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Logistic function as a characteristic of multipactor development

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Abstract

Simulations of multipacting with or without space charge effect bring out a different behavior of particle number growth, namely, the exponential growth of particle number in the simulations without space charge effect and the saturation of particle number (or collision and emission currents) when space charge is considered. That creates a certain confusion in evaluation and comparison of overall danger of multipactor between the approaches. On the other hand, both growth rate and total multipactor current loading at saturation are important for multipactor barriers evaluation. It was noticed and then verified that the logistic function, widely used in chemistry, biology, and ecosystem study, reproduces the particle number growth curves remarkably well. The function contains the parameters, which can be interpreted as particle number growth rate and multipactor current saturation level, so both become correlated and obtained simultaneously in one run. In this work it is shown how the logistic function can be used for characterization of the multipactor barriers and how it can be used for possible reduction of simulation time in the simulations with space charge effect.

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Introduction

Multipactor is an electron discharge driven by secondary electron emission that occurs in microwave components, accelerator structures, space communication systems, and other related electronics [1,2]. In this paper, the simulations of multipactor in the superconducting accelerating cavities will be used for consideration since majority of the multipactor simulations at FNAL were performed during design and production of the accelerating cavities for PIP-II project [3]. The character of avalanche growth of number of particles during multipacting is essentially the same for any vacuum device and type of multipactor, so this limitation does not result in the loss of generality.

The main characteristic of multipactor (MP) in the accelerating cavities is multipacting activity as a function of radiofrequency electromagnetic (RF EM) field level associated with acceleration rate. The field level may be expressed naturally as accelerating voltage V (magnitude of accelerating field component E_z integrated along the cavity axis Z), effective voltage V_{eff} which additionally takes into account transit time factor, energy gain of synchronous particle or accelerating gradient defined as $G=V_{eff}/(\beta\lambda)$, where β is relative particle velocity for which a cavity is designed and λ is a RF field wavelength. The evaluation of the multipacting activity or its intensity is not straightforward though because of two different ways to perform the simulations.

The simulations of multipactor with Particle-in-Cell solver can be performed with or without consideration of space charge effects. In the simulations without space charge effect the number of particles increases exponentially, when the conditions for multipacting are fulfilled, while the number of particles reaches saturation in the simulations with space charge effect (see Fig.1).

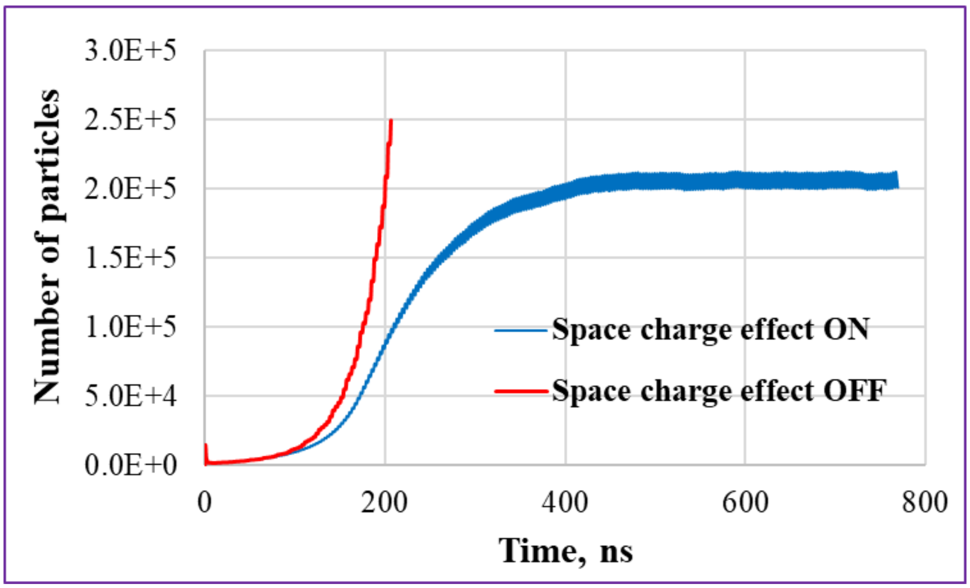


Figure1: The result of multipactor simulations in SSR2 accelerating cavity (f=325 MHz, β=0.47) of PIP-II accelerator performed with and without space charge.

