

Prospects for New Physics in Kaon Decays

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Abstract. In these proceedings, we give an overview of the current status and future prospects of new physics searches using rare kaon decays. We discuss several promising observables whose experimental or theoretical precisions can be reasonably—and in several cases substantially—reduced in the near future. We emphasize the complimentarity of charged and neutral kaon programs in their ability to both probe a wide array of new physics scenarios and test qualitatively different physics.

1 Introduction

In the history of particle physics—flavor physics in particular—kaons have played a critical role in the search for and discovery of new physics (NP). The study of kaon physics led to the discovery of some of the most interesting aspects of the standard model (SM), including quark mixing via the CKM matrix, suppression of flavor-changing neutral currents (FCNCs) by the GIM mechanism, and CP -violation by weak interactions, not to mention the direct or indirect discovery of nearly a quarter of the standard model particles.

Still today, kaons provide a useful tool in the search for beyond standard model (BSM) physics, complementary to other mesons such as the B and D . This is for two main reasons:

1. *CKM Suppression.* Since the kaon is composed of only light valence quarks, FCNC kaon decays feature significantly stronger suppression than that of their heavy-quark counterparts. Neglecting mass effects, the $s \rightarrow d$ transition in FCNC kaon decays, mediated by the weak interaction, is equivalent to the corresponding $b \rightarrow d(s)$ transition in $B_{(s)}$ mesons aside from CKM factors: $|V_{ts}^* V_{td}| \sim \lambda^5$ for the former and $|V_{tb}^* V_{td(s)}| \sim \lambda^{3(2)}$ for the latter, with the Wolfenstein parameter, $\lambda \approx 0.22$.
2. *Decay Width Suppression.* The width of weak meson decays generically scale as $\Gamma_X^{\text{weak}} \sim M_X^5/M_W^4$, where X is the meson and M_W is the mass of the W^\pm -boson. Therefore, weak kaon decays are parametrically substantially suppressed compared to similar D and B decays, again allowing for a relative enhancement of NP effects. This is particularly relevant for light NP, where the branching ratios will generally scale as $\mathcal{B}_{\text{LNP}} \sim (M_W/M_X)^n$ with n depending on the particular model in question.

The purpose of these proceedings is to outline the future prospects of using rare kaon decays for NP searches. The document is strucured as follows: in Sec. 2, we discuss several promising observables that can be expected to see reasonable improvement on either the

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experimental or theoretical side. In Sec. 3, we exemplify the future discovery potential of rare kaon decays in the case where the NP is heavier than the electroweak scale, focusing on $K \rightarrow \pi \bar{\nu}\nu$ decays. In Sec. 4, we give a brief overview of possible light new physics searches that can be performed using charged and neutral kaons. We present our conclusions in Sec. 5.

2 Promising Observables

In this section, we give a general outline of the theoretical and experimental status of some of the most promising kaon observables relevant to the search for physics beyond the standard model. We emphasize that this is by no means an exhaustive list of interesting kaon decay modes, but instead a collection of observables which can effectively probe BSM physics and whose experimental determinations or theoretical predictions can feasibly be improved in the near future.

2.1 $K \rightarrow \pi \bar{\nu}\nu$

The two complimentary decays, $K_L \rightarrow \pi^0 \bar{\nu}\nu$ and $K^+ \rightarrow \pi^+ \bar{\nu}\nu$, provide extremely theoretically clean tests of the SM due to the fact that they are generated almost exclusively by short-distance, electroweak-scale physics. Below the electroweak scale, this decay is described by the effective Hamiltonian [1, 2]

$$\mathcal{H}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} \frac{\alpha}{2\pi \sin^2 \Theta_W} \sum_{\ell=e,\mu,\tau} (\lambda_c X_c^\ell + \lambda_t X_t)(\bar{s}_L \gamma^\mu d_L)(\bar{\nu}_L^\ell \gamma_\mu \nu_L^\ell) + \text{h.c.}, \quad (1)$$

where X_c^ℓ and X_t denote the Wilson coefficients corresponding to the charm- and top-quark contributions, respectively and $\lambda_i = V_{is}^* V_{id}$ is a combination of CKM matrix elements.

