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PROCEEDINGS OF SCIENCE

Scalar QED with Rydberg atoms

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We review recent suggestions to quantum simulate scalar electrodynamics (the lattice Abelian Higgs model) in $1 + 1$ dimensions with rectangular arrays of Rydberg atoms. We show that platforms made publicly available recently allow empirical explorations of the critical behavior of quantum simulators. We discuss recent progress regarding the phase diagram of two-leg ladders, effective Hamiltonian approaches and the construction of hybrid quantum algorithms targeting hadronization in collider physics event generators.

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1. Introduction

In these Proceedings we discuss the possibility of hybrid quantum/classical computing for event generators such as PYTHIA [1, 2]. Our long-term goal is to replace the current hadronization part which turns quarks and gluons into hadrons and is currently implemented with the phenomenological Lund model, by an ab-initio lattice calculation performed by a quantum computing device. More technical details and references can be found in [3].

Recently there has been a lot of interest for quantum simulation for gauge theories [4–9] and in particular for the real-time dynamics. In the following, we consider the possibility where the hadronization part of event generators could be described by a simplified lattice model: the compact Abelian Higgs Model (Scalar QED) in 1 + 1 dimensions. We explain that is (literally!) possible to attempt model building for a quantum simulator with arrays of Rydberg atoms [10–12]: publicly available interfaces [13, 14] allow users to engage in quantum simulations in a rather straightforward way. The matching between the target model and the simulator is non-trivial. However, we show that for some region of the parameters of the simulator, it is possible to construct an effective theory for the simulator which has only one (important) term differing from the target model. More details about this question can be found in a recent preprint [29]. We also briefly discuss practical examples of current work with QuEra.

2. Compact Abelian Higgs Model (CAHM)

The lattice compact Abelian Higgs model is a non-perturbative regularized formulation of scalar quantum electrodynamics (scalar electrons-positrons + photons with compact fields). The partition function reads:

$$Z_{CAHM} = \prod_x \int_{-\pi}^{\pi} \frac{d\varphi_x}{2\pi} \prod_{x,\mu} \int_{-\pi}^{\pi} \frac{dA_{x,\mu}}{2\pi} e^{-S_{gauge}-S_{matter}},$$

with

$$S_{gauge} = \beta_{plaquette} \sum_{x,\mu<\nu} (1 - \cos(A_{x,\mu} + A_{x+\hat{\mu},\nu} - A_{x+\hat{\nu},\mu} - A_{x,\nu})),$$

$$S_{matter} = \beta_{link} \sum_{x,\mu} (1 - \cos(\varphi_{x+\hat{\mu}} - \varphi_x + A_{x,\mu})).$$

It has a local invariance: $\varphi'_x = \varphi_x + \alpha_x$ and where local changes in S_{matter} are compensated by $A'_{x,\mu} = A_{x,\mu} - (\alpha_{x+\hat{\mu}} - \alpha_x)$. φ is the Nambu-Goldstone mode of the original model. The Brout-Englert-Higgs mode is decoupled (heavy). Using methods of Tensor Lattice Field Theory (TLFT) reviewed in [28], one obtains an Hamiltonian and Hilbert space in 1 + 1 dimension in the continuous-time limit:

$$H = \frac{U}{2} \sum_{i=1}^{N_s} \left(L_i^z\right)^2 + \frac{Y}{2} \sum_i (L_{i+1}^z - L_i^z)^2 - X \sum_{i=1}^{N_s} U_i^x$$

with $U^x \equiv \frac{1}{2}(U^+ + U^-)$ and $L^z|m\rangle = m|m\rangle$ and $U^\pm|m\rangle = |m \pm 1\rangle$. m is a discrete electric field quantum number ($-\infty < m < +\infty$). In practice, we need to apply truncations: For a spin-m_{max} truncation we have $U^\pm| \pm m_{max}\rangle = 0$. We focus on the spin-1 truncation ($m = \pm 1, 0$ and

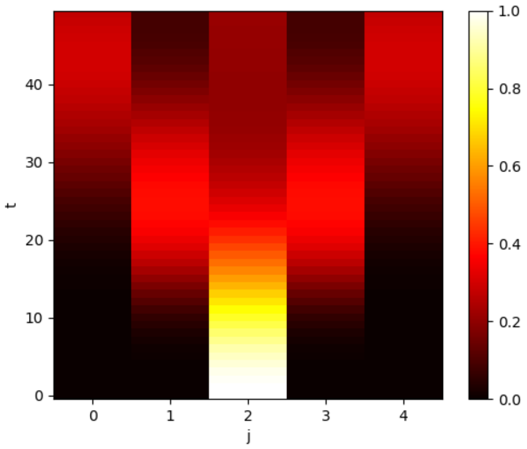

Figure 1: Electric field E_j versus position j for the evolution of a particle-antiparticle pair for the Abelian Higgs model in $1+1$ dimensions for $Y/X = 1$ and $U/X = 0.1$ and 50 time steps with Δt such that $X\Delta t = 0.1$.