Characterization of multipactor

Exponential growth of number of particles in the simulations without space charge is described by equation, which can be derived analytically from the equation of motion in some simple geometries:

$$N(t) = N_0 \cdot e^{\alpha t}, \tag{1}$$

where $N(t)$ is number of the particles in time, N_0 – initial number of particles, t – time and α is a growth rate of particle number. The positive growth rate value indicates development of multipactor in RF device. It is obtained usually by approximation of a simulated curve $N(t)$. This parameter informs how fast is a multipactor development, which is important information to know in case of pulsed RF power. But it makes no evaluation of the multipactor intensity.

To obtain additional information from the simulations without space charge effects the effective secondary emission yield has been introduced in [4]:

$$\langle SEY \rangle = \frac{I_{emission}}{I_{collision}} = e^{\alpha T} \tag{2}$$

where T is RF period, $I_{emission}$ and $I_{collision}$ are emission and collision currents (standard CST PS output) averaged over last 3-5 RF periods of simulation time. The effective secondary emission yield indicates multipactor development when its value is greater than unit (<SEY> >1). Besides that, the parameter evaluates the dynamic conditions for multipacting – the closer <SEY> is to the maximum of secondary emission yield of material, the "better" is the multipactor dynamics. It allows to compare the likelihood of the multipactor barriers when they are obtained in the simulations with the same SEY of material. But still, this is not a direct evaluation of multipactor intensity. A multipacting process may be very fast and easy to develop, but it could be weak at the same time if the area of emission is negligibly small compared to the total inner surface of RF cavity.

The simulations with space charge effect directly evaluate multipactor intensity via emission and collision currents and powers in contrast to those without space charge. Despite this major advantage the PIC simulations with space charge have some weaknesses [5]. Above all, that is much longer simulation time because of lower computational efficiency and necessity to perform simulations until multipactor saturation is reached, since the mentioned MP parameters have sense as the indicators of intensity only at saturation.

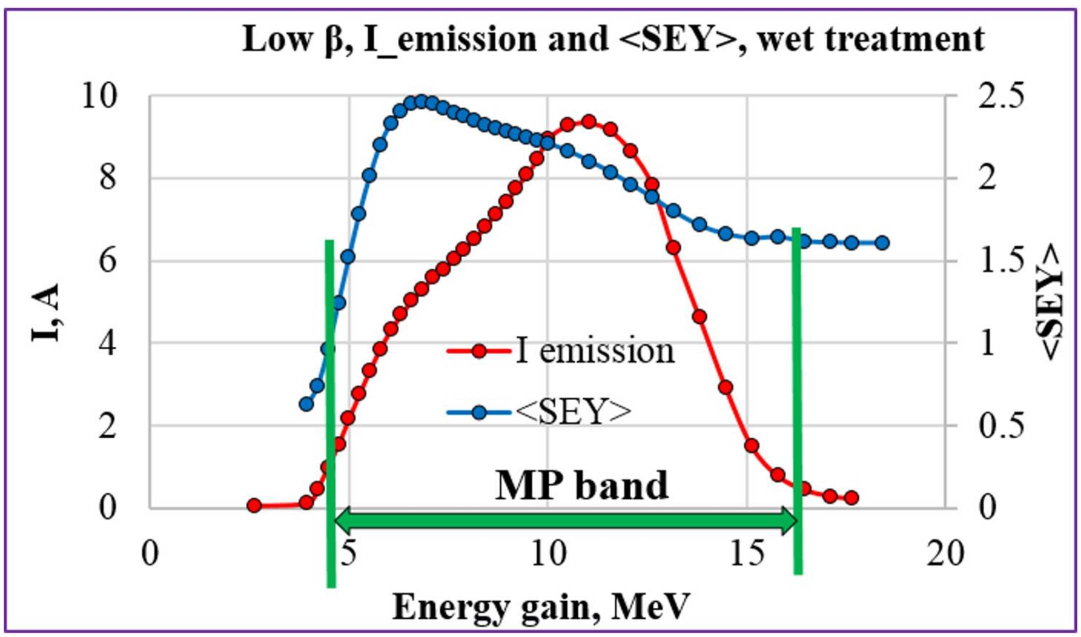
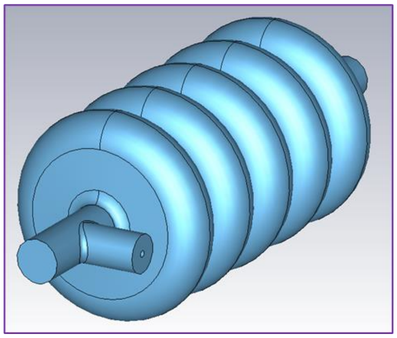


Figure 2: Comparison of MP simulations with space charge (I emission) and without one (<SEY>) performed for PIP-II 650 MHz β = 0.61cavity, SEY of surface corresponds to wet treatment [6].

