Universidade de São Paulo
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Geometrias "Bubbling"
na correspondência AdS/CFT

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Resumo

O escopo deste mestrado é de se familiarizar com a chamada *correspondência AdS/CFT*, que tem sido um dos mais importantes desenvolvimentos na física teórica nas últimas décadas. De acordo com essa correspondência, deformações das geometrias do lado da gravidade (ou lado "AdS") devem ser mapeadas para operadores das teorias de calibre duais (ou lado "CFT").

Em particular, nos temos estado interessados em explorar uma entrada particular no dicionário AdS/CFT, a relação entre os operadores 1/2 BPS em $\mathcal{N} = 4$ super Yang-Mills, e as chamadas *geometrias bubbling* no lado da gravidade.

A fim de fazer isso, apresentamos primeiramente as noções de $\mathcal{N} = 4$ SYM e soluções de Supergravidade. Portanto, podemos expor mais claramente o sentido da correspondência AdS /CFT, e depois mostrar a derivação das geometrias 1/2 BPS duais a estados 1/2 BPS em $\mathcal{N} = 4$ SYM como um exemplo.

Palavras–chave Cordas, Supersimetria, Supergravidade, AdS/CFT, geometrias bubbling.

Abstract

The scope of this Master program was to get acquainted with the so-called *AdS/CFT correspondence*, which has been one of the most important developments in theoretical physics in the last decades. According to this correspondence, deformations of the geometries in the gravity side (or "AdS" side) must be mapped to states of the dual gauge theories (or "CFT" side).

In particular, we have been interested in exploring a particular entry in the AdS/CFT dictionary, namely, the relation between 1/2 BPS operators in $\mathcal{N} = 4$ super Yang-Mills, and the so-called *bubbling geometries* on the gravity side.

In order to do that, we first present the notions of N=4 SYM and Supergravity solutions. In this way, we can expose the statement of the AdS/CFT correspondence, and later show the derivation of 1/2 BPS geometries dual to 1/2 BPS states in N=4 SYM as an example of this one.

Keywords Strings, Supersymmetry, Supergravity, AdS/CFT, bubbling geometries.

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Chapter 1

Introduction

In this work we expose an introductory review to the concept of bubbling geometries. These are based fundamentally in the AdS/CFT correspondence conjectured by Maldacena [1] which is one of the areas of biggest interest in string theory nowadays. In this way, to introduce the correspondence we begin with the theories in which it is based as $\mathcal{N} = 4$ super Yang-Mills and supergravity.

After that we proceed to show arguments by which the conjecture of the AdS/CFT correspondence makes sense, as solutions in type IIB string theory for different limits in coupling constants lead to the two aforementioned theories. This together with symmetry arguments and matching particular cases suggest that they must be related in a way that will be clarified in tha main text.

Finally we arrive to the bubbling geometries as an important application of the AdS/CFT correspondence. They emerge from a particular case of $\mathcal{N} = 4$ SYM, called Matrix Quantum Mechanics. The interesting operators in this theory are $1/2BPS$, to say, they satisfy just half of the supersymmetries, and in this context also $SO(4) \times SO(4) \times R$ bosonic symmetries. In this way, the correspondent type IIB sugra solutions are looked for, and found, in the form of bubbling geometries with the same symmetries.

As said before, the process of obtaining this correspondent geometries serves as a way to understand some details on how the correspondence actually works, and can be used as an enter point to this vast area which is so promising in cases as important as non-perturbative QCD and Quantum Gravity.

Chapter 2

Preliminaries

I start by reviewing some of the ingredients that are necessary to understand what the AdS/CFT correspondence is all about. This will prepare the ground to introduce the bubbling geometries.

2.1 $\mathcal{N} = 4$ super Yang-Mills theory

The field content of this maximally symmetric theory in 4D is given by a gauge field A_μ, $\mu = 0, 1, 2, 3$, four Weyl fermions ψ_α^A (with $A = 1, \ldots, 4$ and $\alpha = 1, 2$), and six real scalars Φ^I [2]. The action (in Euclidean signature) reads

$$\begin{aligned} S \;=\; \frac{1}{g_{YM}^2} \int d^4x \, \mathrm{Tr} \Bigg(& \frac{1}{2} F_{\mu\nu} F^{\mu\nu} + \frac{g_{YM}^2 \vartheta}{8\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu} + D_\mu \Phi^I D^\mu \Phi^I + i \bar{\Psi} \Gamma^\mu D_\mu \Psi \\ & - \frac{1}{2} \left[\Phi^I, \Phi^J \right] \left[\Phi^I, \Phi^J \right] + i \bar{\Psi} \Gamma^I \left[\Phi^I, \Psi \right] \Bigg), \end{aligned} \tag{2.1}$$

where the four fermions have been written as a single Majorana-Weyl fermion in 10D. g_{YM} is the Yang-Mills coupling constant and ϑ is the instanton angle.

It can be checked that this action is conformally invariant at the classical level as all its terms have dimension 4 . With Poincaré invariance these two things combine into the conformal symmetry with $SO(4, 2) \simeq SU(2, 2)$ group. Additionally the combination of this symmetry with the $\mathcal{N} = 4$ Poincaré symmetry form the superconformal symmetry given by the supergroup $SU(2, 2|4)$.

It is important to note that at quantum level the superconformal symmetry remains and it is also believed that the theory is UV finite. As a consequence the coupling constant g_{YM} is actually a non-running parameter which can be fixed to the desired value. Then $\mathcal{N}=4$ SYM is a unique theory defined only by the value of g_{YM} and the rank of the gauge group N.

2.2 Anti de Sitter space

The AdS_5 space is a 5-dimensional space with constant negative curvature. One way to describe it [2] is to take it as an isometric embedding on a flat space of one more dimension, in this case 6. Now if we take the flat space to have coordinates X_i (with $i=-1,0,\dots,4$) then the AdS_5 space is defined by the hyperboloid in $\mathbb{R}^{4,2}$

$$-X_{-1}^2 - X_0^2 + \sum_{k=1}^{4} X_k^2 = -R^2\,, \tag{2.2}$$

where R is the radius of the space. This equation highlights that the isometry group of AdS_5 is $SO(4,2)$. Other useful description of this space comes when we change the coordinates to:

$$X_{-1} + X_4 = \frac{R}{z}\,, \qquad X_\mu = \frac{R}{z} x_\mu\,, \qquad \mu = 0,\dots,3\,, \tag{2.3}$$

so the metric of this space (2.2) becomes

$$ds^2 = \frac{R^2}{z^2}\left(dz^2 + d\vec{x}^{\,2}\right)\,. \tag{2.4}$$

This is called *Poincaré patch* metric with $z \in [0,\infty)$ being the *radial* coordinate of AdS_5. A different description is known making the next parametrization of the $AdS_5 \times S^5$ space, known as *global coordinates*:

$$X_{-1} = R\cosh\rho\cos t\,, \qquad X_0 = R\cosh\rho\sin t\,, \qquad X_k = R\sinh\rho\,\Omega_k\,, \tag{2.5}$$

with $\sum_{k=1}^4 \Omega_k^2 = 1$. In this way the metric reads:

$$ds^2 = R^2\left(-\cosh^2\rho\, dt^2 + d\rho^2 + \sinh^2\rho\, d\Omega^2\right)\,, \tag{2.6}$$

where $t \in \mathbb{R}$ is the *global time* of AdS_5.

2.3 Supergravity in 10 and 11 dimensions

There are various ways to construct supergravity theories. One way is the so called *Noether method* which I will present here. Later I will discuss supergravity in 11 dimension and show how one can dimensionally reduce to get supergravity in 10 dimensions which is important in the AdS/CFT correspondence. Here we follow [3, 4, 5].

2.3.1 The Noether method

Any non linear theory with a gauge symmetry can be constructed by a Noether procedure, starting form the linearized theory. An example is non-Abelian Yang-Mills theory which can be obtained from the linearized free theory. In this method, one takes the linearized theory invariant under local abelian and rigid non-abelian transformations, and through some steps, arrives to the non linear theory. The steps are first to convert the rigid transformation to local (so that the linear lagrangian is no longer invariant) and then adding successively terms to the action and to the transformation law, to get an action which is invariant and transformations which satisfy a closed algebra.

