Fermi National Accelerator Laboratory

TURTLE with MAD Input
(Trace Unlimited Rays Through Lumped Elements)
A Computer Program for Simulating Charged Particle Beam Transport Systems and DECAY TURTLE Including Decay Calculations

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How to Get TURTLE

This manual describes the third-order version with MAD input capability and graphics. *It is not necessarily applicable to other versions of TURTLE.* Copies of this manual may be obtained from
The Reports Office, Fermi National Accelerator Laboratory, P. O. Box 500, Batavia, IL 60510 (Ref. Fermilab-Pub-nonumber).

This program may be obtained from:

> D. C. Carey
> Fermi National Accelerator Laboratory
> P. O. Box 500
> Batavia, IL 60510
>
> Phone: (630) 840-3639
> TRANSPORT_SUPPORT@FNAL.GOV

To get the TURTLE source from the web, follow these instructions:

1. Go to the Fermitools web page, which is at: http://www.fnal.gov/fermitools

2. Click on the word, software, at the top of the page. That will bring you to a list of software titles that are in alphabetical order.

3. Click on Turtle. This will bring you to the Turtle abstract web page.

4. Click on the word, Software, at the top of the page. This will bring You to a Fermitools Terms and Conditions page.

5. After reading this page, click on: I have read and accept the Terms and Conditions. This will be at the bottom of the page.

6. This will bring you to the turtle directory: ftp://ftp.fnal.gov/pub/turtle/

7. Click on the highest version number.

8. This will bring you to that subdirectory and you will see a turtle tar file along with some documentation files.

9. After you click on the file that you want to copy, you get a prompt asking where you would like the file and what it should be called. After you answer the question, it should copy the file.

Introduction

TURTLE is a computer program useful for determining many characteristics of a particle beam once an initial design has been achieved. Charged particle beams are usually designed by adjusting various beam line parameters to obtain desired values of certain elements of a transfer or beam matrix. Such beam line parameters may describe certain magnetic fields and their gradients, lengths and shapes of magnets, spacings between magnetic elements, or the initial beam accepted into the system. For such purposes one typically employs a matrix multiplication and fitting program such as TRANSPORT [1]. TURTLE is designed to be used after TRANSPORT. For convenience of the user, the input formats of the two programs have been made compatible.

The use of TURTLE should be restricted to beams with small phase space. The lumped element approximation, described below, precludes the inclusion of the effect of conventional local geometric aberrations (due to large phase space) or fourth and higher order. A reading of the discussion below will indicate clearly the exact uses and limitations of the approach taken in TURTLE.

TURTLE vs TRANSPORT

TURTLE was designed so that the input data could be the same as that for TRANSPORT. A given set of data could be run through TRANSPORT. Subsequently, it could be run <u>without modification</u> through TURTLE, to explore in more detail the behavior of the beam line.

Because of the compatability between the two programs, there will be a certain amount of duplication in the manuals for the programs. When the specification of an element is identical in the two programs, we have let the appropriate sections of the two manuals be identical also. Therefore any difference in the descriptions corresponds to a real difference between the two programs. The user does not have to scratch his head and wonder if some nuance of phrasing indicates a real difference in usage or method of calculation.

There are elements in TURTLE, such as histograms, that have no meaning to TRANSPORT. Similarly, there are elements in TRANSPORT, such as fitting instructions, that have no meaning to TURTLE. In either case, when some item of data has meaning for one of the programs, but not the other, it will be ignored by the program for which it has no meaning. In other words, histogram specifications will simply be ignored by TRANSPORT. Similarly, fitting instructions will be ignored by TURTLE.

TURTLE vs DECAY TURTLE

Original TURTLE (sometimes referred to simply as TURTLE) was designed originally to transport particles of fixed identity through a charged particle optical system. The aberrations at a single location were included up to third order, and cumulative effects of much higher order were included. The lumped element approach meant that the transformation through a physical element could be calculated ahead of and separate from the ray tracing. In the ray tracing, the already-calculated transformation was simply applied to a particle. For instance, for a nonuniform field bending magnet, the transfer matrix could be calculated to third order. It could then be used to transform the individual particles in the ray tracing.

The decay of a primary particle into secondary particles was inserted in TURTLE by Christoph Iselin. When a decay instruction was included in the data, the program would first calculate the longitudinal position of the decay of each particle. Then it would trace each particle through the optical system. Except for the element in which the decay occurs, the ray tracing would be much like that in original TURTLE. The element in which the decay occurs would be segmented into two parts by the decay. The primary (or parent) particle would be traced through the element to the point of decay. Then each of the secondaries would be traced from the point of decay to the end of the element. From there the tracing would be continued element by element.

Because of the possible longitudinal segmentation of the element in which the decay occurs, the transformations through the elements cannot be completely calculated ahead of time, and then simply applied to the particles as they are traced through the system. Instead, the transformation has to be calculated separately for each particle. For this reason complete third-order transformations are included in TURTLE, but not in DECAY TURTLE. Any third-order terms omitted in DECAY TURTLE are typically unimportant in decay problems, so there is no significant loss of accurary.

Original TURTLE and DECAY TURTLE have the main program and most of the subroutines in common. For a small number of the subroutines, there is an original TURTLE version and a DECAY TURTLE version.

TURTLE and DECAY TURTLE have a built-in histogram package. This histogram package is also a separate set of subroutines. Alternative histogram or plotting packages can also be used with TURTLE. An alternative set of subroutines exists if it is desired to use TURTLE with HBOOK. These subroutines act as a link to the HBOOK subroutines, which must be separately included. Other packages can be used with the substitution of appropriate sets of subroutines.

Mathematical Formulation of TURTLE

Uses and Limitations of the Matrix Approach

We can represent the position and direction of travel of a particle entering a beam line via a vector with six coordinates

$$X = \begin{pmatrix} x \\ x' \\ y \\ y' \\ \ell \\ \delta \end{pmatrix} \tag{1}$$

The coordinates x and y represent respectively the horizontal and vertical displacements of the particle from the reference trajectory. Each transverse displacement, along with the tangent to the reference trajectory, defines a transverse plane. The angles x' and y', are between the particular trajectory and the reference trajectory in the two transverse planes.

The quantity ℓ represents the longitudinal position of the particle relative to a particle travelling on the magnetic axis of the system with the central momentum designed for the system. The remaining quantity $\delta = \frac{\Delta p}{p}$ gives the fractional deviation of the momentum of the particle from the central design momentum of the system.

The effect of the passage of particle across a magnetic element or a drift space may be represented to first order by a transfer matrix R. The coordinates $X(1)$ of the particle at the end of the element are then given in terms of those at the beginning $X(0)$ as

$$X(1) = RX(0). \tag{2}$$

The effect of successive elements, each with its own R matrix may be given by a total R matrix $R(t)$, which is equal to the product of the individual R matrices.

$$R(t) = R(n)R(n-1)...R(2)R(1), \tag{3}$$

where the integer in parentheses indicates the element to which the transfer matrix applies. The first columns of such a matrix are obtained by solving a differential equation of the form

$$g'' + k^2 g = 0 \tag{4}$$

with appropriate boundary conditions, and taking either the value or the derivative of the solution. The differentiation is with respect to distance along the reference trajectory and k^2 is a function only of that quantity. The last two columns are obtained by solving the equation

$$g'' + k^2 g = f \tag{5}$$

3

where f is a driving term given in terms of quantities which depend only on the position along the beam line and single factors of solutions to either equation (4) or (5). Equation (5) may be solved by a Green's function by writing

$$g(t) = \int_0^t G(t, \tau) f(\tau) d\tau. \tag{6}$$

If we treat equation (2) as the first term of a power series expansion of $X(1)$ in terms of $X(0)$, we may write further terms as follows:

$$X(1) = \sum_j R_{ij} X_j(0) + \sum_{ijk} T_{ijk} X_j(0) X_k(0). \tag{7}$$

As in the case of the R matrix, the T matrix for a succession of elements may be written in terms of products and sums of the R and T matrices of the individual elements. The T matrix $T(t)$ for a succession of two elements is written in terms of the individual R and T matrices as follows

$$T_{ijk}(t) = \sum_\ell R_{i\ell}(2) T_{\ell jk}(1) + \sum_{\ell m} T_{i\ell m}(2) R_{\ell j}(1) R_{mk}(1) \tag{8}$$

Notice that at no point in the evaluation of a T matrix for a beam line are two individual T matrices multiplied together. Such a multiplication would yield terms of order higher than second. Since in a matrix approach, one characterizes the effects of the beam line according to order, and we are here expanding to second order, we must consistently truncate our results to second order.

In a matrix approach, the beam itself may also be represented by a matrix σ. The phase space occupied by the beam is taken to be an ellipsoid in six dimensions. Usually, this matrix is initially taken to be diagonal with the square roots of the diagonal elements being equal to the semimajor axes of the ellipsoid. The beam ellipsoid at a later point is obtained, using the R matrix, as

$$\sigma_1 = R \sigma_0 R^T \tag{9}$$

The diagonal elements are now the squares of half the maximum extent of the ellipsoid in a given dimension. The off-diagonal elements give the correlation between the coordinates, as for a tilted ellipse. A phase space envelope which is initially ellipsoidal continues to be so only when we limit our consideration to first-order effects. When higher orders are considered, the elements of the beam matrix may be taken as second moments of a distribution, but give us no information as to the actual shape of the phase space envelope.

The matrix method is indispensible for fitting and is the only reasonable way to obtain an initial design of the beam line. It is also unsurpassed as a tool to determine individual second- and third-order influences on particle trajectories and minimizing such aberrations. Its limitations are in the representation of the beam phase space and the fact that the results are limited by the order of the Taylor's series expansion.

Ray Tracing Methods

A true ray-tracing program computes the trajectory of a particle through a magnetic field directly. It, therefore, does not distinguish among different order effects on a ray, but represents all orders to the accuracy of the numerical integration of the equations of motion. Such an approach is also useful for determining individual contributions to aberrations. If, however, if one wishes to represent the phase space occupied by the beam particles, it is necessary to run large numbers of rays through the optical system to obtain a reasonable population. For a complicated system, this approach can prove time consuming. In addition, it is often unnecessary, as in the case of beams possessing a small phase space volume.

Theory of Ray Tracing Through Lumped Elements

In order to describe the procedure employed in TURTLE, we must explain what is meant by the distinction between local and global classification of aberrations according to order. If we once again let X be a vector giving the coordinates and direction of a particle (henceforth referred to as a ray), and expand the differential equation of motion of a particle passing through a magnetic field in powers of X, we obtain, deleting subscripts

$$DX + EX^2 + FX^3 + ... = 0 \tag{10}$$

The coefficients D, E, and F are matrices, with the terms D_{xx} and D_{yy} being differential operators:

$$Dx = \left(\frac{d^2}{dt^2} + k^2 \right) x \tag{11}$$

The first-order transfer matrix is obtained by solving the differential equation to first order, i.e.,

$$DX = 0 \tag{12}$$

The second-order transfer matrix is now obtained from the second-order coefficients in the differential equation via a Green's function

$$T(t,0) = \int_0^t G(t,\tau)E(\tau)X^2(\tau)d\tau \tag{13}$$

where $x(t)$ is obtained from the first-order transformation

$$X(t) = R(t,0)X(0) \tag{14}$$

We see that to second order, the coefficients of a given order in the differential equation of motion give rise to transfer matrices of the same order. If, however, we wish to go further and asked for third-order transfer matrices $U_{ijk\ell}$, then we find that

$$U = \int_0^t G(t,\tau)F(\tau)X^3(\tau)d\tau \tag{15}$$

$$+ \int_0^t G(t, \tau_1) E(\tau_1) X(\tau 1) \int_0^{\tau_1} G(\tau_1, \tau_2) E(\tau_2) X^2(\tau_2) d\tau_2 d\tau_1$$

$$+ \int_0^t G(t, \tau_1) E(\tau_1) \int_0^{\tau_1} G(\tau_1, \tau_2) E(\tau_2) X^2(\tau_2) d\tau_2 X(\tau_1) d\tau_1$$

The first term contains the third-order coefficients in the differential equation while the second and third contain the second-order coefficients. Elements of transfer matrices beyond second order therefore involve not only coefficients of the differential equation of the same order, but also accumulations of lower-order terms. We accordingly refer to a classification by order of aberrations according to their appearance in the equation of motion as a local classification. A classification of terms by appearance in a transfer matrix will be called global.

A matrix approach as defined on page 3 classifies aberrations by order both locally and globally. Such an approach is necessary for any order if one is interested in the correction of aberrations of that order. A true ray tracing program as explained on page 5 need not classify aberrations by order at all.

In the lumped-element approach to ray tracing, we classify aberrations by order locally, but not globally. The passage of a ray across an individual element is given by a transformation which yields the output ray directly from the input ray. A large number of rays can then be passed through a system in a small amount of computer execution time.

The coordinates of the rays can be collected at any point in the beam line and histograms can be generated. Using histograms, we can exhibit the phase space occupied by the beam explicitly, and do not have to depend on an ellipsoid formalism.