Both types of simulations are close enough for practical purposes in evaluating of the multipactor barriers expressed in terms of accelerating gradients, energy gain, RF power or other characteristics of RF field level (see Fig.2). At the same time this example in Fig.2 also shows the issue with the simulations without space charge effect that was mentioned above. The lower edges of the multipactor barrier are almost the same for both cases, they just reflect that the collision energy reaches first crossover W_1 of SEY in both simulations. But the upper edges are in full disagreement – the simulations with space charge effect shows fading multipacting, while multipacting continues at significant level in the simulations without space charge effect. The discrepancy is explained as follows. In the cavities with elliptical cells the MP is located on the cell equator (Fig.3). The area occupied by MP shrinks with increasing of RF field amplitude resulting in drop of the emission current. The effective secondary emission <SEY> (i.e., growth rate) remains high indicating that the average energy of collision still provides a corresponding number of secondaries per one impact.

The discrepancy may be eliminated by using emission current $I_{emission}$ in the simulations without space charge as well. The emission current without space charge is higher, since space charge suppresses emission, but the multipactor barrier, 4 to 15 Mev, and the barrier shape is the same [7].



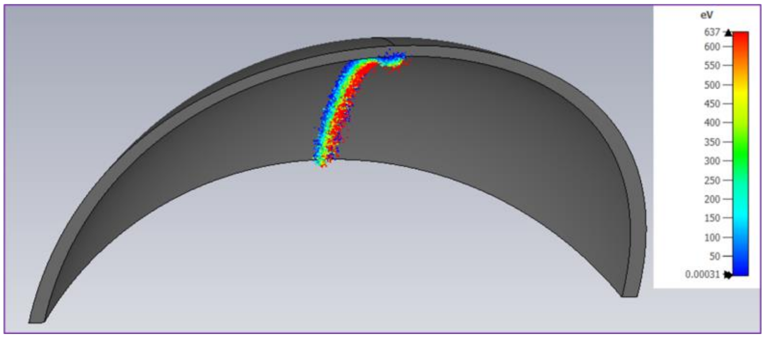
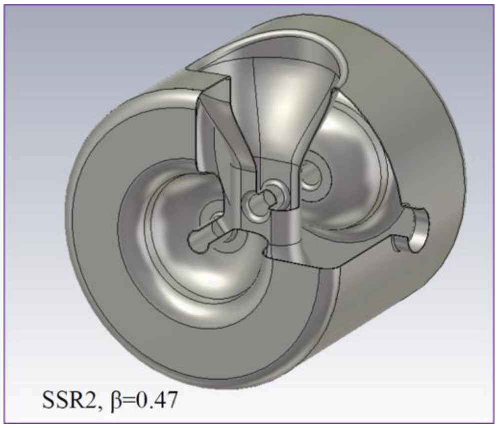


Figure 3: Snapshot of steady state multipacting with space charge effect in PIP-II 650 MHz β = 0.61cavity. Particle colors indicate their energy.

Though the simulations with space charge effect are more physically correct, the saturation level alone is not a complete characteristic of MP development. The simulations of multipactor in the Single Spoke Resonator (SSR) show that the MP barriers at different field levels in this cavity have not only the different saturation levels, but also the different rates of development (see Fig.4). In practice multipactor often does not reach saturation and a weak but fast process can be more problematic than potentially more powerful one.



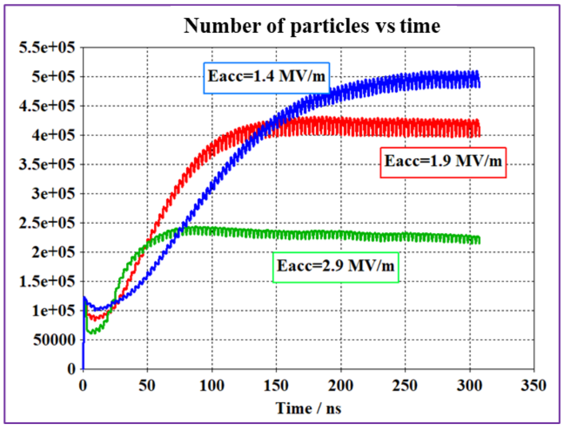


Figure 4: The SSR2 cavity (on the left) and the multipactor developments in the cavity at different accelerating field levels.

