff parameter in this scheme – typically, of order of a few hundred MeV (to ease notations, we suppress spin-isospin indices). This potential leads to the formation of a shallow bound state in the $^3S_1$ channel (deuteron), with the wave function $\Psi_d(\mathbf{p};\Lambda)$.

c) Three-body interactions in the kaon-two-nucleon sector are again described by a local Lagrangian of the type

$$\mathcal{L}_{\bar{K}NN} = \tilde{f}_0\, \psi^\dagger\psi\, \chi^\dagger\chi\, (K^-)^\dagger K^- + \cdots\,, \quad (5)$$

where ellipses stand for the terms with derivatives.

d) The kaon-deuteron scattering amplitude can be calculated in a standard manner, evaluating the six-point function $\bar{K}NN \to \bar{K}NN$ and extracting the double pole, corresponding to the initial and final deuterons (see, e.g. [17]). The multiple scattering series similar to that of Ref. [11] are obtained under usual approximations:
   - Diagrams, containing iterations of V_{NN} within the loops, are omitted.
   - The Fixed Center Approximation (FCA) is used to simplify propagators in the Feynman diagrams: the kinetic energies of the nucleons are neglected.
   - In addition, derivative interactions in the $\bar{K}N$ and $\bar{K}NN$ sectors are neglected.
   - Relativistic corrections for kaons are not included.

   Note that the validity of FCA been studied both in the potential scattering theory (see e.g. [9,19]) and in the EFT approach [20]. In Ref. [11] (see also references therein) it is argued that FCA can be a reasonable approximation even for $M_K/m_p \simeq 0.5$. This fact should be related to the peculiar cancellations at second order, which are discussed in Refs. [19,20]. We could also observe a clear pattern of such cancellations in our exploratory calculations. Further, even if there exists no proof of cancellations beyond second order, from e.g. the comparison to the exact solution of Faddeev equations [7] (see also the discussion in Ref. [11]) one may conclude that FCA works reasonably well also for the re-summed multiple-scattering series (Note, however Ref. [9], where it has been pointed out that large corrections to FCA might emerge due to the presence of the nearby sub-threshold resonance in the $\bar{K}N$ channel.).

e) We wish to note that in the case of the pion-deuteron scattering, the first few terms of the above multiple-scattering expansion can also be derived by using ChPT with non-perturbative pions up to and including $O(p^4)$ [21]. Additional small terms (e.g. boost correction) correspond to the derivative contribution in the meson-nucleon Lagrangian. In the case of the $\bar{K}N$ scattering, such a comparison to ChPT is not possible for obvious reasons.

f) As already discussed in Ref. [10], a crucial difference between the pion-deuteron and kaon-deuteron cases is that the multiple scattering series diverges for the latter whereas it converges for the former (the $\bar{K}N$ scattering lengths are large). For this reason, one has to perform a (partial) re-summation of the multiple-scattering series. Under the approximations listed below, this can be easily done.

Finally, we arrive at the following expression for the pion-deuteron scattering length

$$\left(1 + \frac{M_K}{M_d}\right) A_{\bar{K}d} = \int_0^\infty dr\, (u^2(r) + w^2(r))\, \hat{a}_{\bar{K}d}(r)\,, \quad (6)$$

where M_d is the deuteron mass, $u(r)$ and $w(r)$ denote the usual $S-$ and $D-$wave components of the deuteron wave function $\Psi_d(\mathbf{r};\Lambda)$ and the normalization condition is given by $\int_0^\infty dr\, (u^2(r) + w^2(r)) = 1$. Further,

$$\hat{a}_{\bar{K}d}(r) = \frac{\tilde{a}_p + \tilde{a}_n + (2\tilde{a}_p\tilde{a}_n - b_x^2)/r - 2b_x^2\tilde{a}_n/r^2}{1 - \tilde{a}_p\tilde{a}_n/r^2 + b_x^2\tilde{a}_n/r^3} + \delta\hat{a}_{\bar{K}d} \quad (7)$$