The use of the lumped-element approach permits the inclusion of many sorts of aberrations, but effectively precludes the use of others. To understand how TURTLE works, we must examine the different types of local aberrations. We characterize local aberrations into four types: chromatic effects, geometric effects, magnetic field effects, and mixtures of any of the first three types.

Chromatic effects are due to a deviation of the momentum of a ray from the central momentum of the beam line. An example would be the chromatic aberration of a quadrupole, where the focussing strength depends on the momentum of the particle.

Geometric effects are associated with the monochromatic phase space accepted by the beam line. A trajectory entering a bending magnet at an angle to the central trajectory traverses a different path length in the field and is therefore bent through a different angle than is the central trajectory. This effect gives rise to second- and higher-order geometric aberrations for bending magnets.

Another example of a geometric effect occurs in the normal treatment of quadrupoles. The harmonic oscillator approximation for the motion of a charged particle in a quadrupole is based on a small angle approximation. Corrections for this approximation involve geometric aberrations of third and higher order.

Magnetic field effects are due to nonlinearities appearing in the expansion of the magnetic field in terms of the transverse coordinates of the beam reference system. An example is the effect of a sextupole for on-momentum rays.

Mixed effects represent combinations of the above effects. One example might be the momentum dependence of the effect of a sextupole.

The intent of creating TURTLE was to examine chromatic aberrations and the effect of nonlinearities in magnetic fields to all orders, to evaluate the effect of slits and apertures, and to represent the beam phase space distribution. The distributions represented would then include effects of second and higher orders. Geometric effects are considered locally only to third order, but higher-order global effects will appear due to the accumulation of second-order effects. It was not ppossible to achieve this goal rigorously, so we explain below what was done in each case, and why the approximations used should be valid for beams possessing a small monoenergetic phase space volume.

Transfer matrix elements for quadrupoles and sextupoles are evaluated directly for each ray from the actual momentum of the ray, and are, therefore, exact to all orders in chromatic effects. However, in bending magnets, chromatic effects are evaluated only to third order. But in high-energy separated-function beam lines, the net focussing effects of bending magnets are small compared to quadrupole contributions, so it is usually sufficient to include only second-order chromatic corrections for bending magnets.

Because of the small phase space volume occupied by typical high-energy beams, the third and higher-order geometric effects of a given element will be small. However, higher-order global terms may occur due to cumulative effects of second-order terms. The large lever arms for aberrations in beams at high energies will enhance the importance of such cumulative effects relative to local higher-order geometric effects.

The inclusion of apertures and slits and the representation of the beam phase space with histograms allow a further step in the realistic representation of a beam.

Representation of Magnetic Fields

The normal magnetic field of any magnet, except a solenoid, is assumed to have midplane symmetry. This means that the scalar potential expanded in transverse coordinates about the reference trajectory is taken to be an odd function of the vertical coordinate. If a coordinate rotation is included, then the potential is odd in the coordinate to which the vertical has been rotated. For a bending magnet this will always be in the nonbend plane.

We begin our discussion with the field of the combined-function bending magnet. It is the one element which, in principle, contains all the magnetic multipoles. The expansion of the magnetic field on the nominal magnetic midplane has historically been given in terms

of unitless quantities n, β, and γ. The expansion may also be given in terms of multipole coefficients K_n. The two representations are as follows:

$$
\begin{aligned}
B_y &= B_0(1 - nhx + \beta\ h^2x^2 + \gamma\ h^3x^3 + \ldots) \\
&= (B_0\rho)\sum K_n x^n \\
B_x &= 0
\end{aligned}
\tag{16}
$$

The quantity h is the curvature of the reference trajectory (the reciprocal of the radius of curvature ρ). The expression $B_0\rho$ is equal to the momentum p_0 of the reference particle divided by its charge q.

The dipole component in the expansion is represented by the coefficient K_0 defined as

$$
K_0 = \frac{\alpha}{L} = \frac{B_y}{B_0\rho}
\tag{17}
$$

where α is the total bending angle of a dipole magnet and L is its total length as measured along the reference trajectory. The quadrupole component K_1 is related to the coefficient n by

$$
K_1 = -\frac{n}{\rho^2} = \frac{1}{B_0\rho}\frac{\partial B_y}{\partial x}
\tag{18}
$$

The sextupole component K_2 is related to the coefficients β and ϵ by

$$
K_2 = \frac{\epsilon}{\rho} = \frac{\beta}{\rho^3} = \frac{1}{2B_0\rho}\frac{\partial^2 B_y}{\partial x^2}
\tag{19}
$$

The octupole component K_3 is related to the cubic variation of the magnetic field by.

$$
K_3 = \frac{\epsilon_3}{\rho} = \frac{\gamma}{\rho^4} = \frac{1}{6B_0\rho}\frac{\partial^3 B_y}{\partial x^3}
\tag{20}
$$

The coefficients n, ϵ, and ϵ_3 originally served as the only form for the input data for TRANS-PORT and therefore TURTLE. The coefficients β and γ are also shown above, since they appear directly in the algebraic expressions for the higher order transfer matrix elements. The multipole components K_1, K_2, and K_3 are used as input to the MAD program and are also understood by the present TRANSPORT and TURTLE.

The definition of the multipole components K_0, K_1, K_2, and K_3 allows the orbit of a particle to be computed without knowledge of its momentum. The normalization used here is such that a multipole of unit strength will give unit angular deflection per unit magnet length to a trajectory one transverse unit from the reference trajectory. In the MAD program the values of K_2 and K_3 are defined as the normalized derivatives of the field. The value of K_2 in MAD is then a factor of 2 greater than the TRANSPORT value defined above. The

MAD value of K_3 is a factor of 6 $(= 3!)$ greater than that used here. The normalization of K_n in various computer programs can be found in a table on the following page. TURTLE can be instructed via a UMAD command to observe the MAD conventions so as to ensure consistency of results. The `UMAD` command is described on page 34.

Physical quadrupole, sextupole, and octupole elements are single multipoles and are known as "separated function" components. The reference trajectory through such elements is a straight line, so that the radius of curvature ρ becomes infinite. The appropriate measures of the strength of these multipole components are the coefficients K_0, K_1, K_2, and K_3, since these coefficients are defined directly as field derivatives divided by $B_0\rho$. The quantity $B_0\rho$, being defined in terms of the reference momentum, is not affected by the curvature of the reference trajectory.
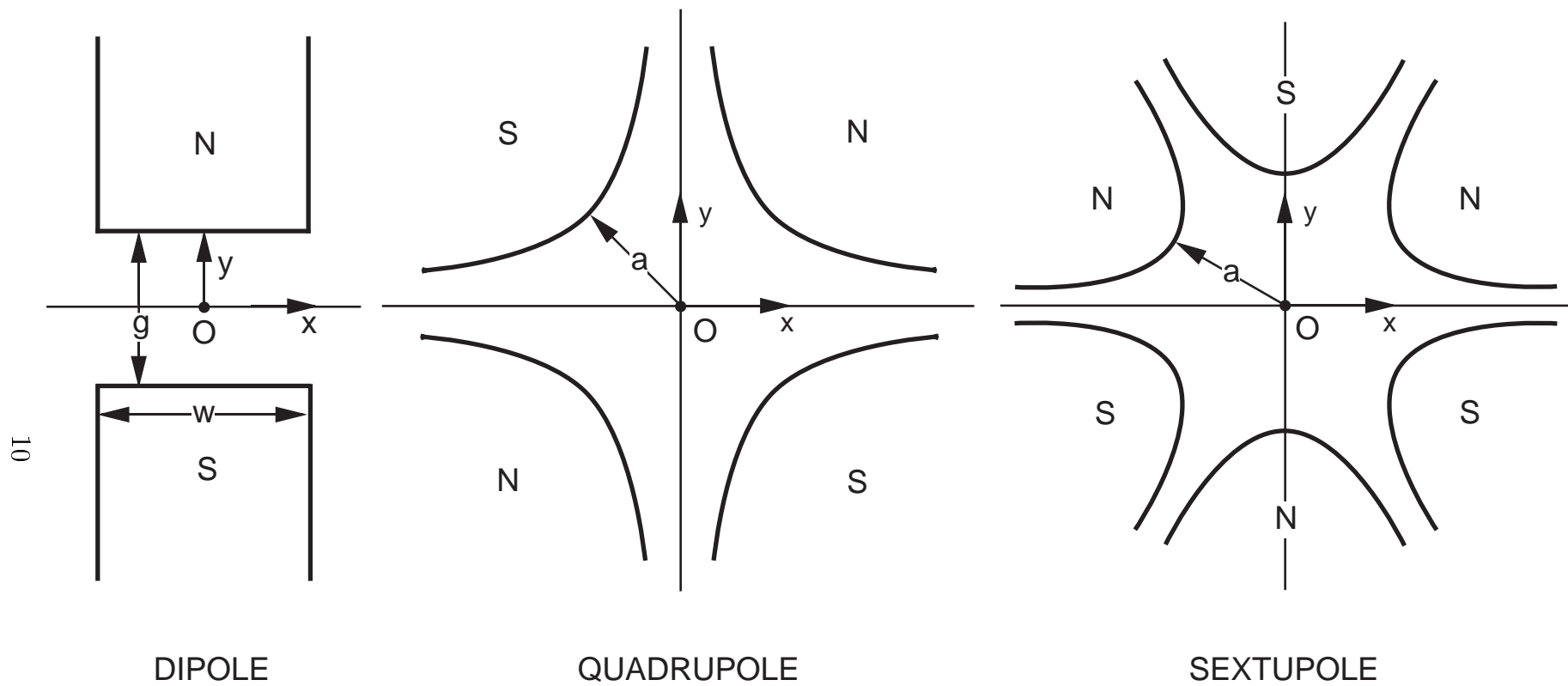
Illustration of the magnetic midplane (x axis) for dipole, quadrupole and sextupole elements. The magnet polarities indicate multipole elements that are positive with respect to each other. The coordinate system is right-handed, so that the beam particles will be emerging from the paper and moving into the face of the reader.

In TRANSPORT (and TURTLE) the multipole strength has traditionally been defined in terms of a pole tip field B_0 and a half aperture a. The half aperture is illustrated in the preceding figure. It is the distance from the optic axis (reference trajectory) to the nearest point on the pole tip.

For a quadrupole the gradient g of the field is also often used. The quantities K_1 and g are related to the pole tip field B_0 and the half aperture a by:

$$K_1 = \frac{g}{(B_0\rho)} = \frac{1}{(B_0\rho)}\frac{B_o}{a} \tag{21}$$

For a sextupole the quantity K_2 is related to the pole tip field B_o and the half aperture a by:

$$K_2 = \frac{1}{(B_0\rho)}\frac{B_o}{a^2} \tag{22}$$

For an octupole the quantity K_3 is related to the pole tip field B_o and the half aperture a by:

$$K_3 = \frac{1}{(B_0\rho)}\frac{B_o}{a^3} \tag{23}$$

Again, the definitions in the MAD program are greater by a factor of 2 for the sextupole term and 6 for the octupole term.

DEFINITION OF MULTIPOLE STRENGTHS
IN VARIOUS COMPUTER PROGRAMS

TRANSPORT TURTLE DIMAD	MAD SYNCH TEAPOT
$B_y(x,0,s) = (B_0\rho)\sum K_n x^n$ $K_n(s) = \frac{1}{B_0\rho}\frac{1}{n!}\frac{\partial^n B_y}{\partial x^n}$ $K_n = \frac{1}{(B_0\rho)}\frac{B_o}{a^n}$	$B_y(x,0,s) = (B_0\rho)\sum \frac{1}{n!}K_n x^n$ $K_n(s) = \frac{1}{B_0\rho}\frac{\partial^n B_y}{\partial x^n}$ $K_n = \frac{n!}{(B_0\rho)}\frac{B_0}{a^n}$

So $K_n(\text{TRANSPORT}) = \frac{1}{n!}K_n(\text{MAD})$

where $n = 0$ Dipole
$n = 1$ Quadrupole

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$$n = 2 \qquad \text{Sextupole}$$
$$n = 3 \qquad \text{Octupole}$$

Below we explain how to use the program and further describe each element available.

Input Format for TURTLE

General Description of Input

The Input Data

TURTLE is designed to be run using the same set of data as was used for TRANSPORT. When using TRANSPORT, one typically starts with a set of data containing initial values of the parameters used in obtaining a fit to some desired constraints. Once the fitting process is complete, and the final parameter values are known, one alters the data to include them for purposes of studying second- and third-order aberrations and misalignments. With a few changes, this data set can be used as input for TURTLE. The data format and the data set structure are the same. Below we discuss the structure and details of the data. For completeness, we include many things that are unchanged from TRANSPORT.

A data set contains three kinds of data: the TITLE, the INDICATOR, and the ELEMENTs. The TITLE is on its own line and contains a string of characters and blanks enclosed by single quotes. Whatever is between the quotes will be used as a heading in the output of a TURTLE run.

