

Black holes, information and holography

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Abstract. We review some aspects of the information problem in black hole physics, from the point of view holographic ideas, and the AdS/CFT correspondence in particular.

1. Introduction

The information paradox envisioned by Hawking in the mid seventies continues to represent a crucial playground for ideas, methods and theories of quantum gravity [1, 2]. The most important departure from conventional thinking in recent years, the *holographic principle*, finds one of its origins in the challenge posed by Hawking to the dogma of quantum unitary evolution.

A concrete mathematical model of holography is given by the AdS/CFT correspondence [3]. In its strongest incarnation, it provides a *definition* of quantum gravity on asymptotically AdS spacetimes, in terms of a conformal field theory without gravity, living on a spacetime of one dimension less. For particular models, such as the basic template relating the maximally supersymmetric Super Yang–Mills theory in 3+1 dimensions, with the type IIB string theory on $\text{AdS}_5 \times \mathbf{S}^5$, one can define expansion parameters on both sides of the duality, such as the $1/N$ expansion of 't Hooft, as dual to the loop expansion in the string theory. These expansions can be used to set up a number of concrete tests of the duality, which may be qualitative or even quantitative in some cases, all of them coming out positive so far.

To the extent that we can mimic the occurrence of a long-lived black-hole resonance in scattering inside AdS, the dual CFT description guarantees that the whole process is unitary. Still, the original argument of Hawking, perhaps refined with a more modern parlance, remains remarkably resilient, and it has not been easy to point out what particular step is to be held responsible for the erroneous conclusion that unitarity will be violated.

2. The paradox

Let us recall Hawking's argument by considering a typical history of formation-and-evaporation for a large semiclassical black hole, as in figure 1.

Viewed as a scattering history, we have the evolution of a state from the *in* Hilbert space \mathcal{H}_{in} in the asymptotic past at spatial infinity, to the *out* Hilbert space \mathcal{H}_{out} at the asymptotic future, again at spatial infinity. If the Carter–Penrose diagram drawn in the picture is a good approximation to the effective geometry, it is clear that \mathcal{H}_{out} is not complete, for \mathcal{H}_{in} is actually in the past of the complete Hilbert space $\mathcal{H}_{\text{out}} \otimes \mathcal{H}_{\text{sing}}$, where we have denoted $\mathcal{H}_{\text{sing}}$ the space of all states that arise formally as the limit of Hamiltonian evolution into the singularity from states localized *inside* the black hole horizon.

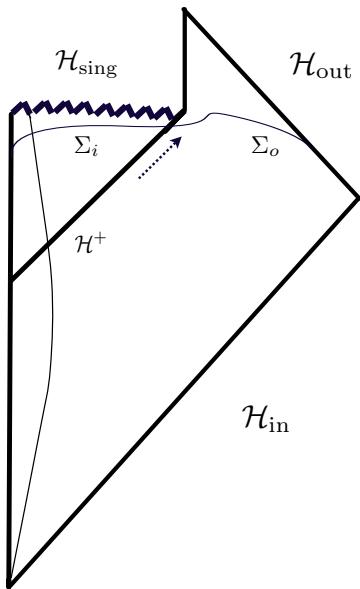


Figure 1. Carter–Penrose diagram for a scattering event with a black-hole intermediate state. The formation of the event horizon \mathcal{H}^+ , whose Hamiltonian evolution hits a spacelike singularity, renders generic *out* states *mixed*. The spacelike Cauchy surface $\Sigma_i \cup \Sigma_o$ is complete, but contains degrees of freedom that never make it to \mathcal{H}_{out} .

