Comparison of different models for gravitational and electromagnetic radiation of binary neutron star mergers

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Abstract. The neutron star binary merger is one of the most energetic phenomena in our Universe. Based on the calculation of gravitational radiation (GR) with the separate consideration of revolution and stellar rotation, and electromagnetic radiation (ER) with unipolar induction DC circuit model and magnetic dipole model, the results are compared to analyse the extent to which the stellar rotation will affect the total gravitational radiation power. Besides, the relationships between radiation power and the related parameters (e.g., orbital radius and stellar mass) are investigated. Furthermore, the feasibility of different types of binary star merger as the progenitor of fast radio bursts and gamma-ray bursts were studied based on the two models of ER, obtaining that binary neutron star and neutron star-white dwarf systems are among the possible progenitors. Finally, the radiation power of both GR and ER are compared under the same conditions. According to the results, the energy dissipation of the system is dominated by gravitational radiation. These results shed lights on further studies on the radiation processes of binary neutron star mergers and binary star systems.

1. Introduction

1.1. Overview of Neutron Stars

A neutron star is a special celestial body with extreme physical conditions in the Universe. Specifically, the interior of it is almost entirely composed of degenerate neutrons, and it has extremely high density with strong gravitational and magnetic fields around. These extreme physical conditions make the neutron star natural laboratories to investigate the laws of physics under extreme conditions, which can provide a lot of valuable information for theoretical research, i.e., neutron stars have attracted much attention of the scientific community.

The study of neutron stars can be traced back to the 1930s. Less than two years after Chadwick discovered neutrons, Baade and Zwicky proposed that supernova explosions might produce an extreme celestial body composed of dense neutrons and named them neutron stars [1]. Then in 1967, Hewish and Bell detected periodic radio pulse signals from PSR B1919+21, which was the first time that a neutron star was observed [2]. This periodic radiation is originated from the azimuth of the beam-shaped radio radiation at its poles, changing with the rapid rotation of neutron star.

It is generally believed that a star with an initial mass greater than eight times the mass of the sun ($8M_\odot$) may collapse into a neutron star after its nuclear fuel is exhausted. Specifically, if the core mass exceeds the Chandrasekhar limit during the collapse, the electronic degeneracy pressure is not enough

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to support the core, i.e., the core will collapse further. Electrons and protons will react as $p + e^- \rightarrow n + v$ to become neutrons and neutrinos that will completely escape later. Furthermore, the combined effect of strong interaction and neutron degeneracy pressure stops the collapse process and forms a stable star [3].

1.2. Neutron star binary mergers

A binary star system that meets certain conditions can form a binary neutron star system after a complex and long evolution, as schematically shown in Figure 1. The evolution of a binary star system is divided into three stages: inspiral, merger, and ringdown. This is a rather complicated process, partly because four basic interactions are involved. The binary neutron star will produce a variety of energy radiation during the merge phase.

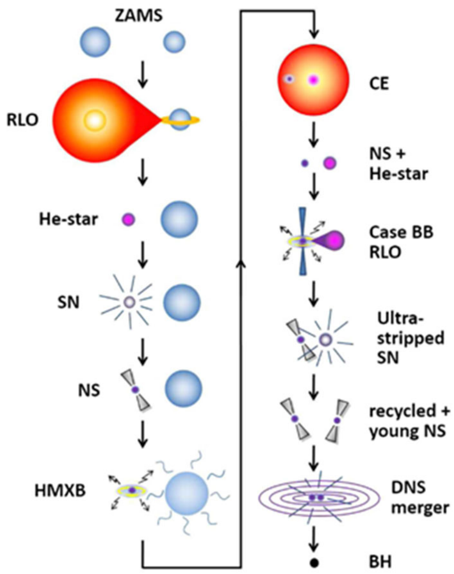


Figure 1. Illustration of the formation of a double neutron star system, from reference [4]

Firstly, according to General Relativity, a pair of objects rotating with each other will cause regular oscillations in the surrounding space-time. This oscillation carries energy and can propagate infinitely, i.e., a gravitational wave. Due to the large mass of the binary neutron star system, the generated gravitational wave signal is extremely strong. Therefore, in 2017, LIGO and Virgo have detected the gravitational wave generated by the binary neutron star merger, GW170817 [5].

Secondly, the binary neutron stars merger will produce a variety of electromagnetic radiation, the most well-known types include gamma-ray bursts (GRB) and afterglow radiation. GRB was accidentally discovered by the Vela satellite of the US in 1967 [6]. It is the highest-energy electromagnetic event known to mankind, which is a typical gamma storm releases energy in a few seconds equivalent to the total energy released by the sun during its entire life cycle [7]. Afterglow radiation are caused by the synchrotron radiation of shock waves accelerated electrons in terms of the interaction of GRB jets with the interstellar medium. In addition, in 2007, Lorimer discovered a special kind of high-energy electromagnetic radiation and named it as Fast Ray Bursts (FRB) [8]. Currently, there is no final conclusion about the mechanism of GRB and FRB, nor the progenitors of these phenomena.

Thirdly, the Kilonova radiation, the mechanism of which is that a large amount of neutron-rich material will be ejected when neutron stars merge. In addition, heavy elements are synthesized in these ejections through fast neutron capture reactions. Subsequently, they undergo radioactive decay to heat the ejection, thus generating radiation [9]. The product of the binary neutron star merger may form supermassive neutron stars (SMNS), black holes, or some theoretical celestial bodies (e.g., quark stars and millisecond magnetars) owing to the different initial masses of the binary stars.

1.3. Arrangement for this article

This paper focuses on electromagnetic and gravitational radiation of neutron star binary mergers. The former includes gamma-ray bursts and fast radio bursts, and the latter involves gravitational wave radiation. The rest part of the paper is organized as following. The Sec. 2 will discuss all four models used in detail with corresponding calculation formula. In Secs. 3 and 4, the calculation results of different models of GR and ER are compared separately, and a number of relevant issues as well as the limitations of the models are discussed. Finally, Sec. 5 gives a brief summary of this paper and offers the outlook for the future work.

2. Analytical models

2.1. Gravitational radiation (GR)

The pulsar binary stars will radiate gravitational energy in the form of gravitational waves during inspiral. This subsection introduces two models for investigating gravitational radiation, where one considers the energy dissipation caused by the revolution of binary stars while the other considers the gravitational radiation caused by the rotation of neutron stars on the basis of the former. On this basis, the latter model can be regarded as an extension and supplement of the former one.