The second line of the input gives the number of particle trajectories to be run through the optical system. If there is a single number on this line, then that single number is the number of rays traced. If there are two numbers on this line, then the first is taken to be the indicator, which is used in TRANSPORT. The second integer on the same line is taken then to be the number of trajectories.

The remainder of the data for a given problem step consist of the ELEMENTs. These data describe the units to be used, the physical objects making up the beam line, and the histograms. Unless otherwise specified, the order of the elements is the same as encountered as one proceeds down the beam line. Provision also exists for listing the elements first, then using other commands to assemble them into beam lines. Each element specifies a magnet or other piece of equipment, a drift space, the initial beam phase space, or a histogram.

The input format of the data is "free-field," which means that exact positioning of data items is not important. A tabbing mark is also taken as a space. The input is case-insensitive, which means that both upper case and lower case alphabetic characters can be used, and no distinction is made between them. More information is given below.

The data for a given problem step are terminated by the word `SENTINEL`, which need not be entered on a separate line.

Notation

Two different input data formats have been used for TRANSPORT as it was developed over the years. The MAD (Methodical Accelerator Design) program of Christoph Iselin (see refs) has an input data format which is both easy to use and clear in its meaning. However, since the program TRANSPORT existed long before MAD was originated, TRANSPORT had its own original format. Both formats are recognized by TURTLE and will continue to be. There is no possibility that at some future point users will be required to convert old data sets to a new format.

The input format in MAD notation is described in this section and in most of the remainder of the manual. The original notation is described in a section entitled "Original TRANSPORT Notation," beginning on page 172.

The MAD program also uses a certain set of parameters to describe a physical element. Strict MAD compatability requires using the same set of parameters. However, since the original TRANSPORT used a different set of parameters, the current version has been made to recognize both possibilities, as well as several others. Details are given both in this section and in the sections for the individual elements.

The TRANSPORT or TURTLE input data for a single element can be described as follows.

1. Each element is given a proper name, henceforth to be known as a label. The label is the first item in the element specification and is followed by a colon. If the elements are to be assembled into a beam line by additional program instructions, then the label must be present. If the beam line consists simply of the sequence of elements as listed, then, in TRANSPORT or TURTLE, the label is optional.

2. The next item in the specification of an element is a mnemonic which identifies what sort of entity (such as a bending magnet, quadrupole, or drift space) is represented. The mnenomic follows the label if one is present, or begins the element if there is no label. The mnenomic is an alphanumeric character string, usually a recognizable name such as BEND or QUAD. The mnemonic is followed by a comma.

3. The element parameters are designated by keywords . The parameters are identified by name (such as L and ANGLE). If a value is to be assigned to the parameter, the parameter name is followed by an equals sign, which, in turn, is followed by the value. If another parameter keyword is to follow, the value must be followed by a comma. The entire sequence is terminated by a semicolon or the end of a line.

Thus a typical element representing a bending magnet might look like.

```
        BM1:    BEND,   L = 10.,   B = 20.,   N = 0.5   ;
```

The meaning of the parameters `L` and `B` is obvious, while `N` represents a normalized field gradient.

In MAD the elements are specified first, then assembled into an optical system by additional program instructions. TRANSPORT (and therefore TURTLE) also allows the option of having the optical system consist of the elements as listed.

These aspects of TURTLE input are explained in greater detail, with examples, in the immediately following sections. The choice of a particular notation for the specification of the elements does not dictate how the beam line is to be assembled. For example, in TRANSPORT and TURTLE, elements specified in MAD keyword notation may simply be listed sequentially to describe a beam line. *However, if a beam line specification is to be read by the MAD program, it must be entirely in MAD notation.* The method of assembling a beam line in MAD is used because accelerators typically have a high degree of repetition and symmetry. Secondary beam lines sometimes have very little repetition or symmetry.

Types of Elements

The mnemonic identifies the element, indicating what sort of entity (such as a magnet, drift space, constraint, etc.) is represented. If a minus sign is placed immediately before the mnemonic with no intervening spaces, the element will be ignored. Leaving such an element in the data might be useful in the future for making additional TRANSPORT or TURTLE runs.

The mnemonic names chosen for the elements are the same as occur in the printed output of the TRANSPORT program. These names for elements which are MAD compatible are the same as used in the MAD program. Some of the names occuring in the printout have been changed slightly from older versions of TRANSPORT or TURTLE. The following is a list of mnemonics recognized by TURTLE. Also included are the meaning of the mnemonics, and whether they are MAD compatible.

Mnemonic	MAD Compatible	Element
UNITS		Input-output units
SPECIAL		Special input parameters
ORDER		Zeroeth, first, second, or third order
RANDOM		Random errors on physical parameters
OBSERVE		Observance of specified apertures
REFERENCE		Charge of reference particle
BEAM		Input phase space dimensions and reference trajectory momentum.
CORRELATION		Beam rotated ellipse
CENTROID		Shift in the beam centroid

PARAMETER	Yes	Parameter value
DRIFT	Yes	Drift space – a field free region
BEND		Sector bending magnet (without fringing fields)
ROTATION		Fringing fields and pole-face rotations for bending magnets
RBEND	Yes	Rectangular bending magnet (with fringing fields)
SBEND	Yes	Sector bending magnet (with fringing fields)
HKICKER	Yes	Horizontal steering magnet
VKICKER	Yes	Vertical steering magnet
KICKER	Yes	Steering magnet in both planes
QUADRUPOLE	Yes	Quadrupole magnet
SEXTUPOLE	Yes	Sextupole magnet
OCTUPOLE	Yes	Octupole magnet
PLASMALENS		Plasma (lithium) lens
SOLENOID	Yes	Solenoid magnet
ACCELERATOR	Yes	Travelling wave linear accelerator
SEPTUM		Electrostatic septum
HORN		Neutrino horn
MARKER	Yes	Position marker
MATRIX		Arbitrary matrix input
SROT	Yes	Coordinate Rotation
SHIFT		A shift in the reference coordinate system
REALIGN		Realignment of the reference trajectory on the centroid.
ALIGN		Magnet alignment tolerances
REPEAT		Repetition of a segment of a beam line
SECTION		Defined section of beam line
UPDATE		Transfer matrix update
ALMARK		Alignment marker
STORE		Storage of matrix element values
HIST		One dimensional histogram
AHIST		Horizontally displayed coordinate for two-dimensional histogram.
DHIST		Vertically displayed coordinate for two-dimensional histogram.
PROFILE		Second variable for mean value plot.
FLAG		Flag for histogram entry.
DECAY		Initiates particle decay (for DECAY TURTLE only).

The first four letters of a mnemonic are necessary for its specification. More may be used for clarity if desired. For example, the mnemonic SECTION may provide greater clarity than just SECT, which, at first glance, might be taken to mean sector magnet. If more than four characters are used, they must be correct. A mnemonic of SECTIO will be recognized but SECTOIN will not. The same restrictions apply to the keywords. The use of parentheses in mnemonics in this report is to show the characters which are non-essential.

A MAD specification for a sector bending magnet with fringing fields and with the individual name of BM1 might then appear as:

```
BM1:    SBEND,  L = 10.,  ANGLE = 0.01,  K1 = 0.5E-6 ;
```

Here, the only physical parameter whose meaning is not obvious is `K1`, which is a measure of field gradient. Expressed in terms of an alternate set of physical parameters (length and magnetic field instead of length and bending angle), the same element might now be:

```
BM1:    SBEND,  L = 10.,  B = 20.,  N = 0.5  ;
```

The quantity indicated by the keyword `N` is an alternate measure of field gradient. More details about these various notations are given in the sections below.

Notation for individual elements

Strictly MAD-Compatible Notation

The strict MAD input notation, which is compatible with the MAD program of Christoph Iselin, can be used in TRANSPORT (or TURTLE) only for the specification of the charged-particle optical system. Other instructions such as units specifications, methods of calculation, printing instructions, and histograms may be specified in a keyword notation which is similar to MAD notation. The options available depend on the type of element.

The strictly MAD notation for the specification of elements has four distinctive characteristics:

1. As explained above, some TRANSPORT (or TURTLE) elements are compatable with MAD while others aren't. Only physical elements (such as an `SBEND` or a `QUAD`) are MAD compatable. TRANSPORT (and TURTLE) recognize those MAD elements which are used to specify the optical system. However, TRANSPORT and TURTLE also have some other elements (such as `BEND`) used for the same purpose but are not in MAD. Items such as units specifications or histograms are also not MAD compatable.

2. There may be several equivalent sets of keywords which can be used to describe a given element. Strict MAD notation requires the use of a particular set of parameters for the specification of any physical element. The parameters to be used for any particular type of element are described in the section for that element. An example might be a sector bending magnet. In MAD notation a sector bending magnet is specified in terms of its length, bending angle, and normalized field gradient K_1. The normalized field gradient K_1 is defined in the earlier section on the mathematical formulation of TURTLE.

3. Strict MAD notation also requires the use of a particular set of units for the specification of any physical parameter. For a sector bending magnet the length is given in

meters, the bending angle in radians, and the normalized field gradient K_1 in inverse meters squared.

4. In MAD the individual elements are defined first, then assembled into beam lines. In TRANSPORT and TURTLE, the beam lines may also be defined by simply listing the elements.

A complete specification for a sector bending magnet with fringe fields would then look like:

```
BM1:   SBEND,  L = 10.,  ANGLE = 0.01,  K1 = 0.5E-6 ;
```

In strict MAD notation the units for length L are meters, those for the bend angle ANGLE are radians, and those for K1 are in inverse square meters.

TRANSPORT and TURTLE will also accept data using MAD parameters which are not in MAD units. For example, an SBEND can be parameterized in terms of L, ANGLE, and K1, while using feet for distance, degrees for angle and inverse squared centimeters for K1. In the sections describing the individual elements, we shall refer to the parameterization of elements in terms of MAD variables as being MAD notation. However, strict MAD notation requires the use of MAD units, while TRANSPORT (and therefore TURTLE) can use alternative sets of units even for MAD parameters.

MAD notation has the advantage that it does not require a separate specification of the beam momentum in order to calculate the transfer matrix. The beam momentum is incorporated in the definition of K_1.

Alternate Parameters

Physical elements may be specified in several different ways in TRANSPORT (and TUR-TLE) by sets of physical parameters. One such element is the sector bending magnet we have been using as an example. Instead of the length and bend angle, we might use the length and magnetic field. We might also use a different normalized gradient n, defined previously. The use of the magnetic field as part of the magnet description now requires knowledge of the momentum to calculate the transfer matrix. For a 600 GeV beam, our bend magnet would then be specified as

```
BM1:   SBEND,  L = 10.,  B = 20.,  N = 0.5  ;
```

In the original TRANSPORT there is provision for a sector bending magnet which does not contain the fringing fields. Such an element is useful in the longitudinal segmentation of a bending magnet to obtain the transfer matrix or beam profile at interior points. The following BEND element is physically the same as the above SBEND except that the fringe fields are not included:

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```
        BM1:    BEND,  L = 10.,  B = 20.,  N = 0.5  ;
```

The order of the parameters is not significant in keyword notation. For example, the above specification is equivalent to

```
        BM1:    BEND,  B = 20.,  L = 10.,  N = 0.5  ;
```

The BEND element as described above, with no fringing fields, is not found in the MAD program. However, it can be described by the same parameters as the RBEND (rectangular bending magnet with fringe fields) or the SBEND (sector bending magnet with fringe fields), which are found in MAD.

In the specification of a bending magnet still other sets of parameters may be substituted for or added to those mentioned. Other elements such as quadrupoles, sextupoles, solenoids, etc. also have alternate or additional sets of parameters. Complete lists will be found in the sections describing each individual element. Not all elements or possible sets of keyword parameters are compatible with the MAD program. The MAD compatible elements or sets of keyword parameters are indicated in the sections describing the individual elements.

The Use of Labels

Labels are available for identification of individual elements. When using explicit LINE commands in the MAD format to assemble beam lines from previously defined components, labels are required. In the LINE command, the element or subline to be included in a beam line is identified by its label. Labels may also be inserted for the user's convenience. In this case the placement of a label on a given element is optional.

The label is placed before the mnemonic. It is then identified by being immediately followed by a colon. The maximum length of a label is fifteen non-blank characters. It may contain letters, numerals, underscore, percent (%), and dollar sign ($).

As an example, a drift space labelled DRF would appear as:

```
        DRF:  DRIFT,  L = 1.5  ;
```

The use of labels to assemble elements into a beam line or subline is completely described on page 148.

Examples

A MAD-Compatible TURTLE Input Deck

An example can be written with the specification of the optical lattice entirely in MAD notation. The elements – the SBEND, the DRIFTs, – are first given, then assembled into an optical lattice by use of the LINE command. The USE statement specifies the lattice to be used for the calculation. The other TURTLE commands now appear either before or after the part of the data where the optical lattice is given. The complete set of data is as follows:

```
'EXAMPLE OF A TURTLE DATA SET'
10000
UMAD
BEAM  X = .5,  XP= 1,  Y =.5,  YP=1,  L= .5,  DEL =1,  P0 = 1  ;
SEC1:    -ORDER  ;   PRINT,   BEAM,   ON   ;
DRI: DRIFT,   L  =  2.745   ;
SBEND,   L  =  9.879,   ANGLE  =  2.96,   K1  =  ..0449   ;
MRKA:   MARKER   ; EXAMPLE:   LINE  =  (DRI,SB,DRI,MRKA)   ;
USE,    EXAMPLE   ;
PRINT,   TRANS,   LOCATION  =  MRKA   ;
HIST,   X,   LOWER = -1.0,   UPPER = 1.0,   STEP = 0.02,   LOCATION  =  MRKA   ;
SENTINEL
```

Since MAD elements are being used, the parameters are in MAD units. The element UMAD then specifies that MAD units (meters, radians) are to be assumed throughout.

The element SBEND includes the fringing fields and assumes that the pole-face-rotation angle is zero unless explicitly specified otherwise. The two drifts are given by a single specification.

The USE command indicates that the line to be used is the one with the proper name EXAMPLE. Several different lines may be assembled in a single set of data. If there is no USE command, TURTLE will run through the elements in order as they occur in the data.

A TURTLE Input Data Set Using MAD Elements

The physical elements making up the beam line can be specified using MAD notation. However, the beam line is specified by simply listing the elements in order. The result is a set of data which appears as:

```
'EXAMPLE OF A TURTLE DATA SET'
10000
UTRANS
BEAM  X = .5,  XP = 1,  Y = .5,  YP = 1,  L = .5,  DEL = 1,  P0 = 1  ;
SEC1:   -ORDER   ;   PRINT,   BEAM,   ON   ;
DRI: DRIFT,   L  =  2.745   ;
SBEND,   L  =  9.879,   ANGLE  =  2.96,   K1  =  .0449   ;
DRI:   DRIFT,   L  =  2.745   ;
PRINT,   TRANS   ;
HIST,   X,   LOWER = -1.0,   UPPER = 1.0,   STEP = 0.02   ;
SENTINEL
```

A TURTLE Input Data Set with Keyword Notation

The same set of data can also be represented in an alternate keyword notation. The variables used here are the same as with original TRANSPORT notation. The complete set of variables available for specifying elements is described in the sections for the individual elements.

```
'EXAMPLE OF A TURTLE DATA SET'
10000
UTRANS
BEAM  X = .5,  XP = 1,  Y = .5,  YP = 1,  L = .5,  DEL = 1,  P0 = 1  ;
SEC1:   -ORDER   ;   PRINT,   BEAM,   ON   ;
DRI: DRIFT,   L  =  2.745   ;
ROTAT,   ANGLE  =  0.0   ;
BEND,   L  =  9.879,   B  =  10,   N  =  .5   ;
ROTAT,   ANGLE  =  0.0   ;
DRI:   DRIFT,   L  =  2.745   ;
PRINT,   TRANS   ;
HIST,   X,   LOWER = -1.0,   UPPER = 1.0,   STEP = 0.02   ;
SENTINEL
```

The combination `ROTAT`, `BEND`, `ROTAT` replaces the element `SBEND`. The element `BEND` can be specified in MAD variables, but it is not a MAD element, and will not be recognized by the MAD program.

Summaries of TURTLE Elements

Table 1. Summary of TURTLE Elements in MAD Notation

Physical Element	Mnemonic	Keywords					
Drift	DRIFT	L					
Bending Magnets with Fringing Fields	RBEND or SBEND	L	ANGLE	K1	K2	K3	TILT
Horizontal and Vertical Steering Magnets	HKICK or VKICK	L	KICK				
Steering Magnet in Both Planes	KICKER	L	HKICK	VKICK			
Quadrupole	QUAD	L	K1	TILT			
Sextupole	SEXT	L	K2	TILT			
Octupole	OCTU	L	K3	TILT			
Plasma Lens	PLAS	L	K1				
Solenoid	SOLE	L	KS				
Accelerator	ACCE	L	VOLT	LAG	FREQ		
Coordinate Rotation	SROT	ANGLE					
Marker	MARKER						

The meanings of the various keywords are given in more detail in the sections of this manual describing the individual physical elements

Table 2. Summary of Other TURTLE Elements in Keyword Notation

Preliminary Specifications in Keyword Notation

Physical Element	Mnemonic	Keywords		
Units	UNIT	Unit type	Unit name	SIZE
Apertures	OBSERVE	APERTURES		
Reference charge	REFERENCE	CHARGED	NEUTRAL	
Mass of Particles in beam	SPEC	PMASS		
Half Aperture of Bending Magnet in x Plane	SPEC	HWIDTH		
Half Aperture of Bending Magnet in y Plane	SPEC	HGAP		
Length of System	SPEC	LENGTH		
Reference Trajectory Momentum	SPEC	P0		
Random Number	SPEC	RANNO		
Focal Plane Rotation	SPEC	FOTILT		
Reference Momentum for Magnetic Fields	SPEC	PREF		
Parent Particle Name and Charge	SPEC	PARENT		
First Daughter Particle Name and Charge	SPEC	DAUGHTER1		
Second Daughter Particle Name and Charge	SPEC	DAUGHTER2		
Order of Calculation	ORDER	N1	N2	N
Random Errors	RANDOM	Element Type	Parameter Name	VARIATION

Beam Description in Keyword Notation

Physical Element	Mnemonic	Keywords									
Beam	BEAM	X	XP	Y	YP	L	DEL	PO	GAUSS	SHELL	READ
RMS Addition	BEAM	X	XP	Y	YP	L	DEL	PO	RMS	particle name	
Beam (Rotated Ellipse)	CORR	C21, C31, C41, C51, C61, C32, C42, C52, C62, C43, C53, C63, C54, C64, C65									
Beam Centroid Shift	CENT	X	XP	Y	YP	L	DEL				

Physical Elements in Keyword Notation

Physical Element	Mnemonic	Keywords				
Drift Space	DRIFT	L				
Wedge Bending Magnet without Fringing Field	BEND	L L L L B RADIUS	ANGLE ANGLE B RADIUS ANGLE ANGLE	K1 N N N N N	K2 EPS EPS EPS EPS EPS	K3 EPS3 EPS3 EPS3 EPS3 EPS3
Pole Face Rotation including Fringing Field	ANGLE	HGAP	FINT	H		
Bending Magnets with Fringing Fields	RBEND or SBEND	L L L B RADIUS	ANGLE B RADIUS ANGLE ANGLE	N N N N N	EPS EPS EPS EPS EPS	EPS3 EPS3 EPS3 EPS3 EPS3
Horizontal and Vertical Steering Magnets	HKICK or VKICK	L L	B ANGLE			
Quadrupole	QUAD	L L	B GRAD	APER TILT	TILT	
Sextupole	SEXT	L	B	APER	TILT	
Octupole	OCTU	L	B	APER	TILT	
Plasma Lens	PLAS	L L	B GRAD	APER TILT	TILT	
Solenoid	SOLE	L	B			
Accelerator	ACCEL	L	VOLT	LAG	WAVEL	
Electrostatic Septum	SEPTUM	L L	VOLTAGE E	APER	TILT	
Slit	SLIT	Coordinate	HWIDTH			
Elliptical Slit	SLIT	Coordinate	Coordinate	HWIDTH	HHEIGHT	

Other Transformations in Keyword Notation

Physical Element	Mnemonic	Keywords						
Alignment	ALIGN	X	RX	Y	RY	Z	RZ	Other Keywords
Arbitrary Matrix	MATRIX	Rij	Tijk	Uijkℓ				
Coordinate Rotation	SROT	ANGLE						
Coordinate System Shift	SHIFT	X	XP	Y	YP	L	DEL	

Assembling Beam Lines

Physical Element	Mnemonic	Keywords			
Repeat control	REPEAT	N			
Defined Section	SECTION	BEGIN	END	FORWARD	BACKWARD

Operations in Keyword Notation

Physical Element	Mnemonic	Keywords				
Update	UPDATE	R1	R2			
Alignment Marker	ALMARK	R1	R2			
Storage of Parameters	STORE	Parameter Name				
One-dimensional Histogram	HIST	Coordinate	LOWER	UPPER	STEP	FLAG
Horizontal Coordinate of Two-Dimensional Histogram	AHIST	Coordinate	LOWER	UPPER	STEP	FLAG
Vertical Coordinate of Two-Dimensional Histogram	DHIST	Coordinate	LOWER	UPPER	STEP	FLAG
Mean Value Plot	PROFILE	Coordinate	LOWER	UPPER	STEP	FLAG
Histogram Flag	FLAG					

Output Format

Initial Listing

TURTLE begins by printing out the user's input data. The listing is identical to that produced by TRANSPORT for the same data. The total number of elements in the beam line after each line is interpreted is also printed out. The number may increase by more than one after certain lines for two reasons:

1. Several elements may be placed on a single line, separated by semicolons.

2. A data element may result in several stored elements. One example of that is an element containing an algebraic expression. The instructions to retrieve the quantities in the expression and to perform the algebraic operations are stored in the program as elements.

The only other output is made up of the histograms. In the printed output, each histogram takes up a separate page. If a histogram is large enough, it may take up two or more pages. Examples from TURTLE of one- and two-dimensional histograms and a mean value plot follow. Descriptions of the data format to specify histograms is found on page 160.

Examples of Histogram Output

```
HISTOGRAM NO  1
DISTRIBUTION OF    X IN CM    546.750 FT   FROM THE START


            INTERVAL                          SCALE FACTOR:  100 X'S EQUAL    309 RAYS

LESS THAN     5.800         0

   5.800 TO   5.820         0
   5.820 TO   5.840         0
   5.840 TO   5.860         0
   5.860 TO   5.880         0
   5.880 TO   5.900         0
   5.900 TO   5.920         0
   5.920 TO   5.940         0
   5.940 TO   5.960         0
   5.960 TO   5.980         0
   5.980 TO   6.000         6   X
   6.000 TO   6.020        19   XXXXX
   6.020 TO   6.040        42   XXXXXXXXXXXX
   6.040 TO   6.060        78   XXXXXXXXXXXXXXXXXXXXXXXX
   6.060 TO   6.080       110   XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   6.080 TO   6.100       137   XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   6.100 TO   6.120       183   XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   6.120 TO   6.140       215   XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   6.140 TO   6.160       249   XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   6.160 TO   6.180       249   XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   6.180 TO   6.200       305   XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   6.200 TO   6.220       287   XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   6.220 TO   6.240       298   XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   6.240 TO   6.260       302   XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   6.260 TO   6.280       309   XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   6.280 TO   6.300       305   XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   6.300 TO   6.320       290   XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   6.320 TO   6.340       283   XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   6.340 TO   6.360       278   XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   6.360 TO   6.380       253   XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   6.380 TO   6.400       213   XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   6.400 TO   6.420       178   XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   6.420 TO   6.440       150   XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   6.440 TO   6.460       114   XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   6.460 TO   6.480        78   XXXXXXXXXXXXXXXXXXXXXXXXX
   6.480 TO   6.500        44   XXXXXXXXXXXXX
   6.500 TO   6.520        20   XXXXX
   6.520 TO   6.540         5   X
   6.540 TO   6.560         0
   6.560 TO   6.580         0
   6.580 TO   6.600         0
   6.600 TO   6.620         0
   6.620 TO   6.640         0
   6.640 TO   6.660         0
   6.660 TO   6.680         0
   6.680 TO   6.700         0

GREATER THAN   6.700         0

        TOTAL NUMBER OF ENTRIES =       5000     INCLUDING UNDERFLOW AND OVERFLOW

        CENTER =      6.261   RMS HALF WIDTH =      0.111


DISTRIBUTION OF    X IN CM    546.750 FT   FROM THE START
```

Example of a histogram for the quantity x.