$U^x = L^x/\sqrt{2}$). The U-term represents the electric field energy, the Y-term, the matter charges (determined by Gauss's law) and the X-term: currents inducing temporal changes in the electric field. Using this Hamiltonian, we have evolved an intial state with a bit of electric field in the middle. By Gauss's law, this is equivalent to a pair of bosonic particle-antiparticle separated by one lattice spacing. An example is shown in Fig. 1. and various stages can be reinterpreted in terms of patterns such as string breaking, discussed in Refs. [1, 2].

3. The simulator

In [18], we adapted the optical lattice construction [29] using arrays of ^{87}Rb atoms [10–14] separated by controllable (but not too small) distances, coupled to the excited Rydberg state $|r\rangle$ with a detuning Δ. The ground state is denoted $|g\rangle$ and the two possible states $|g\rangle$ and $|r\rangle$ can be seen as a qubit. We have the occupations $n|g\rangle = 0,\ n|r\rangle = |r\rangle$. The Hamiltonian reads

$$H = \frac{\Omega}{2}\sum_i (|g_i\rangle\langle r_i| + |r_i\rangle\langle g_i|) - \Delta\sum_i n_i + \sum_{i<j} V_{ij} n_i n_j,$$

with

$$V_{ij} = \Omega R_b^6/r_{ij}^6,$$

for a distance r_{ij} between the atoms labelled as i and j. Note that when $r = R_b$, the Rydberg radius, $V = \Omega$. This repulsive interaction prevents two atoms close enough to each other to be both in the $|r\rangle$ state. This is the so-called blockade mechanism which can be used to produce an effective spin-1 local Hilbert space. In order to shortcut the discussion of the staggered interpretation of the electric field, we have plotted its square $E_j^2 = (n_j^{up} - n_j^{down})^2$ (see [18] for a discussion) in Fig. 2 which also exhibit interesting behavior with a similar interpretation as for Fig. 1.

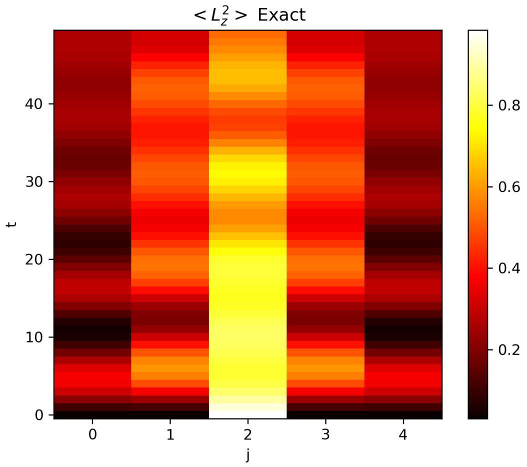

Figure 2: Time evolution for $E_j^2 = (n_j^{up} - n_j^{down})^2$ for a 5-rung ladder (ten ^{87}Rb atoms). At initial time, all the atoms are in the ground state except for the top atom in the middle rung. $\Omega = 2\pi \times 2 * 10^6$ MHz, $\Delta = 2\Omega$, $a_x = R_b$ $a_y = 0.5R_b$.

4. The effective Hamiltonian and phase diagram

As reported in a recent preprint [29], we constructed a translation-invariant effective Hamiltonian by integrating over the simulator high-energy states produced by the blockade mechanism. Remarkably, for all the simulators considered (ladders, prisms, and others), the effective Hamiltonians have the three types of terms present for the CAHM (Electric field, matter charge and currents energies) but, in addition, terms quartic in the electric field. For positive detuning, the new terms create degenerate vacua resulting in a very interesting phase diagram.

The phase diagram of the ladder simulator with rung size twice the lattice spacing has been investigated experimentally. Evidence for an incommensurate phase between crystalline commensurate phases will be reported in an upcoming preprint. It is relatively straightforward for theoretical users to perform analog simulations with Rydberg arrays using publicly available interfaces [13, 14]. We hope that this exploration will improve our understanding of inhomogeneous phases and the Lifshitz regime of lattice quantum chromodynamics [30, 31], as well as the examination of "chiral spiral" condensation [32].

5. Conclusions

We have considered ladder-shaped Rydberg arrays with two atom per rung as simulators for the compact Abelian Higgs model. Ultimately the matching between simulator and target model should be understood in the continuum limit (universal behavior). Effective Hamiltonians for the simulator were found with same three types of terms as the target model plus an extra quartic term. The two-leg ladder has a very rich phase diagram. Explorations of the phase diagram with AWS/QuEra are ongoing as well as the possibility of an hybrid interface with PYTHIA are pursued.

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