2.1.1. Gravitational radiation powered by binary star revolution. The motion of the binary star satisfies the Einstein field equation

$$R^{\mu v} - \frac{1}{2} g^{\mu v} R = k T^{\mu v} \tag{1}$$

where $R^{\mu v}$ is the Rich tensor, $g^{\mu v}$ is the metric tensor, and $T^{\mu v}$ is the energy - momentum tensor of the matter field. Taking the weak field linear approximation to the field equation (i.e., ignoring the nonlinear effect at the field source) and substituting the mass quadrupole $Q_{kj} = \int \mu x_k x_j dV$ (μ is the particle mass), the gravitational radiation power can be derived as [2]

$$P_{gra} = -\frac{dE}{dt} = \frac{G}{5c^5}\left(\dddot{Q}_{ij}{}^2 - \frac{1}{3}\dddot{Q}_{kk}{}^2\right) \tag{2}$$

Subsequently, Newtonian dynamics will be used to solve the motion of the binary star. This approximation is reasonable since the strong field at the field source is not considered. Supposing that the binary stars revolve around each other in the x-y plane, and ignoring the effect of their rotation and internal structure and shape, the binary stars can be regarded as two mass points with masses m1 and m2, respectively. Taking the centroid coordinate system, one obtains $d_1 = \frac{m_2}{m_1+m_2} d$, $d_2 = \frac{m_1}{m_1+m_2} d$, where $d = d_1 + d_2$ is the binary distance. Therefore, according to Newton's law of gravitation, the orbital equation of the binary star motion can be easily obtained as: $d = \frac{a(1-e^2)}{1+ecos\varphi}$. Apparently, it is a typical ellipse equation, where a is the semi-major axis of the ellipse, e is the eccentricity, and the angular velocity of the revolution of binary star

$$\dot{\varphi} = \frac{[\mathrm{G}(m_1 + m_2)\mathrm{a}(1-\mathrm{e}^2)]^{1/2}}{\mathrm{d}^2} \tag{3}$$

Since the motion of the double star is limited to the plane, the mass quadrupole moment of the system has only four non-zero components, which can be expressed as:

$$Q_{ij} = \begin{pmatrix} \mu d^2 cos^2\varphi & \mu d^2 sin\varphi cos\varphi \\ \mu d^2 sin\varphi cos\varphi & \mu d^2 sin^2\varphi \end{pmatrix} \tag{4}$$

in which the converted mass has been taken in the formula $\mu = \frac{m_1 m_2}{m_1+m_2}$. Then, the third derivative of the mass quadrupole is calculated and substituted into the angular velocity formular

$$\dddot{Q}_{ij} = \begin{pmatrix} \beta A(\varphi) & -\beta B(\varphi) \\ -\beta B(\varphi) & -\beta C(\varphi) \end{pmatrix} \tag{5}$$

where $\beta = 2Gm_1m_2(\frac{m_1+m_2}{a^5(1-e^2)^5})^{1/2}$, substituting into the radiant power expression, one derives

$$P_{rev} = \frac{8G^4}{15c^5} \frac{m_1^2 m_2^2 (m_1 + m_2)}{a^5 (1 - e^2)^5} (1 + ecos\varphi)^4 [12(1 + ecos\varphi)^2 + e^2 sin^2\varphi] \tag{6}$$

In order to find the average radiated power of a binary star in a revolution period, one ought to first obtain the revolution period according to Kepler's third law

$$\mathrm{T} = \sqrt{\frac{4\pi^2 a^3}{G(m_1 + m_2)}} \tag{7}$$

and one knows

$$< P_{rev} > = -\int \frac{dE}{T} = \frac{(1 - e^2)^{2/3}}{2\pi} \int_0^{2\pi} \frac{P}{(1 + ecos\varphi)^2} d\varphi \tag{8}$$

Substitute Eq. (6), the expression of P is simplified as

$$< P_{rev} > = \frac{32G^4}{5c^5} \frac{m_1^2 m_2^2 (m_1 + m_2)}{a^5} f(e) \tag{9}$$

where $f(e) = \frac{1 + \frac{73}{24}e^2 + \frac{37}{96}e^4}{(1 - e^2)^{\frac{7}{2}}}$ is the correction factor caused by orbital eccentricity. The binary star system will radiate gravitational waves according to this formula. Obviously, the total energy of the system will gradually decrease over time. This directly leads to the reduction of the distance between the binary stars and the shortening of the revolution period, which is the main reason for the inspiral. This part will be studied in detail in Secs. 3 and 4.

2.1.2. Gravitational radiation powered by stellar rotation. In model one, the rotation of the neutron star is neglect because the model ignores the structure and shape of the neutron star and treats it as a mass point. However, in fact, neutron stars are not true spheres, newborn neutron stars generally rotate rapidly attributed to their extremely small radius and conservation of angular momentum, which also radiates gravitational waves. This model mainly takes this part of energy radiation into consideration.

Since the interior of a neutron star is very dense and the difficulty for external conditions to change its shape, it is reasonable to regard it as a rigid body. Supposing the circular frequency of the neutron star's rotation is ω and introduction the following coordinate system x'^{μ} that rotates with the neutron star, its relationship with the laboratory coordinate system x^{μ} is

$$x^{\mu} = \begin{pmatrix} cos\omega t & -sin\omega t & 0 & 0 \\ sin\omega t & cos\omega t & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} x'^{\mu} \tag{10}$$

Taking the ellipsoid of inertia in x'^{μ} system, the moment of inertia can be expressed in the form of a diagonal matrix diag(I_{11}, I_{22}, I_{33}). Set x_1', x_2', x_3' for the three principal axes of inertia ellipsoid, the mass quadrupole moment

$$Q_{11}(t) = \left(\int \mu x_1'^2 dV'\right) cos^2\omega t + \left(\int \mu x_2'^2 dV'\right) sin^2\omega t = I_{11} cos^2\omega t + I_{22} sin^2\omega t \tag{11}$$

Similarly, the other components of the quadrupole can be obtained, and the matrix of the mass quadrupole moment is

$$Q_{ij}(t) = \begin{pmatrix} \frac{1}{2}[(I_{11} + I_{22}) + (I_{11} - I_{22})cos2\omega t] & \frac{1}{2}(I_{11} - I_{22})sin2\omega t & 0 \\ \frac{1}{2}(I_{11} - I_{22})sin2\omega t & \frac{1}{2}[(I_{11} + I_{22}) + (I_{11} - I_{22})cos2\omega t] & 0 \\ 0 & 0 & I_{33} \end{pmatrix} \tag{12}$$

Introducing the equatorial ellipticity $\delta = \frac{I_{11} - I_{22}}{I}$, $I = I_{11} + I_{22}$, the energy radiation power produced by the rotation of the neutron star can be obtained from the Eq. (2) and (12) as

$$P_{rot} = \frac{G}{5c^5}\left(\dddot{Q}_{ij}^{\,2} - \frac{1}{3}\dddot{Q}_{kk}^{\,2}\right) = \frac{32}{5}\frac{G}{c^5}\omega^6 I^2 \delta^2 \tag{13}$$

Adding the subscripts 1 and 2 to the two neutron stars respectively, one derives the total power of the gravitational radiation generated by binary star system due to their rotation

$$P = \sum P_{rot} = \frac{32}{5}\frac{G}{c^5}\sum_{i=1}^{2}\omega_i^6 I_i^2 \delta_i^2 \tag{14}$$

2.2. Electromagnetic radiation (ER)

2.2.1. Unipolar induction model. The Unipolar induction DC circuit model was originally proposed by Goldreich and others during the study of the electromagnetic radiation of Jupiter-Io system. Since the model is concise with clear physical images, it is widely used in astrophysics researches. Figure 2 shows the fundamental ideas of this model.