Therefore, it seems practically useful to characterize MP development by these two parameters – saturation level and growth rate. The problem is how to define and calculate them. Saturation level can be obtained directly from the simulations with space charge, but it is not so straightforward for growth rate.

In the simulations with space charge the growth of number of particles in time can be assumed almost exponential in the beginning of the process while the space charge is not strong. It seemed that this part of curve can be approximated by the exponential function and used to calculate growth rate. But the attempts to use this approach were not successful because of the difficulties to define right time interval for the approximation.

A growth rate obtained directly from the simulations without space charge is not correct since in this case the growth rate values are not the same as with space charge. The space charge pushes the particles out from the interval of stable phase motion and possibly out from the area where favorable field distribution exists. Due to these additional losses the growth rate in the simulations with space charge is typically lower. Besides this way to obtain growth rate automatically doubles the number of simulations because the simulations both with and without space charge are required.

It was noticed that the logistic function, widely used in chemistry, biology, and ecosystem study, reproduces the particle number growth curves from the simulations with space charge remarkably well. The function expressed in terms of number of particle multiplication has form:

$$N(t) = \frac{N_S}{1+(\frac{N_S}{N_0}-1)\cdot e^{-\alpha t}} \;, \qquad (3)$$

where $N(t)$ is number of particles as a function of time, N_0 – initial number of particles, Ns – number of particles at saturation, α – growth rate. The key interest in this function is that it integrates the saturation and the growth rate.

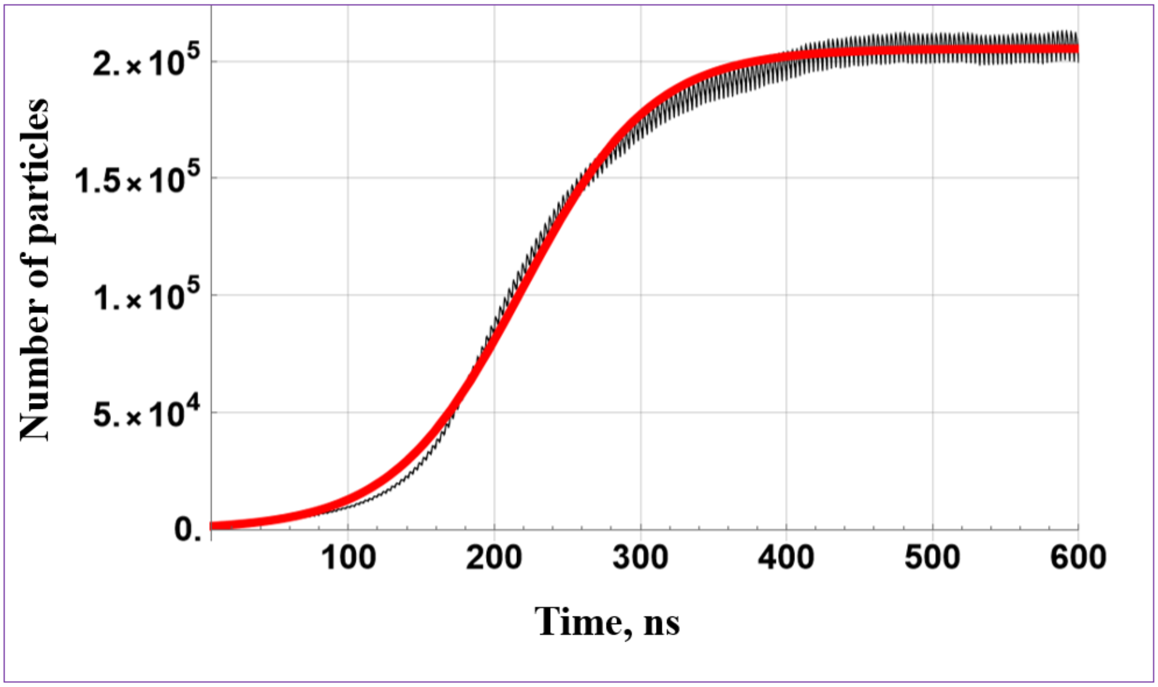


Figure 5: The multipactor development in SSR2 cavity at Eacc=0.9 MV/m (black) and the fitted logistic function. The irregular 50 ns part in the beginning of the development is not shown. The parameters of the logistic function are N_0 =1434, Ns = 205348 and $\alpha = 0.0227$ ns^{-1}, the fitting was performed in 6-769 ns interval.

