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Some Warnings About Quantum Space Gravimetry Enhance Earth Observations Project

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Abstract. In this paper, we discuss in brief some basic issues of quantum space gravimetry, related to standard approach of geodesy which is based on the Newton model of gravity and Euclidean geometry. We emphasize the need to apply relativistic gravity in practical high-precision geodesy.

Here we do not intend to solve the existing hard experimental and theoretical problems, being essential for the topic: development of quantum gravity, physics of dark matter and dark energy, novel physical principles of extended general relativity, in particular, a nonlinear superposition principle in general relativity and its extensions, and so on.

Rather, we point out the fundamental unsolved problems, which are substantial for quantum space gravimetry and future practical high-precision geodesy. We outline the possible ways for their study and decision. Thus, to some extend, the present paper is a program for further developments, not a presentation of the final solutions.

Our goal is to warn corresponding scientific community about the ultimate necessity for going outside the frameworks of the formulated more than three century ago, and used up to now in geodesy, Newton gravity, together with Euclidian geometry.

At present, in the emerging high-precision geodesy one must replace them with modern models of gravity and corresponding non-Euclidean geometry.

Without using and further development of those issues, the interpretation of data obtained from high-precision measurements by satellites for geodetic use seems to be quite problematic, uncertain, and may be misleading for practitioners.

1. Introduction

Nowadays, the theory of gravity and its experimental verifications are under close scrutiny.

The main reasons are: Newton gravity, with its linear principle of superposition of gravitational fields of arbitrary sources, is not adequate in the case of modern high-precision-experiments and observations in celestial mechanics, astrophysics, cosmology, etc.

1. For many high-precision practical uses, the Newton model of gravity is replaced by General Relativity (GR), possibly with proper cosmological constant Λ.

2. On the one hand, GR describes very well a lot of phenomena at very different scales and we are still waiting for first indications of serious discrepancies between GR and observations and/or experiments.

3. On the other hand, it is obvious that GR cannot be the final theory of gravity, at least because after an almost a century of different intensive efforts and attempts, we are still unable

to make GR compatible with Quantum Mechanics (QM), as well as with some firmly established, but still not well-understood phenomena like Dark Matter (DM) and Dark Energy (DE).

4. There exists a striking discrepancy between high-precision data of measurements of the Newton constant G in various places on the Earth surface, see [1] and references therein, Fig.1, and also [2]. The cause may be in different systematic errors and different precisions of variety of devices, used for measuring of the Newton constant G.

The main unsolved question: "Is the Newton constant G really a constant or it is some kind of field?" must be solved experimentally by measurements on the Earth surface and in the space with the same mobile equipment, the same precision and the same systematic errors.

Note that one has to make difference between measurements of the Newton constant G, and the measurements of the gravitational acceleration g at given place, since for them one needs quite different experimental equipment [4, 5].

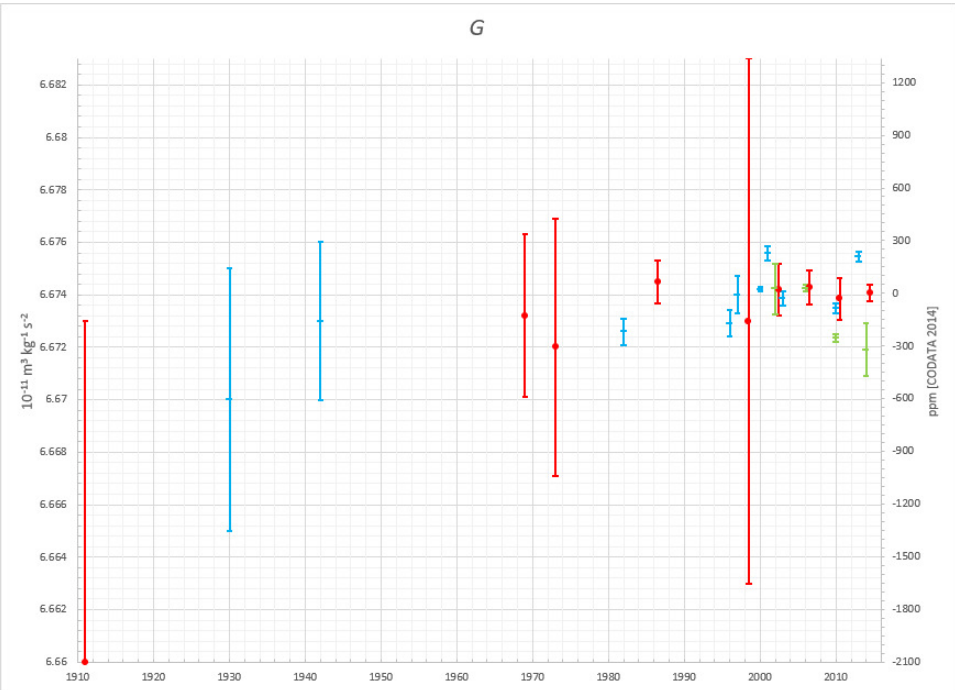

Figure 1. The results of measurements of the Newton constant G in different places on the Earth surface. The clear discrepancy between high-precision measurements during the last decades is still a serious mystery.