with $b_x^2 = \tilde{a}_x^2/(1+\tilde{a}_u/r)$. Furthermore,

$$\left(1+\frac{M_K}{m_p}\right)a_{p,n,x,u} = \tilde{a}_{p,n,x,u}\,, \tag{8}$$

where $a_{p,n,x,u}$ denote the threshold scattering amplitudes for $K^-p \to K^-p$, $K^-n \to K^-n$, $K^-p \to \bar{K}^0n$ and $\bar{K}^0n \to \bar{K}^0n$, respectively (these quantities are proportional to pertinent $\tilde{d}_i,\ i=1\cdots 4$ from Eq. (4)). Finally, the quantity $\delta\hat{a}_{\bar{K}d}$ is proportional to the three-body LEC $\tilde{f}_0$ from Eq. (5). The value of this constant is completely unknown at present. Different estimates (e.g. the dimensional estimate or the study of the Λ-dependence) lead to the conclusion that the uncertainty introduced by this term is not very large. For this reason, we further neglect this contribution altogether. In this case, Eqs. (6) and (7) become formally identical to ones derived in Ref. [11].

4 Isospin breaking

The equation (7) contains four different combinations of the threshold amplitudes. Consequently, one has first to express these amplitudes in terms of two scattering lengths a_0 and a_1, which should then be determined from the analysis of the combined data on kaonic hydrogen and deuterium. Avoiding this step and working in the particle basis is not possible: in this case, number of unknowns a_p, a_n, a_x, a_u exceeds the number of independently measured observables.

We take into account the leading-order isospin-breaking corrections in the kaon-nucleon scattering amplitudes which are due to the unitary cusps [16]. The re-summation of the bubble diagrams leads to the following simple parameterization

$$a_p = \frac{\frac{1}{2}(a_0+a_1)+q_0a_0a_1}{1+\frac{q_0}{2}(a_0+a_1)}\,, \qquad a_n = a_1\,,$$
$$a_x = \frac{\frac{1}{2}(a_0-a_1)}{1-\frac{iq_c}{2}(a_0+a_1)}\,, \qquad a_u = \frac{\frac{1}{2}(a_0+a_1)-iq_ca_0a_1}{1-\frac{iq_c}{2}(a_0+a_1)}\,, \tag{9}$$

where

$$q_c = \sqrt{2\mu_c\Delta}\,, \quad q_0 = \sqrt{2\mu_0\Delta}\,,$$
$$\Delta = m_n + M_{\bar{K}^0} - m_p - M_K\,,$$
$$\mu_c = \frac{m_pM_K}{m_p+M_K}\,, \quad \mu_0 = \frac{m_nM_{\bar{K}^0}}{m_n+M_{\bar{K}^0}}\,. \tag{10}$$

It can be checked that the isospin-breaking corrections to the individual amplitudes in Eq. (9) are large. However, the isospin-breaking correction to the kaon-deuteron scattering length, which is determined from Eqs. (6) and (7), turns out to be much smaller (see Ref. [22] for a detailed discussion on this issue).

5 Numerical results and discussion

The comparison of the results for kaonic hydrogen to experiment is displayed in Fig. 1. Here, we have used the values of a_0 and a_1 given in Refs. [23–27] as theoretical input in Eq. (2). On the basis of this comparison, one may conclude that the scattering lengths that are obtained from the fit to the data above threshold, in most of the cases are not compatible with the DEAR measurement. It is also seen that the isospin-breaking corrections are huge and can not be neglected even at the present accuracy of the experiment.

The compatibility of different values of a_0 and a_1 with the experiment can be made more transparent with the help of the following picture. From Eq. (9) it is seen that, for a given a_p the (complex) quantities a_0 and a_1 obey the equation

$$a_0 + a_1 + \frac{2q_0}{1-q_0a_p}\,a_0a_1 - \frac{2a_p}{1-q_0a_p} = 0\,. \tag{11}$$

Together with the requirement $\mathrm{Im}\,a_I \geq 0$, which stems from unitarity, Eq. (11) defines a circle in the ($\mathrm{Re}\,a_I$, $\mathrm{Im}\,a_I$)–plane. Part of this circle is shown in Fig. 2 (note that, bearing in mind the preliminary character of the DEAR data [2], we use only central values in order to illustrate the construction of the plot and do not provide a full error analysis). In order to be consistent with the DEAR data, both a_0 and a_1 should be on the right of this universal DEAR circle. For comparison, on the same figure we also indicate the (much milder) restrictions, which arise, when the KpX data are used instead of DEAR data. As we see, in most of the approaches it is rather problematic to get a value for a_0 which is compatible with DEAR. This kind of analysis may prove useful in the near future, when the accuracy of the DEAR is increased that might stir efforts on the theoretical side, aimed at a systematic quantitative description of the $\bar{K}N$ interactions within the unitarized ChPT.