The premise of applying this model is that there must be a magnetosphere with a strong magnetic field around the binary star. Since all existing observations and theoretical studies have shown that there is a very strong magnetic field on the surface of a neutron star (10^4~10^{11} T), the premise is naturally satisfied. In this way, when the neutron star revolves in the magnetosphere, it will cut the lines of magnetic force and generate the induced electromotive force (EMF). Setting the magnetic field strength in the magnetosphere as B and in terms of Faraday's law of electromagnetic, one obtains

$$E = \int (v \times B) \cdot dl\, P_{rot} \cong 2vBR_c \tag{15}$$

It is easy to know that the speed of a star's motion is $v = (\omega - \omega_s) \times \vec{a} = \Delta\omega \times \vec{a} \cong a\Delta\omega$, where ω is the revolution angular velocity, ω_s is the rotation angular velocity, and the direction of $\vec{a}$ is from the host star to the companion star. This electromotive force will generate a DC circuit loop in the magnetosphere. Substituting the current expression based on Ohm's law into the above equation, one derives

$$I = \frac{E}{R_{tot}} \cong \frac{2aBR_c}{R_{tot}}\Delta\omega \tag{16}$$

where R_{tot} is the total resistance of the circuit, the magnetic resistor R_{meg} and resistor of neutron star R_{NS}. Nevertheless, neutron star, as a compact neutron object, its resistor should be much less than the resistance of the magnetic fluid resistance. Hence, it can be considered as $R_{tot} = 2R_{meg}$, where factor '2' is caused by the DC circuit of both the upper and lower layer.

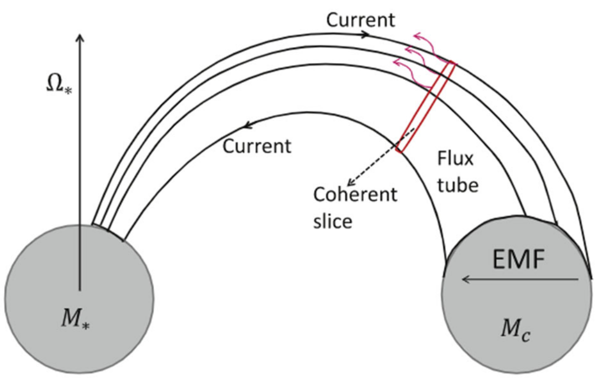


Figure 2. Schematic picture of unipolar inductor model, from reference [10]

In this way, it is obvious that the power lost by the circuit due to internal energy dissipation is

$$P_H = -\frac{dE}{dt} = 2I^2 R_{tot} = \frac{8a^2B^2R_c^2}{R_{tot}}\Delta\omega^2 \tag{17}$$

If the contribution of the companion star's magnetic field to the magnetosphere is neglected, the main star's magnetic field can be regarded as the magnetic field produced by the magnetic dipole. Since a>>R*, the magnetic field strength of the magnetosphere at the companion star is approximately $B = B_*\frac{R_*^3}{a^3}$.

Assuming that the magnetospheric resistance is equal to the impedance of free space, i.e., $R_{meg} = \mu_0 c$, the electromagnetic radiation power of the binary star system is

$$P_H = \frac{v^2}{\mu_0 c} \frac{B_*^2 R_*^6 R_c^2}{a^6} \tag{18}$$

In fact, the actual situation of this process is much more complicated than the above calculation, partly because the rotation of the neutron star will produce a toroidal field. As a consequence, it will interfere with the above-mentioned polar field and make the loop unstable. Lai did a detailed study on this and suggested using a quasi-cyclic circuit to describe this process. He proposed that the increase of the toroidal field will lead the circuit to be cut off, and magnetic reconnection will occur to release the magnetic energy, and then the circuit will be reconnected. Magnetic energy is continuously released through this cycle. Based on the above considerations, Lai obtains the energy dissipation power as [11]

$$\dot{E}_{diss} = -T\Delta\Omega = \zeta_\varphi \Delta\Omega \frac{\mu^2 R_c^2}{2a^5} = \left(\frac{v_{rel}}{c}\right)^2 \frac{B_*^2 R_*^6 R_c^2}{\pi a^6} \tag{19}$$

After converting its unit system to UI, the result is numerically equal to Eq. (18). Furthermore, if it is assumed that the dissipated energy is uniformly radiated in all directions in the form of electromagnetic waves, i.e., spherical waves are radiated by the field source, the Poynting vector at the position R from the field source is

$$\bar{S} = \frac{P_H}{4\pi R^2} = 1.4 \times 10^{43} \left(\frac{B_*}{10^{13} GR}\right)^2 \left(\frac{a}{30km}\right)^{-7} \vec{n}\ ergs^{-1} m^{-2} \tag{20}$$

This is the electromagnetic radiation energy that can be received per unit area at the position R from the field source.

2.2.2. Magnetic dipole model. In this model, it is assumed that binary neutron stars carry a certain amount of charge. In other words, when the binary stars rotate rapidly, a current loop will be generated, which is a standard magnetic dipole model as illustrated in Figure 3. Therefore, the equation of ER of the system can be derived from the results of magnetic dipole radiation.