```
HISTOGRAM NO  3
HORIZONTAL AXIS    X IN CM     546.750 FT   FROM THE START
VERTICAL AXIS      Y IN CM     546.750 FT   FROM THE START
                   5.800     6.000     6.200     6.400     6.600      TOTALS
                   I**---**---**---**---**---**---**---**---**---**I--------
 -0.360 TO  -0.340 I                                             I      0
 -0.340 TO  -0.320 I                                             I      0
 -0.320 TO  -0.300 I                                             I      0
 -0.300 TO  -0.280 I                                             I      0
 -0.280 TO  -0.260 I                    3     2    1             I      6
 -0.260 TO  -0.240 I                   13   112221 3             I     16
 -0.240 TO  -0.220 I                 151323341454 3              I     39
 -0.220 TO  -0.200 I               2 3435275955545311            I     69
 -0.200 TO  -0.180 I              121516478944778363231          I     92
 -0.180 TO  -0.160 I             21 84358A58F6857752531          I    118
 -0.160 TO  -0.140 I            1236579FE79ACBCID863423          I    180
 -0.140 TO  -0.120 I            1732B9CEJJD84FCE67998631         I    212
 -0.120 TO  -0.100 I            3377ABBAFGFA7A8CCEC8323          I    209
 -0.100 TO  -0.080 I          11696BALDDI9IHLFLKF8993421         I    281
 -0.080 TO  -0.060 I          247ABCHDGDIDMQEGEKA9CA852          I    304
 -0.060 TO  -0.040 I         1253C9CCIHGDMOEJKJJGEB9D6311        I    331
 -0.040 TO  -0.020 I         425EBDHEFNHIMKGKJEIE7BE7631         I    345
 -0.020 TO   0.000 I         326BJAIEMMIJEJLJFLGFH96644          I    350
  0.000 TO   0.020 I         1379C9DGHFOMHOEIEIHFBE768332        I    339
  0.020 TO   0.040 I         2 4869JIK7DFGGLADGJFFD86641         I    300
  0.040 TO   0.060 I         1 555ABAAHI9IERJGELGHG9D3531        I    313
  0.060 TO   0.080 I         131467DIFCJIJFKMNGBGD9A971          I    308
  0.080 TO   0.100 I          155688EC7EBGLBIGGBEA8B6112         I    253
  0.100 TO   0.120 I         2371667C6DLKEBFG6CAEC6253           I    230
  0.120 TO   0.140 I         1 4597DFIB6GJBCCJ7954441            I    212
  0.140 TO   0.160 I         2137569A8G8477CF9B776222            I    166
  0.160 TO   0.180 I          11 563727D77A57855572              I    113
  0.180 TO   0.200 I          1 1 39546987732523311              I     80
  0.200 TO   0.220 I           1143318452442434331               I     60
  0.220 TO   0.240 I            1 2 36414462133                  I     40
  0.240 TO   0.260 I              221345 24211                   I     27
  0.260 TO   0.280 I                 1114                        I      7
  0.280 TO   0.300 I                                             I      0
  0.300 TO   0.320 I                                             I      0
  0.320 TO   0.340 I                                             I      0
  0.340 TO   0.360 I                                             I      0
                   I**---**---**---**---**---**---**---**---**---**I--------
                   I                                             I
                   I                                             I
                   I            11122232233322222111             I
                   I            14713814408900098751751742        I
           TOTALS  I 000000000692807359957829503833804840500000000 I   5000

           TOTAL NUMBER OF ENTRIES =      5000 INCLUDING UNDERFLOW AND OVERFLOW AS FOLLOWS


                             LEFT                      RIGHT
           ABOVE              0           0              0
                              0         5000             0
           BELOW              0           0              0


  SUM OF SQUARES =        64988.
          CENTER =     6.261   RMS HALF WIDTH =     0.111
          CENTER =    -0.001   RMS HALF WIDTH =     0.106
          CORRELATION =   -0.0011


NO    3 TWO DIMENSIONAL PLOT OF
HORIZONTAL AXIS    X IN CM     546.750 FT   FROM THE START
VERTICAL AXIS      Y IN CM     546.750 FT   FROM THE START
```

Example of a two-dimensional histogram of y vs x.

```
HISTOGRAM NO  4
VERTICAL AXIS    DP/P IN PC       0.000 FT   FROM THE START
HORIZONTAL AXIS     X IN CM       0.000 FT   FROM THE START

                                            VERTICAL SCALE FROM -20.00000 TO  20.00000
            INTERVAL              MEAN    ERROR   HORIZONTAL SCALE FROM  -0.20000 TO   0.20000

LESS THAN    -20.000      0     0.000    0.000

 -20.000 TO  -19.000      0     0.000    0.000
 -19.000 TO  -18.000      0     0.000    0.000
 -18.000 TO  -17.000      0     0.000    0.000
 -17.000 TO  -16.000      0     0.000    0.000
 -16.000 TO  -15.000      0     0.000    0.000
 -15.000 TO  -14.000    345    -0.001    0.002                                    X
 -14.000 TO  -13.000    342     0.001    0.002                                     X
 -13.000 TO  -12.000    322    -0.002    0.002                                    X
 -12.000 TO  -11.000    346     0.001    0.002                                     X
 -11.000 TO  -10.000    330     0.000    0.002                                    X
 -10.000 TO   -9.000    326     0.001    0.002                                     X
  -9.000 TO   -8.000    342     0.002    0.002                                     X
  -8.000 TO   -7.000    347    -0.002    0.002                                    X
  -7.000 TO   -6.000    307     0.000    0.002                                    X
  -6.000 TO   -5.000    312    -0.002    0.002                                    X
  -5.000 TO   -4.000    320     0.000    0.002                                     X
  -4.000 TO   -3.000    328     0.001    0.002                                     X
  -3.000 TO   -2.000    359     0.000    0.002                                    X
  -2.000 TO   -1.000    353    -0.003    0.002                                    X
  -1.000 TO    0.000    302    -0.001    0.002                                    X
   0.000 TO    1.000    351    -0.001    0.002                                    X
   1.000 TO    2.000    364     0.002    0.002                                     X
   2.000 TO    3.000    338     0.001    0.002                                     X
   3.000 TO    4.000    316     0.000    0.002                                    X
   4.000 TO    5.000    300     0.002    0.002                                     X
   5.000 TO    6.000    351     0.000    0.002                                     X
   6.000 TO    7.000    322    -0.003    0.002                                    X
   7.000 TO    8.000    304     0.002    0.002                                     X
   8.000 TO    9.000    372     0.000    0.002                                     X
   9.000 TO   10.000    336     0.001    0.002                                     X
  10.000 TO   11.000    321     0.000    0.002                                    X
  11.000 TO   12.000    344    -0.003    0.002                                    X
  12.000 TO   13.000    322     0.000    0.002                                     X
  13.000 TO   14.000    355     0.002    0.002                                     X
  14.000 TO   15.000    323     0.001    0.002                                     X
  15.000 TO   16.000      0     0.000    0.000
  16.000 TO   17.000      0     0.000    0.000
  17.000 TO   18.000      0     0.000    0.000
  18.000 TO   19.000      0     0.000    0.000
  19.000 TO   20.000      0     0.000    0.000

GREATER THAN  20.000      0     0.000    0.000

        TOTAL NUMBER OF ENTRIES =     10000     INCLUDING UNDERFLOW AND OVERFLOW
        CENTER =     -0.007   RMS HALF WIDTH =     8.679
        CENTER =      0.000   RMS HALF WIDTH =     0.031
        CORRELATION =    0.0037

MEAN VALUE PLOT NO  4
VERTICAL AXIS    DP/P IN PC       0.000 FT   FROM THE START
HORIZONTAL AXIS     X IN CM       0.000 FT   FROM THE START
```

Example of a mean value plot of x vs $\delta = \Delta p / p_0$.

The mean value plot is perhaps the only type of histogram needing some explanation in addition to what is to be found in the section on the histogram specification. The mean value plot is like a two-dimensional histogram in that two coordinates to be represented are specified, along with the total range and bin size for each. The rays are binned according to the first coordinate. The bins are spread out vertically, as with a two-dimensional histogram. Within each bin, the average value of the second coordinate is calculated. Its value is represented by the horizontal position of the X on the line corresponding to a given bin.

Title, Number of Rays, and Comments

Title

The title occupies the first line in every problem step of a TURTLE data set. *The title is always required and must be followed by a second line with the number of rays* (see next section).

The title must be enclosed within quotation marks ('). The string may begin and end in any column (free field format), for example

        `'High-Resolution 200 GeV Beam Line'`

Example of a TURTLE DATA SET

        Title
        Number of rays
        Elements

        ————

        ————

        `SENTINEL`

Number of Rays

The second line of the input contains the number of trajectories to be traced through the system. If there are two integers on the second line, then the first one is the TRANSPORT indicator number. It has no meaning in TURTLE. The second integer is the number of trajectories. Thus the second line, indicating 100,000 trajectories are to be run through the beam line might be

        100000

where only the number of trajectories is indicated, or

        0 100000

where both the TRANSPORT indicator and the number of trajectories are present.

After the line containing the number of rays is read, TURTLE can either continue reading the data of which the title and number rays are a part. Alternatively, TURTLE can read a binary file created by TRANSPORT. This binary file gives the description of the beam line and any histograms to be created. The histograms must be inserted in the data when running TRANSPORT, since there is no further editing in TURTLE.

The keyword which indicates that the remainder of the data is to be read from a binary file is BREAD. This keyword follows the number of rays and is separated from that number by a comma. Thus the indicator line, in this case, would be

        0 100000, BREAD

The common blocks are then read from FORTRAN logical unit 7 on a disk. The data need consist of only a line giving the TITLE and the line giving the number of rays along with the mnemonic BREAD. All the data, including all the elements and the SENTINEL, are read from the disk file.

Comments

Comments may be introduced anywhere in the deck where an element would be allowed. Comments are identified differently in MAD and in original TRANSPORT. Both methods of identifying comments are available in the present versions of TRANSPORT and TURTLE. The comments are not stored, but appear only in the initial listing of the beam line.

In MAD anything on a line after an exclamation mark (!) is considered to be a comment. For example, the bending magnet element described earlier can have an attached comment as follows:

```
BM1:   SBEND,  L = 10.,  ANGLE = 0.01,  K1 = 0.5E-6 ; !  THIS IS A COMMENT
```

In original TRANSPORT the comments made on any line are enclosed within single parentheses. No parentheses are allowed within the parentheses containing a comment.

Example of the Use of Comments in a Data Set

```
'TITLE'
0
(THIS IS A TEST PROBLEM TO ILLUSTRATE THE)
(USE OF COMMENTS)
elements
(COMMENTS MAY ALSO BE MADE BETWEEN ELEMENTS)
elements
SENTINEL
```

Preliminary Specifications

Input-Output Unit Sets –
UMM, UMETER, UMICR, UMAD, UTRANS

TRANSPORT (and therefore TURTLE) allows a great deal of flexibility in the choice of units for representation of physical quantities. Several different sets of units may be specified by means of single unit set commands. The choice of units may be made independently of the choice of notation for the elements.

Further changes may be made in individual units changes. The unit set commands are described in this section. The format for individual unit changes is given in the next section.

The unit sets `UMETER` and `UMAD` use meters for transverse dimensions and radians for angles. The unit set `UMM` uses millimeters for transverse dimensions and milliradians for angles. The unit set `UMICRON` uses microns (not micrometers – that is a measuring instrument, not a unit) for transverse dimensions and microradians for angles.

Since transfer matrix elements are ratios of transverse measures of the beam, the elements of the transfer matrix are numerically the same in the four unit sets `UMETER`, `UMAD`, `UMM`, and `UMICRON`. The accelerator phase space parameters beta and alpha will also be the same in all four of these unit sets.

The unit set `UMM` has transverse distances in millimeters and transverse angles in milliradians. It also has longitudinal distance in meters. These units are on the order of the dimensions usually found in accelerators and beam lines. The unit set `UMM` is therefore a very convenient unit set to work with. All further examples in this manual will be given in the unit set `UMM`, unless there is good reason to mention more than one unit set.

The command `UMAD` allows compatability with the MAD program and facilitates the performance of accelerator calculations. Both unit sets `UMAD` and `UMETER` cause all lengths to be expressed in meters and all angles (except for accelerator phase advance) to be expressed in radians. The fractional momentum deviation δ is expressed as a fraction, not as a percent. One reason these units are preferred by accelerator designers is that they are self-consistent. The angular measure milliradians is intrinsically self-inconsistent since it is the ratio of two lengths expressed in different units. The unit percent is self-inconsistent since a quantity in percent must be divided by 100 to be used in any equation.

The use of a particular set of units, such as `UMAD` is specified by placing the appropriate command, such as `UMAD`, at the beginning of the sequence of elements in the data deck. This command should immediately follow the title and indicator and precede any `UNIT` command, described in the following section.

Example of the Use of UMAD in a Data Set
_____