The neutral mode, $K_L \rightarrow \pi^0 \bar{\nu}\nu$, is nearly pure CP -violating, depending to a high degree of accuracy only on the imaginary part of Eq. (1), and is therefore entirely dominated by short-distance physics at the electroweak scale. The hadronic matrix element is extracted from the isospin-related $K^+ \rightarrow \pi^0 e^+ \nu$ branching ratio [3] and contributions from indirect CP -violation are also taken into account [4]. Including next-to-leading order QCD and electroweak corrections to X_t , the predicted SM branching ratio is [5]

$$\mathcal{B}(K_L \rightarrow \pi^0 \bar{\nu}\nu)_{\text{SM}} = (2.59(6)_{\text{SD}}(2)_{\text{LD}}(28)_{\text{param}}) \times 10^{-11}, \quad (2)$$

where the short-distance uncertainty (SD) is computed from the residual renormalization scale variation of X_t , the long-distance uncertainty (LD) arises from the extraction of the hadronic matrix element from the K_{l3} decay, and the parametric uncertainty (param) is primarily due to CKM matrix elements with a small contribution from the uncertainty in the top-quark mass.

Although the neutral decay mode contains nearly no long-distance contaminations, it is very challenging to measure experimentally due to its fully neutral initial and final states. Currently, the best upper bound on this decay is set by the KOTO experiment at J-PARC with a preliminary value of [6]

$$\mathcal{B}(K_L \rightarrow \pi^0 \bar{\nu}\nu) < 2.0 \times 10^{-9} \text{ (90\% CL)}, \quad (3)$$

a $> 30\%$ improvement on the previous upper bound in Ref. [7]. The future successor of the KOTO experiment, KOTO-II, has already been proposed [8] and could be expected to reach $\sim 25\%$ sensitivities given a SM branching ratio, corresponding to a 5σ discovery of the $K_L \rightarrow \pi^0 \bar{\nu}\nu$ decay.

The charged mode, $K^+ \rightarrow \pi^+ \bar{\nu}\nu$, unlike the neutral mode, obtains non-negligible contributions from the charm-quark due to the fact that it depends on the real part of the effective Hamiltonian in Eq. (1). In particular, despite being parametrically suppressed by a factor of m_c^2/M_W^2 , the real part of the charm contribution is much less CKM suppressed: $\text{Re}\lambda_c \sim \lambda$ versus $\text{Re}\lambda_t \sim \text{Im}\lambda_t \sim \lambda^5$ (the imaginary part of the charm contribution is still negligible since $\text{Im}\lambda_c \sim \lambda^5$). These contributions are generated via bilocal charged-current operator insertions, and are known up to next-to-next-to-leading order in QCD [9] and next-to-leading order in QED [10]. Additional non-perturbative long-distance effects from charm- and up-quarks have been estimated in Ref. [11] and amount to a roughly 10% correction to the charm contribution. Preliminary work has also been done recently to compute these effects on the lattice [12]. Altogether, the SM prediction for the charged mode decay is [5]

$$\mathcal{B}(K^+ \rightarrow \pi^+ \bar{\nu}\nu)_{\text{SM}} = (7.73(16)_{\text{SD}}(25)_{\text{LD}}(54)_{\text{param}}) \times 10^{-11}. \quad (4)$$

The current best measurement of the $K^+ \rightarrow \pi^+ \bar{\nu}\nu$ decay has been performed by the NA62 collaboration at CERN with the full Run 1 dataset [13]

$$\mathcal{B}(K^+ \rightarrow \pi^+ \bar{\nu}\nu) = (10.6^{+4.0}_{-3.4} \Big|_{\text{stat.}} \pm 0.6 \Big|_{\text{sys.}}) \times 10^{-11}, \quad (5)$$

where “stat.” and “sys.” refer to statistical and systematic errors, respectively. Data-taking is ongoing for NA62 Run 2 and shows a significant improvement over Run 1, with the number of signals observed in 2022 alone comparable to the full Run 1 dataset [14]. Given a similar beam delivery up to long shutdown 3, the charged-mode branching ratio can be feasibly measured to $\sim 15\%$ precision [14].