The current generated from revolution of two neutron starts with charge of q_1, q_2 and binary revolution period T is $I = \frac{q_1+q_2}{T}$. Substituting Eq. (7) into the formula, one obtains

$$I = \frac{\sqrt{G(m_1+m_2)}}{2\pi} \frac{q_1+q_2}{a^{3/2}} \tag{21}$$

The current loop is approximately regarded as a circular ring, and the radius is equal to the semi-major axis a of the ellipse. Therefore, the magnetic moment corresponding to the current loop is

$$\vec{m} = \pi a^2 I = \frac{\sqrt{G(m_1+m_2)}}{2} (q_1+q_2) a^{1/2}\ \vec{n} \tag{22}$$

The electric field intensity E and the magnetic field intensity B generated by the magnetic dipole oscillation at the distance from the field point are [12]:

$$\begin{cases} \vec{E} = \dfrac{1}{4\pi\varepsilon_0 c^3 r} \hat{e}_r \times [\ddot{m}] \\ \vec{B} = \dfrac{\mu_0}{4\pi c^2 r} \hat{e}_r \times (\hat{e}_r \times [\ddot{m}]) \end{cases} \tag{23}$$

Substituting Eq. (22) into the above two formulas, one obtains the current loop Poynting vector of the electromagnetic radiation at the distance of r:

$$\bar{S} = \frac{1}{2\mu_0} Re(\vec{E} \times \vec{B}^*) = \frac{\mu_0^2 |\ddot{m}|^2}{32\pi^2 c^3 R^2} \tag{24}$$

Since the distance of the binary stars will be reduced due to energy radiation, i.e., the radius of the current loop a is a function of t:

$$\bar{S} = \frac{\mu_0^2 G(m_1+m_2)}{128\pi^2 c^3 R^2} \frac{q_1+q_2}{4} a^{-1} (-\frac{1}{2} a^{-1} \frac{da}{dt} + \frac{d^2 a}{dt^2})^2 \tag{25}$$

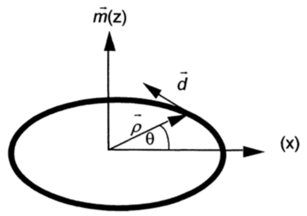


Figure 3. Schematic picture of magnetic dipole model, from reference [13]

It can be seen that in order to obtain the analytical solution of the Poynting vector, one must find the specific form of the function a(t) that exhibits how the distance of binary star evolves with time, which will be discussed in the next section.

3. Comparison of models for GR

3.1. Radiated power of two models

From Eq. (9), the gravitational radiation power generated by the revolution of the double neutron star is $< P_{rev} > = \frac{32G^4}{5c^5}\frac{m_1^2 m_2^2(m_1+m_2)}{a^5}f(e)$. Following four binary neutron star systems are selected and listed in Table 1.

Table 1. Some properties of four binary neutron star systems

Name	M_m ($M_\odot$)	M_c ($M_\odot$)	D(kpc)	e
GW170817	1.46	1.28	40000	≤0.024
GW190425	2.02	1.35	15900	≤0.048
PSR B1913+16	1.441	1.387	6.4	0.617
PSR J0737−3039	1.337	1.250	1.15	0.088

* The data is derived from LIGO Scientific Collaboration

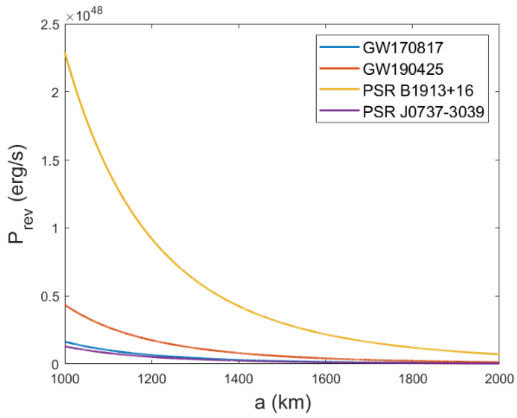


Figure 4. Relation between radiated power of GR and radius of orbit

Figure 4 depicts the relation graph of P_{rev} and a when a ranges from 1000km to 2000km. It can be seen that the radiated energy flow is very high when the binary stars revolve, and the radiant power increases rapidly when the binary stars approach. In order to obtain the relationship between the radiant power of the neutron star's rotation and its mass m, radius r and other parameters, the expression of the moment of inertia needs to be obtained. Although Model 2 assumes that the neutron star is not a sphere, it can be approximately applied to the sphere's moment of inertia formula when calculating the moment of inertia, which is $I = 0.4mr^2$, thus formula (13) turns

$$P_{rot} = \frac{128}{125}\frac{G}{c^5}\omega^6 m^2 r^4 \delta^2 \tag{26}$$

Similarly, four pulsar systems are selected and listed in Table 2.

Table 2. Some properties of four different pulsars

Name	M (M⊙)	R (km)	T_{rot} (ms)	D(kpc)
PSR B1257+12	1.4	10.4	6.22	0.71
PSR J0348+0432	2.01	13	39.1	2.1
PSR J1614-2230	1.908	13	3.15	1.2
PSR J0737-3039A	1.337	~25	22.7	1.15

* The data is derived from Centre de Données astronomiques de Strasbourg (CDS)

From the rotation period of pulsars, the circular frequency can be obtained. Among the above four pulsars, the maximum circular frequency is of order $\omega_{max} = \frac{2\pi}{T_{rot}} = \frac{2\pi}{6.22ms} \sim 10^3$, and the minimum circular frequency is of order $\omega_{min} = \frac{2\pi}{39.1ms} \sim 10^2$. Therefore, the circular frequency is set in the range of 100 s^{-1} to 1000 s^{-1}. Figure 5 presents the relation of radiated power and circular frequency.

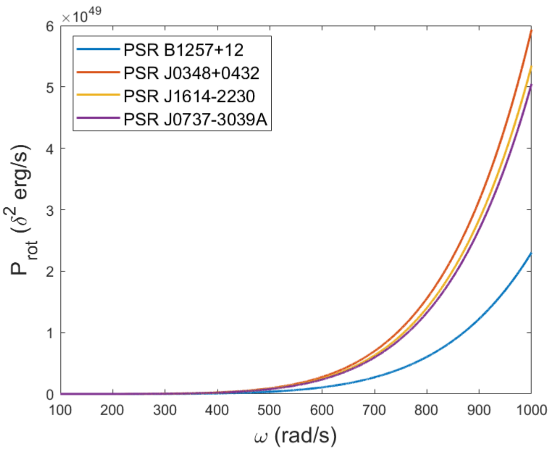


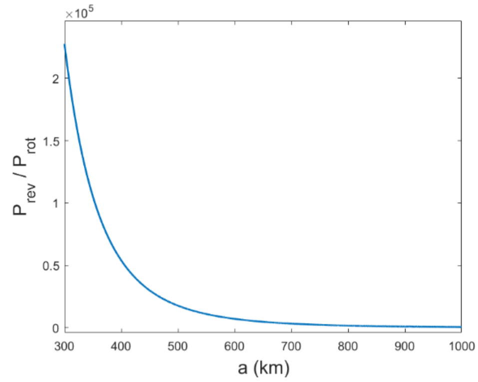
Figure 5. Relation between radiated power of GR and frequency.