The condition that all measurements on \mathcal{H}_{out} are effectively *inclusive*, so that we always trace over degrees of freedom (d.o.f) supported on $\mathcal{H}_{\text{sing}}$, assumes implicitly that all d.o.f. on spacelike surfaces like $\Sigma = \Sigma_i \cup \Sigma_o$ are independent. This condition is essentially the *locality* of the effective Quantum Field Theory (QFT), i.e. for local operators supported respectively on the inside, Σ_i , or the outside Σ_o portions of a spacelike Cauchy surface, we have

$$[\mathcal{O}_{\Sigma_i}, \mathcal{O}_{\Sigma_o}] = 0. \quad (1)$$

Hence, locality of the low-energy QFT on this effective geometry leads to the evolution of pure states into mixed states in the map $\mathcal{H}_{\text{in}} \rightarrow \mathcal{H}_{\text{out}}$. It remains to check how reliable is the geometry of the whole Carter–Penrose diagram, and the hypersurface Σ in particular. On general grounds, the low-energy effective field theory will be a good approximation whenever the effective loop expansion parameter

$$\lambda_{\text{eff}} = \frac{G}{R_s^2} \quad (2)$$

is small, where G is Newton's constant and $R_s = 2GM$ is the Schwarzschild radius of the black hole, controlling the value of the local curvature at the horizon. Taking into account that the Beckenstein–Hawking entropy $S_{\text{BH}} = A_H/4G \sim R_s^2/G$, we have $\lambda_{\text{eff}} \sim 1/S_{\text{BH}} \ll 1$ even for a moderately microscopic black hole. Hence, for $M > M_{\text{P}}$ the low-energy QFT treatment should be *very* accurate.

The Cauchy surface Σ can be chosen to be a ‘nice slice’, i.e. one that stays clear of high-curvature regions near the singularity and whose extrinsic curvature is also small in Planck units [4]. Furthermore, it is possible to place Σ in such a way that Σ_o intersects most of the Hawking radiation on its way to \mathcal{H}_{out} . This means that local measurements on Σ_o can test whether the state of the Hawking radiation corresponding to almost all the mass M , is approximately pure or not. Local QFT is a good approximation on Σ until most of the Hawking radiation has passed through Σ_o .

Thus, we are forced to settle for one of two options:

- $\mathcal{H}_{\text{sing}}$ is not measurable and the information stored there literally disappears; quantum mechanics must be amended, and the scattering matrix is not unitary. A variant of this

would have $\mathcal{H}_{\text{sing}}$ supported on ‘another universe’, the singularity being smoothed out into a wormhole passage.

- $\mathcal{H}_{\text{sing}}$ is accessible as part of \mathcal{H}_{out} in the form of a *remnant*; a Planck-mass ‘particle’ capable of storing arbitrarily large amounts of information.

Each of these alternatives has notorious problems. Violations of quantum coherence are constrained by rather general arguments [5]. Remnants pose formidable problems at the phenomenological level [6]. On the other hand, none of these alternatives seems compatible with the AdS/CFT correspondence, which guarantees full unitarity of the final S-matrix.

If we insist on enforcing the unitarity, another basic principle must yield, and the natural candidate is QFT locality (1). This is the basic tenet of the ‘black-hole complementarity’ scenario, according to which the S-matrix questions can be addressed in an effective model using only the exterior geometry (the time reversal of \mathcal{H}_{out}) [7]. The mathematical surface at \mathcal{H}^+ is replaced by the ‘stretched horizon’, of Planckian thickness, storing a quantum system with one bit per Planck area that thermalizes all infalling states and radiates back in the form of Hawking radiation.

Of course, such a picture is grossly inconsistent with locality and the Equivalence Principle [8]. On the other hand, low-energy QFT should not be completely ‘wrong’. So one question we can ask is the size of ‘nonlocality’ corrections to (1) that are needed to restore unitarity.

3. Information retrieval and non-locality

The time scale for the retrieval of information, t_{bit} , has been estimated by Page to be of the order of the evaporation time [9, 10]. The argument rests on the operational definition of information as the difference between the entanglement entropy and the coarse grained thermal entropy. For a bipartite partition of a system, $A \cup B$, in a pure state $|\psi\rangle$, we define the marginal entanglement density matrix by $\rho_A = \text{Tr}_B |\psi\rangle\langle\psi|$, and the corresponding entanglement entropy as $S_A = -\text{Tr}_A \rho_A \log(\rho_A)$. On the other hand, the coarse-grained thermal entropy is defined as $S_{T,A} = \log(D_{T,A})$, where $D_{T,A}$ is the dimension of the space of states with fixed energy E_A on the A side. This definition assumes that the total energy of the state $E = \langle\psi|H|\psi\rangle$ is well approximated by an extensive partition into A and B , i.e. $E \approx E_A + E_B$. In practice, this requires demanding that both A and B are macroscopic. With these preliminaries, one can define the information retrieval as