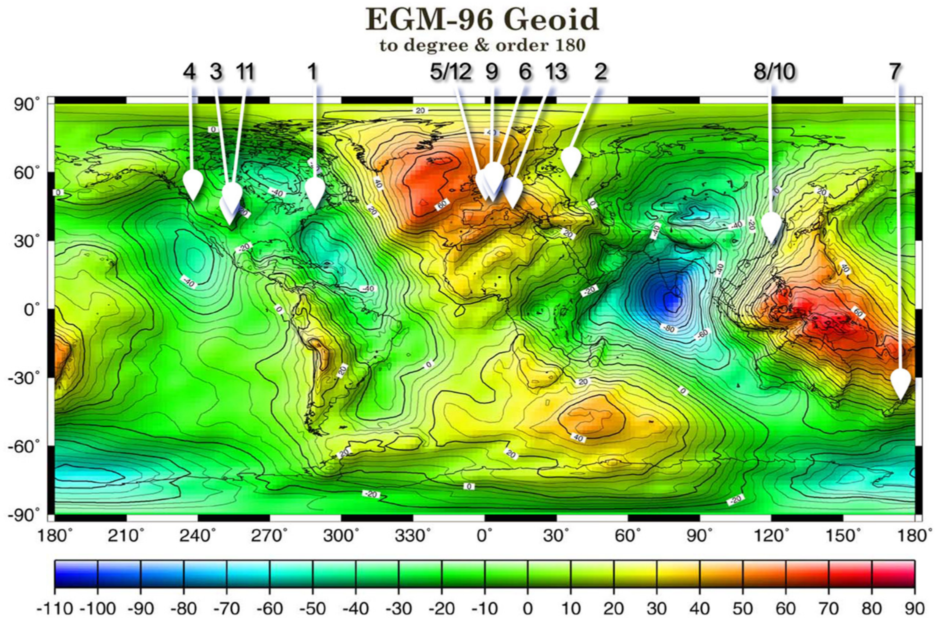

Figure 2. The pointers show the places where the data from Table 1 were obtained. As we see, these places are in a very narrow band on the Earth surface and do not give a representable picture of the values of Newton constant around the globe.

Table 1. The numerical results of the known high-precision measurements of the Newton constant G in units $G \times 10^{11}\,\mathrm{m}^3\,\mathrm{kg}^{-1}\,\mathrm{s}^{-2}$.

No	*G*	*Uncertainty*	*Altitude*	*Geoid*	*Latitude*	*Longitude*
1	6.67248(43)	6.4×10^{-5}	134.976	-30	39.13	-77.22
2	6.6729(5)	7.5×10^{-5}	151.785	10	55.76	37.62
3	6.6739(70)	1.0×10^{-4}	2160.594	-20	35.84	-106.29
4	6.674255(92)	1.4×10^{-5}	28.618	-30	47.66	-122.30
5	6.67559(27)	1.4×10^{-5}	89.050	50	48.82	2.21
6	6.67422(98)	1.5×10^{-4}	259.822	50	51.25	7.15
7	6.67387(27)	1.5×10^{-4}	0.000	20	-41.21	174.91
8	6.67228(87)	1.3×10^{-4}	38.477	10	30.51	114.41
9	6.67425(12)	1.9×10^{-5}	447.939	50	47.37	8.55
10	6.67349(18)	2.7×10^{-5}	38.477	10	30.51	114.41
11	6.67234(14)	2.1×10^{-5}	134.976	-20	40.01	-105.27
12	6.67545(18)	2.7×10^{-5}	89.050	50	48.82	2.21
13	6.67191(99)	1.48×10^{-4}	50.875	50	43.78	11.26
14	6.674484	1.161×10^{-5}	–	–	–	–
15	6.674184	1.164×10^{-5}	–	–	–	–

For search of time-variations of gravitational constant G see the recent paper [3] and the references therein.