2.2 ϵ_K

The measure of indirect CP -violation in the neutral kaon system, ϵ_K , is the only observable considered here where the experimental determination enjoys a higher precision than the theoretical prediction. Experimentally, ϵ_K has been measured to per-mil precision [15]

$$|\epsilon_K| = (2.228 \pm 0.011) \times 10^{-3}. \quad (6)$$

The theoretical calculation has received substantial improvements in the last few years, with uncertainty being reduced to $O(1\%)$, aside from parametric uncertainties [5, 16]. On the perturbative side, the top-quark contribution to ϵ_K has been computed up to next-to-leading order in QCD and electroweak interactions [5, 16, 17], and the charm-quark contribution is known to next-to-next-to-leading order in QCD and next-to-leading order in QED [5, 16, 18]. Furthermore, the bag parameter, \hat{B}_K has been computed on the lattice to few-percent-level precision [19]. Incorporating the charm-mass power corrections calculated in Ref. [20] yields the most up-to-date theoretical prediction

$$|\epsilon_K|_{\text{SM}} = (2.170(65)_{\text{pert}}(76)_{\text{nonpert}}(153)_{\text{param}}) \times 10^{-3}, \quad (7)$$

where the errors refer to the residual renormalization scale dependence in the perturbative calculation, the uncertainty arising from the computation of the hadronic matrix element, and parametric uncertainty (predominantly from V_{cb}), respectively. All three of these sources of error can reasonably be reduced in the future with calculations of the three-loop QCD correction to the top-quark contribution, updated lattice results and next-to-leading order matching between RI/SMOM and $\overline{\text{MS}}$ schemes, and improved determinations of V_{cb} from modern B -factories.

2.3 $K^0 \rightarrow \mu^+ \mu^-$

Unlike the previously discussed neutral kaon decays, the $K^0 \rightarrow \mu^+ \mu^-$ decay receives large long-distance contaminations from the CP -even parts of the decay. However, as discussed in Ref. [21], the interference effects of these decays arise solely from short-distance physics and are therefore theoretically very clean. This point was further expanded upon in Ref. [22] by studying only the nearly pure CP -violating decay of the K_S into the $\ell = 0$ angular momentum state of the dimuon pair. Experimentally, it is very challenging to discriminate between the $\ell = 0$ and $\ell = 1$ states, however it was shown in Ref. [22] that, under reasonable assumptions this decay can instead be extracted from interference effects as

$$\mathcal{B}(K_S \rightarrow (\mu^+ \mu^-)_{\ell=0}) = \mathcal{D}_F \mathcal{B}(K_L \rightarrow \mu^+ \mu^-) \frac{\tau_S}{\tau_L} \left(\frac{C_{\text{int}}}{C_L} \right)^2. \quad (8)$$

The three assumptions leading to Eq. (8) are further discussed and justified in Ref. [22]. In Eq. (8), τ_S and τ_L are the lifetimes of the K_S and K_L , respectively, \mathcal{D}_F is a dilution factor, and C_{int} and C_L are coefficients which can be extracted from the time-dependent decay width of an asymmetric $K^0 - \bar{K}^0$ beam.