```
        'TITLE'
        0
        UMAD
        other elements
        SENTINEL
```


A different set of units was originally considered standard in TRANSPORT. This original set of units is still available and is specified by substituting the command UTRANS for UMAD in the above example. Previous manuals were written in terms of this original set of units. Now there is a choice among several basic sets. The units to be used in specifying the physical parameters of the various elements are given in the sections describing those elements. Appropriate units are given for UMM and sometimes UMETER.

The UMAD command is very similar to UMETER. The two specifications differ in that with UMAD the accelerator phase advance is in radians/2π while the coordinate rotation about the longitudinal axis is specified in radians. In all other unit sets, both quantities are specified in degrees.

With UMETER the multipole strengths are also specified in the standard TRANSPORT convention as the normalized field derivative divided by $n!$, where n is the order of the multipole. Compared to UMETER, the UMAD sextupole strength should be multiplied by a factor of two. The UMAD octupole strength should be multiplied by a factor of six. Details may be found in the section of this manual on Representation of Magnetic Fields under the Mathematical Formulation of TURTLE.

Any of the five unit sets, UMAD, UTRANS, UMETER, UMM, or UMICRON can be used for any of the physical parameters on any element. The units of a specific type of quantity, such as length, transverse distance, angle, magnetic field, particle mass can also be set individually. See the next section describing the UNIT command. Units can be anything the user desires, such as furlongs, parsecs, or cubits for length, arc seconds for angles, stone or solar masses for mass, and Webers per square foot for magnetic field.

The TRANSPORT (and therefore TURTLE) units and their meanings are:

Keyword	Quantity	Unit in Unit set UMM	Symbols used in SLAC-75
X	horizontal and vertical transverse dimensions, magnet apertures, and misalignment displacements	mm	x,y
XP	horizontal and vertical angles and misalignment rotation angles	mr	θ, ϕ
DL	pulsed beam length and wave length in accelerator	mm	ℓ
DP	momentum spread	permil	δ
ANGLE	bend, pole face rotation, and coordinate layout angles	mr	
L	length (longitudinal) of elements, layout coordinates, and bending magnet pole face curvatures	meters	t
B	magnetic fields	kG	B
MASS	mass	electron mass	GeV
P	momentum and energy gain in accelerator section	GeV/c GeV	p(0) ΔE
PHASE	Betatron phase shift	degrees	ψ
ROLL	Rotations about the reference trajectory	degrees	
V	Electric fields and voltages	MV	

The units that are specified by the various unit sets are as follows. Any further units changes should refer to these units:

Code Digit	UMM	UMETER	UMICR	UMAD	UTRANS
X	mm	meters	μ	meters	cm
XP	mr	radians	μr	mradians	mr
DL	mm	meters	μ	meters	cm
DEL	pm	fraction	pμ	fraction	pc
ANGLE	mr	radians	μr	radians	degrees
L	meters	meters	meters	meters	meters
B	kG	kG	kG	kG	kG
MASS	GeV	GeV	GeV	GeV	Electron mass
P	GeV/c	GeV/c	GeV/c	GeV/c	GeV/c
PHASE	degrees	degrees	degrees	radians/2π	degrees
ROLL	degrees	degrees	degrees	radians	degrees
V	MV	MV	MV	MV	MV

The meaning of the abbreviations used above is as follows:

cm	=	centimeters
mm	=	millimeters
μ	=	microns
mr	=	milliradians
μr	=	microradians
pc	=	percent
pm	=	permil (one part in a thousand)
pμ	=	permic (one part in a million)
kG	=	Kilogauss
MV	=	Megavolts

The command UMAD also affects the definition of the multipole component (see above).

UNIT – Individual Units Changes

Strict MAD input requires no individual units changes. It requires merely the specification of the global unit set `UMAD`, described on the previous few pages. Elements described in terms of `MAD` parameters, in other keyword notation, or in original TRANSPORT notation may require individual units changes from one of the global unit set specifications.

The individual units changes are placed in the data at the beginning of the ELEMENTS. They follow only the global units specifications.

The individual units changes are written in keyword notation. A `UNIT` element which changes the `UMM` transverse unit of meters to inches would look like:

```
UNIT,  X,  IN  ;
```

Since TURTLE recognizes the mnemonic `IN`, a units conversion factor is not needed. If TURTLE did not recognize the mnemonic `IN`, then the above units conversion element would have to be written as:

```
UNIT,  X,  IN,  SIZE = 25.4  ;
```

In addition to the element name `UNIT`, there are three necessary items to be specified:

1. The unit type (`X`, `XP`, `P`, etc.) as indicated on the charts on the previous two pages.

2. The abbreviation of the new unit (see examples below). This will be printed on the output listing. It is written as a simple keyword.

3. The scale factor (if needed). The scale factor is needed only if TURTLE does not recognize the abbreviation of the new unit. If a new unit (such as furlongs or stone) is introduced, TURTLE needs to be told the size of the new unit. The scale factor is the size of the new unit in terms of the unit in the prevailing unit set. It is indicated by the keyword `SIZE`. Consider, for example, the case when the new transverse unit is inches and the unit set `UMM` has been specified. The `UMM` transverse unit is mm, and an inch is 25.4 mm, so the scale factor is 25.4.

Examples

In the following examples, it will be assumed that the prevailing unit set is `UMM`. Thus, unless otherwise specified, longitudinal units are in meters, transverse units in millimeters and milliradians, and momentum in GeV/c.

To change length to feet, width to inches, and momentum to MeV/c, add to the front of the data set the elements

```
UNIT,  L,  FT  SIZE = 0.3048;
UNIT,  X,  IN  SIZE = 25.4;
UNIT,  P,  MEV  SIZE = 0.001;
```

The scale factor, 0.3048, gives the length of the new unit, feet, in terms of the reference unit, meters, etc. Since the unit names FT, IN, and MEV are all recognized by TRANSPORT, the size specification is unnecessary, and may be omitted.

Qualifications

The keywords ELEMENT, BEAM, FLOOR, ALIGN, INPUT, and OUTPUT may be used to restrict the units change to a limited application. The keywords are placed on the units element, and are separated by commas. The keyword ELEMENT indicates that the units change is to be used only for the description of a physical element. The application can be further restricted to the input data alone or the printed output alone by use of the additional keywords INPUT or OUTPUT. There is no output of the input data in TURTLE, but the keyword OUTPUT can still be in the input.

The data may then be input in one set of units and output in another. Two beam lines may also be concatenated longitudinally, even if they are specified in different units. A new set of units specifications must be placed between the two beam line descriptions.

The keyword BEAM refers to the beam dimensions and the transfer matrix elements. The keyword FLOOR refers to the floor coordinates. The keyword ALIGN refers to the misalignment parameters. These three keywords apply both to input and output and are not subject to further restriction by the INPUT and OUTPUT keywords.

The unit elements are the first elements in the data (immediately following the title and the number of trajectories) and should not be inserted in any other location. The unit set specification (UTRANS, UMAD, UMETER, UMM, or UMICR) should appear first, followed by any UNIT elements. Units specifications produce no printed output during the calculation, their effect being visible only in the output from other elements.

More Examples

If the linear measures for the units changes above were to apply only to the input data for the physical elements, then the example would be modified to:

```
UNIT,  L,  FT,  SIZE = 0.3048,  ELEMENT,  INPUT ;
UNIT,  X,  IN,  SIZE = 25.4,  ELEMENT,  INPUT ;
UNIT,  P,  MEV,  SIZE = 0.001  ;
```

If the beam dimensions are to be expressed in microns and microradians, the following elements should be added to the front of the data set.

```
        UNIT,  X,  MU,  SIZE = 1.0E-3 ,  BEAM ;
        UNIT,  XP,  MUR,  SIZE = 1.0E-3 ,  BEAM ;
```

Finally, if the misalignment parameters are to be expressed in mils (thousandths of an inch) and arc minutes, then the following units changes should be inserted.

```
        UNIT  X,  MIL,  SIZE = .0254,  ALIGN ;
        UNIT  XP,  MIN,  SIZE = .290888,  ALIGN ;
```

The specifications of sets of units is equivalent to a set of UNIT specifications, with one small caveat. The specification of a unit set (UTRANS, UMAD, etc.) not only specifies the units, but also the reference for further unit changes. The specification of the unit size on the UNIT element is done relative to the prevailing unit set.

For the conventional units listed below, it is sufficient to stop with the unit name (the conversion factor is automatically inserted by the program). If units other than those listed below are desired, then the unit name and the appropriate conversion factor must be included. The conversion units below are based on the assumption that the prevailing unit set is UMM.

Input-output units: UNIT element – For use with UMM
(Conversion factors for dimension changes versus unit type mnemonic and unit)

Unit	Unit Type Mnemonic											
	X	XP	DL	DEL	ANGLE	L	B	MASS	P	PHASE	ROLL	V
M	1000.	--	1000.	--	--	1.	--	--	--	--	--	--
CM	10.	--	10.	--	--	.01	--	--	--	--	--	--
MM	1.	--	1.	--	--	.001	--	--	--	--	--	--
MICR	.001	--	.001	--	--	.001	--	--	--	--	--	--
FT	304.8	--	304.8	--	--	.3048	--	--	--	--	--	--
IN	25.4	--	25.4	--	--	.0254	--	--	--	--	--	--
R	--	1000.	--	--	1000.	--	--	--	--	(a)	(a)	--
MR	--	1.	--	--	1.	--	--	--	--	(b)	(b)	--
MUR	--	.001	--	--	.001	--	--	--	--	(c)	(c)	--
DEG	--	(d)	--	--	(d)	--	--	--	--	1.	1.	--
PC	--	--	--	10.	--	--	--	--	--	--	--	--
PM	--	--	--	1.	--	--	--	--	--	--	--	--
PMIC	--	--	--	.001	--	--	--	--	--	--	--	--
N	--	--	--	1000.	--	--	--	--	--	--	--	--
MEV	--	--	--	--	--	--	--	--	.001	--	--	--
GEV	--	--	--	--	--	--	--	--	1.	--	--	--
KG	--	--	--	--	--	--	1.	--	--	--	--	--
G	--	--	--	--	--	--	.001	--	--	--	--	--
TUNE	--	--	--	--	--	--	--	--	--	$1/2\pi$	--	--

(a) The size of a radian in degrees is $180./\pi$
(b) The size of a milliradian in degrees is $.180/\pi$
(c) The size of a microradian in degrees is $.000180/\pi$
(d) The size of a degree is milliradians is $1000.*\pi/180.$
PC is an abbreviation for percent
PM means permil which is one part in a thousand or one-tenth of a percent
PMIC means permic which is one part in a million or one-ten-thousandth of a percent
N means fractional measure or parts per 100 percent

OBSERVE — Observe Apertures

The insertion of the

```
        OBSERVE,   APERTURES   ;
```

element in the data, will cause all specified apertures to be taken into account. If a particle trajectory passes outside a given aperture, it is terminated and a new trajectory is initiated at the beginning of the system.

The quadrupole aperture is normally taken to be circular and the half aperture is given by the keyword `APERTURE` for the `QUAD` element. One can specify a different aperture for quads and make it elliptical through the use of the `SPECIAL` element. The apertures for bending magnets are given by the (`SPECIAL, HGAP`) and (`SPECIAL, HWIDTH`) elements. All aperture constraints are applied both at the entrance and exit faces of the element to which they pertain. See the section describing the use of the `SPECIAL` element for further details on aperture specification.

In DECAY TURTLE the apertures for each of the three particle types can be activated separately. The default names for the three particle types are `PARENT`, `DAUGHTER1`, and `DAUGHTER2`. These names can be changed via the `SPEC` element. A popular choice is `PION`, `MUON`, and `NEUTRINO`.

The name of the particle for which the apertures are to be effective is included in the element activating the apertures. Thus if the command were to apply to muons only, the element would be written:

```
        OBSERVE,   APERTURES,   MUON   ;
```

If an aperture activating element is not present for a given kind of particle, the only apertures affecting those particles are slits.

REFERENCE – Charge of Reference Trajectory

The reference trajectory is normally taken to be a possible path of a particle with unit positive charge and the central design momentum of the beam line. If the charge of the parent particle is respecified to be neutral, then this may no longer be the case. Sweeping magnets may be used to separate neutral parents from directly produced charged particles. The reference trajectory through the sweeping magnets will be that of a neutral particle.

The charge of the reference trajectory is neutral following a

```
        REFERENCE,   NEUTRAL   ;
```

element. It is restored to unit positive charge by a

```
        REFERENCE,   CHARGED   ;
```

element.

SPECIAL — Special Input Parameters

A number of constants are used by the program which do not appear as parameters of any element. Many of these are initial values, such as floor coordinates, to be set at the beginning of the beam line. Others are the default values of certain parameters which do appear in the description of physical elements. A special element has been provided to allow the designer to set the values of these various constants.

Most of these special parameter elements will appear at the beginning of the data, after any units changes. In any case, the special parameters must always precede the physical element(s) to which they apply. *Once introduced, they apply to all succeeding elements of the appropriate type in the beam line unless reset to zero or to new values.* Many of the `SPECIAL` parameters apply to bending magnets. In such cases, the value of a parameter as given by the `SPECIAL` element may be overridden for a single magnetic element by including the parameters in the description of the element itself.

There is no strictly MAD format for the `SPECIAL` element. However, any `SPECIAL` element can be written in keyword notation.

Keyword Notation

A large number of keywords are available for use with the `SPECIAL` element and are listed below. However, each use of the `SPECIAL` element involves a single keyword. Separate special parameters must be set by separate uses of the `SPECIAL` element. An example might be the specification of the quadratic dependence of a bending magnet field. Using keyword notation, it could also be written as