Similar is the situation with the cosmological constant Λ at different places in the Universe [6], see Fig 3:

The main unsolved question: "Is the cosmological constant Λ really a constant, or it is some kind of field?", may be solved experimentally by proper measurements using high precision QSG

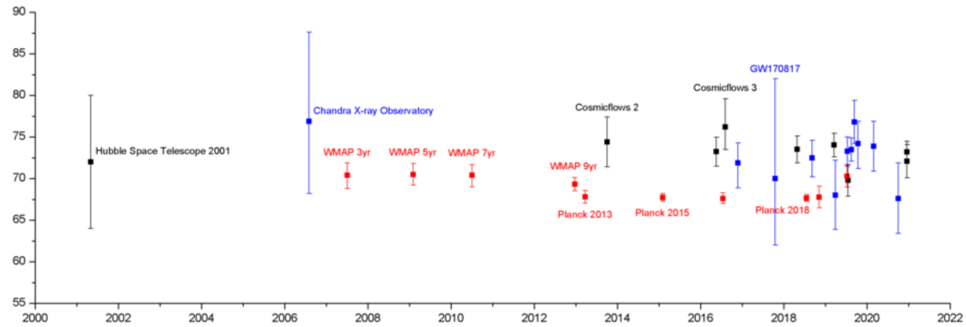

Figure 3. The results of measurements of the cosmological constant Λ in different places in the Universe. The observed in the last years discrepancy is also known as H_0-problem, where $H = \sqrt{\frac{\Lambda c^2}{3\Omega_\Lambda}}$ is the Hubble constant.

on satellites with large-altitude orbits and looking for post-Newtonian exponential tails in the gravitational potential V_{grav}:

$$V_{grav} = -\frac{Gm}{r}\left(1 - a_{vector}e^{-r/\lambda_{vector}} + a_{scalar}e^{-r/\lambda_{scalar}}\right).$$

For a recent review on this topic see [7, 8] and the references therein.

A more general and detailed consideration of the variation of constants can be found in [9].

The origin of the above experimental discrepancies is completely unclear.

It is not excluded that it lies in the properties, e.g. statics and dynamics of Dark Matter and Dark Energy, including their influence on the interior of the Earth, the Moon, the Solar system planets, as well as the Sun. Such influence is currently not sufficiently studied.

5. Absence of a more precise model than PREM [10] of the Earth interior as well as precise models of the interior of the Moon, the planets and the Sun.

6. The obvious conclusion is that today we know much more about the surrounding space than about the interiors of the Earth, the planets, their satellites and the Sun.

Very recently, some progress has been made in describing the Mars' interior [11].

2. Theoretical models

As a result of the circumstances, described in the points 1-6 of the Introduction, during the last several dozens of years a lot of different kind of modifications and extensions of GR were proposed and partially studied in the existing large amount of literature.

All these models cannot be considered to be true physical models of gravity. Instead, they present some mathematical speculations, which aim at the search of a future theory of gravity.

The main physical problem of these mathematical models is the absence of new physical principles for their construction.

In contrast, one can explain the vitality of GR by its deep and experimentally well-established physical principles on which GR was build.

Unfortunately, the basic GR principles are no longer sufficient for further development of the theory of gravity. For this purpose, novel physical principles are needed urgently. The most important issue is their experimental high-precision verification and confirmation.

Just as an example of extended GR model, we present here some preliminary results for possible DM and DE effects on the Earth's gravitational field, obtained in the framework of the simplest scalar-field-modification of GR, called Minimal Dilatonic Gravity (MDG) [12, 16, 17]. It is a further development and physical concretization of the O'Hanlon model of gravity [13].

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In MDG, the Newton constant is replaced by an effective variable relativistic field $G_{eff}(\Phi)$, using a single scalar field Φ of Brans-Dicke type [14]. This field can be related with the known DM-effects. It is introduced according to the relations $G_{eff}(\Phi) = G/\Phi = G\, g(\phi)$.

In addition, the cosmological constant Λ, which is related with DE, together with a cosmological potential $U(\Phi)$, is introduced in MDG [1].

The action of the model is

$$\mathcal{A}_{g_{\mu\nu},\Phi} = \frac{c^3}{16\pi G}\int d^4x\sqrt{|\det(g_{\mu\nu})|}\,(\Phi R - 2\Lambda U(\Phi))$$

Just to illustrate the novel effects of DM and DE in the framework of MDG we can consider a simple model of the Earth, as a spherically symmetric non-rotating body with mass distribution, given by the experimentally established PREM [10].

This physical approximation is good-enough for qualitative consideration of the MDG-model of the Earth. For the first time, such model was applied to the gravitational field of the Earth in [2].

The hydrostatic equations of the static spherically symmetric bodies in MDG are a nontrivial generalization of the well-known Tolman-Oppehneimer-Volkov equations in GR, which, in its turn, generalize the hydrostatic Newton equations for static spherically symmetric objects, see for example, [15] and the references therein.