Thus, the gravitational radiation caused by the rotation of a neutron star is also quite considerable. Even if the ellipticity of the equator is only 10^{-2}. When the rotation frequency is of order 10^3, the radiation power can still reach the order of 10^{45}. In contrast, for the four selected systems, when the orbit radius of the revolution is 10^3 km, the gravitational radiation caused by the revolution of the binary star is on the order of 10^{46} to 10^{48}. However, the energy will decrease sharply with the shortening of the orbital radius, reaching 10^{56} erg/s when the radius is on the order of 10^2; while the neutron star will reduce its rotation frequency due to energy dissipation after it is formed. The pulsar PSR J1748-2446ad with the shortest rotation period has a circular frequency of only 10^3 s^{-1}. Therefore, for the middle and late stages of the binary star precession, that is, when the orbital radius is less than 10^3 km, the neutron star's rotation relative to the gravitational force generated by the revolution of the binary star radiation is generally extremely small and can be ignored. To further quantify this effect, the PSR J0737-3039 binary star system is chosen as an example. The relevant data is listed in Table 3, and the relationships are shown in Figure 6 (assuming the equatorial ellipticity δ=0.1).

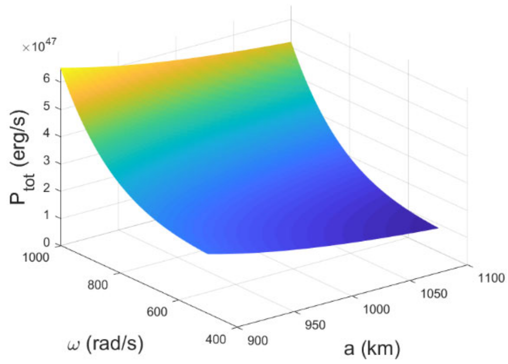
Table 3. Some properties of the PSR J0737-3039 binary star system

Name	M_m (M⊙)	M_c (M⊙)	R (km)	T_{rot} (ms)	e
PSR J0737−3039	1.337	1.250	—	—	0.088
PSR J0737-3039A	1.337	—	~25 [14]	22.7	—
PSR J0737-3039B	1.250	—	~25*	2770	—

* The data is derived from Centre de Données astronomiques de Strasbourg (CDS)



(a)



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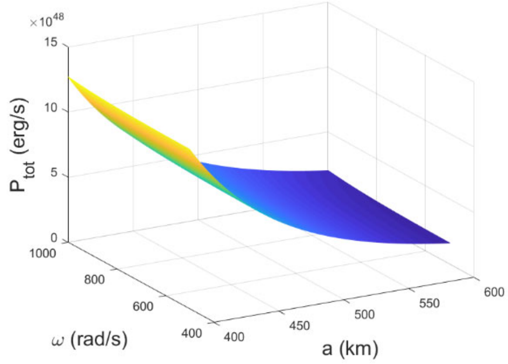

(c)

Figure 6. Figure (a) shows the change of ratio, P_{rev} / P_{rot} with the orbit radius a. Figures (b) and (c) show how the total radiated power is affected by the radius a and the rotation frequency ω. The value of a in Figure (b) is (900km, 1100km), and for Figure (c), a is (400km, 600km). Since the rotation frequency of PSR J0737-3039B is quite small, which has little effect on total radiated power, its rotation frequency change is not considered.

In the early stage of the inspiral (900km<a<1100km), and the rotation frequency of the star is of order 10^3. Their rotation frequency will have a certain impact on the gravitational radiation power. When 400km<a<600km, the gravitational radiation power is already dominated by the orbital radius.

3.2. Evolution of orbit and angular frequency

Next, the precession phenomena caused by energy dissipation in the binary star system are analyzed. According to classical mechanics, the total energy of a binary star system is

$$E = -\frac{Gm_1m_2}{2a} + \frac{1}{5}\sum_{i=1}^{2} m_i\omega_i^2 r_i^2 \tag{27}$$

where the first term on the right side of the equation is the total energy of revolution, and the second term is the rotation energy of the binary star system. The evolution of the orbit is only related to the energy of the revolution. To prove this, it can be assumed that the energy loss caused by the rotation of the neutron star has a proportion of η that causes the orbit radius to change, then

$$\frac{da}{dt} = \frac{2a^2}{Gm_1m_2}\left(\frac{dE}{dt} - \frac{2}{5}\eta\sum_{i=1}^{2} m_i r_i^2 \omega_i \frac{d\omega_i}{dt}\right) \tag{28}$$

In order to find the relationship between the radius of the orbit and time, the angle term should not appear in the equation. Thus, the average radiant power in one revolution period is used. Hence, substituting the equation (9) into equation (28), one derives

$$<\frac{da}{dt}> = -\frac{64G^3}{5c^5}\frac{m_1 m_2(m_1+m_2)}{a^3}f(e) - \frac{4a^2}{5G}\eta\left(\frac{\sum_{i=1}^{2} m_i r_i^2 \omega_i \frac{d\omega_i}{dt}}{m_1 m_2}\right) \quad (29)$$

The first term on the right side of the formula is the radiation caused by the revolution of a binary star, denoted as $<\frac{da}{dt}>_{rev}$; the second term is the radiation caused by the rotation of neutron star, denoted as $<\frac{da}{dt}>_{rot}$. The first term can be directly integrated

$$< a >_{rev} = \{a_0^4 - 4[\frac{G^3}{5c^5}m_1 m_2(m_1+m_2)f(e)t]\}^{1/4} \quad (30)$$

where 't' is an integer multiple of the period T, and 'a' is the radius of the orbit of the binary star after n cycles. According to the above assumptions, the frequency of the neutron star's rotation circle satisfies

$$P_{rot} = I\omega\frac{d\omega}{dt} = -\frac{32}{5}\frac{G}{c^5}(1-\eta)\omega^6 I^2\delta^2 \quad (31)$$

The corresponding solution is

$$\omega(t) = \omega_0[1 + \frac{256}{25}\frac{G}{c^5}(1-\eta)\omega_0^4 m r^2\delta^2 t]^{-1/4} = \omega_0[1 + 2(1-\eta)f(m,r)\omega_0^4 t]^{-1/4} \quad (32)$$

where $f(m,r) = \frac{128}{25}\frac{G}{c^5}mr^2\delta^2$, is the function related to the moment of inertia of the neutron star, and ω0 is the frequency of the rotation circle corresponding to t=0. It can be solved by the above formula $< a >_{rot} \sim t^{-1/4}$, this formula diverges at t=0, i.e., it has no physical meaning. The only solution is to order $\eta = 0$, which proves that the energy lost by the rotation of double stars will not affect the length of the revolution orbit. Therefore, the expression of the rotation angle frequency is

$$\omega(t) = \omega_0[1 + 2f(m,r)\omega_0^4 t]^{-1/4} \quad (33)$$

4. Comparison of models for ER

4.1. Radiated power of two models

According to the unipolar induction model, the power of electromagnetic waves radiated by the binary star system is

$$P_H = \frac{v^2}{\mu_0 c}\frac{B_*^2 R_*^6 R_c^2}{a^6} = \frac{(2\pi)^2}{\mu_0 c T^2}\frac{B_*^2 R_*^6 R_c^2}{a^4} \quad (34)$$