Finally, we turn to our main goal of determining both a_0 and a_1 from the simultaneous analysis of the kaonic hydrogen and kaonic deuterium data. To this end, we determine a_1 from Eq. (11) and substitute it into Eqs. (6), (7) and (9). In a result one arrives at a non-linear equation[1] for determining a_0 with a given input value of $A_{\bar{K}d}$. In the absence of experimental data on kaonic deuterium, we have scanned the ($\mathrm{Re}\,A_{\bar{K}d}$, $\mathrm{Im}\,A_{\bar{K}d}$)–plane in the interval -2 fm $< \mathrm{Re}\,A_{\bar{K}d} < 0$ and 0.5 fm $< \mathrm{Im}\,A_{\bar{K}d} < 2.5$ fm and tried to find solutions, using in addition DEAR input data. The results of this investigation, which are displayed in Fig. 3, are very interesting: it turns out that the solutions exist only if $\mathrm{Im}\,A_{\bar{K}d} \lesssim 1$ fm and moreover, if $\mathrm{Im}\,A_{\bar{K}d} \simeq 1$ fm then one finds solutions only in a very small interval around $\mathrm{Re}\,A_{\bar{K}d} \simeq -1$ fm. We wish to also note that all this agrees with the scattering data analysis, carried out in Ref. [29]. Note that if $\mathrm{Im}\,A_{\bar{K}d}$ crosses the

[1] In Eq. (6) we use the NLO wave function of the deuteron with the cutoff parameter $\Lambda = 600$ MeV [28]. The final results however show a very weak dependence on the cutoff.

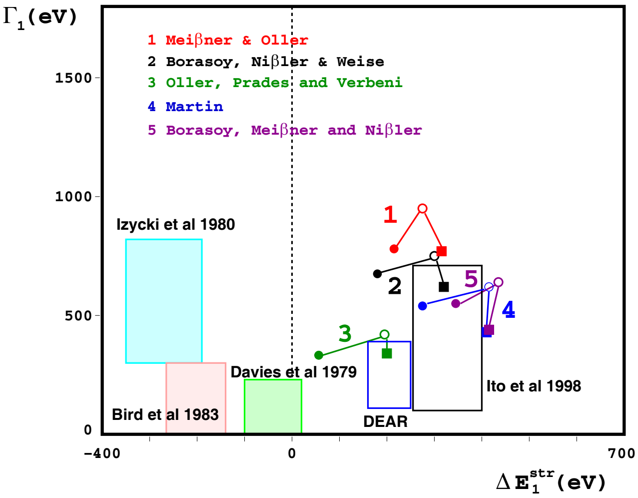

Fig. 1. Comparison of the calculated values of the strong shift and width in kaonic hydrogen [23–27] with existing experimental data. Filled circles correspond to completely neglecting isospin-breaking corrections. Empty circles correspond to including isospin breaking in a_p, according to Eq. (9) but neglecting the Coulomb corrections in Eq. (2) (the term, proportional to $\alpha(\ln\alpha - 1)$ in curly brackets). Finally, filled squares correspond to the full result in Eq. (2).

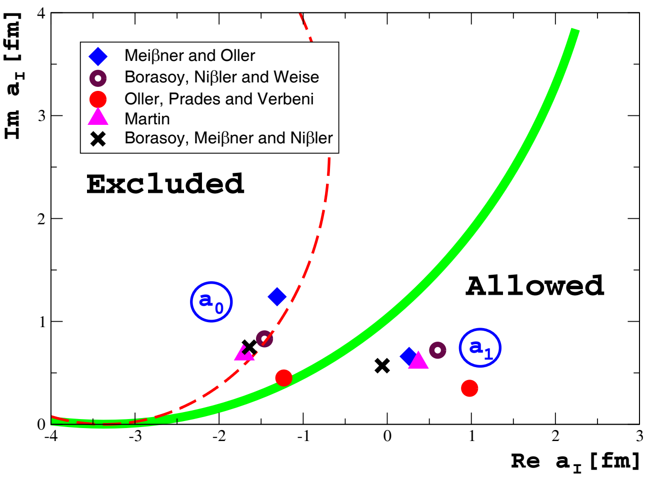

Fig. 2. Restrictions set by the DEAR data on the values of the scattering lengths a_0 and a_1 (thick solid line). For comparison, we give the scattering length calculations from different analyzes: 1) Meißner and Oller [23], 2) Borasoy, Nißler and Weise, fit u [24], 3) Oller, Prades and Verbeni, fit A4 [25], 4) Martin [26], 5) Borasoy, Meißner and Nißler [27]. The dashed line corresponds to the restrictions, obtained by using KpX data instead of DEAR data.

border of the shaded area in Fig. 3 continuously from below, then on the same branch one gets the solution with $\mathrm{Im}\, a_1 \leq 0$ that is forbidden by unitarity.