While the branching ratio in Eq. (8) is in principle a theoretically clean observable, long-distance contaminations from indirect CP -violation are not fully negligible, as pointed out in Ref. [23]. This is due to an enhancement from the $K_L \rightarrow (\mu^+ \mu^-)_{\ell=0}$ amplitude, leading to a shift in Eq. (8) of at most a few percent, depending on the relative phase of the K_L and K_S amplitudes. A larger, practical difficulty is the fact that one requires an asymmetric $K^0 - \bar{K}^0$ beam to observe the interference effects, where a smaller asymmetry leads to a larger dilution factor, \mathcal{D}_F , in Eq. (8). Current experiments which are well-equipped for high-luminosity neutral kaon physics such as KOTO and LHCb either use nearly pure K_L beams or produce equal amounts of K^0 and \bar{K}^0 . Several methods of circumventing this issue which could be incorporated into future experiments such as KOTO-II are presented in Ref. [22].

2.4 $K \rightarrow \pi \bar{\ell} \ell$

Similar to the $K^0 \rightarrow \mu^+ \mu^-$ decay, the $K \rightarrow \pi \bar{\ell} \ell$ decay receives large contributions from long-distance physics, namely from non-local operator insertions in chiral perturbation theory (ChPT). Such contributions depend on form factors which can only be theoretically calculated on the lattice or extracted from experimental data. Preliminary lattice studies have been performed for the computation of these form factors and are expected to reach uncertainties $\lesssim 10\%$ within the next decade (see Refs. [24–26] for more details).

Without a theoretical determination of the ChPT form factors, the prediction for the charged decay, $K^\pm \rightarrow \pi^\pm \bar{\ell} \ell$, suffers from large long-distance uncertainties. However, the electron-channel and muon-channel form factors measured by the E865 [27] and NA62 [28] collaborations, respectively, can be compared as a test of lepton flavor universality violation in the $K^\pm \rightarrow \pi^\pm \bar{\ell} \ell$ decay [29]

$$\text{LFUV}(a_+^{\mu\mu} - a_+^{ee}) = -0.014 \pm 0.016. \quad (9)$$

On the other hand, in the case of the neutral decay, both the $K_S \rightarrow \pi^0 \bar{\ell} \ell$ and $K_L \rightarrow \pi^0 \bar{\ell} \ell$ decays depend on the same ChPT form factor. This form factor can then be extracted from the measured, mostly CP -conserving K_S decay in order to refine the theoretical prediction for the mostly CP -violating K_L decay, which is highly sensitive to BSM physics. The current best determination of $\mathcal{B}(K_S \rightarrow \pi^0 \bar{\ell} \ell)$ comes from the NA48/1 collaboration [30] with an $O(50\%)$ statistical uncertainty, a result which can be significantly improved upon by LHCb [31].

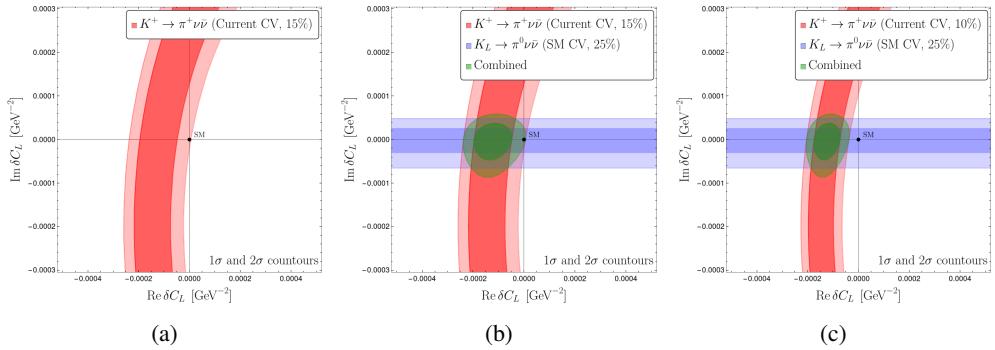


Figure 1: Prospective favored regions for the real and imaginary parts of the new physics Wilson coefficient in Eq. (10) given future measurements of $\mathcal{B}(K \rightarrow \pi \bar{\nu} \nu)$ if the current NA62 central value, Eq (5), remains unchanged. In all plots, 1σ and 2σ contours are shown. Figure (a) shows only the projected 15% sensitivity of NA62 after Run 2. Figure (b) also shows a 15% sensitivity NA62 result (red) with the projected final measurement of the neutral mode with sensitivity of 25% from KOTO-II (blue) along with combined favored region (green). Figure (c) shows the same as figure (b) but instead with the optimistic final NA62 sensitivity of 10% in red.

