

Studies of Jet Production Rates in e^+e^- -annihilation at $E_{cm} = 29 \text{ GeV}^*$

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Abstract. Production rates of multijet hadronic final states are studied in e^+e^- annihilation at 29 GeV center of mass energy. QCD shower model calculations with exact first order matrix element weighting at the first gluon vertex are capable of reproducing the observed multijet event rates over a large range of jet pair masses. The method used to reconstruct jets is well suited for directly comparing experimental jet rates with parton rates calculated in perturbative QCD. Evidence for the energy dependence of α_s is obtained by comparing the observed production rates of 3-jet events with results of similar studies performed at higher center of mass energies.

Introduction

Within the framework of Quantum Chromodynamics (QCD), a renormalizable gauge theory of the strong interactions [1], multihadronic final states in e^+e^- annihilation reactions are described by the production of quarks, antiquarks and gluons, which eventually fragment into hadrons. At high enough energies, these hadrons appear as jets of particles which reflect the dynamics of the initial quarks and gluons. In the past, detailed studies of multijet event production have been performed by the JADE [2;3] and the TASSO [4] collaborations in the e^+e^- center of mass energy range of 14 to 46.7 GeV. The comparison of these data with QCD and fragmentation model calculations revealed that $O(\alpha_s^2)$ QCD models underestimate the production rates of 4-jet events. This deficiency was shown to be caused by an apparent lack of 4-parton events in the underlying $O(\alpha_s^2)$ QCD calculations and could not be accounted for by varying the fragmentation parameters of these models [2;4]. Models based on leading logarithmic approximation, which include contributions from higher ($\geq 3^{rd}$) order perturbation terms, were demonstrated to provide better descriptions of the observed jet rates, especially if the model includes correction terms according to the exact $O(\alpha_s)$ matrix element [4]. Furthermore, it was shown that by using the jet reconstruction procedure introduced by JADE [2], experimental jet rates may directly be compared to theoretical calculations of parton (i.e. quark and gluon) production [3;5]. This allows one, as a significant proof of the validity of QCD, to perform direct tests of theoretical predictions in a way which largely avoids the usual systematic uncertainties as they appear, for instance, in determinations of α_s carried out to date [6].

In this paper, we present jet production rates from hadronic data collected with the upgraded Mark II detector [7] at PEP at 29 GeV center of mass energy. The jet reconstruction algorithm introduced by JADE is employed in this analysis. To the extent that this algorithm indeed provides a valid comparison between experimental jet production rates and theoretical parton production rates, the experimental data provide an important resource for the testing of both presently available and future QCD calculations and QCD models. As an example we compare our 3-jet event production rates and the results of other experiments at different center of mass energies to perturbative QCD calculations of $O(\alpha_s^2)$ in order to investigate the energy dependence of α_s .

Theoretical and Experimental Definition of Jets.

In second order perturbation theory, the relative production rates R_n of 2-, 3- and 4-jet events are quadratic functions of the coupling strength of the strong interactions, α_s :

$$\begin{aligned} R_2 &\equiv \frac{\sigma_2}{\sigma_{tot}} = 1 + C_{2,1} \cdot \alpha_s + C_{2,2} \cdot \alpha_s^2 \\ R_3 &\equiv \frac{\sigma_3}{\sigma_{tot}} = C_{3,1} \cdot \alpha_s + C_{3,2} \cdot \alpha_s^2 \\ R_4 &\equiv \frac{\sigma_4}{\sigma_{tot}} = C_{4,2} \cdot \alpha_s^2, \end{aligned} \quad (1)$$

where σ_{tot} is the total hadronic cross section and the σ_n are the corresponding cross sections for n -parton event production. $C_{n,k}$ are the k -th order coefficients for n -jet production and are functions of the parton resolution criteria chosen in the theoretical calculations. Criteria to define resolvable partons are introduced in order to calculate finite parton production cross sections. The detailed choice of resolution criteria is, to a certain extent, arbitrary. A commonly used method is to require the square of the scaled invariant mass of any pair of partons i and j ,

$$y_{ij} = \frac{M_{ij}^2}{E_{cm}^2}, \quad (2)$$

to satisfy the relation

$$y_{ij} \geq y_{min}, \quad (3)$$

where y_{min} is the cut-off parameter defining *resolvable* partons.