As I have mentioned, Yang-Mills theory is a simple example. Let us consider an invariance under a rigid and a local abelian transformations with parameters T and Λ, to say

$$\delta A_a^i = s^i_{jk} T^j A_a^k \,, \qquad \delta A_a^i = \partial_a \Lambda^i \,. \tag{2.7}$$

Clearly we have

$$[\partial_\Lambda, \partial_T] = \partial_a (s^i_{jk} T^j A_a^k) = s^i_{jk} T^j \partial_a \Lambda^k \,. \tag{2.8}$$

So if we consider an action

$$A^{(0)} = \int \mathrm{d}^4 x \left(-\frac{1}{4} f^i_{ab} f^{abi} \right) \,, \tag{2.9}$$

with

$$f^i_{ab} = \partial_a A^i_b - \partial_b A^i_a \,, \tag{2.10}$$

this is invariant under the above transformations. Now the procedure continues making the rigid transformations local, so that the parameter T becomes space-time dependent: $T^i(x)$. This has the consequence that the original action is no longer invariant under this transformation but instead we have:

$$\delta A^{(0)} = \int \mathrm{d}^4x\, \partial_a T^k(x) j^a{}_k\,, \tag{2.11}$$

with $j^{ai} = s_{jl}{}^i A^l_b f^{abj}$. Now in order to cancel this variation we redefine the action having a new term

$$A^{(1)} = A^{(0)} - \frac{1}{2} g \int \mathrm{d}^4x \left(A^i_a j^{ai} \right) . \tag{2.12}$$

To make this action invariant up to zero order in g we also redefine the transformation law making the initial rigid transformation now proportional to the initial abelian one ($\Lambda^i = \frac{1}{g} T^i(x)$) so the initial separate transformation get reunited into one only:

$$\delta A^i_a = \frac{1}{g} \partial_a T^i(x) + s_{jk}{}^i T^j(x) A^k_a(x)\,, \tag{2.13}$$

Again the given action is not invariant completely under this transformation because of the variation under the second term of eqn. 2.13 which is:

$$\delta A_1 = \int \mathrm{d}^4x \left(-g(A^i_a A^j_b s_{ij}{}^k)(A^l_b \delta_a T^m s_{lm}{}^k) \right) . \tag{2.14}$$

Because of this, we have to continue the process of adding a new term in the action and the transformation. Interestingly in this case it is enough to add the additional term in the action:

$$A^{(2)} = A^{(1)} + \int \mathrm{d}^4x \frac{g^2}{4} \{ (A^i_a A^j_b s_{ij}{}^k)(A^{bl} A^{am} s_{lm}{}^k) \} = \frac{1}{4} \int \mathrm{d}^4x F_{ab}{}^k F^{abn} s_{kn} \tag{2.15}$$

where $F_{ab}{}^i = \partial_a A^i_b - \partial_b A^i_a - g s_{jk}{}^i A^j_a A^k_b$. This action is invariant under eqn. (2.13) to all orders. We have found the action of Yang-Mills theory.

Finally we must note that the algebra of the transformations closes as

$$\begin{aligned} [\delta_{T_1}, \delta_{T_2}] A^i_a &= s_{jk}{}^i T^j_2 \left(\frac{1}{g} \partial_a T^k_1 + s_{lm}{}^k T^l_1 A^m_a \right) - (1 \leftrightarrow 2) \\ &= \frac{1}{g} \partial_a T^i_{12} + s_{jk}{}^i T^j_{12} A^k_a \end{aligned} \tag{2.16}$$

This is again a transformation of the type of eqn. (2.13). We have used $T^i_{12} = s_{jk}{}^i T^j_2 T^k_1$ and the Jacobi identity of the structure constants.

This is an example of how the process works and it can be applied in the same way to construct supergravity theories even tough the technical aspects are more complicated, but the essential steps are the same. These steps are: establishing the linear non-interacting theory with its corresponding action and a pair of transformations, one local abelian and other rigid global. Then make local the rigid transformations and change the lagrangian and the proper transformations order by order to obtain an invariant action with a closed algebra in the transformations.

2.3.2 Supergravity in 11 dimensions

It has been shown that supergravity (SUGRA) exists in up to 11 dimensions [6] and in that dimension there is only one SUGRA theory [7]. It is possible to construct SUGRA theories in dimensions less than 11 through the procedure of dimensional reduction [3].

Supergravity in 11D was constructed by Cremmer, Julia and Scherk [7] having a field content:

g_{MN}	metric
A_{MNP}	3-form potential
ψ_M	gravitino .

Let us start with some notation. I will use M, N, P as curved indices and A, B, C as flat indices. The gamma matrices and the gravitino satisfy:

$$\{\Gamma_A, \Gamma_B\} = 2\eta_{AB}, \quad \overline{\psi}_M = \psi_M^T C^{-1}, \quad C^{-1}\Gamma_A C = -\Gamma_A^T,$$

where the signature is $(- + + ... +)$. The spin connection is $\omega_M{}^{AB} = \omega_M{}^{AB}(e) + K_M{}^{AB}$ and the torsion:

$$K_M{}^{AB} = -\frac{1}{16}(\overline{\psi}_N \Gamma_M{}^{ABNP}\psi_P + 4\overline{\psi}_M \Gamma^{[A}\psi^{B]} + 2\overline{\psi}^A \Gamma_M \psi^B)\,.$$

Having established that, the Lagangian is:

$$
\begin{aligned}
e^{-1}L =& R(\omega) - \frac{1}{2\cdot 4!}F^2_{MNPQ} + \frac{1}{6}\left(\frac{1}{4!4!3!}\varepsilon^{MNPQRSTUVWX}F_{MNPQ}F_{RSTU}A_{VWX}\right) \\
& - \frac{1}{2}i\bar{\psi}_M\Gamma^{MNP}\nabla_N\left(\frac{1}{2}\left(\omega+\hat{\omega}\right)\right)\psi_P \\
& - \frac{1}{8\cdot 4!}i\left(\bar{\psi}_M\Gamma^{MNPQRS}\psi_N + 12\bar{\psi}^P\Gamma^{QR}\psi^S\right)\left(\frac{1}{2}\left(F+\hat{F}\right)\right)_{PQRS} ,
\end{aligned}
\tag{2.17}
$$

where F is the field strength of A. Here we have introduced some other fields:

$$
\hat{\omega}_M{}^{AB} = \omega_M{}^{AB} + \frac{1}{16}\bar{\psi}_N\Gamma_M{}^{ABNP}\psi_P\,, \tag{2.18}
$$

$$
\hat{F}_{MNPQ} = F_{MNPQ} + \frac{3}{2}i\bar{\psi}_{[M}\Gamma_{NP}\psi_{Q]}\,. \tag{2.19}
$$

We arrive at the following equations of motion:

$$
\begin{aligned}
R_{MN}(\hat{\omega}) - \frac{1}{2}g_{MN}R(\hat{\omega}) &= \frac{1}{12}\left(\hat{F}_{MPQR}\hat{F}_N{}^{PQR} - \frac{1}{8}g_{MN}\hat{F}^2\right) \\
\Gamma^{MNP}\hat{\nabla}_N(\hat{\omega})\psi_P &= 0 \\
\nabla_M(\hat{\omega})\hat{F}^{MNPQ} + \frac{1}{2}\left(\frac{1}{4!\cdot 4!}\varepsilon^{NPQRSTUVWXY}\hat{F}_{RSTU}\hat{F}_{VWXY}\right) &= 0
\end{aligned}
\tag{2.20}
$$

where the supercovariant derivative is defined as

$$
\hat{\nabla}_M = \nabla_M + \frac{1}{288}\left(\Gamma_M{}^{NPQR} - 8\delta^N_M\Gamma^{PQR}\right)F_{NPQR}\,. \tag{2.21}
$$

This system as we stated before has one local supersymmetry with transformation laws:

$$
\begin{aligned}
\delta e_M{}^A &= \frac{1}{4}i\bar{\epsilon}\Gamma^A\psi_M \\
\delta A_{MNP} &= -\frac{3}{4}i\bar{\epsilon}\Gamma_{[MN\psi P]} \\
\delta\psi_M &= \hat{\nabla}_M(\hat{\omega})\epsilon
\end{aligned}
\tag{2.22}
$$

and the algebra of its transformation closes. The importance of supergravity with respect to string theory is that supergravity can be seen as the low energy limit of string theory, when we neglect all massive modes of the string spectrum and just keep the massless level.

2.3.3 10D Supergravity

As was explained earlier, the SUGRAs in lower dimensions than 11 can be obtained through the process of dimensional reduction. Making a Kaluza-Klein reduction we divide our 11D Minkowski space in this way:

$$M_{11} = M_{10} \times S^1 \qquad x^M = (x^\mu, y) \tag{2.23}$$

The compactification is going to be made on a circle (S^1) because it is the simpler way to work with it. We choose the fields to not depend on the internal coordinates (in this case y). So the 11D fields can be split this way (just the bosonic part for now)

$$g_{MN} \to g_{\mu\nu}, g_{\mu 11}(C_{(1)}), g_{11\,11} \tag{2.24}$$

$$A_{MNP} \to A_{\mu\nu\rho}(C_{(3)}), A_{\mu\nu 11}(B_{(1)}) \tag{2.25}$$

With this fields it can be done an ansatz for the solution:

$$ds^2_{11} = e^{-\frac{2}{3}\phi} ds^2_{10} + e^{\frac{4}{3}\phi}(dy + C_{(1)})^2 \tag{2.26}$$

$$A_{(3)} = C_3 + B_{(2)} \wedge dy \tag{2.27}$$

which can be inserted into the (bosonic) 11D Lagrangian

$$\mathcal{L}_{11} = R * 1 - \frac{1}{2} F_{(4)} \wedge * F_{(4)} + \frac{1}{6} F_{(4)} \wedge F_{(4)} \wedge A_{(3)} \tag{2.28}$$

to give

$$\begin{aligned} \mathcal{L}_{IIA} \quad = \quad & e^{-2\phi}\left[R * 1 + 4 d\phi \wedge * d\phi - \frac{1}{2} H_{(3)} \wedge * H_{(3)}\right] \\ & -\frac{1}{2} F_{(2)} \wedge * F_{(2)} - \frac{1}{2} \widetilde{F}_{(4)} \wedge * \widetilde{F}_{(4)} \\ & -\frac{1}{2} F_{(4)} \wedge * F_{(4)} \wedge B_{(2)} \end{aligned} \tag{2.29}$$

for 10D Type IIA SUGRA. Here $H_{(3)} = dB_{(2)}, F_{(2)} = dC_{(1)}, F_{(4)} = dC_{(3)}$ and