Fig.5 shows the logistic function fitted to the simulated number of particles evolution during multipactor development in SSR2 cavity. The fitting to the raw simulation data was performed in the full-time interval from 0 to 600 ns with the fitting procedure from Mathematica. The parameters of the logistic function that provided fitting are N_0 =1434, Ns = 205348 and $\alpha = 0.0227$ ns^{-1}. The growth rate calculated from the exponential growth of number of particles in the simulations at the same conditions, but without space charge is $\alpha = 0.0285$ ns^{-1}, which is higher as expected. Therefore, the logistic function is a promising way to characterize multipactor development in general. But the accuracy of the fitting is not high and the whole procedure is tedious in such uninvolved way to perform it. Fortunately, there are ways to make the characterization of multipacting with logistic function simpler and more accurate.

Practical application of the logistic function.

Before consideration the ways to define the logistic function parameters let us smooth and reduce the simulation data using the averaging in moving maps as shown in Fig.6. This smoothing is not a principal requirement, it just makes the data manipulation and visualization easier.

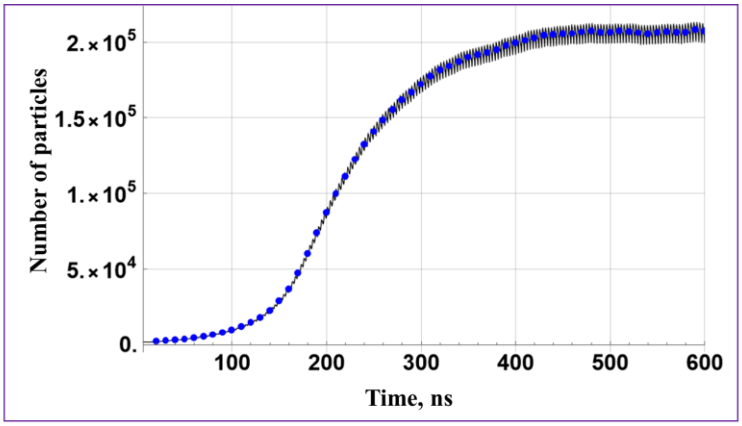


Figure 6: Smoothing N(t) of multipactor development in SSR2 cavity at Eacc=0.9 MV/m (black) and reducing the number of points using averaging in moving maps. The original simulation data is shown in black; the result of averaging is shown in blue.

The most straightforward way to use the logistic function for the characterization of MP process is the direct fitting of the function to a whole experimental data including sufficient part of saturation as shown in Fig.7 (the same fitting from Fig.5 is shown here along with the averaged data for easier comparison). This direct way is especially useful when the simulations were performed in a complex structure. In the complex structures MP often is a mix of one- and two-side multipacting modes, the multipacting area may change and migrate from the point of origin during the MP development. As a result, growth rate changes in time and a particle number growth does not follow the logistic function exactly. Nevertheless, the advantage of this general approach is that the mentioned variations of the particle number growth are effectively averaged. The drawback of this usually effective approach is a necessity to perform simulations until saturation is reached which means longer simulation times.

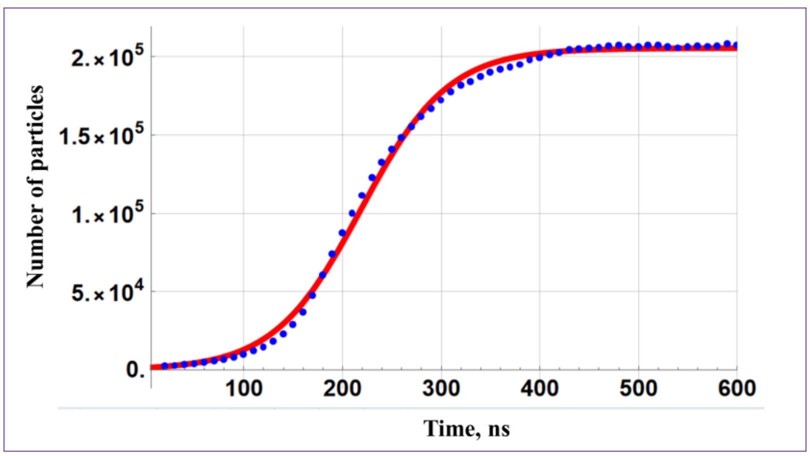


Figure 7: The multipactor development in SSR2 cavity at Eacc=0.9 MV/m (blue) and the logistic function fitted in the time interval of 6-769 ns. The fitted parameters are N_0=1434, N_s=205348 and α =0.0227 ns^{-1}.