In conclusion we note that the region, where the solutions exist, is much larger in the case of the KpX input than for the DEAR input for the kaonic hydrogen. Namely, in the case of the KpX input the shaded area in Fig. 3 covers the most part of the plot.

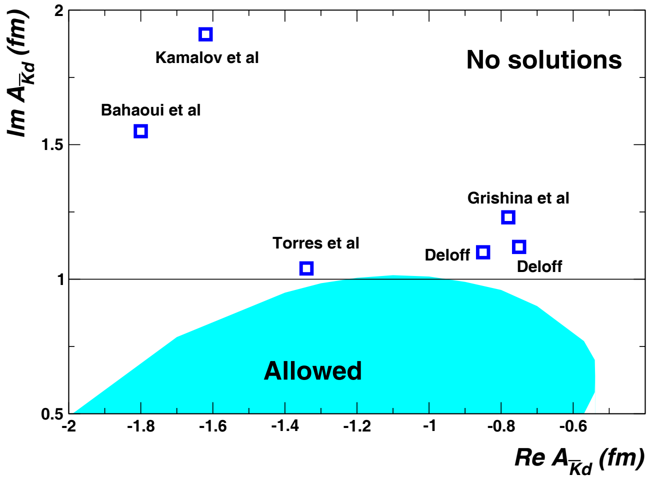

Fig. 3. The region in the $(\mathrm{Re}\, A_{\bar{K}d}, \mathrm{Im}\, A_{\bar{K}d})$–plane where solutions for a_0 and a_1 exists. For comparison, we also show the results of calculations of the kaon-deuteron scattering length from Refs. [6, 7, 9, 11, 30].

6 Conclusions

Up to now, in the theoretical description of the deuteron we have restricted ourselves to the approximations outlined above: FCA, no derivative interactions, no relativistic corrections, no iterations of V_{NN} in the loops. From the comparison with the potential model calculations one may expect that these approximations provide a good starting point for the description of the kaon-deuteron scattering length.

Within these approximations it turns out that the combined analysis of DEAR/SIDDHARTA data on kaonic hydrogen and deuterium is more restrictive than one would *a priori* expect. In particular, we see that solutions exist only in a rather small area of the $(\mathrm{Re}\, A_{\bar{K}d}, \mathrm{Im}\, A_{\bar{K}d})$–plane. Due to this fact, in certain cases it might be possible to pin down the values of a_0 and a_1 at a reasonable accuracy, even if $A_{\bar{K}d}$ itself is not measured very accurately. This statement constitutes our main result.

Finally, we wish to emphasize that, in the view of the forthcoming SIDDHARTA experiment, the question of the corrections to the leading-order approximate result acquires crucial importance – even if, as expected, they are moderate and do not change the qualitative picture. In our opinion, it will be very useful to carry out a systematic calculation of these corrections in the effective field theory framework, e.g. in the one described in the present work.