The detailed derivation of hydrostatic equations for four unknown functions $m(r)$, $\Phi(r)$, $p_{\Phi}(r)$, and $p(r)$ of the static spherically symmetric bodies in MDG:

$$\frac{dm}{dr} = 4\pi r^2\epsilon_{eff}/\Phi, \quad (1)$$

$$\frac{d\Phi}{dr} = -4\pi r^2 p_{\Phi}/\Delta, \quad (2)$$

$$\frac{dp_{\Phi}}{dr} = -\frac{p_{\Phi}}{r\Delta}\left(3r - 7m - \frac{2}{3}\Lambda r^3 + 4\pi r^3\epsilon_{eff}/\Phi\right) - \frac{2}{r}\epsilon_{\Phi}, \quad (3)$$

$$\frac{dp}{dr} = -\frac{p+\epsilon}{r}\,\frac{m + 4\pi r^3 p_{eff}/\Phi}{\Delta - 2\pi r^3 p_{\Phi}/\Phi}, \quad (4)$$

together with corresponding boundary conditions at the center of the Earth $r = 0$:

$$m(0) = m_c = 0, \quad \Phi(0) = \Phi_c, \quad p(0) = p_c,$$
$$p_{\Phi}(0) = p_{\Phi c} = \frac{2}{3}\left(\frac{\epsilon(p_c)}{3} - p_c\right) - \frac{\Lambda}{12\pi}V'(\Phi_c), \quad (5)$$

at the surface of the Earth with mean radius $r_{Earth} \approx 6371.0$ km:

$$m_{Earth} = m(r_{Earth}; p_c, \Phi_c), \quad \Phi_{Earth} = \Phi(r_{Earth}; p_c, \Phi_c), \quad p_{\Phi_{Earth}} = p_{\Phi}(r_{Earth}; p_c, \Phi_c). \quad (6)$$

and at the cosmological horizon in the emerging de-Sitter Universe with huge unknown radius $r_{Universe} \sim 1/\sqrt{\Lambda} \approx 10^{22}$ km (in presence of only the Earth in it – as a crude approximation to the real Universe with huge total mass $M_{Universe} \gg m_{Earth}$):

$$\Delta(r_{Universe}; p_c, \Phi_c) = 0, \quad \Phi(r_{Universe}; p_c, \Phi_c) = 1 \quad (7)$$

[1] DM can be related either with some kind if new matter, outside the Standard Model of Particles (SMP), or by some modification of the theory of gravity in the style of MDG. The existence of DM is now well-established observationally in cosmology and astrophysics. Despite the intensive efforts and significant financial support, new massive matter particles, which spread with small velocities in comparison with velocity of light, were not found during the last decades, using the methods of particle physics.

Being also well established observationally in cosmology and astrophysics, DE is related with the very small cosmological constant $\Lambda \approx 10^{-44} km^{-2}$. From field-theoretical point of view, DE must constitute from particles, moving with the light velocity.

of the very nontrivial moving-boundary problem which appears in presence of a positive cosmological constant $\Lambda > 0$.

The last relation in Eq. (5) is a specific additional MDG condition of type $F_\Phi(p_{\Phi c} p_c, \Phi_c) = 0$ which ensures the finiteness of the DM pressure p_Φ at the center of the Earth.

The elimination of the unknown quantity $r_{Universe}$ from Eqs.(7) leads to second MDG-centre-values-relation in the form $F_\Lambda(p_c, \Phi c) = 0$. This procedure must be done numerically with high precision.

The dilatonic potential $V(\Phi)$ is defined by its derivative with respect to dilaton field Φ by the relation $V' = \frac{2}{3}\left(\Phi U'(\Phi) - 2U(\Phi)\right)$.

The general physical conditions on the cosmological potential $U(\Phi)$ are described in detail in [16]. Here we use the simplest cosmological potential $U(\Phi) = \Phi^2 + \frac{3}{16\Pi^2}\left(\Phi - 1/\Phi\right)$. In MDG. It plays the role of the harmonic oscillator of classical mechanics. This cosmological potential has a single physical vacuum of dilaton field, being a good approximation for any other cosmological potential $U(\Phi)$ in vicinity of its physical vacuum.

The parameter Π is the dimensionless Compton length (measured in cosmological units) of the dilaton and may have an extremely small values: $\Pi = \sqrt{\Lambda}\,\hbar/(m_\Phi c) \leq 10^{-30}$, due to the smallness of cosmological constant Λ and Planck constant $\hbar$.

There is no matter outside the Earth in above MDG model. Hence, there $p \equiv 0$ and $\epsilon \equiv 0$. In this domain we have a specific pure dilaton-sphere. Its structure is determined by the shortened system of differential equations in which the Eq.(4) has to be omitted.