The idea to stop simulations before saturation is reached and find the logistic function parameters using a truncated curve is very attractive, since it reduces simulation times. But without saturation the initial part of *N(t)* curve becomes a serious source of inaccuracy in the logistic function fitting. The initial part of the particle growth curve could be simply removed from the fitting procedure, but the problem is that there is no formal indication at which point a cut out should be done. On the other hand, the result of fitting is very sensitive to the choice of starting point on the curve, so the found parameters of the logistic function may significantly differ even for moderately different cut out points.

The situation can be improved if an initial number of particles is preliminary estimated. The estimation is performed by fitting of the exponential function to the initial part of the particle number growth curve as shown in Fig.8. The evaluation does not have to be very precise, since it is just the virtual initial number of particles for the following fitting of the logistic function. The Table 1 shows the parameters of the logistic function obtained with different virtual initial number of the particles and different time intervals used for fittings.

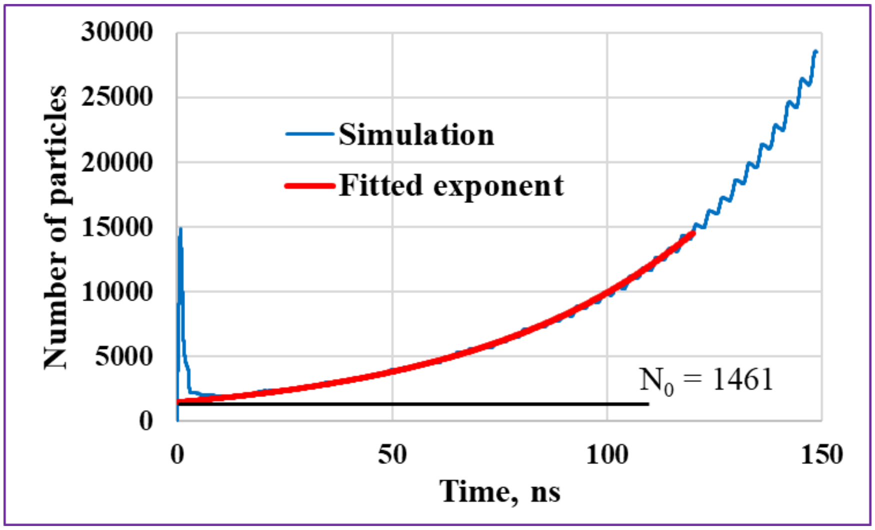


Figure 8: Defining virtual initial number of particles N_0.

Table 1.

N_0	N_s	α	Time interval, ns
1484	205348	0.0227	6 - 769
1481	205388	0.0225	100 - 760
1600	201334	0.0224	6 - 312
1500	203288	0.0227	100 - 290
2000	201664	0.0214	100 - 380
760	192007	0.0263	100 - 375

The results in the Table 1 indicate that the parameters of the logistic function are not very sensitive to the estimated initial number of particles, but nevertheless it is better to keep it close to the found virtual value (see the last row in the Table). The logistic function with the parameters found by fitting in the shortest time interval 100-290 ns is shown in Fig.9. The minor inconsistencies between the logistic function and the simulations data maybe are due to that the multipactor development in this particular accelerator single spoke cavity design at the given field level is a mix of one- and two-point multipactor modes which have basically different growth rates (the two-point mode dominates).

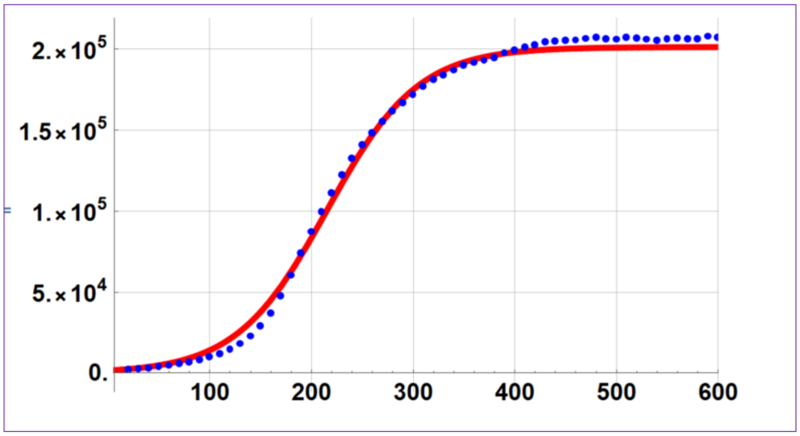


Figure 9: The logistic function (red line) obtained by fitting to the simulations data (blue dots) in the time interval 100-290 ns.