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References

1. U.-G. Meißner, U. Raha and A. Rusetsky, Eur. Phys. J. C **47** (2006) 473 [arXiv:nucl-th/0603029].
2. G. Beer *et al.* [DEAR Collaboration], Phys. Rev. Lett. **94** (2005) 212302.
3. M. Iwasaki *et al.*, Phys. Rev. Lett. **78** (1997) 3067; T. M. Ito *et al.*, Phys. Rev. C **58** (1998) 2366.
4. J. H. Hetherington and L. H. Schick, Phys. Rev. **137** (1965) B935.
5. G. Toker, A. Gal and J. M. Eisenberg, Nucl. Phys. A **362** (1981) 405.
6. M. Torres, R. H. Dalitz and A. Deloff, Phys. Lett. B **174** (1986) 213.
7. A. Deloff, Phys. Rev. C **61** (2000) 024004.
8. R. C. Barrett and A. Deloff, Phys. Rev. C **60** (1999) 025201.
9. A. Bahaoui, C. Fayard, T. Mizutani and B. Saghai, Phys. Rev. C **68**, 064001 (2003) [arXiv:nucl-th/0307067].
10. A. Gal, arXiv:nucl-th/0607067.
11. S. S. Kamalov, E. Oset and A. Ramos, Nucl. Phys. A **690**, 494 (2001) [arXiv:nucl-th/0010054].
12. P. Labelle and K. Buckley, arXiv:hep-ph/9804201; X. Kong and F. Ravndal, Phys. Rev. D **59** (1999) 014031; Phys. Rev. D **61** (2000) 077506 [arXiv:hep-ph/9905539]; B. R. Holstein, Phys. Rev. D **60** (1999) 114030 [arXiv:nucl-th/9901041]; D. Eiras and J. Soto, Phys. Rev. D **61** (2000) 114027 [arXiv:hep-ph/9905543].
13. A. Gall, J. Gasser, V. E. Lyubovitskij and A. Rusetsky, Phys. Lett. B **462** (1999) 335 [arXiv:hep-ph/9905309]; Phys. Rev. D **64** (2001) 016008 [arXiv:hep-ph/0103157]; J. Gasser, V. E. Lyubovitskij and A. Rusetsky, Phys. Lett. B **471** (1999) 244 [arXiv:hep-ph/9910438];
14. J. Schweizer, Phys. Lett. B **587** (2004) 33 [arXiv:hep-ph/0401048]; Eur. Phys. J. C **36** (2004) 483 [arXiv:hep-ph/0405034].
15. V. E. Lyubovitskij and A. Rusetsky, Phys. Lett. B **494** (2000) 9 [arXiv:hep-ph/0009206]; J. Gasser, M. A. Ivanov, E. Lipartia, M. Mojžiš and A. Rusetsky, Eur. Phys. J. C **26** (2002) 13 [arXiv:hep-ph/0206068].
16. U.-G. Meißner, U. Raha and A. Rusetsky, Eur. Phys. J. C **35** (2004) 349 [arXiv:hep-ph/0402261].
17. U.-G. Meißner, U. Raha and A. Rusetsky, Eur. Phys. J. C **41** (2005) 213, Erratum *ibid* C **45** (2006) 545 [arXiv:nucl-th/0501073].
18. E. Epelbaum, Prog. Part. Nucl. Phys. **57** (2006) 654 [arXiv:nucl-th/0509032].
19. G. Fäldt, Phys. Scripta **16** (1977) 81.
20. V. Baru, C. Hanhart, A. E. Kudryavtsev and U.-G. Meißner, Phys. Lett. B **589** (2004) 118 [arXiv:nucl-th/0402027].
21. S. R. Beane, V. Bernard, E. Epelbaum, U.-G. Meißner and D. R. Phillips, Nucl. Phys. A **720** (2003) 399 [arXiv:hep-ph/0206219].
22. R. Chand and R. H. Dalitz, Annals Phys. **20** (1962) 1.
23. J. A. Oller and U.-G. Meißner, Phys. Lett. B **500** (2001) 263 [arXiv:hep-ph/0011146].
24. B. Borasoy, R. Nißler and W. Weise, Phys. Rev. Lett. **94** (2005) 213401 [arXiv:hep-ph/0410305]; Eur. Phys. J. A **25** (2005) 79 [arXiv:hep-ph/0505239].
25. J. A. Oller, J. Prades and M. Verbeni, Phys. Rev. Lett. **95** (2005) 172502 [arXiv:hep-ph/0508081].
26. A. D. Martin, Nucl. Phys. B **179** (1981) 33.
27. B. Borasoy, U.-G. Meißner and R. Nißler, Phys. Rev. C **74** (2006) 055201 [arXiv:hep-ph/0606108].
28. E. Epelbaum, W. Glöckle and U.-G. Meißner, Eur. Phys. J. A **19** (2004) 401 [arXiv:nucl-th/0308010]; E. Epelbaum, private communication.
29. A. Sibirtsev, M. Büscher, V. Y. Grishina, C. Hanhart, L. A. Kondratyuk, S. Krewald and U.-G. Meißner, Phys. Lett. B **601** (2004) 132 [arXiv:nucl-th/0406061].
30. V. Y. Grishina, L. A. Kondratyuk, M. Büscher and W. Cassing, Eur. Phys. J. A **21** (2004) 507 [arXiv:nucl-th/0402093].