In the above formulas (1) - (7), written down in so called geometrical units $c = 1, G = 1$ for simplicity, we have $\Delta = r - 2m - \frac{1}{3}\Lambda r^3$, and the very important for understanding of present article expression for the effective energy density and effective pressure

$$\epsilon_{eff} = \epsilon + \epsilon_{\Lambda} + \epsilon_{\Phi}, \quad p_{eff} = p + p_{\Lambda} + p_{\Phi}. \tag{8}$$

Here the energy density of the DE ϵ_Λ and the pressure p_Λ of the DE are

$$\epsilon_{\Lambda} = \frac{\Lambda}{8\pi}\Big(U(\Phi) - \Phi\Big), \quad p_{\Lambda} = -\frac{\Lambda}{8\pi}\Big(U(\Phi) - \frac{1}{3}\Phi\Big), \tag{9}$$

and obey the following specific MDG-equation-of-state of DE

$$\epsilon_{\Lambda} + p_{\Lambda} + \frac{\Lambda}{12\pi}\Phi = 0. \tag{10}$$

It explains very well the well-known empirical result of observational cosmology $\epsilon_\Lambda + p_\Lambda \approx 0$, because of the very small observed value of the cosmological constant Λ.

Using the area $A = 4\pi r^2$ of the surrounding the center of the Earth sphere of radius r, and the true infinitesimal geometrical distance $dl = dr\Big/\sqrt{1 - \frac{2m}{r} - \frac{1}{3}\Lambda r^2}$ in the three dimensional curved space, in geometrical units we obtain compact expressions for DM energy density and DM pressure:

$$\epsilon_{\Phi} = \frac{1}{8\pi}\frac{1}{A}\frac{d}{dl}\left(A\frac{d\Phi}{dl}\right), \quad p_{\Phi} = \frac{1}{8\pi}\frac{1}{A}\frac{dA}{dl}\frac{d\Phi}{dl}. \tag{11}$$

As it should be expected, these quantities do not contain explicitly the cosmological constant Λ. Instead, they are related to the gradients of the effective gravitational constant $G_{eff}(\Phi) = 1/\Phi$.

The MDG-equation-of-state of DM is much more complicated:

$$\epsilon_{\Phi} = p - \frac{1}{3}\epsilon + \frac{\Lambda}{8\pi}V'(\Phi) + \frac{p_{\Phi}}{2}\,\frac{m + 4\pi r^3 p_{eff}/\Phi}{\Delta - 2\pi r^3 p_{\Phi}/\Phi}. \tag{12}$$

It strongly depends on the equation of state of the usual mater of the Earth $\epsilon = \epsilon(p)$ which relates energy density ϵ and pressure p. See also the nontrivial relation between Figs. 4 and 7.

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We actually do not need to know the explicit form of the equation of state $\epsilon = \epsilon(p)$. Indeed, to solve the problem, we can use in Eqs. (1)-(4) directly the PREM empirical data for the matter density $\rho(r)$ of the Earth, presented in Fig.4. Note that in geometrical units $\epsilon(r) = \rho(r)$. This is an essential simplification in our calculations.

Thus, in MDG one can conclude that the empirical knowledge of the function $\rho(r)$ from PREM is equivalent to knowledge of the complicated matter equation of state of the Earth.

Now, we have a well defined moving boundary problem for our model of the Earth structure in MDG.

The analytical and numerical analysis for neutron stars, which defers from the above MDG model of the Earth only by mater equation of state, can be found in [16, 17]. In realistic models of neutron stars, in the framework of this model, one has to assume the mass of the scalar field to be $m_\Phi c^2 \approx 4.3 \times 10^{-13}$ eV [17, 18]. This value of $m_\Phi c^2$, gives for Compton length of the dilaton $\lambda_\Phi \approx 3000$ km. Here we adopt this value of the Compton length of the dilaton in the MDG model of the Earth, assuming that in the two cases the dilaton is the same particle.

The preliminary results for the Earth are shown in the four figures 4 – 8 below:

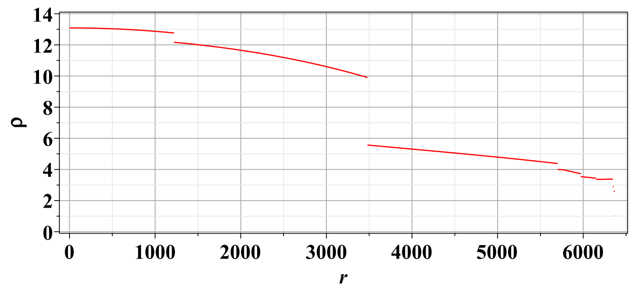

Figure 4. The density $\rho(r)$ in unites $\frac{g}{cm^3}$ of the matter inside the Earth according to the PREM [10]. The distance r from the Earth center is in units km