$$\widetilde{F}_{(4)} = F_{(4)} - C_{(1)} \wedge H_{(3)} \tag{2.30}$$

Through manipulations the $D = 10, \mathcal{N} = 1$ supergravity theory can be constructed from here but the Type IIB is quite different and cannot be obtained this way because it is a chiral theory, *i.e.*, its two fermions have the same chirality, and this is a characteristic which cannot be inherited from SUGRA in 11D. Neither the Noether method can be applied to construct this theory but a variant of it [4]. The bosonic part of the Lagrangian for type IIB supergravity turns out to be:

$$\begin{aligned}\mathcal{L}_{IIB} &= e^{-2\phi}\left[R*1+4d\phi\wedge *d\phi-\frac{1}{2}H_{(3)}\wedge *H_{(3)}\right] \\ &\quad -\frac{1}{2}F_{(1)}\wedge *F_{(1)}-\frac{1}{2}\widetilde{F}_{(3)}\wedge *\widetilde{F}_{(3)}-\frac{1}{4}\widetilde{F}_{(5)}\wedge *\widetilde{F}_{(5)} \\ &\quad -\frac{1}{2}C_{(4)}\wedge *H_{(3)}\wedge F_{(3)}\end{aligned} \tag{2.31}$$

where $H_{(3)} = dB_{(2)}, F_{(1)} = dC_{(0)}, F_{(3)} = dC_{(2)}, F_{(5)} = dC_{(4)}$ and

$$\begin{aligned}\widetilde{F}_{(3)} &= F_{(3)} - C_{(0)}\wedge H_{(3)} \\ \widetilde{F}_{(5)} &= F_{(5)} - \frac{1}{2}C_{(2)}\wedge H_{(3)} + \frac{1}{2}B_{(2)}\wedge F_{(3)}\end{aligned}$$

Additionally the $F_{(5)}$ self duality must be imposed by hand. More explicitly the field content of this theory is:

$$\begin{aligned} g_{\mu\nu} &\qquad \text{metric-graviton} \\ C_{(0)}+\imath\Phi &\qquad \text{axion-dilaton} \\ B_{(2)}+\imath C_{(2)} &\qquad \text{rank 2 antisymmetric} \\ C_{(4)} &\qquad \text{rank 4 antisymmetric} \\ \Psi^I_{\mu\alpha}\ \ I=1,2 &\qquad \text{Majorana-Weyl gravitinos} \\ \lambda^I_\alpha\ \ I=1,2 &\qquad \text{Majorana-Weyl dilatinos}\end{aligned} \tag{2.32}$$

Chapter 3

The statement of the AdS/CFT correspondence

The AdS/CFT correspondence conjecture assumes that there is an equivalence or duality between type IIB string theory (which has as low energy limit a SUGRA theory with the same supersymmetries) on $AdS_5 \times S_5$ background and $\mathcal{N} = 4$ SYM in 4 dimensions. The duality is to be understood as the existence of a map between states and fields on the string side to the local gauge invariant states of the $\mathcal{N} = 4$ SYM theory, and also a correspondence between the correlators of the two theories [2]. In this conjecture the quantum field theory would "live" on the boundary of the background of the string theory.

A first way to check the validity of this conjecture is to take note of the global symmetries of the theories involved. To say, $\mathcal{N} = 4$ SYM have $SU(2,2|4)$ as its transformation group, with bosonic subgroup $SO(4,2) \times SO(6)$ which is in fact the isometry group of $AdS_5 \times S^5$. We can identify specifically the $SO(4,2)$ isometry of AdS_5 with the four dimensional conformal of $\mathcal{N} = 4$ SYM on the boundary where this theory lives, and the $SO(6)$ isometry of S_5 corresponds to R-symmetry of $\mathcal{N} = 4$ SYM. Additional to it both theories obey to the discrete Montonen-Olive $SL(2, \mathbb{Z})$ duality.

Another way to see where this conjecture comes from is comparing systems of D-branes from both sides. On the side of strings these are known to be the endpoints of open strings, and on the side of SUGRA as solitonic solutions to the equations of motion.

More explicitly let us take type IIB strings in 10D in which we have N D3-branes, in the regime where $g_s N$ is small. The action takes the form:

$$S = S_{brane} + S_{bulk} + S_{int} \,, \tag{3.1}$$

where the terms in the action corresponds to the two theories involved in the correspondence *i.e.* S_{brane} corresponds to $\mathcal{N} = 4$ SYM on four dimensions with gauge group $U(N)$ (thanks to the N D-branes),[1] S_{bulk} corresponds to type IIB supergravity and the other term is for the interactions.

The interaction terms can be thrown away in the limit $l_s = \sqrt{\alpha'} \to 0$ since $\kappa \propto \alpha'^2 g_s$. Also higher derivative terms disappear and the bulk action becomes quadratic making the closed strings there free. Meanwhile the open string sector remain interacting since the gauge theory cupling is $g_{YM}^2 = 4\pi g_s$.[2] Note that the low energy limit now have two decoupled theories $\mathcal{N} = 4$ SYM on the branes and free gravity in the bulk.

Taking the another route, we can analize the system of branes in a different way (now valid for $g_s N \gg 1$). As we wrote before a D3-brane is also a solitonic solution to the equations of motion:

$$ds^2 = H^{-1/2} \sum_{\mu,\nu=0}^{3} \eta_{\mu\nu} dx^\mu dx^\nu + H^{1/2} \sum_{I=1}^{6} \left(dy^I\right)^2 \,, \tag{3.2}$$

$$H = 1 + \frac{R^4}{r^4} \,, \tag{3.3}$$

where H is harmonic in the transverse coordinates, and assuming spherical symmetry on them, r is the radial coordinate. Here

$$R^4 \equiv 4\pi g_s N \alpha'^2 \tag{3.4}$$

is the “charge” of the brane.

There are two main regions in this space: the first is when $r \gg R$ so $H \simeq 1$ and this is basically Minkowsky space in 10D; and the second when $r < R$, region which is called the *throat*. Here the axion C_0 and the dilaton are constants ($e^\varphi = g_s$), while the NS-NS 2-form B_2 and the R-R 2-form C_2 are zero. But the self-dual five form is:

[1] The $U(1)$ factor can be shown to decouple so that one is eventually left with $SU(N)$.

[2] This follows from comparing the Yang-Mills action (2.1) with the D3-brane DBI action.

$$F_5 = (1+\star)\, dx_0 \wedge dx_1 \wedge dx_2 \wedge dx_3 \wedge dH^{-1}\,. \tag{3.5}$$

If we make the change of variable $z \equiv R^2/r$ and take the so called near horizon limit $z \to \infty$. The metric (3.2) becomes

$$ds^2 = \frac{R^2}{z^2}\left(dz^2 + \sum_{\mu,\nu=0}^{3} \eta_{\mu\nu} dx^\mu dx^\nu\right) + R^2 d\Omega_5^2\,, \tag{3.6}$$

which can be recognized as the $AdS_5 \times S^5$ geometry. To complete the idea, Maldacena [1] took the limit $\alpha' \to 0$. The importance of this limit is that in it the flat part of the D-brane decouples so just the $AdS_5 \times S^5$ part contributes to the dynamics. To say, similar to the former case, the system decouples in two parts: type IIB strings in the *throat* which is asymptotically $AdS_5 \times S^5$, and free gravity in flat space.

It raises the idea that as with the two schemes presented we obtained first a system decoupled into $\mathcal{N}=4$ SYM and free gravity and second a system decoupled into type IIB strings on $AdS_5 \times S^5$ and free gravity, then it is reasonable to postulate that the parts decoupled of free gravity in both schemes are correspondent. The identification among the parameters are:

$$g_{YM}^2 = 4\pi g_s\,, \qquad R = (4\pi g_s N)^{1/4}\sqrt{\alpha'} = \lambda^{1/4}\sqrt{\alpha'}\,, \tag{3.7}$$

and the rank of the gauge group N corresponds to the 5-form flux threading the S^5. In this way the statement of the conjecture is that given the identification of parameters already mentioned, the theories are equivalent in the sence explained at the beginning of the section.

3.1 Practical restrictions

Due to the difficulties in quantizing strings in background Ramond-Ramond fields, it has become necessary to take certain limits in order to obtain quantitative results from the correspondence. The first limit to be discussed will be *supergravity* remembering that string theory coincides with supergravity, with the same supersymmetries, at low energy.

This limit is obtained in a two step process, first we take $g_s \to 0$ with R constant, and then the limit of large string tension putting $R^2/\alpha' = \sqrt{4\pi g_s N} \to \infty$. This let to decouple the high energy modes from the supergravity fields on $AdS_5 \times S^5$.

For Yang-Mills, the first limit (called t'Hooft limit) is to take $g^2_{YM} \to 0$ and $N \to \infty$, leaving λ fixed (where $\lambda \equiv g^2_{YM} N = g_s N$). And the second limit is to take to infinity the only remaining parameter $\lambda \to \infty$, which correspond to the strong coupling in the gauge theory. Therefore the AdS/CFT correspondence is matching a theory on weak coupling to a theory in strong coupling, which is of great interest since strong coupling very difficult to work with.

3.2 Matching the spectra

In this section we will see how are actually related the spectrum of $\mathcal{N} = 4$ SYM and type IIB strings on $AdS_5 \times S^5$, showing that we have a one-to-one correspondence among states on $AdS_5 \times S^5$ and operators on the gauge side.