The JADE collaboration adapted the above definition of resolvable partons to an experimental jet finding algorithm [2;8], which works as follows. In each hadronic event, the squares of the scaled pair masses,

$$y_{kl} = \frac{M_{kl}^2}{E_{vis}^2}, \quad (4)$$

are calculated for all pairs of particles k and l , where E_{vis} is the visible energy of the event. Both charged and neutral particles are used; charged particles are assumed to be pions and neutrals to be photons. Those particles i and j with the smallest pair mass are replaced by a pseudoparticle with four-momentum $(\mathbf{p}_i + \mathbf{p}_j)$. The procedure is then repeated until the scaled masses of all particle or pseudoparticle pair-combinations exceed a certain threshold value y_{cut} :

$$y_{kl} \geq y_{cut}, \quad (5)$$

and the remaining clusters are called “jets”. To calculate the pair mass M_{kl} , the expression

$$M_{kl}^2 = 2 \cdot E_k \cdot E_l \cdot (1 - \cos \Theta_{kl}) \quad (6)$$

is used, where E_k and E_l are the energies of particles k and l and Θ_{kl} is the angle between their momentum vectors. Studies with Monte Carlo generated events showed that this choice

of M_{kl} provides the closest agreement between jet- and parton-multiplicities at comparable values of y_{cut} , the experimental cutoff in the jet finding algorithm, and y_{min} , the QCD cutoff parameter for massless partons in the perturbative QCD calculations [2-5].

In the following analysis, this jet algorithm will be used to analyse the relative production rates R_n of n-jet events at 29 GeV center of mass energy in the dynamic range of $0.015 \leq y_{cut} \leq 0.140$. After comparing experimental results with the predictions of a QCD plus fragmentation model, detailed model studies will be presented in order to further investigate the correspondence between jet- and parton-multiplicities.

Experimental Jet Rates and Comparison with QCD Model Calculations.

The integrated luminosity of the event sample analysed in this study totals 27 pb^{-1} and was accumulated between November 1985 and February 1986 at an e^+e^- center of mass energy of 29 GeV at the PEP storage ring at SLAC, using the upgraded version of the Mark II detector [7]. The basic particle selection criteria are described elsewhere [9]. In addition, the total missing momentum of an event, $|\sum \vec{p}_i|$, is required not to exceed 30 % of the e^+e^- center of mass energy, the angle θ_S of the event sphericity axis [10] with the beam direction must satisfy $|\cos\theta_S| < 0.8$, the total visible energy of the event, E_{vis} , is required to exceed 40 % of E_{cm} and the event momentum imbalance along the beam direction must not exceed 40 % of E_{vis} . These criteria suppress events from the processes $e^+e^- \rightarrow \gamma\gamma \rightarrow \text{hadrons}$ and $e^+e^- \rightarrow \tau^+\tau^-$ as well as events where a large fraction of the particles is lost near the beampipe. We verified that the resulting jet production rates do not depend on small variations of any or all of the selection criteria. After these cuts, 7482 well contained hadronic events remain to be analysed.

The relative production rates R_n of n-jet events, reconstructed with the jet algorithm described above, are presented in Table 1 for y_{cut} values ranging from 0.015 to 0.140. These jet production rates are also shown in Fig. 1 as a function of y_{cut} . The statistical errors given in Table 1 are omitted in the figure since they are smaller than the printed symbols.

In Fig. 1, the data are compared to jet production rates determined in model calculations using the Lund QCD shower and string fragmentation model (version 6.3) with exact first order matrix element weighting [11]. This model was shown to provide the best overall description of hadronic e^+e^- data [12-14] and of jet production rates in detail [4]. The fragmentation parameters of the model were chosen according to the results of our previous studies [12]. The QCD parameter Λ_{LLA} , which determines the magnitude of the strong coupling-constant in the underlying QCD calculations, was chosen such that the experimental jet rates are well described, resulting in $\Lambda_{LLA} = 340 \text{ MeV} \pm 20 \text{ MeV}$. Note that Λ_{LLA} must be regarded as a parameter of this specific QCD model. It can generally not be compared with numerical results of $\Lambda_{\overline{MS}}$, which is the QCD parameter defined in second (or higher) order perturbative QCD calculations in the $\overline{MS}$ renormalization scheme. The model calculations include the effects of initial state photon radiation and a full software simulation of the

Mark-II detector and they underwent the same selection criteria and analysis steps as the real data.