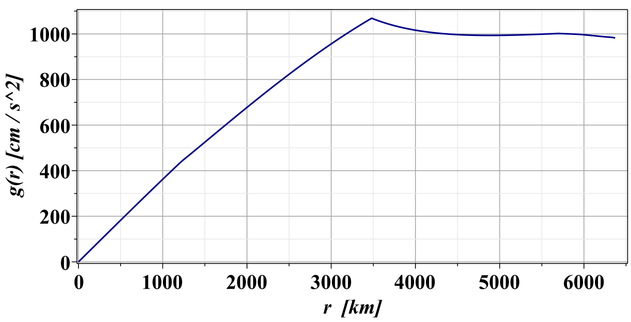

Figure 5. The PREM-gravitational-acceleration inside the Earth in Newton gravity.

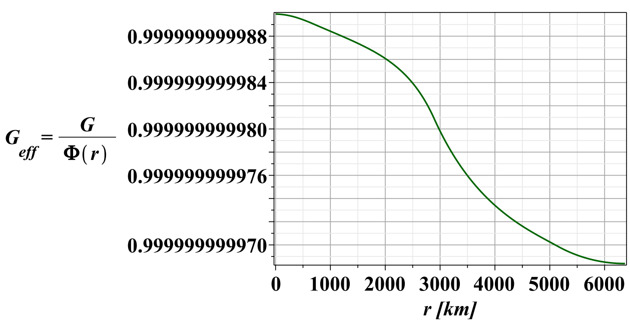

Figure 6. The effective field G_{eff} inside the Earth, which replaces the Newton constant G in the MDG model.

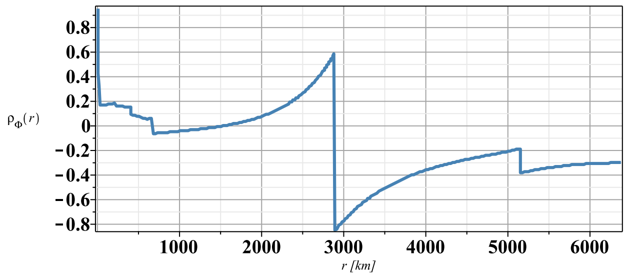

Figure 7. The MDG-Dark-Matter-profile inside the Earth. Note that it follows the PREM-matter-density-profile in Fig. 4 in some specific way.

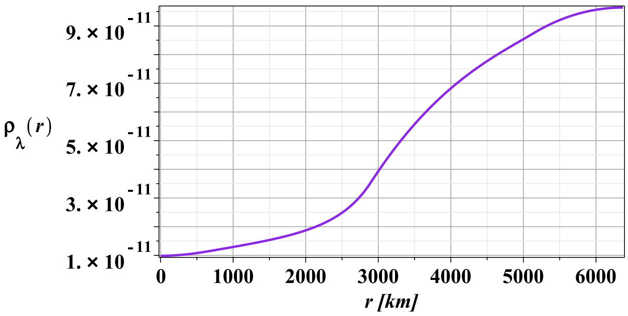

Figure 8. The MDG-Dark-Energy-profile inside the Earth. Note that it is completely different from MDG-Dark-Matter-profile in Fig. 7.

As seen in Figs. 4-8, in framework of the MDG model, the effective gravitational constant G_{eff} , as well as effects of DM and DE indeed depend on the place in three-dimensional space. In our crude MDG-model of the Earth we assume exact spherically symmetric distribution of the usual matter inside a non-rotating Earth and absence of other bodies in the Universe, outside the Earth. As a result, we obtain that G_{eff} , as well as DM and DE depend only on the distance to the center of the Earth.

It is obvious, that any inhomogeneities inside the Earth, as well as presence of other bodies around the Earth will destroy spherical symmetry. In this case, the dependence of G_{eff} on the place in three dimensional space, as well as DM and DE will be much more complicated. These dependencies may be similar to dependencies, shown in Figs. 1 and 2, and in Table 1, at least to some extend.

For now, our present consideration shows the real possibility to explain the empirical data by proper new model of gravity in the style of MDG-model. For this purpose two basic things are needed:

1. Experimental hi-precision data – to find the physical principles for building new physically valuable model of gravity, as extension of GR.

2. Much more complicated numerical analysis of the model of the Earth, taking into account the real inhomogeneities inside the Earth, its rotation, real form, hydrosphere and atmosphere, as well as surrounding the Earth space environment.

The solution of these two hard problems we leave for future investigation in collaboration with proper scientific organizations.

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3. Conclusion

1. In the present-days real state of affairs, the interpretation of data obtained from high-precision measurements by satellites for geodetic use seems to be quite problematic, uncertain, and may be misleading for practitioners.