3.2.1 Gauge theory operators

On the gauge side *i.e.*, $\mathcal{N} = 4$ SYM the spectrum is the collection of all local gauge invariant operators $\mathcal{O}(x)$ which are also polynomial in the fields of the theory. As said before these theory have a symmetry $SU(2,2|4)$ so its operators can be organized according to the infinite dimensional irreducible unitary representations of this group and can be labeled upon its bosonic subgroup $SO(3,1) \times SO(1,1) \times SU(4)_R$ [8][9]. The labels are respectively: a pair (s_+, s_-) of integers or half-integers, the conformal dimension Δ and Dynkin labels of the representations of $SU(4)_R$ $[r_1, r_2, r_3]$ ([2]).

The conformal dimension Δ is the eigenvalue of the dilatation operator and can be obtained from two-point functions

$$\langle \mathcal{O}(x)\mathcal{O}(0)\rangle \sim \frac{1}{x^{2\Delta}}\,, \tag{3.8}$$

where Δ has a dependence on the 't Hooft coupling of the form $\Delta = \Delta_0 + \gamma(\lambda)$, being Δ_0 the classical dimension and γ the *anomalous dimension.*

For future use it is important to introduce some terms: *Conformal primary operators* are the ones annihilated by the generators K^μ of special conformal transformations, and *superconformal primary operators* are annihilated by the conformal supercharges S. Another way to define these last operators is that they have the lowest dimension among operators in a given superconformal multiplet or representation. These operators are composed just by symmetrized products of scalar fields. [3] *Chiral primary operators (CPO)* are the superconformal primaries that are annihilated by some combination of supercharges Q. These operators are very important since they are protected from quantum corrections so, for example, its dimension Δ remains the same to any order.

These operators are 1/2 BPS so preserves 8 Q's and 8 S's. More explicitly a single-trace CPO operator with conformal dimension Δ can be written as:

$$\mathcal{O}_\Delta(x) = C_{I_1\cdots I_\Delta}\mathrm{Tr}(\Phi^{I_1}\ldots\Phi^{I_\Delta})\,, \tag{3.9}$$

where $C_{I_1\cdots I_\Delta}$ is a $SO(6)$ symmetric traceless tensor.

3.2.2 Bulk modes

There are arguments affirming that in the supergravity limit of the Yang-Mills theory the operators which are not protected by supersymmetry decouple from the protected ones since their conformal dimension tend to infinity, leaving just the last in the spectrum. This spectrum is arranged in multiplets of $SU(2,2|4)$ and given a dimensional reduction over S_5, the chiral primary operators correspond to the KK modes:

$$\varphi(x,y) = \sum_{\Delta=0}^{\infty}\varphi_\Delta(x)\,Y_\Delta(y)\,, \tag{3.10}$$

where φ is a supergravity field, (x,y) are the coordinates on AdS_5 and S_5 respectively and $Y_\Delta(y)$ are spherical harmonics which expands S^5. Due to the compactification on S_5 the

[3] This is easy to see by analyzing the transformation properties of the $\mathcal{N}=4$ fields under Poincaré supercharges. Schematically

$$\{Q,\psi\} = F + [\Phi,\Phi]\,, \qquad \{Q,\bar\psi\} = D\Phi\,, \qquad [Q,\Phi] = \psi\,, \qquad [Q,F] = D\psi\,.$$

Then the only fields that are not Q-exact are the Φ^I. Moreover they have to enter in a symmetrized combination because the commutator between two Φ^I appears in the first transformation above.

supergravity field acquire mass. A complete account of the masses of the fields is found in [2].

3.3 Matrix model description of half-BPS operators

In this section we will overview how we can describe the dynamics of the Half-BPS sector of $\mathcal{N}=4$ SYM in terms of a gauged matrix quantum mechanics model as seen for example in [10][11]. In this article the authors take $\mathcal{N}=4$ SYM on $\mathbb{R}\times S^3$ where the action reads:

$$S=\frac{2}{g_{YM}^2}\int d^4x\sqrt{|g|}\mathrm{tr}\ \Big[-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}-\frac{1}{2}D^{\mu}\phi_iD_{\mu}\phi_i-\frac{R}{12}\phi_i^2+\frac{1}{4}[\phi_i,\phi_j]^2-2\imath\lambda_A^{\dagger}\sigma^{\mu}D_{\mu}\lambda^A \\ +(\rho_i)^{AB}\lambda_A^{\dagger}\imath\sigma^2[\phi_i,\lambda_B^*]-(\rho_i^{\dagger})_{AB}(\lambda^A)^T\imath\sigma^2[\phi_i,\lambda^B]\Big]. \qquad (3.11)$$

where $x^{\mu}=(t,\theta,\psi,\chi)$ with $\mu,\nu,...=0,1,2,3.$; $\mathrm{x}^a=(\theta,\psi,\chi)$ with $a,b=1,2,3$. The gauge covariant derivative is $D_{\mu}=\nabla_{\mu}-\imath[A_{\mu},\quad]$, and the field strength, $F_{\mu\nu}=\partial_{\mu}A_{\nu}-\partial_{\nu}A_{\mu}-\imath[A_{\mu},A_{\nu}]$. The metric is taken as:

$$ds^2=g_{\mu\nu}dx^{\mu}dx^{\nu}=-dt^2+R^2(d\theta^2+sin^2\theta d\psi^2+sin^2\theta sin^2\psi d\chi^2) \qquad (3.12)$$

From this point we can make the dimensional reduction and obtain the matrix model. This is done expanding on spherical harmonics on S^3.

$$\phi_i(x)=\sum_{l=0}^{\infty}\sum_{m=1}^{(m+1)^2}\phi_i^{lm}(t)Y_{(0)}^{lm}(\mathrm{x})$$

with correspondent expansions for the spinors and the gauge field. These are not shown because from now on we will just work with the scalar part.
Truncating this expansion just to the zero modes, which correspond to the 1/2 BPS condition, we have:

$$\phi_i(x)=X_i(t), \qquad (3.13)$$

then the action 3.11 can be truncated consistently to (just the scalar part):

$$S=\int\mathrm{tr}\ \left[\frac{1}{2}(D_tX_i)^2-\frac{1}{2}\left(\frac{m}{6}\right)^2X_i^2\right], \qquad (3.14)$$

where the mass parameter is related to the SYM coupling constant through

$$\left(\frac{m}{3}\right)^3 = \frac{32\pi^2}{g_{SYM}^2}, \tag{3.15}$$

The BPS condition says that the operators are going to be composed by only one complex scalar field $Z = X_1 + \imath X_2$

In this way the action becomes, as expected, a matrix quantum model in one dimension:

$$S[Z(t)] = \frac{1}{2}\int dt \,\mathrm{Tr}\, \left(|D_t Z|^2 - |Z|^2\right) . \tag{3.16}$$

Finally, if we go to the eigenvalue basis, naming the eigenvalues as z_i, $i = 1, ..., N$, and make a gauge choice ($A_t = 0$), the Hamiltonian can be written as:

$$H = \frac{1}{2}\sum_{i=1}^{N} \left(|\dot{z}_i|^2 + |z_i|^2\right) . \tag{3.17}$$

Before the physical analysis about of the Hamiltonian, it is important to note that this system is not directly equivalent to the matrix model as the matrices here are just complex but not necessarily Hermitian. The match is actually achieved taking into account that the creation operators for $Z^\dagger$ decouple in the half-BPS sector, leaving just the creation operators for Z, and recovering in this way the usual quantum mechanics [10]. Changing to the eigenvalue basis have the effect to introduce a Vandermonde determinant making that the model describes fermions in a harmonic potential [12]. In this way the description of the system for large N can be done in terms of incompressible "droplets" of fermions in the phase space.

Chapter 4

“Bubbling” geometries

The AdS/CFT conjecture states that deformations on the AdS geometries have correspondence with operators in the dual CFT which lies at the boundary of the AdS space-time. As an interesting example of this was found in the form of a map for the 1/2 BPS operators of $\mathcal{N} = 4$ SYM. It can be shown that these operators form a decoupled sector in $\mathcal{N} = 4$ SYM which can after certain manipulation be described by a series of harmonics oscillators on a gauged quantum mechanics matrix model. The matrix model is well known to be completely integrable. That this happen comes from the fact that, seen in the eigenvalue basis, the eigenvalues behave as fermions in a harmonic potential. As explained in the last section the 1/2 BPS states can be seen as fermions in the classical limit, so the ground state describes a Fermi sea of eigenvalues which correspond to droplets in the phase space of the system.

This facts have been used by Lin, Lunin and Maldacena [13] which have constructed explicitly the full moduli space of 1/2 BPS IIB supergravity solutions.

So each configuration of droplets in the phase space of the gauged quantum mechanics (which is obtained after dimensional reductions of $\mathcal{N} = 4$ SYM) has a correspondent geometry on the AdS side and the other way around. Since the droplets are formed by fermions, these are incompressible, this is matched on the gravity side by the fact that the Ramond-Ramond five-form flux is quantized.