Substituting Eq. (7) into this equation, one finds

$$P_H = \frac{G(m_1+m_2)}{\mu_0 c}\frac{B_*^2 R_*^6 R_c^2}{a^7} \quad (35)$$

Since the unipolar induction model requires the magnetic field of the host star to be much stronger than the magnetic field of the companion star, it is not suitable for the analysis of general binary neutron star systems. However, since the magnetic field of white dwarfs is generally much smaller than that of neutron stars, this model should be suitable for analyzing the neutron star - white dwarf system. The parameters of four binary star system are listed in Table 4.

Table 4. Some properties of different binary star system

Name	M_c ($M_\odot$)	Rc ($R_\odot$)	T_{rev} (s)
4U 1820-303	0.0648	0.0314	685
4U 0513-40	0.0435	0.0358	1020
M15 X-2	0.0325	0.0395	1356
XTE J1751-305	0.0171	0.0489	2544

* The data is derived from Reference [15], and the radius of companion star is calculated according to the radius-mass relations of white dwarfs $R \sim M^{-1/3}$, as shown in Figure 7.

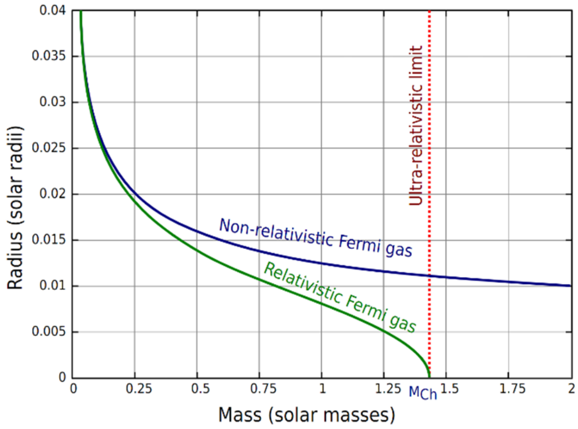

Figure 7. The relationship between radius and mass for a white dwarf star consisting of a cold Fermi gas in two cases whether relativistic effect is considered, plotted by AllenMcC [16].

Assuming that the radius of the neutron star is 10 km and the magnetic field is 106 T, which are reasonable for general neutron stars. Then, the relation graph of the radiation power with the radius of the orbit is exhibited in Figure 8.

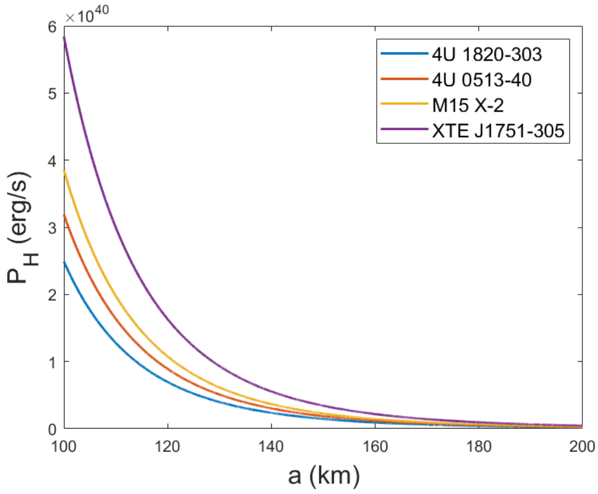

Figure 8. Relation between PH and radius of orbit a based on unipolar induction model.

Therefore, the radiation power in this model is more sensitive to the change of a. When a=100km, the above systems are all of order 10^{40}. Substituting the formula (29) for the evolution of the orbital radius 'a' with 't' into formula (25), the Poynting vector of the binary star system radiating electromagnetic waves is

$$\bar{S} = \frac{\mu_0^2 G(m_1 + m_2)}{128\pi^2 c^3 R^2} \frac{q_1 + q_2}{4} f(m,e)^2 [a^{-9} - 8f(m,e)a^{-12} + 16f(m,e)^2 a^{-15}] \quad (36)$$

where $f(m,e) = \frac{64G^3}{5c^5} m_1 m_2 (m_1 + m_2) f(e)$. Assuming that the radiation energy in all directions at the position R away from the field source is uniform, the total power of electromagnetic waves radiated by the binary stars under this model is

$$P_M = \frac{\mu_0^2 G(m_1 + m_2)}{128\pi c^3} (q_1 + q_2) f(m,e)^2 [a^{-9} - 8f(m,e)a^{-12} + 16f(m,e)^2 a^{-15}] \quad (37)$$

For model two, three binary neutron star systems GW170817, GW190425, PSR J0737-3039 are selected, and the neutron star - white dwarf system NGC1851 and the binary white dwarf system WD J0651+2844 are selected for comparison. The parameters of these five systems are listed in Table 5.

Table 5. Parameters of five different binary star systems

Name	M_m ($M_\odot$)	M_c ($M_\odot$)	D(kpc)	e
GW170817	1.46	1.28	40000	≤0.024
GW190425	2.02	1.35	15900	≤0.048
PSR J0737−3039	1.337	1.250	1.15	0.088
4U 0513-40	1.4	0.0435	12	0.71
WD J0651+2844	0.50	0.26	0.66	0.383

According to the magnetic dipole model, the relation graph of electromagnetic waves radiated by the above five systems are exhibited in Figure 9.

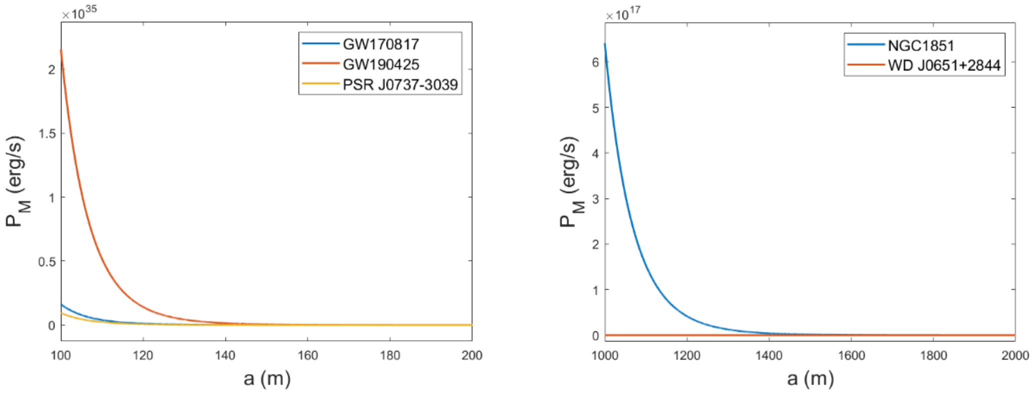

Figure 9. The two Figures shows the relation between P_M and radius of orbit based on magnetic dipole model. Binary neutron stars are compared in Figure (a), with other two systems shown in Figure (b).