2. In physics we have no absolute truth and absolute theories.

All we can do, is to create physical models and compare them with the physical reality with the available precision at given time instant.

When the precision increases by 2-3 orders of magnitude, due to the technological progress, as a rule, we observe novel phenomena, previously unknown.

This explains the boundaries of applicability of any old model. As a result, the old model must be replaced with a new one.

3. Let me point out a known example, directly related with geodesy:

In the framework of measurements of distances with a relative precision of 10^{-4}, the geometry of space around the Earth is Euclidian one.

It was Gauss, who checked with this precision that the sum of the angles in a triangle is 180^0, using three mountain peaks around Geneva. His result supported the Euclidian geometry.

Therefore, Gauss stopped developing the geometry of curved space, leaving this issue for his PhD student Riemann.

Due to more recent developments of technology (Mössbouer effect) the measurements with precision 10^{-7} prove the non-Euclidean geometry of space around the Earth surface and supported GR [19, 20].

It turns out that the sum of the angles in a geodesic triangle on the Earth surface is larger than 180^0.

Note that for such basic observation on the surface of a neutron star, it is enough to have precision of 10^{-2}.

4. At small enough distances and velocities, the motion of elementary particles is described by QM, not by Classical Mechanics (CM). Quantitative measure for the new physics is given by a comparison of CM action with the Planck constant, which is very small from a macroscopic point of view. Such quantitative measure needs high-precision experiments.

Decisive condition for applicability of CM- or QM-description of assembles of particles in macroscopic scales (Bose condensate, assemble of cold atoms etc.), included in Quantum Space Gravimetry (QSG), is the absence or presence of coherent-macro-state. It is necessary that for its creation there are physical conditions realized with high precision.

5. There is no any theoretical basis that could guarantee that after increasing by 3-4 orders of magnitude of QSG precision for geodetic purposes, we still would be able to use the Newton model of gravity created more than three centuries ago, and based on linear equations (Laplace equation) and linear superposition principle (Poison equation).

Note that the Poison equation is a basis for analysis of data from satellites in geodesy since it is assumed that any deviations of the gravitational field of the Earth from the average one are caused by local space or time variations of the matter density in the Earth interior, in the oceans and seas, in the polar ice, in the Earth atmosphere, etc. [21].

This assumption cannot be true in nonlinear models of gravity like GR and its modifications. Without careful analysis, we cannot be sure that the routine approach to this problem correctly describes reality by the linear superposition principle of the Newton model.

Just the opposite, it is physically natural to expect that the high-precision observational data will not be correctly interpreted using the oldest Newton model of gravity, with the well-known limits of applicability.

6. Based on the Newton model of gravity is the interpretation of the modern satellite data in large amount of important real cases [22]:

- Ice-sheet loss at North Pole

- Glacier and ice-cup loss in North America
- Precipitation increase in North America
- Glacier retreating in West-North America
- Surface water drying in Central North America
- Progression from dry to wet period Central North America
- Groundwater depletion and drought West Cost of North America
- Drought in Texas
- Recovery from early-period drought North-South America
- Recent drought in Central-South America
- Patagonian ice-field melt
- Ice-sheet loss at South Pole
- Decline the Aral Sea
- Decline the Caspian Sea
- Precipitation increase in Asia
- Glacier melt, surface water in Asia
- Groundwater depletion in Cine
- Water depletion and precipitation decrease in India
- Groundwater depletion in South Asia and Iran
- Progression from dry to wet period in East Australia
- Return to normal after wet period in North Australia
- Groundwater depletion in North Africa
- Increasing lake levels and groundwater in East Africa
- Precipitation increase in the West Africa
- Precipitation decrease in the South Africa
- Precipitation increase in the South-East Africa

etc.

After achieving higher precision in several orders of magnitude, the new satellite data may be misleading for practitioners, because of the wrong interpretation based on Newton model of gravity and use of still standard in geodesy Euclidean geometry.

7. Let us mention some of the basic achievements of modern relativistic geodesy [23, 24], which have to be taken into account in high-precision satellite measurements:

- Time and Frequency Metrology in Context of Relativistic Geodesy
- Chronometric Geodesy
- Measuring Gravitational Field in GR
- GR Gravity Gradiometry
- Reference -Ellipsoid End Normal Gravity Field in Post-Newtonian Geodesy
- Use of Geodesy and Geophysics Measurements to Probe Gravitational Interactions
- Use of Space-Time Curvature in Future Navigation Systems
- Measurement of Frame Drugging with Geodetic Satellites (CHAMP, GRACE and beyond)
- Test of GE with LARES Satellites

We stress that the problem of the nonlinear principle of superposition in GR and its modification is a completely open issue. Some initial attempts to formulate this problem were performed in [25].