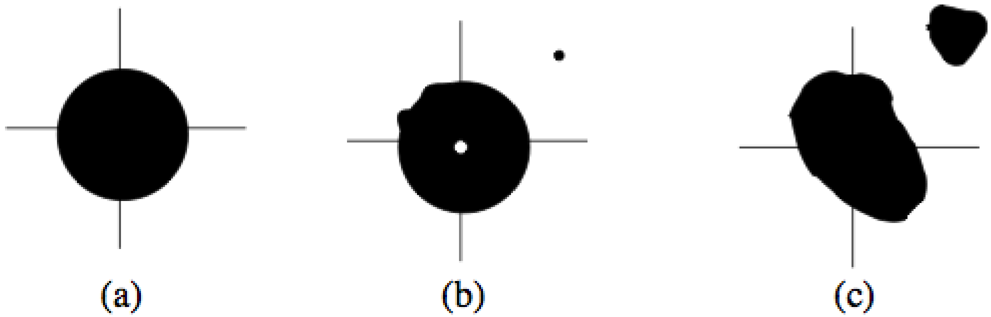

Figure 4.1: (a) Droplet distribution corresponding to $AdS_5 \times S^5$. (b) Gravitons on $AdS_5 \times S^5$ (ripples on the circle), “giant graviton” and “dual giant graviton” (hole and dot). (c) Generic distribution corresponding to a “bubbling” geometry. Figure from [13].

4.1 Review of the LLM construction

This section is based mainly on the article by [13] in which are constructed 1/2 BPS IIB supergravity backgrounds. The dual of these background on the gauge side can be described as fermions as in [10] as we explained before, and satisfy the condition $\Delta = J$, where Δ is the conformal dimension and J is a particular $U(1)$ R-charge in the R-symmetry group. In order to obtain the geometries on the IIB side we have in consideration that the BPS states preserve $SO(4) \times SO(4) \times \mathbb{R}$ bosonic symmetries so the solution (assuming just the five-form field strength varies) must be of the type:

$$\begin{aligned} ds^2 &= g_{\mu\nu}dx^\mu dx^\nu + e^{H+G}d\Omega_3^2 + e^{H-G}d\tilde{\Omega}_3^2\,, \\ F_{(5)} &= F_{\mu\nu}dx^\mu \wedge dx^\nu \wedge d\Omega_3 + \tilde{F}_{\mu\nu}dx^\mu \wedge dx^\nu \wedge d\tilde{\Omega}_3\,, \end{aligned} \tag{4.1}$$

where $\mu, \nu = 0, \ldots, 3$. To say, the geometry contain two spheres, but the time symmetry is still not explicit. The two three-spheres S^3 and $\tilde{S}^3$ in the metric make the $SO(4) \times SO(4)$ isometries manifest. The additional $\mathbb{R}$ isometry corresponds to the Hamiltonian $\Delta - J$.

As described in the article by LLM, requiring that the geometry preserves the killing spinor leads to reveal the form of a general half BPS IIB supergravity background (see next section for a derivation):

$$ds^2 = -h^{-2}(dt + V_i dx^i)^2 + h^2(dy^2 + dx^i dx^i) + ye^G d\Omega_3^2 + ye^{-G} d\tilde{\Omega}_3^2\,, \tag{4.2}$$

$$h^{-2} = 2y \cosh G\,, \qquad z = \frac{1}{2}\tanh G\,, \tag{4.3}$$

$$y\partial_y V_i = \epsilon_{ij}\partial_j z\,, \qquad y(\partial_i V_j - \partial_j V_i) = \epsilon_{ij}\partial_y z \tag{4.4}$$

$$F = dB_t \wedge (dt + V) + B_t dV + d\hat{B}\,,$$

$$\tilde{F} = d\tilde{B}_t \wedge (dt + V) + \tilde{B}_t dV + d\hat{\tilde{B}}\,, \tag{4.5}$$

$$B_t = -\frac{1}{4}y^2 e^{2G}\,, \qquad \tilde{B}_t = -\frac{1}{4}y^2 e^{-2G}\,, \tag{4.6}$$

$$d\hat{B} = -\frac{1}{4}y^3 *_3 d\left(\frac{z+\frac{1}{2}}{y^2}\right)\,, \qquad d\hat{\tilde{B}} = -\frac{1}{4}y^3 *_3 d\left(\frac{z-\frac{1}{2}}{y^2}\right)\,, \tag{4.7}$$

where $i = 1, 2$, and $\star_3$ is the Hodge dual operator for the flat three-dimensional space parametrized by x_1, x_2, and y. It can be seen that the variable y is equal to the product of the radii of the two spheres in the geometry.

It is important to note that all the solution is given in terms of a single function z which satisfies the equation:

$$\partial_i\partial_i z + y\partial_y\left(\frac{\partial_y z}{y}\right) = 0\,. \tag{4.8}$$

As y is equal to the product of the radii of the two spheres, there are singularities at $y = 0$ unless z take some special values. In this case must be $z = \pm 1/2$. For these values of z in the plane $y = 0$ interestingly the radii of one of the spheres go to zero while the other remains finite (this happens symmetrically irrespective of which sphere go to zero and which not). The regions on the plane $y = 0$ with $z = -1/2$ and $z = 1/2$ correspond to fermions and holes, and the x_1, x_2 plane correspond to the phase space. Knowing that, to get the solution, the $\Phi = z/y^2$ replacement is made and eqn. (4.8) becomes Laplace equation in six dimensions with spherical symmetry on four dimension being y the radial coordinate of the last four. Here solving this equation by Green's functions, the values of z at the boundary $y = 0$ acts as charge sources and the solution reads:

$$z(x_1, x_2, y) = \frac{y^2}{\pi}\int_{\mathbb{R}^2} \frac{z(x_1', x_2', 0)dx_1' dx_2'}{[(\mathbf{x}-\mathbf{x}')^2 + y^2]^2}\,. \tag{4.9}$$

and

$$V_i(x_1, x_2, y) = \frac{\epsilon_{ij}}{\pi}\int_{\mathbb{R}^2} \frac{z(x_1', x_2', 0)(x_j - x_j')dx_1' dx_2'}{[(\mathbf{x}-\mathbf{x}')^2 + y^2]^2} = \frac{\epsilon_{ij}}{\pi}\int_{\mathcal{D}} \frac{(x_j - x_j')dx_1' dx_2'}{[(\mathbf{x}-\mathbf{x}')^2 + y^2]^2}. \tag{4.10}$$

The energy of the solutions can be calculated as:

$$\Delta = J = \int_{\mathcal{D}} \frac{d^2x}{2\pi\hbar}\,\frac{1}{2}\frac{(x_1^2+x_2^2)}{\hbar} - \frac{1}{2}\left(\int_{\mathcal{D}} \frac{d^2x}{2\pi h}\right)^2 . \tag{4.11}$$

Being $\mathcal{D}$ the region (fermion region) where $z = -\frac{1}{2}$. This coincides with the energy of fermions in an harmonic oscillator potential minus the energy of the ground state N fermions, just the same as the case of the description of BPS operators by the gauged matrix quantum mechanics exposed before, showing the match between the gravity and gauge part.

To obtain the flux generated by a droplet let us have the quantization rule for the area

$$\frac{\mathcal{A}}{2\pi\hbar} = N\,, \tag{4.12}$$

with $\hbar = 2\pi l_p^4$, where N coincides with the number of fermions. The basis state of this system is just a black circle of radius $R = \sqrt{2\hbar N}$. Here the S_5 background can be obtained by fibering the $\tilde{S}^3$ sphere on a two-dimensional surface Σ which cover the droplet.

4.2 Derivation of the type IIB solutions

In this solution (making the ansatz 4.1)we are just assuming that the axion and the dilaton are constant, and the two three-form field strengths (see section 2.3.3) are zero. It is important to note that the self-duality condition in ten dimensions imply also that there is just one independent gauge field in four dimensions:

$$F = e^{3G} *_4 \tilde{F}, \qquad \tilde{F} = -e^{-3G} *_4 F, \qquad F = dB, \qquad \tilde{F} = d\tilde{B} \tag{4.13}$$

In order to find the solutions we don't use the EOM but the killing spinor equations which are the equations describing unbroken supersymmetry. These equations are known to imply in certain cases the equations of motion. As these equations are in first order it is preferable to work with them:

$$\nabla_M \eta + \frac{\imath}{480}\Gamma^{M_1M_2M_3M_4M_5} F^{(5)}_{M_1M_2M_3M_4M_5}\Gamma_M \eta = 0 \tag{4.14}$$

In order to separate this equation into component blocks we select a basis for the gamma matrices:

$$\Gamma_\mu = \gamma_\mu \otimes 1 \otimes 1 \otimes 1, \qquad \Gamma_a = \gamma_5 \otimes \sigma_a \otimes 1 \otimes \hat{\sigma}_1, \qquad \Gamma_{\tilde{a}} = \gamma_5 \otimes 1 \otimes \tilde{\sigma}_a \otimes \hat{\sigma}_2 \tag{4.15}$$

where the accents just distinguish position on the otherwise usual Pauli Matrices. The chirality condition in this basis reads:

$$\Gamma_{11}\eta = \gamma^5 \hat{\sigma}_3 \eta = \eta \tag{4.16}$$

where the intermediate result come from:

$$\Gamma_{11} = \prod \Gamma_\mu \prod \Gamma_a \prod \Gamma_{\tilde{a}} = \gamma^5 \hat{\sigma}_3, \qquad\qquad \gamma^5 = \imath \Gamma_0 \Gamma_1 \Gamma_2 \Gamma_3 \tag{4.17}$$

Now, lets consider spinors on a unit sphere, obeying the equation:

$$\nabla_c \chi = a \frac{\imath}{2} \gamma_c \chi, \qquad a = \pm 1 \tag{4.18}$$