3 Heavy New Physics

In this section, we give a general sketch of the prospective discovery reach of current/future kaon experiments, assuming that new physics is heavier than the electroweak scale. Of the observables presented in Sec. 2, we focus on $K \rightarrow \pi \bar{\nu} \nu$ since significant experimental progress can be made at NA62 and potentially KOTO-II.

The sensitivity of an observable to heavy new physics can be analyzed in a model-independent¹ way by allowing for new physics corrections to the SM Wilson coefficients. In particular, we consider the effective Hamiltonian

$$\mathcal{H}_{\text{eff}} = (C_L^{\text{SM}} + \delta C_L)(\bar{s}_L \gamma^\mu d_L)(\bar{\nu}_L \gamma_\mu \nu_L) + \text{h.c.}, \quad (10)$$

where the SM contribution, C_L^{SM} , is given in Eq. (1). As discussed in Sec. 1, the heavy GIM/CKM suppression of FCNC kaon decays make high-scale and/or weakly coupled new physics more competitive with the SM observables. This is exemplified by the SM Wilson coefficient²

$$C_L^{\text{SM}} \sim \frac{4G_F}{\sqrt{2}} \frac{\alpha}{2\pi \sin^2 \Theta_W} \lambda_t \approx -(130 \text{ TeV})^{-2} + i(200 \text{ TeV})^{-2}. \quad (11)$$

Currently, the charged-mode decay is consistent with the SM prediction at 1σ , placing upper bounds on the real and imaginary parts of the new physics Wilson coefficient

$$\text{NA62 (Current):} \quad |\text{Re} \delta C_L| \lesssim (120 \text{ TeV})^{-2}, \quad |\text{Im} \delta C_L| \lesssim (70 \text{ TeV})^{-2}, \quad (12)$$

¹In practice, to avoid a huge proliferation of relevant effective operators one typically still makes model-dependent assumptions, e.g. minimal flavor violation, lepton flavor universality, tree-level third-generation couplings, etc.

²We emphasize that it is important to not conflate the values of inverse Wilson coefficients with the actual energy scales being probed: the scale of physics contributing to C_L^{SM} is clearly the electroweak scale, but the extremely small couplings involved in the FCNC process suppress the numerical value of the Wilson coefficient in Eq. (10). In short, bounds on δC_L do not directly correspond to bounds on possible NP scales, but instead a combination of NP scales with NP couplings to the SM.

at the 90% confidence level. The neutral decay is only sensitive to the imaginary part of the Wilson coefficient and the KOTO upper bound gives

$$\text{KOTO (Current): } |\text{Im}\delta C_L| \lesssim (50 \text{ TeV})^{-2}, \quad (13)$$

again at 90% confidence.

Including only future NA62 measurements with anticipated 15% precision and assuming a SM-like branching ratio gives a substantial improvement on the bound of $\text{Re}\delta C_L$ and a marginal improvement on $\text{Im}\delta C_L$

$$\text{NA62 (15\%): } |\text{Re}\delta C_L| \lesssim (225 \text{ TeV})^{-2}, \quad |\text{Im}\delta C_L| \lesssim (100 \text{ TeV})^{-2}. \quad (14)$$

If instead the central value of the branching ratio remains unchanged from the current NA62 value in Eq. (5), the result is still compatible with the SM prediction at the 2σ level, as shown in Fig. 1a.