As can be seen in Fig.1, the model provides an excellent description of the jet rates observed in the entire range of jet masses analyzed. This observation is in agreement with similar investigations performed at 35 GeV center of mass energy [4]. Together with the results of our previous studies on the qualitative description of data by various QCD models [12], it emphasizes that the Lund QCD shower model is currently the most reliable model to describe hadronic final states in the PEP and PETRA e^+e^- energy range [12-14].

This situation, however, is sometimes regarded as unsatisfactory, since the Lund shower model is only based on leading logarithmic approximations plus a first order correction at the first gluon vertex [11], rather than on complete next-to-leading order perturbation theory (e.g. in 2^{nd} order) where the QCD parameter Λ is defined in a certain renormalization scheme (usually the $\overline{MS}$ scheme). Models based on 2^{nd} order QCD calculations, on the other hand, were seen to provide a poorer description of the data in general [12] and especially of the observed multijet production rates [2;4]. This disagreement was explained by an apparent lack of 4- and higher multi-parton events in the $O(\alpha_s^2)$ calculations. Therefore the QCD shower model is the preferred model to determine corrections to the experimental data for the effects of initial state photon radiation, hadronization and the finite acceptance and resolution of the detector.

The Correspondence between Jet and Parton Rates.

The data presented so far are not corrected for initial state radiation, detector acceptance and fragmentation. In previous investigations, however, it was shown that experimental jet rates, defined by the jet algorithm described above, are closely related to the underlying parton rates and can thus be directly compared to perturbative QCD calculations [3;5]. This possibility shall now be further investigated for our data at 29 GeV center of mass energy.

In Fig. 2, the relative 2-, 3- and 4-jet production rates¹, determined in model calculations using the Lund QCD shower and fragmentation model as shown in Fig. 1 and as discussed in the previous section, are presented at 3 different stages of model calculations. First, they are calculated from the original quarks and gluons by using the jet algorithm described above. We verified that this procedure is equivalent to terminating the QCD shower process and counting parton rates at the corresponding values of y_{min} . In a second step, jets are reconstructed after fragmenting quarks and gluons into final particles. Finally, the effects of initial state photon radiation as well as the finite resolution and acceptance of the Mark II detector are simulated and the same event selection criteria as used for the final data analysis are applied. As can be seen in Fig. 2, with the inclusion of the effects of fragmentation as well as initial state photon radiation and detector resolution, the reconstructed jet production

¹For reasons of clarity, the small rate of 5-jet events observed at the smallest values of y_{cut} are omitted.

rates faithfully reproduce the parton rates.

While $O(\alpha_s^2)$ string fragmentation models result in similarly small corrections for 2- and 3-jet events above $y_{cut} = 0.04$, larger deviations are obtained in regions where they do not provide a good description of the data, as for instance at low values of y_{cut} or for 4-jet rates in general [2;4]. However, if these models are modified to accommodate the possibility of using small energy scales ($\mu^2 \ll E_{cm}^2$) in the $O(\alpha_s^2)$ calculations according to [15], we find similar small correction factors in the entire range of y_{cut} as for the shower model shown in Fig. 2 [16].

Independent fragmentation models mostly predict larger deviations between partons and fragmented jets. This, however, is mainly caused by the fact that these models do not conserve energy and momentum in the fragmentation process [17]. In addition, independent jet fragmentation models are not compatible with many other experimental observations [12-14]. They are therefore not considered to provide realistic estimates of fragmentation corrections to jet production rates.