This together with the full form of the metric (4.1) give us the following spin connection in the sphere directions

$$\nabla_a = \nabla'_a - \frac{1}{4} \Gamma^\mu{}_a \partial_\mu (H+G), \qquad \nabla_{\tilde{a}} = \nabla'_{\tilde{a}} - \frac{1}{4} \Gamma^\mu{}_{\tilde{a}} \partial_\mu (H-G) \tag{4.19}$$

where ∇' contains the spin connection on a unit sphere. The ten dimensional spinor can be decomposed as:

$$\eta = \epsilon_{a,b} \otimes \chi_a \otimes \tilde{\chi}_b \tag{4.20}$$

where $\chi_a, \tilde{\chi}_b$ obey equation (4.18). With the aforementioned basis the contraction of the five form with the gamma matrices in (4.14) can be written as:

$$\begin{aligned}
M &\equiv \tfrac{\imath}{480} \Gamma^{M_1 M_2 M_3 M_4 M_5} F^{(5)}_{M_1 M_2 M_3 M_4 M_5}\,, \\
M &= \tfrac{\imath}{48} (e^{-\frac{3}{2}(H+G)} \Gamma^{\mu\nu} F_{\mu\nu} \epsilon_{abc} \Gamma^{abc} - e^{-\frac{3}{2}(H-G)} \Gamma^{\mu\nu} \tilde{F}_{\mu\nu} \epsilon_{\tilde{a}\tilde{b}\tilde{c}} \Gamma^{\tilde{a}\tilde{b}\tilde{c}})\,, \\
M &= \tfrac{\imath}{8} e^{-\frac{3}{2}(H+G)} (\Gamma^{\mu\nu} F_{\mu\nu} \imath \gamma^5 \hat{\sigma}_1 + \tfrac{1}{2} \varepsilon_{\mu\nu}{}^{\lambda\rho} \Gamma^{\mu\nu} F_{\lambda\rho} \imath \gamma^5 \hat{\sigma}_2)\,, \\
M &= -\tfrac{1}{4} e^{-\frac{3}{2}(H+G)} \gamma^{\mu\nu} F_{\mu\nu} \gamma^5 \hat{\sigma}^1
\end{aligned} \tag{4.21}$$

where in the last step we have used $\varepsilon_{\mu\nu}{}^{\lambda\rho}\Gamma^{\mu\nu}\gamma^5 = 2\imath\Gamma^{\lambda\rho}$ and the chirality condition. So equation (4.14) can be decomposed as:

$$(\imath a e^{-\frac{1}{2}(H+G)}\gamma_5\hat{\sigma}_1 + \frac{1}{2}\gamma^\mu\partial_\mu(H+G))\epsilon + 2M\epsilon = 0\,, \tag{4.22}$$

$$(\imath b e^{-\frac{1}{2}(H-G)}\gamma_5\hat{\sigma}_2 + \frac{1}{2}\gamma^\mu\partial_\mu(H-G))\epsilon - 2M\epsilon = 0\,, \tag{4.23}$$

$$\nabla_\mu\epsilon + M\gamma_\mu\epsilon = 0 \tag{4.24}$$

In order to calculate the metric and the fields it is convenient to define some bilinears, calculate their variation and proceed the calculation with the aid of this information. The bilinears are:

$$f_1 = \imath\bar{\epsilon}\hat{\sigma}_1\epsilon \qquad f_2 = \imath\bar{\epsilon}\hat{\sigma}_2\epsilon \qquad K_\mu = -\bar{\epsilon}\gamma_\mu\epsilon \qquad L_\mu = \bar{\epsilon}\gamma^5\gamma_\mu\epsilon \qquad Y_{\mu\nu} = \bar{\epsilon}\gamma_{\mu\nu}\hat{\sigma}_1\epsilon \tag{4.25}$$

Now, with the aid of (4.24) and the identities:

$$\gamma^\mu\gamma^\alpha\gamma^\beta = g^{\mu\alpha}\gamma^\beta + g^{\alpha\beta}\gamma^\mu - g^{\mu\beta}\gamma^\alpha - \imath\varepsilon^{\sigma\mu\alpha\beta}\gamma_\sigma\gamma^5 \tag{4.26}$$

$$\begin{aligned}
[\gamma_\mu, \not{F}] &= (\gamma_\mu\gamma^{\alpha\beta} - \gamma^{\alpha\beta}\gamma_\mu)F_{\alpha\beta} \\
&= (\gamma^{\alpha\beta}\gamma_\mu + 2(g^\alpha_\mu\gamma^\beta - g^\beta_\mu\gamma^\alpha) - \gamma^{\alpha\beta}\gamma_\mu)F_{\alpha\beta} \\
&= 2(F_{\mu\beta}\gamma^\beta - F_{\alpha\mu}\gamma^\alpha) \\
&= 4F_{\mu\beta}\gamma^\beta
\end{aligned} \tag{4.27}$$

The variation of these can be calculated as:

$$\begin{aligned}
\nabla_\mu f_1 &= \imath[(\nabla_\mu\bar{\epsilon})\hat{\sigma}_1\epsilon + \bar{\epsilon}\hat{\sigma}_1\nabla_\mu\epsilon] \\
&= -\frac{\imath}{4}e^{-\frac{3}{2}(H+G)}F_{\alpha\beta}\bar{\epsilon}(\gamma_\mu\gamma^5\gamma^{\alpha\beta}\hat{\sigma}_1\hat{\sigma}_1 - \hat{\sigma}_1\hat{\sigma}_1\gamma^{\alpha\beta}\gamma^5\gamma_\mu)\epsilon \\
&= -\frac{\imath}{4}e^{-\frac{3}{2}(H+G)}F_{\alpha\beta}\bar{\epsilon}(\gamma_\mu\gamma^{\alpha\beta} + \gamma^{\alpha\beta}\gamma_\mu)\gamma^5\epsilon \\
&= -\frac{\imath}{4}e^{-\frac{3}{2}(H+G)}F_{\alpha\beta}\bar{\epsilon}\frac{1}{2}(g_\mu{}^\alpha\gamma^\beta + g^{\alpha\beta}\gamma_\mu - g_\mu{}^\beta\gamma^\alpha - \imath\varepsilon^\sigma{}_\mu{}^{\alpha\beta}\gamma_\sigma\gamma^5 \\
&\qquad + g^{\alpha\beta}\gamma_\mu + g^\beta{}_\mu\gamma^\alpha - g^\alpha{}_\mu\gamma^\beta - \imath\varepsilon^{\sigma\alpha\beta}{}_\mu\gamma_\sigma\gamma^5 - \alpha \rightleftharpoons \beta)\gamma^5\epsilon \\
&= \frac{1}{2}e^{-\frac{3}{2}(H+G)}\varepsilon_{\sigma\mu\alpha\beta}F^{\alpha\beta}K^\sigma \\
&= -e^{-\frac{3}{2}(H-G)}\tilde{F}_{\mu\sigma}K^\sigma
\end{aligned} \tag{4.28}$$

$$
\begin{aligned}
\nabla_\mu f_2 &= \imath[(\nabla_\mu\bar\epsilon)\hat\sigma_2\epsilon + \bar\epsilon\hat\sigma_2\nabla_\mu\epsilon] \\
&= -\frac{\imath}{4}e^{-\frac{3}{2}(H+G)}F_{\alpha\beta}\bar\epsilon(\gamma_\mu\gamma^5\gamma^{\alpha\beta}\hat\sigma_1\hat\sigma_2 - \hat\sigma_2\hat\sigma_1\gamma^{\alpha\beta}\gamma^5\gamma_\mu)\epsilon \\
&= -\frac{\imath}{4}e^{-\frac{3}{2}(H+G)}F_{\alpha\beta}\bar\epsilon\imath(\gamma_\mu\gamma^{\alpha\beta} - \gamma^{\alpha\beta}\gamma_\mu)\epsilon \\
&= -e^{-\frac{3}{2}(H+G)}F_{\mu\nu}K^\nu
\end{aligned} \tag{4.29}
$$

$$
\begin{aligned}
\nabla_\mu K_\nu &= -[(\nabla_\mu\bar\epsilon)\gamma_\mu\epsilon + \bar\epsilon\gamma_\nu(\nabla_\mu\epsilon)] \\
&= \frac{1}{4}e^{-\frac{3}{2}(H+G)}F_{\alpha\beta}\bar\epsilon(\gamma_\mu\gamma^5\gamma^{\alpha\beta}\gamma_\nu\hat\sigma_1 - \gamma_\nu\gamma^{\alpha\beta}\gamma^5\gamma_\mu\hat\sigma_1)\epsilon \\
&= -\frac{1}{4}e^{-\frac{3}{2}(H+G)}F_{\alpha\beta}\bar\epsilon(\gamma_\mu\gamma^{\alpha\beta}\gamma_\nu - \gamma_\nu\gamma^{\alpha\beta}\gamma_\mu)\gamma^5\hat\sigma_1\epsilon \\
&= -\frac{1}{4}e^{-\frac{3}{2}(H+G)}F_{\alpha\beta}\bar\epsilon\frac{1}{2}(g_\mu{}^\alpha\gamma^\beta\gamma_\nu + g^{\alpha\beta}\gamma_\mu\gamma_\nu - g_\mu{}^\beta\gamma^\alpha\gamma_\nu - \imath\varepsilon^\sigma{}_\mu{}^{\alpha\beta}\gamma_\sigma\gamma^5\gamma_\nu \\
&\qquad - g^{\alpha\beta}\gamma_\nu\gamma_\mu - g^\beta{}_\mu\gamma_\nu\gamma^\alpha + g^\alpha{}_\mu\gamma_\nu\gamma^\beta + \imath\varepsilon^{\sigma\alpha\beta}{}_\mu\gamma_\nu\gamma_\sigma\gamma^5 - \alpha \rightleftharpoons \beta)\gamma^5\hat\sigma_1\epsilon \\
&= -\frac{1}{4}e^{-\frac{3}{2}(H+G)}F_{\alpha\beta}\bar\epsilon(2g_\mu^\alpha g_\nu^\beta - 2g_\mu^\beta g_\nu^\alpha + 2\imath\varepsilon^\sigma{}_\mu{}^{\alpha\beta}g_{\sigma\nu}\gamma^5)\gamma^5\hat\sigma_1\epsilon \\
&= -\frac{1}{4}e^{-\frac{3}{2}(H+G)}\bar\epsilon[(2F_{\mu\nu} - 2F_{\nu\mu})(-\imath\hat\sigma_2) + 2\imath\varepsilon_{\nu\mu\alpha\beta}F^{\alpha\beta}\hat\sigma_1]\epsilon \\
&= -e^{-\frac{3}{2}(H+G)}(F_{\nu\mu}f_2 - \tfrac{1}{2}\varepsilon_{\nu\mu\alpha\beta}F^{\alpha\beta}f_1)
\end{aligned} \tag{4.30}
$$