If we now also consider a measurement of the neutral mode at KOTO-II with 25% precision (we always consider a SM branching ratio for the neutral mode), the combined bounds with the prospective NA62 measurement are

$$\text{NA62 (15\%) + KOTO-II (25\%): } |\text{Re}\delta C_L| \lesssim (240 \text{ TeV})^{-2}, \quad |\text{Im}\delta C_L| \lesssim (280 \text{ TeV})^{-2}, \quad (15)$$

for a SM value of the charged mode branching ratio. Again, for an unchanged NA62 central value, the results are compatible with the SM at the 2σ level (Fig. 1b).

Finally, we consider the highly optimistic case where the final NA62 sensitivity is reduced to 10% if allowed additional data-taking due to the delay in long shutdown 3 as well as 25% precision from KOTO-II. Assuming SM-like branching ratios, the bound on the real part of the Wilson coefficient is improved by $\sim 20\%$

$$\text{NA62 (10\%) + KOTO-II (25\%): } |\text{Re}\delta C_L| \lesssim (290 \text{ TeV})^{-2}, \quad |\text{Im}\delta C_L| \lesssim (280 \text{ TeV})^{-2}, \quad (16)$$

and a $\sim 3\sigma$ tension is observed in the case of an unchanged NA62 central value (Fig. 1c).

From this, it is clear that substantial progress will be made in the heavy new physics reach of rare kaon decays in the near future; even the most pessimistic scenario of Eq. (14) shows a marked improvement over the current best limits in Eq. (12). For recent, more sophisticated analyses including other weak effective theory operators as well as complimentary kaon observables, we defer to Refs. [6, 29, 32] and references therein.

4 Light New Physics

Unlike heavy new physics, BSM scenarios which include new light degrees of freedom are much more difficult to approach in a model-independent way, leading to a plethora of models which can explain a wide array of problems in the SM (for a recent review, see Ref. [33]). In Ref. [34], eleven benchmark models for light dark-sector portals are presented with interesting parameter spaces that can be tested in current or future experiments, many of which can be effectively probed with kaons.

With the advancements in measurements of $K \rightarrow \pi \bar{\nu}\nu$ over the past decade, most of the kaon bounds on these benchmark models come from $K \rightarrow \pi X_{\text{inv}}$ searches, where X_{inv} is an invisible state which escapes the detector. Due to the generalized Grossman-Nir bound [35]

$$\mathcal{B}(K_L \rightarrow \pi^0 X_{\text{inv}}) \leq 4.3 \mathcal{B}(K^+ \rightarrow \pi^+ X_{\text{inv}}), \quad (17)$$

stronger constraints are nearly always placed by K^+ experiments versus corresponding K_L experiments. Additionally, charged kaon decays can also probe heavy neutral lepton models through measurements of $\mathcal{B}(K^+ \rightarrow \ell^+ \nu)$, where $\ell = e, \mu$. Currently, leading bounds on seven of the eleven benchmark models given in Ref. [34] have already been placed by charged kaon experiments in interesting regions of parameter space [13, 33, 36–39], making charged kaons powerful probes for a large variety of light new physics models.

Furthermore, the assumptions leading to Eq. (17) can be violated in specific models, leading to enhancements of the neutral decay mode over the charged and almost ubiquitously require light new physics (see e.g. Refs. [40–43]). Therefore, the importance of a strong K_L program should not be understated solely based on Eq. (17).

5 Conclusions

When discussing future prospects for discovering new physics, it is important to keep in mind that we do not control the nature or the energy scales of new physics. It is therefore beneficial to study systems which can test a large variety of BSM scenarios. Due to their strong CKM and GIM suppressions within the SM, rare kaon decays cast a wide net in the search for new physics, efficiently testing heavy and light new physics models while simultaneously giving insight into some of the most theoretically well-understood hadronic decays in the SM. Furthermore, charged and neutral kaon physics programs provide highly complimentary information, often probing qualitatively different physics. Despite the recent cancellation of the HIKE program at CERN, the remainder of Run 2 at NA62 as well as the potential of KOTO-II at J-PARC still paint an optimistic picture for future BSM searches using kaons.

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