As a further step in the data analysis presented above, the jet production rates observed for $y_{cut} = 0.04, 0.06$ and 0.08 are explicitly corrected for fragmentation, initial state radiation and detector effects. The corrections are determined from the model calculations presented in Fig. 2 and are applied in two steps. The migration of m -parton events into classes of n -jet events, caused by fragmentation processes, is corrected for by multiplying the vector of the uncorrected jet data with the inverse of a 3×3 migration matrix given by the model calculations. Initial state radiation and detector resolution effects are taken into account by a vector multiplication method. In Table 2, the results are listed and compared to the uncorrected data. As can be seen, the corrected “ m -parton” rates of the data are, within the statistical errors, identical to the uncorrected n -jet event rates. We verified that these results are independent of reasonable parameter changes within the model calculations.

From these studies we conclude that it is feasible, at least in the region of $y_{cut} \geq 0.04$, to compare experimental jet production rates directly with theoretical calculations on parton production cross sections. Possible corrections for fragmentation effects are small and can be absorbed in small readjustments ($\approx 6\%$) of α_s . These findings are in good agreement with previously performed studies [2-5], which also demonstrated that these corrections are energy independent above $E_{cm} \approx 25$ GeV. They are also supported by the theoretical conjecture of “Local Parton Hadron Duality” [18;19]. The close agreement between jets and partons opens a wealth of possibilities of verifying the predictions and dynamics of perturbative QCD in existing and future e^+e^- experiments, without the usual reliance on fragmentation models.

A Test of the Running α_s .

As an example of the possibilities indicated above and as a unique test of the non-Abelian structure and the validity of QCD, investigations of jet production rates provide an oppor-

tunity to test the energy dependence of the strong coupling strength without determining explicit values of α_s [3-5;8;20]. For constant values of y_{min} and according to Eq. 1, the energy dependence of jet production rates is only determined by the energy evolution of α_s . Therefore in Fig. 3 we compare our 3-jet production rates with those from JADE [3], TASSO [4] and AMY [20] in the center of mass energy range from 22 GeV to 56 GeV for various values of y_{cut} . The data are also compared to the most recent QCD calculations complete to $O(\alpha_s^2)$ performed by Gottschalk and Shatz (GS) [21] and by Kramer and Lampe (KL) [22]. The QCD parameter $\Lambda_{\overline{MS}}$ was optimized¹ to describe the data for $y_{cut} \geq 0.06$.

The data are compatible with each other and with the theoretical expectations based on an energy dependent coupling strength. The assumption of an energy independent coupling strength, however, is not compatible with the data. In such a case, R_3 is expected not to depend on the center of mass energy. We also note that the predictions of the two theoretical calculations agree quite well with each other and that, except for $y_{cut} \leq 0.04$, the calculations describe the data in the entire range of jet masses with one consistent value of $\Lambda_{\overline{MS}}$. Further investigations [16] show, however, that even this discrepancy vanishes if the data are compared with $O(\alpha_s^2)$ calculations which use smaller energy scales ($\mu^2 \ll E_{cm}^2$) [15].

In order to quantitatively express the significance of these observations, we present results of fits obtained at $y_{cut} = 0.08$, where the most data are available. The QCD calculations shown in Fig. 3 result in $\chi^2 = 11.5$ for 9 degrees of freedom, while a comparison with an energy independent 3-jet rate only yields $\chi^2 = 56.7$. Neglecting the data points at $E_{cm} = 22$ GeV, which might be affected by fragmentation fluctuations [3;4], $\chi^2 = 29.7$ for 7 degrees of freedom for the hypothesis of an energy independent coupling constant (corresponding to a confidence level (CL) of 10^{-4}), while $\chi^2 = 6.1$ for the QCD calculations (CL = 0.5). The case of an energy independent α_s is clearly ruled out by these numbers.

Summary

Production rates of multijet hadronic final states, observed in e^+e^- annihilations at $E_{cm} = 29$ GeV, are studied by using a jet finding algorithm that defines resolvable jets in terms of minimum required jet pair masses. The data are compared with QCD shower model calculations including a first order matrix element weighting method at the first gluon vertex. The model reproduces the observed relative production rates of 2-, 3-, 4- and 5-jet events over a large range of jet pair masses. Further studies show that the applied method of defining and reconstructing jets in hadronic events ensures that corrections due to fragmentation, initial state photon radiation and acceptance and finite resolution of the Mark-II detector are sufficiently small to directly compare experimental rates of n-jet events to theoretical calculations