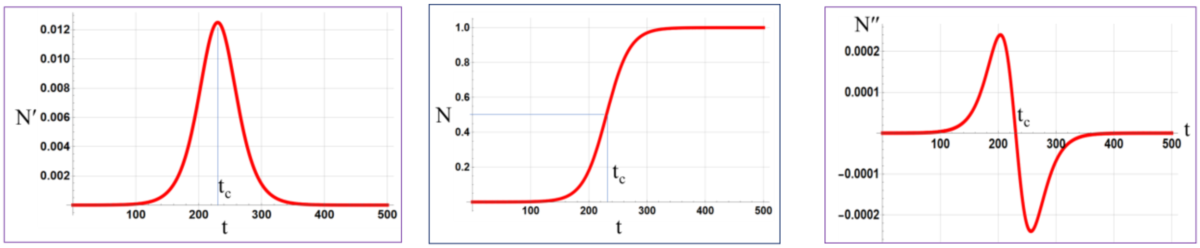


Figure 10: The properties of the logistic function and its derivatives.

The properties of the logistic function allow to find saturation level and growth rate without knowledge N_0 at all. The behavior of the logistic function (1) and its derivatives are shown in Fig.7 for the random convenient parameters $N_s = 1$, $N_0 = 10^{-3}$ and $\alpha = 0.05$. The argument t_c indicates a point, where concavity of the function (1) changes. At this point the first derivative of the function (which is the original differential equation of population growth [8] actually) is maximal and the second derivative of the function is equal to zero:

$$N'(t_c) = \frac{\alpha \cdot [N_s - N(t_c)] \cdot N(t_c)}{N_s} = \max \tag{4}$$

$$N''(t_c) = \frac{\alpha \cdot [N_s - 2 \cdot N(t_c)] \cdot N'(t_c)}{N_s} = 0 \tag{5}$$

It is easy to show from (2) and (3) that the saturation level is

$$N_s = 2 \cdot N(t_c). \tag{6}$$

And the growth rate is

$$\alpha = 2 \cdot N'(t_c)/N(t_c). \tag{7}$$

Therefore, to find saturation level and growth rate in multipactor simulations with space charge we need to have only a central part of N(t), big enough to define accurately t_c, $N(t_c)$ and $N'(t_c)$.

To verify this approach the simulations of a simple 500 MHz cavity from [9] were chosen to avoid the complications in multipactor behavior which occur on the complex RF structures. The predominantly two-side multipactor develops in the cavity gap of 1 cm at the RF field level in the gap of 162.6 kV/m. The simulated number of particles vs time for this barrier is shown in Fig.11. This time the approximation of the central part (10-38 ns) of the curve was performed with polynomial up to 9th power with only odd exponents since the logistic function is symmetric relatively to t_c. The derivative of the polynomial is shown also in Fig.11, the derivative reaches its maximum at $t_c = 28.4$ ns. At this point the number of the particles given by approximation is $N(t_c) = 391321$ and the derivative of the polynomial is $N'(t_c) =$ 34137 particles per ns. Therefore, the estimated saturation level according to (6) is $N_s = 782642$, which is very close to the simulated saturation of 7.6975e5, and the growth rate according to (7) is $\alpha = 0.17447$ ns^{-1}.