8. All the above arguments require deep analysis and preliminary study of the real situation with high-precision QSG for geodetic purposes, both theoretically and experimentally.

We definitely need to try to avoid possible serous mistakes when executing expensive projects like QSG Pathfinder under QSG Program of EC and ESA.

In my opinion, for this purpose, it is necessary to perform deep preliminary theoretical and experimental studies and to use existing scientific organizations and structures at national and international level, proper scientific collaboration in the framework of EU as well as a wider international collaboration in the framework of ESA.

A recent survey on quantum physics in space, including some of the above issues, different modern achievements, open problems, perspectives, existing different approaches and results can be found in the very recent and more detailed review [26, 27].

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References

[1] Gravitational constant - Wikipedia

[2] Fiziev P P, Fizieva Ts Ya, 2014, Variability of the Gravitational Constant - Forty-Three Years Later, *Conference Fundamental Physical Constants* **14**, in Conference proceedings

[3] S. Bhagvati, S. Desai, 2021, Search for variability in Newton's constant using local gravitational acceleration measurements, arXiv:2108.05012

[4] G. Rosi, F. Sorrentino, L. Cacciapuoti, M. Prevedelli, G. M. Tino, 2014, Precision measurement of the Newtonian gravitational constant using cold atoms, NATURE, Vol. 510, 518

[5] Absolute Quantum Gravimeter (A free-fall absolute gravimeter based on laser-cooled atoms), https://www.muquans.com/

[6] Hubble's law - Wikipedia

[7] B. Buscaino, D. DeBra, P. W. Graham, G. Gratta, T. D. Wiser, 2015, Testing long-distance modifications of gravity to 100 astronomical units, Phys. Rev. D 92, 104048

[8] Y.-J. Chen, W. K. Tham, D. E. Krause, D. López, 5 E. Fischbach, R. S. Decca, 2016, Stronger Limits on Hypothetical Yukawa Interactions in the 30–8000 nm Range, PRL **116** 221102

[9] Jean-Philippe Uzan, 2011, Varying Constants, Gravitation and Cosmology, Living Rev. Relativity, 14, 2; https://link.springer.com/article/10.12942/lrr-2011-2

[10] Dziewonski A M, Anderson D L, 1981, Preliminary reference Earth model, *Physics of the Earth and Planetary Interiors,* **25** 297

[11] Martin van Driel et. all., 2021 High-Frequency Seismic Events on Mars Observed by InSight, *JGR: Planets,* **126** 2

[12] Fiziev P P, 2000, A minimal model for dilatonic gravity *Mod Phys Lett A,* **15** 32

[13] O'Hanlon, 1972, Phys. Rev. Lett. **29** 137.

[14] C. Brans, R. H. Dicke, Phys. Rev. **124** 925 (1961).

[15] S. L. Shapiro, S. A. Teukolsky, 1983. Black Holes, White Dwarfs and Neutron Stars, A Wiley-Interscience Publication, Cornel University, Itaca, N.Y.

[16] Fiziev P P, 2013, Withholding potentials, absence of ghosts, and relationship between minimal dilatonic gravity and $f(R)$ theories, *Phys. Rev. D* **87** 044053

[17] Fiziev P, Marinov K, 2017, Modeling of nonrotating neutron stars in minimal dilatonic gravity, *Astrophys Space Sci,* **362** 8

[18] Fiziev P, 2017, *Mod Phys Lett A* **32** 1750141

[19] Pound R V, Rebka G A Jr, 1959, Gravitational red-shift in nuclear resonance, *PRL* **3**

[20] Pound R V, Snider J L,1964, Effect of gravity on nuclear resonance *PRL* **13** 2

[21] Geophysical Surveying Using Gravity https://www.ukm.my/rahim/G-Gravity

[22] Rodell et al., 2018, Emerging trends in global freshwater availability. *Nature* **557** *651*

[23] Bjerhammar A, Relativistic Geodesy, NOAA Technical Report NOS 1 18 NGS 36 (1986)

[24] Relativistic Geodesy, Foundations and Applications, 2019, Editors Dirk Puetzfeld, Claus Lämmerzahl, *Fundamental Theories in Physics,* **196** *Springer Nature Switzerland AG*
[25] Fiziev P P, 2007, Static Fundamental Solutions of Einstein Equations and Superposition, Preprint arXiv:gr-qc/0701108
[26] Belenchia A et al, 2021, Quantum Physics in Space, Preprint arXiv:2108.01435
[27] Iv´an Alonso et al, 2022, Cold Atoms in Space: Community Workshop Summary and Proposed Road-Map, arXiv:2201.07789