$$
\begin{aligned}
\nabla_\mu L_\nu &= (\nabla_\mu\bar\epsilon)\gamma^5\gamma_\nu\epsilon + \bar\epsilon\gamma^5\gamma_\nu\nabla_\mu\epsilon \\
&= -\frac{1}{4}e^{-\frac{3}{2}(H+G)}F_{\alpha\beta}\bar\epsilon(\gamma_\mu\gamma^{\alpha\beta}\gamma_\nu + \gamma_\nu\gamma^{\alpha\beta}\gamma_\mu)\hat\sigma_1\epsilon \\
&= -\frac{1}{4}e^{-\frac{3}{2}(H+G)}\bar\epsilon[F_{\alpha\beta}\gamma^{\alpha\beta}(\gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu) + 4F_{\mu\beta}\gamma^\beta\gamma_\nu + 4F_{\nu\beta}\gamma^\beta\gamma_\mu]\hat\sigma_1\epsilon \\
&= -e^{-\frac{3}{2}(H+G)}[\tfrac{1}{2}g_{\mu\nu}F_{\alpha\beta}Y^{\alpha\beta} + F_\mu{}^\rho Y_{\rho\nu} + F_\nu{}^\rho Y_{\rho\mu}]
\end{aligned} \tag{4.31}
$$

Also using Fierz identities it can be obtained:

$$
K \cdot L = 0, \qquad L^2 = -K^2 = f_1^2 + f_2^2 \tag{4.32}
$$

Implications of the equations for the bilinears

With the information we have obtained from the bilinears we are going to develop further how this affects the metric and the fields: It can be shown that the K^μ is a killing vector and $L_\mu dx^\mu$ is a locally exact form, since $\nabla_{(\mu}K_{\nu)} = 0$ and $dL = 0$.

In this way we can define a coordinate $L = dy$, with the other three coordinates taken as orthogonal to y:

$$ds^2 = h^2 dy^2 + \hat{g}_{\alpha\beta} dx^\alpha dx^\beta \tag{4.33}$$

Another coordinate can be set up if we take into account $K \cdot L = 0$, as L now depends only on y, then we have

$$0 = K^y L_y = K^y \tag{4.34}$$

therefore K just depend on the transversal coordinates. So we can choose one of these coordinates to be in the direction of K, lets call this coordinate t, then we can put the metric as:

$$ds^2 = -h^{-2}(dt + V_i dx^i)^2 + h^2(dy^2 + \tilde{h}_{ij} dx^i dx^j) \tag{4.35}$$

with $i, j = 1, 2$. The coeficients of the g_{tt} and g_{yy} terms of the metric were choosen in order to be consistent with the fact $K^2 = -L^2$.

Lets now do some manipulations in order to get the B form. Taking equation 4.29 and using the fact that K just have one component ($K^t = 1$ by convention):

$$\partial_\mu f_2 = -e^{\frac{3}{2}(H+G)} F_{\mu t} = -e^{\frac{3}{2}(H+G)} \partial_\mu B_t \tag{4.36}$$

where we have used the fact that due to the $\mathbb{R}$ isometry the B form is independent of time.

Now, by the definition of K:

$$\partial_\mu B_t = F_{\mu\nu} K^\nu = -F_{\mu\nu} \bar{\epsilon} \gamma^\nu \epsilon = -\frac{1}{4} \bar{\epsilon} [\gamma_\mu, \not{F}] \epsilon \tag{4.37}$$

where in the last equality we have used (4.27). If we replace (4.21) into (4.22):

$$\frac{1}{2} e^{-\frac{3}{2}(H+G)} \not{F} \gamma^5 \hat{\sigma}_1 \epsilon = (\imath a e^{-\frac{1}{2}(H+G)} \gamma^5 \hat{\sigma}_1 + \frac{1}{2} \not{\partial}(H+G)) \epsilon \tag{4.38}$$

Multiplying by $\gamma^5 \hat{\sigma}_1$ we arrive to the expression (together with its adjoint):

$$\begin{aligned} \frac{1}{2} e^{-\frac{3}{2}(H+G)} \not{F} \epsilon &= (\imath a e^{-\frac{1}{2}(H+G)} + \frac{1}{2} \gamma_5 \not{\partial}(H+G) \hat{\sigma}_1) \epsilon, \\ \frac{1}{2} e^{-\frac{3}{2}(H+G)} \bar{\epsilon} \not{F} &= \bar{\epsilon} (\imath a e^{-\frac{1}{2}(H+G)} + \frac{1}{2} \gamma_5 \not{\partial}(H+G) \hat{\sigma}_1) \end{aligned} \tag{4.39}$$

Replacing this results into (4.37) gives:

$$\begin{aligned}
\partial_\mu B_t &= -\frac{1}{4}e^{\frac{3}{2}(H+G)}(\bar{\epsilon}\gamma_\mu\gamma_5\not{\partial}(H+G)\hat{\sigma}_1 - \bar{\epsilon}\gamma_5\not{\partial}(H+G)\hat{\sigma}_1\gamma_\mu\epsilon) \\
&= -\frac{1}{4}e^{\frac{3}{2}(H+G)}\partial_\nu(H+G)\bar{\epsilon}(\gamma_\mu\gamma_5\gamma^\nu - \gamma_5\gamma^\nu\gamma_\mu)\hat{\sigma}_1\epsilon \\
&= e^{\frac{3}{2}(H+G)}\frac{1}{2}\partial_\mu(H+G)\bar{\epsilon}\gamma_5\hat{\sigma}_1\epsilon \\
&= -e^{\frac{3}{2}(H+G)}\frac{1}{2}\partial_\mu(H+G)f_2 \qquad (4.40)
\end{aligned}$$

where in the last step we have used the chirality condition $\epsilon = \Gamma_{11}\epsilon = \gamma_5\hat{\sigma}_3\epsilon$. Replacing this result in (4.36) gives:

$$\partial_\mu f_2 = \frac{1}{2}f_2\partial_\mu(H+G) \qquad (4.41)$$

which has as solution:

$$f_2 = 4\alpha e^{\frac{1}{2}(H+G)}, \qquad B_t = -\alpha e^{2(H+G)} \qquad (4.42)$$

To obtain f_1 the process is basically the same, using (4.28) and (4.23) we get:

$$\partial_\mu f_1 = \frac{1}{4}e^{-\frac{3}{2}(H-G)}\bar{\epsilon}[\gamma_\mu, \tilde{\not{F}}]\epsilon = \frac{1}{2}\partial_\mu(H-G)f_1 \qquad (4.43)$$

with solution:

$$f_1 = 4\beta e^{\frac{1}{2}(H-G)}, \qquad \tilde{B}_t = -\beta e^{2(H-G)}, \qquad 4\beta = 1 \qquad (4.44)$$

Now to find H we begin by adding (4.22) and (4.23), multiplying by $\hat{\sigma}_1$ and using the chirality condition:

$$\hat{\sigma}_1\not{\partial}H\epsilon = (-\imath a e^{-\frac{1}{2}(H+G)}\gamma_5 + b e^{-\frac{1}{2}(H-G)})\epsilon \qquad (4.45)$$

$$\bar{\epsilon}\hat{\sigma}_1\not{\partial}H = -\bar{\epsilon}(-\imath a e^{-\frac{1}{2}(H+G)}\gamma_5 + b e^{-\frac{1}{2}(H-G)}) \qquad (4.46)$$

If we multiply (4.45) by $\frac{1}{2}\imath\bar{\epsilon}\gamma_\mu$ and its adjoint (4.46) by $\frac{1}{2}\imath\gamma_\mu\epsilon$ and sum we get:

$$\begin{aligned}
\frac{1}{2}\imath\bar{\epsilon}\hat{\sigma}_1(\gamma_\mu\gamma^\nu + \gamma^\nu\gamma_\mu)\epsilon\partial_\nu H &= \partial_\mu H\imath\bar{\epsilon}\hat{\sigma}_1\epsilon = \partial_\mu H f_1 \\
&= \frac{\imath}{2}\bar{\epsilon}[\gamma_\mu, -\imath a e^{-\frac{1}{2}(H+G)}\gamma_5 + b e^{-\frac{1}{2}(H-G)}]\epsilon \\
&= -a e^{-\frac{1}{2}(H+G)}\bar{\epsilon}\gamma_5\gamma_\mu\epsilon = -a e^{-\frac{1}{2}(H+G)}L_\mu \qquad (4.47)
\end{aligned}$$

As we already know f_1 (4.44), it is easy to get H:

$$e^H = -a\gamma y = y, \qquad \gamma = -a \qquad (4.48)$$

After this we can fix α multiplying (4.45) by $\bar{\epsilon}\gamma_5\hat{\sigma}_1$ to get:

$$h^{-2}\gamma\partial_y e^H = -aL^2 = -a\left(f_1^2 + \frac{b}{4\alpha a}f_2^2\right) \tag{4.49}$$

To match the condition (4.32) we must have $b = 4\alpha a$. Therefore if we choose $\alpha = \beta$, the condition $b = a$ must hold.