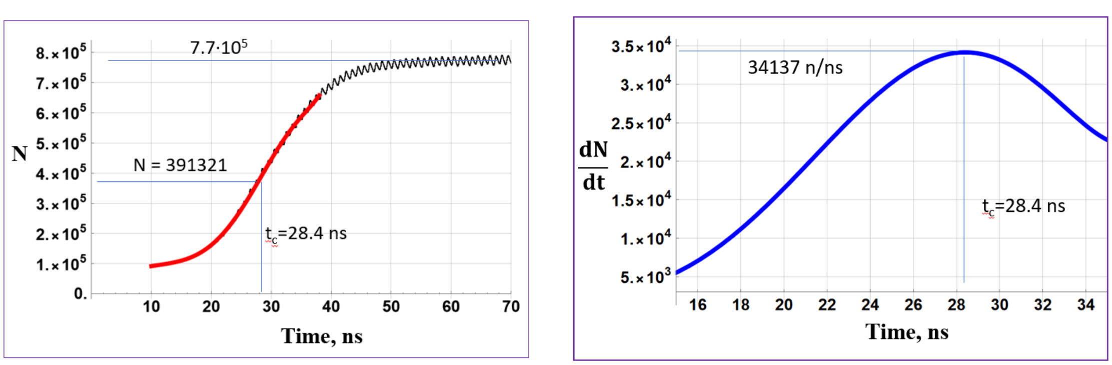


Figure 11: The left plot shows the simulated number of particles vs time (in black) and the polynomial that approximates the central part of the curve (in red). On the right plot the polynomial derivative is shown.

Up to this point of the paper all manipulations with data (averaging, approximating, equation solving etc) were performed with the help of Mathematica. In practice it would be very convenient and effective to calculate the logistic function parameters directly in CST Particle Studio. Unfortunately, the post-processing tools of CST PS are not very powerful. First of all, the author did not succeed in obtaining a polynomial approximation of the central part of the particle growth function. The only sensible result was achieved with the local averaging and the derivative of the result of the averaging as shown in Fig.12. Of course, the result is not accurate and can serve as a rough estimate only. Apparently, the user defined post-processing tools have to be developed.

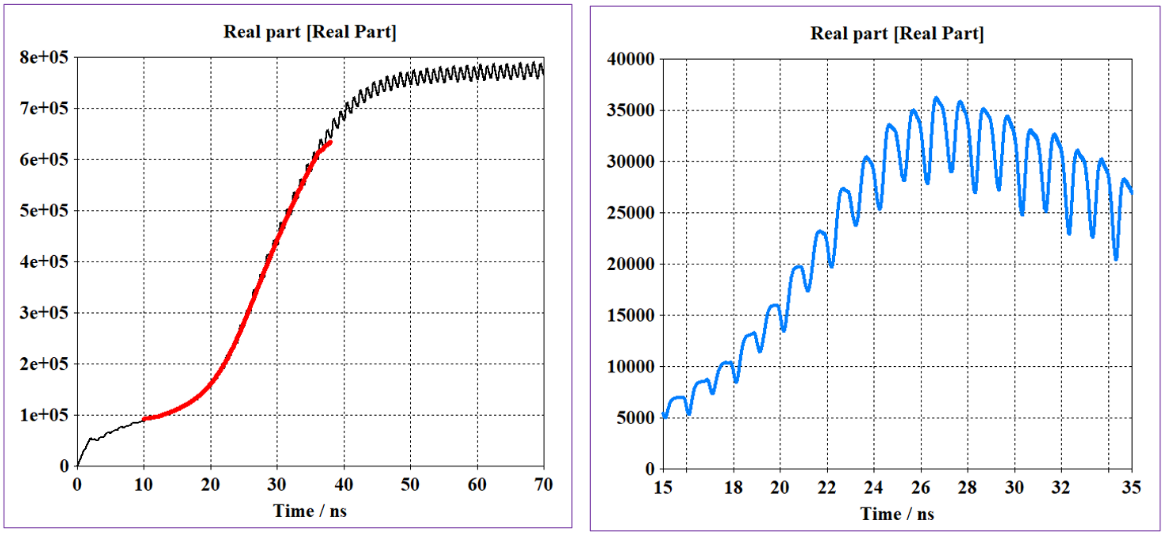


Figure 12: On the left plot the result of CST local averaging is shown, the left plot shows the derivative of the local averaging.

Conclusion

The logistic function accurately reproduces growth of particle number in multipacting process. The function parameters can be interpreted as particle number growth rate and multipactor current saturation level. Therefore, the logistic function parameters indicate how fast a multipactor develops and how intensive it can be.

Important that both parameters are obtained in a single run of simulations with space charge effect active. Moreover, it is not necessary to continue simulations up to complete saturation is reached – a central part of particle number (or emission current) growth curve, that includes a point of concavity change, is sufficient to define the logistic function parameters, if it includes the point of concavity change. Therefore, this is an additional possible reduction of simulation time with space charge effect.

It should be noted that the logistic function as a characteristic of multipactor development is less accurate if overall process is a combination of different multipactor modes, which often happens in the complex RF structures.

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