It can be seen that the radiation power in this model is very sensitive to the change of a. The radiation power increases sharply due to the inspiral. When a=100 m, the radiation power of the dual neutron star system can reach the order of 10^{34}; in addition, it is also combined with the system. There is a big relationship. When a=100 m, the radiant power of the neutron star - white dwarf system is only on the order of 10^{32}, and the binary white dwarf system is only on the order of 10^{28}.

4.2. The production of FRB and GRB

The currently known astrophysical processes involving the highest energy radiation include fast radio bursts (FRB) and gamma-ray bursts (GRB). This section discusses the feasibility of applying the above two models to explain FRB and GRB.

For a typical FRB, its duration is on the order of milliseconds $t\sim 1\ ms$, and the radiated power at the field source is estimated to be of order 10^{40} erg/s. For the unipolar induction model, the above simulation has given that when the radius of the neutron star is 10 km and the surface magnetic field intensity is 10^6 T, the distance between the double stars is on the order of 10^2 km to meet this energy index. Therefore, the neutron star-white dwarf star system has the possibility of being the progenitor of FRB. Furthermore, for the binary neutron star system, although the model cannot give accurate predictions, in principle, the binary neutron star will also produce a magnetosphere. Moreover, the magnetic field strength will not be lower than that of the neutron star-white dwarf system, i.e., it can also be used as progenitors of FRB in theory.

For the magnetic dipole model, when the dipole radius is 100 m, if the system charge value can reach the order of 10^2 C, the energy index required by FRB can be achieved. Since the radiation power in this model is extremely sensitive to changes in a, the total charge value of the system only needs to be no less than 10^{-6} C to meet the requirements when a=20 m.

Compared to FRB, GRB has higher radiation power. Generally, the duration of short gamma-ray bursts is less than 2 s, but the average radiation power of field sources can reach the order of 10^{50} erg/s. The unipolar induction model is different from the magnetic dipole model, since it requires that the distance between the double stars cannot be too small. Otherwise, the DC loop will be unstable and the current loop cannot be formed. Assuming that the limit of the distance between the binary stars is the radius of the neutron star *r*~10 km, in order to make the radiation power reach the magnitude that satisfies the GRB, the magnetic field strength of the host star needs to be no less than 10^8 T, which has reached the magnetic field of the magnetar. Thus, if the neutron star-white dwarf system can be progenitors of GRB, the neutron star should be a magnetar. Since white dwarfs cannot generate such high magnetic fields, the binary white dwarf system cannot be a progenitor of GRB.

There are many conjectures about the mechanism of GRB, among which hot fireball model is one of the mainstream views. The model suggests that a high-temperature and compact photosphere will be formed at the end of supernova explosions and binary star mergers, and then the photosphere will expand to a critical radius, leading to the energy dissipated as thermal radiation. In addition, high-energy gamma rays will be emitted during this process. The thermal radiation power satisfies [17]

$$L_{th} = 4\pi R_{ph}^2 \sigma_B T'^4 \Gamma(R_{p\hbar})^2 \sim 4\pi R_0^2 \sigma_B {T_0}^4 \tag{38}$$

where Γ is the Lorentz factor. According to the magnetic dipole model, if the fireball is to meet the radiation power requirements, the total charge of the binary neutron star needs to be no less than 10^{-4} C when the dipole radius is 10 m, which is easy to meet for most neutron stars. For the neutron star-white dwarf system, the total charge requirement is not less than 0.1 C, and for the double white dwarf system, it is not less than 10^3 C, which is impossible for general white dwarf stars. Therefore, the model indicates that binary neutron stars and neutron star - white dwarf systems may be candidates for the progenitors of GRB, excluding binary white dwarf systems.

Finally, two models in describing the electromagnetic radiation have its own limitations: unipolar induction model cannot accurately describe the magnetic field intensity near the binary system, which greatly limits its applications, it only simulates neutron star - white dwarf system herein as an example. Nevertheless, one can still obtain valuable information based on the simulation results of the model considering the actual physical conditions. The limitation of the magnetic dipole model is that the radiation power is too sensitive to changes in the orbit radius ($P_M \sim a^{-15}$), hence the model is unable to make a reasonable prediction of the electromagnetic radiation law during inspiral. Thereby, the model can only be used to explain the high-energy radiation generated during the binary star fusion stage, e.g., FRB and GRB. On the other hand, it cannot explain the intensity of electromagnetic radiation released during the inspiral of binary neutron star systems.

4.3. Comparison of GR and ER

To quantitatively compare the magnitude of electromagnetic radiation and gravitational radiation, a comparison factor is defined

$$\gamma = \frac{P_{GR}}{P_{ER}} \tag{39}$$

It can be seen from Section 3.2 that when a<500 km or the neutron star's rotation period is long, the gravitational radiation caused by the rotation of the neutron star can be ignored compared to the radiation intensity of the revolution of the binary star. On this basis, only the radiation power generated by the revolution for P_{GR} is considered here. Two models of P_H and P_M were taken to get P_{ER}, of which four systems in Table 4 are still used for unipolar induction model, and their orbital eccentricity are set as 0.1. Besides, other three systems are selected for the magnetic dipole model, as listed in table 6 and Figure 10 demonstrates the result of comparison.