$$I_A = S_{T,A} - S_A . \quad (3)$$

By studying model systems, Page was able to argue that $I_A \approx 0$ until A and B become of comparable size, with the turning point corresponding to $E_A \approx E_B \approx E/2$. Hence, in the black hole case, the first bit will come out roughly at the time of half-evaporation of the black hole. This is, in order of magnitude, the evaporation time itself:

$$t_{\text{bit}} \sim t_{\text{ev}} \sim \frac{R_s^3}{G} \sim R_s S_{\text{BH}} \sim \frac{R_s}{S_{\text{BH}}} . \quad (4)$$

Quantum amplitudes in the background of a black hole typically decay exponentially in time with a characteristic time scale of the order of R_s . Therefore, the size of effects that start at times of order t_{bit} is

$$e^{-t_{\text{bit}}/R_s} \sim e^{-S} , \quad (5)$$

where S is a large number of the order of the black hole entropy S_{BH} . Thus, we conclude that nonlocal effects responsible for the restoration of unitarity should be of order $\exp(-1/\lambda_{\text{eff}})$, nonperturbative in the QFT’s expansion parameter, explaining why it is so difficult to account for the restoration of unitarity in the Hawking description: one would need ‘instanton-like’ effects to achieve the feat.

The utter smallness of these effects explain why the corrections to locality,

$$[\mathcal{O}_{\Sigma_i}, \mathcal{O}_{\Sigma_o}] \sim \exp(-S), \quad (6)$$

remain unnoticed in low-energy QFT, thus only becoming of $O(1)$ for Planck-scale black holes. Another possibility for large non-locality effects would be the consideration of very large operators, [11], so that multiparticle effects exponentiate and cancel the nominal suppression in (6):

$$[\mathcal{O}_S, \mathcal{O}'_S] = O(1), \quad (7)$$

with $\mathcal{O}_S \sim \mathcal{O}_1 \cdot \mathcal{O}_2 \cdots \mathcal{O}_S$ is a product of $O(S)$ local operators. This is in agreement with expectations: the Hawking radiation is emitted in about $M/T_H \sim S$ quanta, so we need an operator of $O(S)$ elementary fields to measure the whole density matrix of the Hawking radiation.

4. Enter AdS/CFT

We can regularize four-dimensional asymptotically flat spaces by a very large Anti-de Sitter spacetime, with curvature radius $R \gg R_s$ much larger than the Schwarzschild radius of any black hole of interest. The basic AdS/CFT dictionary defines quantum gravity on such a AdS₄ with curvature radius R in terms of a three-dimensional conformal field theory on a spatial sphere \mathbf{S}^2 of radius R and with central charge of order N_{dof} with

$$N_{\text{dof}} = \frac{R^2}{G}. \quad (8)$$

A Schwarzschild black hole of radius $R_s \ll R$, formed by scattering from the AdS boundary, will decay back into Hawking radiation, reaching an equilibrium thermal state with $T < 1/R$. The whole process can be understood as some Hamiltonian evolution in the gravity-free CFT on $\mathbf{R} \times \mathbf{S}^2$, which guarantees the unitarity of the process. When considering very massive black holes with $M \sim N_{\text{dof}}/R$, the Schwarzschild radius reaches the curvature radius $R_s \sim R$. At this point the nature of the black holes in AdS changes, for black holes with $R_s \gg R$ are strongly affected by the negative curvature. In particular, they have positive specific heat and can coexist in equilibrium with their own Hawking radiation [12]. This means that these large AdS black holes do not really evaporate, and can be associated with equilibrium thermal states of the CFT,

$$\rho_T = \frac{e^{-H/T}}{\text{Tr } e^{-H/T}}, \quad (9)$$

at temperature $T \gg 1/R$ and with entropy $S \sim N_{\text{dof}}(RT)^2$.

While small black holes with $R_s \ll R$ are more faithful to the original physics question in asymptotically flat spacetimes, very large AdS black holes have the advantage that their CFT dual description is known with good approximation. This fact makes them actually more interesting from the point of view of the information paradox.