With this information we can obtain an expression for the killing spinor. If we remember that H only depends on y we can write (4.45) as:

$$\left(\frac{1}{hy}\hat{\sigma}_1\Gamma^3 + \imath a e^{-\frac{1}{2}(H+G)}\gamma_5 - b e^{-\frac{1}{2}(H-G)}\right)\epsilon = 0 \tag{4.50}$$

which can be simplified using (4.32), (4.42) and (4.44) and factoring $e^{-\frac{1}{2}(H-G)}$ to obtain:

$$\left(\sqrt{1+e^{-2G}}\hat{\sigma}_1\Gamma^3 + a\imath e^{-G}\gamma_5 - a\right)\epsilon = 0 \tag{4.51}$$

Taking $K^t = 1$ and the definition of K (4.25) we know that $\epsilon^\dagger\epsilon = 1$, also taking $L_y = -a$ and the definition of L we get $\epsilon^\dagger\Gamma^0\Gamma^5\Gamma^3\epsilon = -a$. As the operator $\Gamma^0\Gamma^5\Gamma^3$ is unitary and $\mid a \mid = 1$ then we must have:

$$\left[1 + a\Gamma^0\Gamma^5\Gamma^3\right]\epsilon = 0 \qquad or \qquad \left[1 + a\imath\Gamma_1\Gamma_2\right]\epsilon = 0 \tag{4.52}$$

It can be verified that a killing spinor of the form:

$$\epsilon = e^{\imath\delta\Gamma^5\Gamma^3\hat{\sigma}_1}\epsilon_1 = (\cosh\delta + \imath a\sinh\delta\gamma^5)\epsilon_1 \tag{4.53}$$

satisfy the two last projection formulas (4.51) and (4.52) given

$$\Gamma^3\hat{\sigma}^1\epsilon_1 = a\epsilon_1, \qquad \sinh 2\delta = ae^{-G} \tag{4.54}$$

We can know the scale of ϵ_1 putting (4.53) into the definition of f_2 obtaining

$$\epsilon_1 = e^{\frac{1}{4}(H+G)}\epsilon_0, \qquad \epsilon_0^\dagger\epsilon_0 = 1 \tag{4.55}$$

Using equation (4.13) we can separate the components of B and $\tilde{B}$ which are not in the time direction

$$\begin{aligned} B &= B_t(dt+V) + \hat{B} \\ d\hat{B} + B_t dV &= -h^2 e^{3G} *_3 d\tilde{B}_t \\ \tilde{B} &= \tilde{B}_t(dt+V) + \hat{\tilde{B}} \\ d\hat{\tilde{B}} + \tilde{B}_t dV &= h^2 e^{-3G} *_3 dB_t \end{aligned} \tag{4.56}$$

where the $*_3$ operator is the flat 3D hodge dual operator in the y, x_1, x_2 directions. Finally to get the fields B and $\tilde{B}$ we first have to calculate V. This can be done taking the antisymmetric part of (4.30):

$$-\frac{1}{2}d[h^{-2}(dt+V)] = \frac{1}{2}dK = e^{-(H+G)}F + e^{-(H-G)}\tilde{F} \tag{4.57}$$

If we take just the part not depending on dt as separated in (4.56) we get:

$$\begin{aligned}\frac{1}{2}h^{-2}dV &= -e^{-(H+G)}(d\hat{B} + B_t dV) - e^{-(H-G)}(d\hat{\tilde{B}} + \tilde{B}_t dV)\\ &= h^2(e^{-H+2G} *_3 d\tilde{B}_t - e^{-H-2G} *_3 dB_t)\\ dV &= 2h^4 y *_3 dG = \frac{1}{y} *_3 dz, \qquad z \equiv \frac{1}{2}\tanh G \end{aligned}\tag{4.58}$$

where we have used the results (4.42), (4.44) and based on (4.32) we also used:

$$h^{-2} = y(e^G + e^{-G}) \tag{4.59}$$

so

$$2h^4 y dG = \frac{1}{y}\frac{1}{2}\frac{4dG}{(e^G + e^{-G})^2} = \frac{1}{y}\frac{1}{2}\operatorname{sech}^2 G dG = \frac{1}{y}\frac{1}{2}d\left(\frac{1}{2}\tanh G\right) \tag{4.60}$$

So using this result in the components different from time of B and $\tilde{B}$ as in (4.56):

$$\begin{aligned} d\hat{B} &= \frac{1}{4}e^{2(H+G)}\frac{1}{y} *_3 dz - \frac{e^{(H+G)}}{4h^2}\frac{1}{y} *_3 dz + \frac{e^{(H+G)}}{2}h^2 *_3 dy\\ &= -\frac{1}{4}y^3 *_3 d\left(\frac{z+\frac{1}{2}}{y^2}\right)\\ d\hat{\tilde{B}} &= -\frac{1}{4}y^3 *_3 d\left(\frac{z-\frac{1}{2}}{y^2}\right)\end{aligned}\tag{4.61}$$

Also as dV must be closed for z:

$$\frac{1}{y}\partial_i^2 z + \partial_y\left(\frac{1}{y}\partial_y z\right) = 0 \tag{4.62}$$

4.3 Ground state configuration

For future considerations the only droplet configurations to be dealt with will be the ones with radial symmetry, so here we change to polar coordinates $(x_1, x_2) \to (R, \phi)$. With this prescription we have $V_R = V_1 \cos\phi + V_2 \sin\phi = 0$. Defining $V \equiv V_\phi = R(-V_1 \sin\phi + V_2 \cos\phi)$ equation (4.4) becomes:

$$y\,\partial_y V \;=\; -R\,\partial_R\, z\,, \qquad \frac{1}{R}\,\partial_R V \;=\; \frac{1}{y}\,\partial_y\, z\,. \tag{4.63}$$

and the equations (4.9) and (4.10) are now:

$$z(R,y) = -\int z(R',0)\frac{\partial}{\partial R'} z_0(R,y;R')dR'\,, \tag{4.64}$$

$$V(R,y) = \int z(R',0) g_V(R,y;R')dR'\,, \tag{4.65}$$

where

$$z_0(R,y;R') \;=\; \frac{R^2 - R'^2 + y^2}{2\left[(R^2+R'^2+y^2)^2 - 4R^2R'^2\right]^{1/2}}\,, \tag{4.66}$$

$$g_V(R,y;R') \;=\; \frac{-2R^2R'(R^2 - R'^2 + y^2)}{\left[(R^2+R'^2+y^2)^2 - 4R^2R'^2\right]^{3/2}}\,. \tag{4.67}$$

where z_0 is the function that LLM designate to be the circular droplet (ground state), as can be easily verified from eqn. (4.66): $z(R',0) = 1/2\,\mathrm{sign}(R' - R_0)$, which together with eqn. (4.64) gives $z(R,y) = z_0(R,y;R_0)$. As previously anticipated (see fig.4.1) such a configuration gives rise to the $AdS_5 \times S^5$ solution. This can be seen clearer after the change of coordinates [13]:

$$y = R_0 \sinh\rho \sin\theta\,, \qquad R = R_0 \cosh\rho\cos\theta\,, \qquad \phi = \tilde{\phi} + t\,, \tag{4.68}$$

because one recovers the $AdS_5 \times S^5$ metric in standard global form

$$ds^2 = R_0\big(- \cosh^2\rho dt^2 + d\rho^2 + \sinh^2\rho d\Omega_3^2 + d\theta^2 + \cos^2\theta d\tilde{\phi}^2 + \sin^2\theta d\tilde{\Omega}_3^2\big)\,. \tag{4.69}$$

Conclusions

- In the process of this Master program we reviewed basic aspects of string theory, particularly the low energy limit Supergravities correspondent to the "different" string theories, and more especially the type IIB which is the basis to formulate the AdS/CFT conjecture. Also the counterpart, the $\mathcal{N} = 4$ SYM theory, was introduced.

- We also reviewed some of the arguments which are used to propose the Maldacena conjecture as the comparison of symmetries between $\mathcal{N} = 4$ SYM and $Ads_5 \times S^5$, and the argument of similarity of solutions in type IIB superstring theory which (with some assumptions) for $g_s N << 1$ give $\mathcal{N} = 4$ SYM attached to the N D3-branes and for $g_s N >> 1$ give type IIB SUGRA on $AdS_5 \times S^5$ each one decoupled from free gravity on the bulk and the flat side respectively.

- In the scope of the AdS/CFT correspondence, we reviewed in detail the derivation made by LLM of the 1/2 BPS geometries in type IIB string theory. This geometries correspond in the CFT side to the also 1/2 BPS operators in Matrix Quantum Mechanics, which can be described as free fermions.

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