¹The fitted value of $\Lambda_{\overline{MS}} = 210 \text{ MeV} \pm 13 \text{ MeV}$ for the calculations of GS corresponds to $\alpha_s(29\text{GeV}) = 0.144$, which is slightly less than our previous result of $\alpha_s(29\text{GeV}) = 0.158$ obtained in a study of energy-energy correlations [9]. The difference is due to the fact that fragmentation corrections, predicted by the Lund model as shown in Fig. 2, are not taken into account at this point. Such corrections would shift the corresponding values for α_s by about +6% to $\alpha_s(29\text{GeV}) = 0.153$, which is in excellent agreement with the result in [9].

of n-parton events. This possibility allows one to test the predictions of existing and future QCD calculations without too much dependence on phenomenological fragmentation models. In order to verify the energy dependence of the strong coupling strength, α_s , 3-jet rates observed by different experiments in the center of mass energy range from 22 GeV to 56 GeV are compiled and are compared to different QCD calculations in $O(\alpha_s^2)$. The data agree well with each other and with the QCD expectation of an energy dependent α_s , while the possibility of an energy *independent* coupling strength can be ruled out. This evidence and the ability of second order QCD calculations and of QCD shower model calculations to describe experimental jet data in a wide range of jet masses and center of mass energies enhance the confidence in such theoretical predictions for future experiments at higher energies.

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y_{cut}	R_2	R_3	R_4	R_5
0.015	18.1 ± 0.5	47.4 ± 0.6	28.0 ± 0.5	6.44 ± 0.29
0.02	26.2 ± 0.5	51.4 ± 0.6	20.0 ± 0.5	2.49 ± 0.18
0.03	40.2 ± 0.6	49.9 ± 0.6	9.52 ± 0.34	0.40 ± 0.07
0.04	51.4 ± 0.6	43.7 ± 0.6	4.83 ± 0.25	---
0.05	60.2 ± 0.6	37.4 ± 0.6	2.38 ± 0.18	---
0.06	67.2 ± 0.5	31.5 ± 0.5	1.31 ± 0.13	---
0.08	76.7 ± 0.5	22.9 ± 0.5	0.41 ± 0.07	---
0.10	83.5 ± 0.4	16.4 ± 0.4	0.07 ± 0.03	---
0.12	88.0 ± 0.4	12.0 ± 0.4	---	---
0.14	91.1 ± 0.3	8.93 ± 0.33	---	---

Table 1. Observed production rates R_n of 2-, 3-, 4- and 5-jet events at $E_{cm} = 29$ GeV, in % of the total hadronic cross section.

y_{cut}	Obs.	Data uncorr.	Data corr.
0.04	R_2	51.4 ± 0.6	52.9 ± 0.8
	R_3	43.7 ± 0.6	42.2 ± 0.8
	R_4	4.83 ± 0.25	4.96 ± 0.35
0.06	R_2	67.2 ± 0.5	67.0 ± 0.8
	R_3	31.5 ± 0.5	31.7 ± 0.8
	R_4	1.31 ± 0.13	1.34 ± 0.20
0.08	R_2	76.7 ± 0.5	76.2 ± 0.8
	R_3	22.9 ± 0.5	23.4 ± 0.8
	R_4	0.41 ± 0.07	0.36 ± 0.10

Table 2. Production rates R_n of 2-, 3-, and 4-jet events at $E_{cm} = 29$ GeV, before and after correction for fragmentation, initial state radiation and detector resolution effects, in % of the total hadronic cross section.