A version of the paradox can be reformulated for these large black holes by considering perturbations [13]. A small perturbation (like throwing a small object to the black hole) will dissipate at the black-hole horizon and mix through the available phase space. Hence, the time for information retrieval of such perturbation is expected to be of the order of the retrieval time for any other bit of information in the black hole. Although these black holes do not evaporate, i.e. $t_{\text{ev}} = \infty$, the time scale $t_{\text{bit}} \sim S/T$, with T the effective CFT temperature, is still a natural guess for the information retrieval time. The question we want to address is whether one can dig out t_{bit} from natural observables on the CFT side.

The answer is positive. First, the natural physical quantity to monitor is the time correlation function of a generic CFT operator

$$G(t) = \langle \text{Tr } \rho_T \mathcal{O}(t) \mathcal{O}(0) \rangle_{\text{CFT}}. \quad (10)$$

Second, if the matrix elements of \mathcal{O} in the energy basis form a random banded matrix of width Γ , the generic behavior of the correlator is a decay $\exp(-\Gamma t)$ for initial times, and a pattern of Poincaré recurrences at very late times, of order $t_H \sim \Gamma^{-1} \exp(S)$. In typical systems $\Gamma \sim T^{-1}$ times factors of coupling constants. With these constraints, one expects the infinite time average of the correlator to be of order [14, 15, 16, 17, 18]

$$\overline{G(t)} = \frac{\Gamma}{t_H} \sim \exp(-S) \sim \exp(-1/\lambda_{\text{eff}}). \quad (11)$$

We can use these results to draw a sharp divide between the low-energy QFT and the fully nonperturbative treatment of the CFT. When computing $G(t)$ in the bulk gravitational description in AdS, one always finds a quasinormal behavior $G(t) \sim \exp(-\Gamma t)$, with $\Gamma \sim T$ and determined by the lowest quasinormal frequency of the black hole. There is no sign of the Poincaré recurrences in this approximation, and in fact their absence is guaranteed to all orders in powers of λ_{eff} , because one can prove that the mode spectrum of quantum fields in the black hole background is *continuous*. This infrared phenomenon is also the source of the famous horizon divergence of the entropy contributed by quantum fields, as emphasized by 't Hooft, and represents the sharpest 'loophole' in Hawking's treatment of the information propagation in black holes.¹

Finally, we can also confirm that the information retrieval time $t_{\text{bit}} \sim S/T$ may be recovered from the structure of $G(t)$ by looking at the time scale at which the quasinormal behavior becomes dominated by the long-time fluctuation noise (c.f. [17]), given by the time average, i.e.

$$\overline{G(t)} \sim e^{-S} \sim e^{-\Gamma t_{\text{bit}}}, \quad (12)$$

thus recovering $t_{\text{bit}} \sim S/\Gamma \sim S/T$.

5. Outlook

We have argued that the AdS/CFT correspondence offers the best chance of a sharp formulation of the information paradox in black holes, by reducing it to mathematically well-defined criteria for time correlation functions. More recently, these questions have been addressed in matrix models, as simplified versions of the boundary CFT [19, 20, 21]. The results confirm that the quasinormal behavior (or power-like analogs) are robust properties of the $1/N$ expansion in these matrix models.

In another interesting turn, Hayden and Preskill have defined a new time scale relevant to the information processing by the black hole [22]. By considering the status of the no-cloning theorem for an observer that *knows* the exact microscopic state of a black hole, it was concluded that information retrieval cannot be faster than the so-called *scrambling* time $t_{\text{scr}} \sim T^{-1} \log(S)$ (see also [23]). It remains a very interesting open problem to identify the scrambling time in terms of CFT correlation functions, or perhaps more elaborated observables.

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¹ The 'instanton-like' form prompted suggestions of concrete semiclassical tunneling processes that could account for the time average [13]. The time average (11) is correctly estimated by the tunneling between a classical black hole solution and the vacuum AdS manifold filled with thermal gravitons at the same temperature, although the detailed time structure of the correlator is not correctly reproduced, since no sign of the longest Poincaré recurrences is found [17].

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