Table 6. Parameters of three different binary star systems

Name	M_m ($M_\odot$)	M_c ($M_\odot$)	D(kpc)	e
GW170817	1.46	1.28	40000	≤0.024
GW190425	2.02	1.35	15900	≤0.048
PSR J0737−3039	1.337	1.250	1.15	0.088

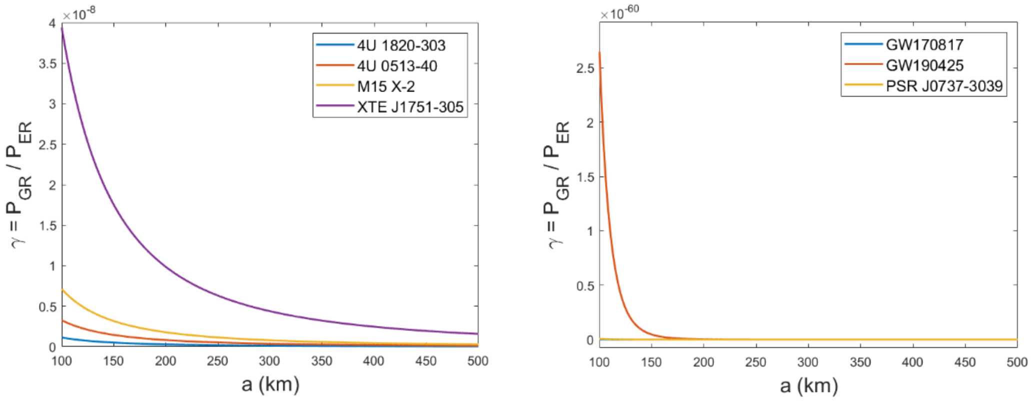

Figure 10. Figure (a) is the comparison of radiated power of GR and ER for unipolar induction model, and the relation between γ and a (km) for magnetic dipole model is shown in Figure (b).

It can be seen that the energy dissipated by gravitational radiation in the two models is far greater than the energy dissipated by electromagnetic radiation, i.e., the latter can be completely ignored in the precession stage. This is the reason for the calculation of the binary star orbit evolution with time only considering the gravitational radiation.

5. Conclusion

In summary, this paper investigates the laws of gravitational radiation and electromagnetic radiation in the process of binary star merging obtained by different models, as well as analyzes and compares these results. Based on the analysis, the following conclusions are obtained. Firstly, the gravitational radiation in the process of binary star precession includes the contributions of the revolution and rotation of the binary star. The gravitational radiation generated by the revolution is related to the mass and radius of the binary star, and is mainly affected by the radius ($P_{rev} \propto a^{-5}$, $P_{rev} \propto m^{3/2}$). Besides, the gravitational radiation produced by the rotation is mainly affected by the rotation frequency ($P_{rot} \propto \omega^6$). In the early stage of inspiral, the gravitational radiation generated by the rotation may have a greater contribution to the total radiant energy. When a<500 km, the gravitational radiation basically comes from the revolution of the binary star.

Additionally, both the monopole induction model and the magnetic dipole model can explain the electromagnetic radiation of a binary star to a certain extent. With regard to high-energy radiation, both models show that the law of electromagnetic radiation will increase rapidly with the decrease of a. Nevertheless, the electromagnetic radiation power in the unipolar induction model is also proportional to the square of the magnetic field strength of the host star. Whereas, in the magnetic dipole model, it is also proportional to the total charge of the binary star.

Moreover, two models describing electromagnetic radiation give the following predictions for high-energy radiation. The dual neutron star system is a possible candidate for the progenitor of FRB and GRB. Besides, the neutron star - white dwarf system may also be the progenitor of the two, but the conditions are more stringent, i.e., the neutron star is required to belong to a magnetar. In addition, the double white dwarf system is unlikely to be the progenitor of the two.

Finally, the electromagnetic radiation power of the binary star system during the precession phase is much smaller than the gravitational radiation power. Therefore, the precession of the binary star orbit can be regarded as completely dominated by the gravitational radiation.

This article has presented the analytical models for the four models describing gravitational radiation and electromagnetic radiation. Therefore, the accuracy and precision of the model can be verified by further calculation and fitting with the data measured in the experiment. In addition, the unipolar model and the magnetic dipole model only give calculation methods from a phenomenological perspective, but cannot answer the generation mechanism of FRB and GRB. It is necessary for understanding these magnificent phenomena in the universe-in short, which are worthwhile exploring in depth in the future. Overall, these results offer a guideline for future study on the gravitational and electromagnetic radiation processes of binary neutron star merger and binary star systems in certain conditions.

References

[1] W. Baade & F. Zwicky. Remarks on Super-Novae and Cosmic Rays [J]. Physical Review, 1934, 46 (1): 76–77.
[2] A. Hewish, S. J. Bell, et al. Observation of a Rapidly Pulsating Radio Source [J]. Nature, 1968, 217 (5130): 709–713.
[3] S. Weinberg: Gravitation and Cosmology, Principles and Application of the General Theory of Relativity [M]. Wiley-VCH, 1972.
[4] T. M. Tauris, M. Kramer, P. C. C. Freire, et al. Formation of Double Neutron Star Systems [J]. The Astrophysical Journal, 2017, 846(2): 170.
[5] B. P. Abbott, R. Abbott, et al. GW170817: Observation of Gravitational Waves from a Binary Neutron Star Inspiral [J]. Physical Review Letters, 2017, 119(16): 161101.
[6] R.W. Klebesadel, I.B. Strong, R.A. Olson. Observations of Gamma-Ray Bursts of Cosmic Origin [J]. Astrophysical Journal Letters, 1973, 182: L85.
[7] Arizona State University. Massive Star's Dying Blast Caught by Rapid-response Telescopes [N]. PhysOrg. 26, 2017.
[8] D. R. Lorimer, M. Bailes, M. A. McLaughlin, et al. A Bright Millisecond Radio Burst of Extragalactic Origin [J]. Science, 2007,318: 777.
[9] D. Xiao, J.J. Geng, Z.G. Dai. Binary Neutron Star Mergers: Gravitational Wave Emission and Electromagnetic Counterparts [J]. Physics, 2019.
[10] J. S. Wang, Y. P. Yang, X. F. Wu, et al. Fast Radio Bursts from the Inspiral of Double Neutron Stars [J]. ApJL, 2016, 822: L7.
[11] D. Lai. DC Circuit Powered by Orbital Motion: Magnetic Interactions in Compact Object Binaries and Exoplanetary Systems [J]. The Astrophysical Journal Letters, 2012.
[12] S.S. Cai, Y. Zhu, J.J. Xu: Electrodynamics [M]. Higher Education Press, 2002.
[13] K.W. Yung, P.B. Landecker, D.D. Villani (1998). An Analytic Solution for the Force between Two Magnetic Dipoles [J]. Magnetic and Electrical Separation, 2012, Vol. 9, pp. 39-52.
[14] H. H. Zhao and L. M. Song. The Radius Estimation of Double Pulsar PSR J0737-3039A [J]. International Astronomical Union, 2013.
[15] Yu S H, Lu Y J. The Gravitational Wave Radiation from Ultracompact X-ray Binaries (in Chinese) [J]. Sci Sin-Phys Mech Astron, 2019, 49: 019501.
[16] Image from https://commons.wikimedia.org/wiki/File:ChandrasekharLimitGraph.svg [OL]
[17] P. Kumar, B. Zhang. The Physics of Gamma-Ray Bursts & Relativistic Jets [J]. Physics Reports, 2015.