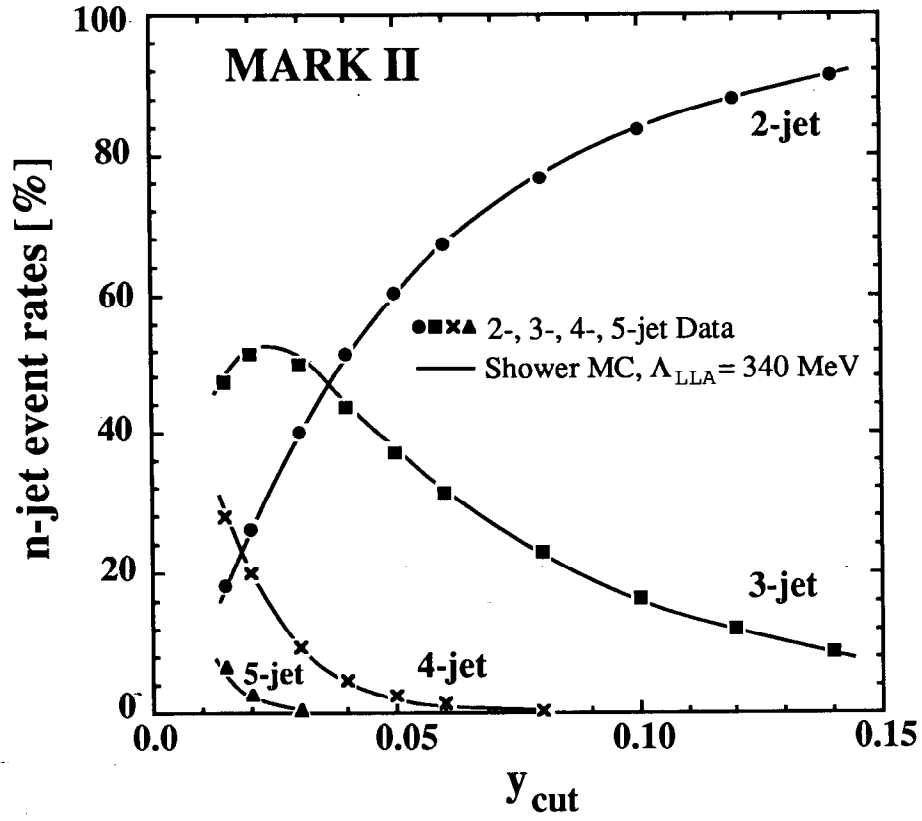


Fig.1 The observed 2-, 3-, 4- and 5-jet event rates for different values of y_{cut} , together with the corresponding results of the Lund QCD shower and fragmentation model.

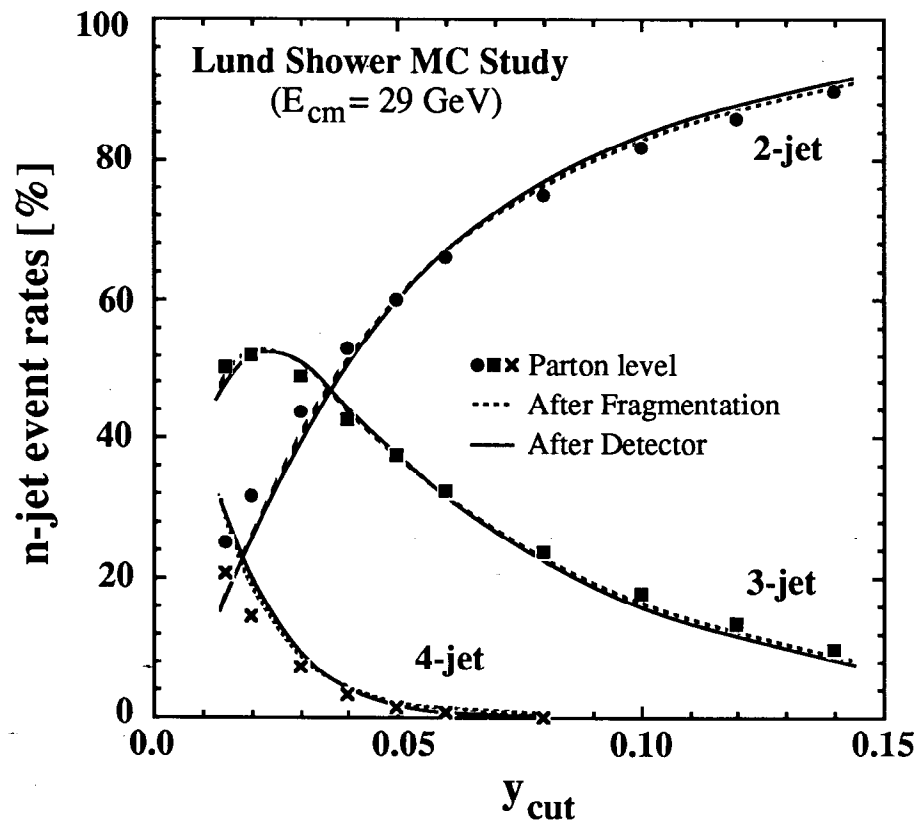


Fig.2 The influence of fragmentation as well as of initial state radiation and detector resolution on n-jet event rates, simulated in model calculations, as a function of y_{cut} .

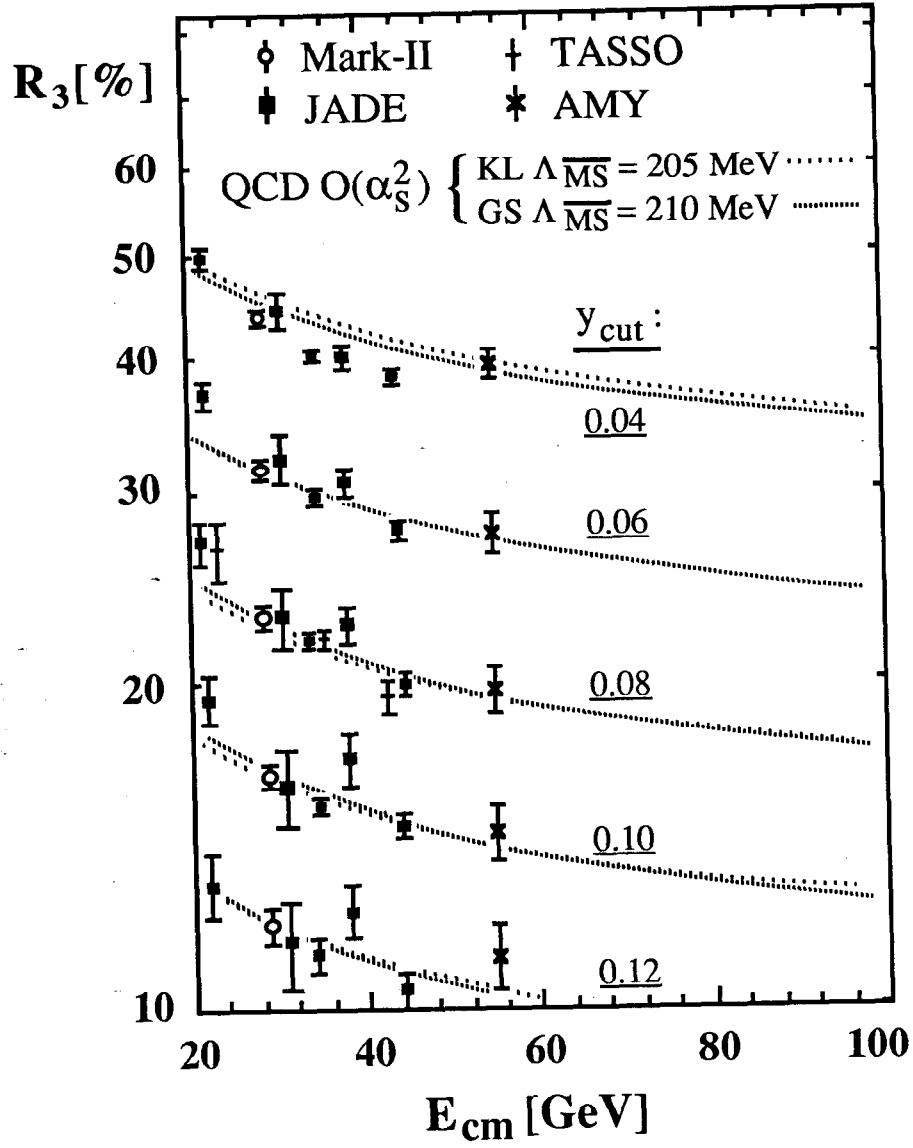


Fig.3 Compilation of 3-jet event production rates at different center of mass energies, compared with $O(\alpha_s^2)$ calculations of Kramer and Lampe (KL) and of Gottschalk and Shatz (GS).