```
SPEC,  EPS = .001  ;
```

A precise definition of this particular special parameter can be found below on page 53.

When `EPS` is specified by the special parameter element, the value given is applied to all subsequent bending magnets. This particular special parameter can also be written directly on any element to which it applies. If so, the value given on the element will momentarily override the value given by the `SPECIAL` element. The `RBEND`, `SBEND`, and `BEND` elements will recognize the special parameter `EPS`. When `EPS` is given on a bending magnet element, it applies only to that bending magnet.

Some of the special parameters are used by TRANSPORT but not by TURTLE. In such a case, TURTLE will accept the parameter in the input data, but it will have no effect for the calculation. Only keywords describing a parameter used by TURTLE are described below.

Keywords and Their Meanings

The keywords which can be used in specifying the special input parameters are given below. First, we have a small table, giving a very short description of the use of each keyword. The table is followed by a set of minisections. In each minisection is given a more complete description of one or more of the special parameters and their use. Included also may be equations, diagrams, sample data, and tables of possible values.

Symbol	Keyword	Description
	N	Multipole components of quadrupoles
	PARAM	
	PHASE	
L	LENGTH	Cumulative length of the System
p_0	P0	Reference Trajectory Momentum
p_{ref}	PREF	Reference Momentum for Magnetic Fields
m	PMASS	Particle Mass
α	FOTILT	Tilt to Focal Plane
	RANNO	Random Number
$W/2$	HWIDTH	Half Apertures of Bending Magnets
$g/2$	HGAP	
κ_1	FINT1	Fringing-Field Integrals
κ_2	FINT2	
$1/R_1$	H1	Pole-Face Curvature
$1/R_2$	H2	
ϵ	EPS	Quadratic term for Magnetic Field
ϵ_3	EPS3	Cubic term
r_s	RMPS	Other Magnetic Field Components
r_a	RNMS	
v_r	VR	
n'	NP	
ϵ'	EPSP	
	QAPA	Quadrupole Apertures
	QAPB	
	QAPH	
	PARENT	Particle Names and Charges
	DAUGHTER1	
	DAUGHTER2	
	CHARGE	

Multipole Aberrations in Quadrupoles

The specifications for a sextupole component, an octupole component, and a dodecapole component in a physical quadrupole might be:

```
SPEC,   N  =  -3, PARAM  =  .001,   PHASE  =  54.   ;
SPEC,   N  =  -4, PARAM  =  .002,   PHASE  =  3.    ;
SPEC,   N  =  -6, PARAM  =  .005,   PHASE  =  -8.   ;
```

The phase angles are given here in degrees. The three parameters used are N, PARAM, and PHASE. The meanings of the parameters are as follows:

A negative code digit N indicates a multipole aberration in a quadrupole. Code digit -N indicates a $2N$ pole. Multipoles up to and including a 40 pole may be included.

The special parameter PARAM is equal to B_N/ga^{N-1}, where B_N is the pole tip field due to the $2N$ pole, a is the half aperture, and g is the unaberrated gradient of the quadrupole.

The special parameter PHASE is the phase angle α_N in degrees.

The spatial dependence of the scalar magnetic potential giving rise to a multipole is then given by $r^N \sin(N\theta - \alpha_N)$. [4] Here the sense of θ is defined such that $x = r \cos \theta$ and $y = r \sin \theta$. Since the multipole is normalized to the gradient, it may be introduced once and allowed to apply to all succeeding quadrupoles, independent of excitation.

Cumulative Length of the System

To set the system length L to 1000 meters at a given point in the beam line, use the element

```
SPEC,  LENGTH  =  1000.0  ;
```

The keyword LENGTH indicates the cumulative length of system (along the reference trajectory), in the same units as the element lengths (normally meters). Possible units changes are described on page 38. The cumulative length is set to zero initially, then increased by the length of each element, and finally printed at the end of the system. This element allows the cumulative length to be reset as desired.

Reference Trajectory Momentum

To reset the reference trajectory momentum p_0 to 200 GeV at a given place in the beam line, use the element

```
SPEC,   P0  =   200.0   ;.
```

The keyword `P0` indicates the momentum of the reference trajectory. This is the same item as is found on the `BEAM` element (normally GeV/c). It may be redefined at any point in the beam. In this sense it is not really a preliminary specification, but is included here because it is given by a `SPEC` element. The momentum of the beam particles are not changed, but just the momentum of the central trajectory. If, for example, the central momentum is increased, then clearly the beam centroid, as expressed in percent, is shifted downward. This element is useful when a beam is transferred from one optical system to another with a different design momentum.

Reference Momentum for Magnetic Fields

A statement setting the magnetic field reference momentum p_{ref} to 100 GeV might be:

```
SPEC,   PREF = 100.     ;
```

where the keyword `PREF` indicates the reference momentum (normally GeV/c) to which the magnetic fields are normalized. If this parameter is omitted it is taken to be the central momentum p_0 on the `BEAM` element. If it is included, all subsequent magnetic fields will be scaled by the ratio p_0/p_{ref}. The purpose of this parameter is to allow the user to change the momentum of the beam without retyping the values of all the magnetic fields.

Particle Mass

The initial specification which determines that the charged particle is a proton is

```
SPEC,   PMASS  =   .9382  ;
```

The keyword `PMASS` indicates the mass m of the particles comprising the beam, (GeV in `UMM` and `UMETER`; default 0). The selection of still other units is described on page 38. A non-zero mass introduces the dependence of pulse length ℓ on velocity, an important effect in low-energy pulsed beams.

Tilt to Focal Plane

Very often it is desired to have a listing of the higher-order aberrations along the focal plane of a system rather than perpendicular to the optic axis, i.e. along the x coordinate. If the focal plane makes an angle α with respect to the x axis (measured clockwise) then provision has been made to rotate to this focal plane and print out the higher-order aberrations. This is achieved by the following procedures:

The keyword used for this element is `FOTILT`. The programming procedure for a tilt in the x(bend)-plane (rotation about y axis) is:

```
SPEC,  FOTILT = α   ;
DRIFT,  L = 0.   ;        (a necessary do-nothing element)
PRINT,  TRANS  ;
SPEC,  FOTILT = −α   ;  (rotate back to zero)
DRIFT,  L = 0.   ;        (a necessary do-nothing element)
SPEC,  FOTILT = 0.   ;  (to turn off rotation element)
```

The programming procedure for a tilt in the y-plane (rotation about x-axis) is:

```
SPEC,  FOTILT = α   ;
SROT,  ANGLE = 90.   ;
DRIFT,  L = 0.   ;
SROT,  ANGLE = -90.   ;
PRINT,  TRANS  ;
SPEC,  FOTILT = −α   ;  (rotate back to zero)
SROT,  ANGLE = 90.   ;
DRIFT,  L = 0.   ;
SROT,  ANGLE = -90.   ;
SPEC,  FOTILT = 0.   ;  (to turn off rotation element)
```

Random Number

A statement resetting the initial value of the random number used for error calculations to NNNN is

```
SPEC,  RANNO = NNNN   ;
```

The keyword `RANNO` indicates this initial value. In the IBM versions of TRANSPORT (and therefore TURTLE), if this element is included but left blank, the input value will be determined by the computer clock. It will also be printed as part of the `SPEC` element in the initial data listing. Its value will then be available for further runs with the same beam line configuration. The user can copy the printed value of NNNN into the `SPEC` element, so that, in all subsequent runs, the random number will have the same initial value.

Half Apertures of Bending Magnets

Setting the horizontal half width $W/2$ of the aperture of a bending magnet to 50.0 millimeters is done with the element:

```
SPEC,   HWIDTH   =   50.0   ;
```

Setting the vertical half aperture $g/2$ to 25 millimeters is done with:

```
SPEC,   HGAP   =   25.0   ;
```

Here the unit set is assumed to be `UMM`, where transverse distances are measured in millimeters.

The keyword `HWIDTH` indicates the horizontal half-aperture of bending magnet, in the same units as horizontal beam width, normally 0 (i.e. effect of horizontal half aperture is ignored). The units of horizontal beam width are millimeters in `UMM` and meters in `UMETER`. Other unit sets or units may be selected as described on pages 34 and 38.

The keyword `HGAP` indicates the vertical half-aperture of bending magnet, in the same units as vertical beam height. *This parameter must be inserted if the effect of the spatial extent of the fringing fields upon transverse focusing is to be taken into account. It is essential for producing an accurate floor layout of the beam line if the quantity $g^2/2\rho$ is a large enough transverse dimension to have a detectable effect on the magnet locations.* Here ρ is the trajectory radius of curvature in the central body of the bending magnet. More details can be found in the section on fringing fields starting on page 87. The `HGAP` parameter can be specified directly on the `ROTATION` element indicating a pole-face rotation for a bending magnet or on the `RBEND` or `SBEND` elements which specify a bending magnet with fringing fields. This parameter must be given a nonzero value in any third-order calculation. (See the `ROTATION` and `BEND` elements as a cross reference.) The units for `HGAP` are determined in the same manner as those for `HWIDTH`. The default value of the half aperture is 0.

Fringing-Field Integrals

These two integrals [*] are denoted by the symbols κ_1 and κ_2 for historical reasons. They should not be confused with the symbols which are used in MAD notation to denote respectively the dipole, quadrupole, and sextupole components of the interior field of a bending magnet. They are all dimensionless.

To set κ_1 to 0.4, use the elements:

---

[*]See page 89 and SLAC-75 [3] page 74 for further explanation.

```
SPEC,  FINT1  =  0.4  ;
```

The keyword `FINT1` denotes an integral related to the extent of the fringing field of a bending magnet. If the (`SPEC,  HGAP =  g/2. ;`) element has been inserted, the program inserts a default value of $\kappa_1 = 1/2$ unless a (`SPEC, FINT1 =  `κ_1` ;`) element is introduced, in which case the program uses the κ_1 value selected by the user. The parameter `FINT1` may be specified directly on the `ROTATION` (described on page 87) element indicating a pole-face rotation for a bending magnet or on the `RBEND` or `SBEND` elements (described on page 91) which specify a bending magnet with fringing fields. The preferred use of the special parameter element in this case is to specify a different default value at the beginning of the beam line. The table below shows typical values for various types of magnet designs. The value of κ_1 is given by an integral along a straight line which is the projection of the reference trajectory from the exterior of a bending to its interior. It is given by

$$\kappa_1 = \int_{-\infty}^{+\infty} \frac{B_y(z)[B_0 - B_y(z)]}{gB_0^2} \, dz$$

The keyword `FINT2` indicates a second integral related to the extent of the fringing field. Default value of $\kappa_2 = 0$ unless specified by a (`SPEC,  FINT2 =  `κ_2` ;`) element. At present, this integral has no effect on any matrix element calculated by TRANSPORT, and therefore also has no effect in TURTLE. Typical values of κ_1 and κ_2 are given below for four types of fringing field boundaries:

      a) a linear drop-off of the field,
      b) a clamped "Rogowski" fringing field,
      c) an unclamped "Rogowski" fringing field,
      d) a "square-edged" <u>nonsaturating</u> magnet.

Model	κ_1	κ_2
Linear	$b/6g$	3.8
Clamped Rogowski	0.4	4.4
Unclamped Rogowski	0.7	4.4
Square-edged Magnet	0.45	2.8

where b is the extent of the linear fringing-field. The distance b is measured parallel to the portion of the reference trajectory outside the field of the bending magnet. [†]

For most applications κ_2 is unimportant. If you find it is important to your result, you

---

[†]See page 98 of Steffen's book, *High Energy Beam Optics*, Wiley, 1965.

should probably be making a more accurate calculation with a differential equation ray-tracing program. A list of such programs is given after the references at the end of this manual.

Pole-Face Curvature

To set R_1 to 2 meters (or $1/R_1$ to 0.5 meters) and R_2 to 5 meters (or $1/R_2$ to 0.2 meters) use the elements:

```
SPEC,  H1  =  0.5  ;
SPEC,  H2  =  0.2  ;
```

The keyword H1 gives the reciprocal $1/R_1$ of the radius of curvature of the entrance face of bending magnets. The units used are of longitudinal length, normally meters. (See the figure on the following page.)

The keyword H2 gives the reciprocal $1/R_2$ is the radius of curvature of the exit face of bending magnets. Again the units used are of longitudinal length, normally meters.

The pole face curvatures $(1/R_1)$ and $(1/R_2)$ affect the system only in second or higher order, creating effective sextupole components at the entrance and exit faces of the magnet. If the parameters are not specified, they are assumed to be zero, i.e. no curvature and hence no sextupole component. The pole-face curvature may also be specified directly on the ROTATION element, indicating a pole-face rotation for a bending magnet, or on the RBEND or SBEND elements, which specify a bending magnet with fringing fields.

Field Boundaries for Bending Magnets

The TRANSPORT sign conventions for x, β, R and h are all positive as shown in the figure. The positive y direction is out of the paper. Positive β's imply transverse focusing. Positive R's (convex curvatures) represent negative sextupole components of strength $S = (-h/2R)\ sec^3\beta$. (See SLAC-75 [4], page 71 or SLAC-PUB-3381 [11], page 22.)

Second-Order Term in the Midplane Field Expansion of Bending Magnets

To set the second-order coefficient ϵ to 0.01, include the element:

```
SPEC,   EPS   =   0.01   ;
```

The keyword EPS indicates a second-order measure of magnetic field inhomogeneity in bending magnets. If

$$B(x) = B(0)[1 - n(x/\rho_0) + \beta(x/\rho_0)^2 + \gamma(x/\rho_0)^3 + \ldots]$$

is the field expansion in the median (y = 0) plane, then $\epsilon(1)$ is defined as

$$\epsilon(1) = \beta(1/\rho_0)^2$$

(where ρ_0 is measured in unit of horizontal beam width – millimeters in UMM, meters in UMETER–for other units choices see pages 34 and 38). The ϵ parameter may also be specified directly on the bending magnet element. This parameter affects second- and higher-order calculations only. The TRANSPORT (and TURTLE) default value for $\epsilon(1)$ is 0.

Third-Order Term

To set the third-order coefficient ϵ_3 to 0.001, include the element:

```
SPEC,   EPS3   =   0.001   ;
```

The keyword EPS3 indicates a cubic term for the field of a bending magnet. The quantity ϵ_3 is given by:

$$\epsilon_3 = \gamma(1/\rho_0)^3.$$

The units are the inverse cubed of the horizontal beam width unit.

Other Magnetic Field Components

The previous description of a magnetic field has been based on two assumptions: the existence of a reference orbit and midplane symmetry. To allow the representation of field errors and deliberate violations of these assumptions, additional field components are introduced. In terms of them, the field expansion on a reference plane (nominally the magnetic midplane) is given by

$$B_y = B_o(1 + r_s)(1 - nhx + \beta\, h^2 x^2 + \gamma\, h^3 x^3 + \ldots)$$

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$$B_x = B_o r_a (v_r - n' h x + \beta' h^2 x^2 + ...)$$

The additional parameters required can be specified by the following SPECIAL elements:

```
SPEC,  RMPS  =  value  ;
SPEC,  RNMS  =  value  ;
SPEC,  VR  =  value  ;
SPEC,  NP  =  value  ;
SPEC,  EPSP  =  value  ;
```

The meanings of these five keywords are:

r_s	RMPS	The fractional excess bend field on a bending magnet. This excess field will not change the reference trajectory, but simply steer the beam about the reference trajectory.
r_a	RNMS	Overall scaling factor for the midplane-symmetry-violating field components of a bending magnet.
v_r	VR	The vertical (midplane-symmetry-violating) bend field of a bending magnet.
n'	NP	The midplane-symmetry-violating normalized gradient of a bending magnet.
ϵ'	EPSP	The midplane-symmetry-violating sextupole component of a bending magnet. The quantity ϵ' is equal to $\beta' h^2$. The units are the inverse square of the horizontal beam width unit. The horizontal beam width unit is millimeters with UMM or meters with UMETER. Other units selections are described on pages 34 and 38.

These parameters all may be specified directly on the BEND element, indicating the central body of a bending magnet, or on the RBEND or SBEND elements, which specify a bending magnet with fringing fields. The preferable use of the special parameter specification is to alter the default values of zero.

Quadrupole Apertures

In TURTLE aperture constraints are turned on by the (OBSERVE, APERTURES ;) element. For DECAY TURTLE, such an element turns on the aperture constraints for only the parent particle. Aperture constraints for the parent particle in DECAY TURTLE can also be activated by the (OBSERVE, APERTURES, PION ;) element, providing that the name PION has been specified for the parent particle. For the first of the decay products, aperture constraints are similarly turned on by (OBSERVE, APERTURES, MUON ;). Here it is assumed that the name of the first decay product has been set to MUON. Finally, the aperture constraints are turned on for the second decay product by (OBSERVE, APERTURES,

`NEUTRINO   ;`). Although neutrinos would be the logical third particle in this situation, they are a poor example of the use of an aperture constraint.

The quadrupoles are assumed to have <u>circular</u> apertures with the radius taken from the `QUADRUPOLE` element. However, if a (`SPEC,   QAPA   ;`) or (`SPEC,   QAPH ;`) element occurs in a beam line, the circular aperture is deactivated for all subsequent quadrupoles and cannot be reactivated. Instead, TURTLE will use the aperture specified on the (`SPEC,   QAPA   ;`) or (`SPEC,   QAPH   ;`) element. The element

```
        SPEC,  QAPA  =  value  ;
```

specifies the horizontal semi-axis of an elliptical aperture. If used alone, the aperture is taken to be circular. If it is followed by a

```
        SPEC,  QAPB  =  value  ;
```

element, the value `QAPB` is taken as the vertical semi-axis of the same elliptical aperture. A circular or elliptical aperture can be produced by the vacuum chamber inside the aperture of the quadrupole.

The element `SPEC,   QAPH` specifies a <u>hyperbolic</u> <u>aperture</u>, where the hyperbola is given by:

$$|xy| \le a^2/2$$

The value of the parameter a is then given by:

```
        SPEC,  QAPH  =  value  ;
```

A hyperbolic aperture can be produced by the pole tips of the quadrupole. Elliptic and hyperbolic apertures may be specified simultaneously for the same magnet.

Particle Names and Charges

In DECAY TURTLE, the charges and names of both the primary and the secondary particles may be specified. The two secondary particles are often referred to in this manual as the "first daughter" and "second daughter" particles. Occasionally we will also use the terminology of "charged secondary" and "neutral secondary" although this terminology is not really appropriate since the charges of the secondaries may be respecified. Designations of "charged daughter" and "neutral daughter" are used in previous versions of this manual.

The default is for the parent and the first daughter particle to have unit positive charge and for the second daughter particle to be neutral. The charges of any of the three particles may be respecified to be positive or negative unit charge or neutral. The particle name

is indicated by the keyword `PARENT`, `DAUGHTER1`, or `DAUGHTER2`. Any portion of the word `DAUGHTER`, or the entire word will be taken as `DAUGHTER1`. These three keywords also serve as the default particle names. The particle charge is indicated by the keyword `CHARGE`. Only charges of +1, -1, or 0 have any meaning to TURTLE.

To change the particle names to `PION`, `MUON`, and `NEUTRINO`, the following three elements are used:

```
SPEC,   PARENT  =  PION  ;
SPEC,   DAUGHTER1  =  MUON  ;
SPEC,   DAUGHTER2  =  NEUTRINO  ;
```

To reverse the roles of the two secondary particles, the following three elements should be used:

```
SPEC,   PARENT  = PION, CHARGE  =  1.0  ;
SPEC,   DAUGHTER1  = NEUTRINO, CHARGE  =  0  ;
SPEC,   DAUGHTER2,  MUON, CHARGE  =  1.0  ;
```

To look at opposite sign muons, use the combination:

```
SPEC,   PARENT  =  PION,  CHARGE =  -1.0  ;
SPEC,   DAUGHTER1  =  MUON,  CHARGE  =  -1.0  ;
SPEC,   DAUGHTER2  =  NEUTRINO,  CHARGE  =  0  ;
```

For a neutral kaon decaying into oppositely charged pions, we use the three elements:

```
SPEC,   PARENT  = KAON,  CHARGE  =  0  ;
SPEC,   DAUGHTER1  = PIP, CHARGE  =  1  ;
SPEC,   DAUGHTER2  =  PIM,  CHARGE  =  -1 ;
```

The two secondary particles must be given different names as they will be referred to by name in the specifications for rms additions to the beam, slits, the instructions to observe apertures, histograms, and histogram flags.

The masses of the particles and the lifetime of the primary particle are given by the `DECAY` element. DECAY TURTLE does not require charge conservation.

ORDER – Higher-Order Calculations

The parameter or parameters associated with the `ORDER` element are integers. These integers are identified by keyword for consistency with the rest of the data.

As an example, a second-order run could be specified by the insertion of the element

```
ORDER,  N = 2.   ;
```

near the beginning of the data, after all `UNIT` elements and before the `BEAM` element. The default is first order if the order element is not present. The default is second order if the `ORDER` element is present with no keywords.

Notation

Three keywords may be specified:

Symbol	Keyword	Description
n	N	The order n of the expansion about the reference trajectory in each element.
n_1	N1	The order n_1 of the expansion about the reference trajectory in each element. The two keywords, N and N1 are identical in meaning. TRANSPORT allows the specification of two order parameters, one for the expansion and another for the display. If there is only one order parameter to be specified, it is more natural simply to call it N. If there are two, then it is more natural to label them N1 and N2.
n_2	N2	The order n_2 of the expansion about the beam centroid that is calculated, accumulated, and displayed. This keyword has no meaning in TURTLE and exists for TRANSPORT.

Usage

The insertion of an `ORDER` element into the data indicates that second-order geometric and magnetic field effects are taken into consideration. All chromatic effects discussed are taken into account whenever $\Delta p/p$ is not zero. Also, all quadrupole multipole moments are effective whenever specified. The `ORDER` element is necessary in order for second-order geometric transfer matrix elements or second order field variations in a bending magnet to have an effect. It is also necessary in order for the magnetic field of a sextupole to have an effect. Without an `ORDER` element, a sextupole is taken to be a drift space.

A third-order specification indicates that third-order geometric and magnetic field effects

are taken into consideration. Without a third-order element, an octupole is taken to be a drift space.

In DECAY TURTLE, a second-order calculation is automatically invoked when a `DECAY` element is present in the data, calling for a decay calculation. The reason for this is that the decay process typically causes the phase space occupied by the decay products to be sufficiently larger than that of the parent particle, so that a higher-order calculation is warranted.

Examples

Let us examine first the case of an on-axis beam. A single number may then be used to specify the order of the calculation. If this number is absent, it is assumed to equal two. The `ORDER` element then reverts to the original TRANSPORT usage of specifying second order. The original TRANSPORT element

        ORDER   ;

is equivalent to

        ORDER,   N = 2.   ;

Second-order matrices are included in the program for combined-function bending magnets (including fringing fields), quadrupoles, sextupoles, solenoids, and the arbitrary matrix. They have not been calculated for the acceleration (`ACCEL`) element.

A third-order calculation is specified by setting the number on the `ORDER` element to three. The command for a third-order run is then

        ORDER,   N = 3.   ;

Third-order matrix elements are now included for quadrupoles (including fringing fields), sextupoles, octupoles, and combined function bending magnets (also including fringing fields).

RANDOM – Random Errors on Physical Parameters

Using a random number generator, random errors may be placed on the physical parameters used to describe a beam line. Any type of physical parameter can be given an error which falls within a specified range. The type of physical parameter and the magnitude of the error are both specified on the `RANDOM` element. There is only keyword notation for this element.

This element differs from most others in that the keywords must be in a certain order. The element begins with the mnemonic `RANDOM`. The first keyword indicates the type of element on which the random error is imposed. It is an element type, such as `QUAD` or `BEND`.

The second keyword indicates the physical parameter to which the error is applied. Typical keywords for the physical parameters of a `QUAD` or `BEND` element might be `L` or `B`.

The final keyword is the word `VARIATION`, which may be shortened to `VARI`. It is used to indicate the size of the random error on the indicated parameter.

For example, a random error in quadrupole excitation might be represented by the element

```
RANDOM,  QUAD,  B,  VARI = .001  ;
```

Such an element will cause the placement of a random error of maximum magnitude .001 kilogauss on the pole tip field of the quadrupoles in the system. All quadrupoles will be affected, and the random errors will be independent.

For an error to be applied to an element, the element specification must be written in terms of the parameters for which an error magnitude is given. If a quadrupole is specified as

```
QUAD,  L = 10.,  GRAD = 5.   ;
```

then the parameter `B` does not appear. The error specification given above in terms of `B` then introduces no error to the gradient of this quadrupole. An error on the gradient of a quadrupole where the gradient is given directly, as above, can be specified by

```
RANDOM,  QUAD,  GRAD,  VARI = .001  ;
```

At present the list of elements and their parameters to which random errors can be applied is as follows.

Element	Parameter Keywords
ROTAT	ANGLE, HGAP, FINT, H
DRIFT	L
BEND	L, B, RADIUS, ANGLE, N, K1, RMPS, RNMS, VR, NP, K1P, EPS, K2, EPSP, K2P, EPS3, K3
QUAD	L, B, APERTURE, GRADIENT, K1
CENTROID	X, XP, Y, YP, L, DEL
ACCELERA	L, VOLT, LAG, FREQ
SEXTUPOL	L, B, APERTURE, K2
SOLENOID	L, B, KS
SROT	ANGLE
OCTUPOLE	L, B, APERTURE, K3
RBEND, SBEND	L, B, RADIUS, ANGLE, N, K1, RMPS, RNMS, VR, NP, K1P, E1, E2, EPS, K2, EPSP, K2P, H1, H2, EPS3, K3, HGAP, FINT
PLASMA	L, B, APERTURE, GRADIENT, K1

If any physical parameter is to receive a random error, then random numbers for all physical parameters are generated. If a second type of error is introduced, the random numbers used for the first type will then be unchanged. This procedure produces the type of results that a user would be likely to expect. If two types of errors are present, and the magnitude of the second is reduced to zero, the result should be the same as if only the first type of error were present. An example might be an investigation of the errors of focusing elements in a system. If both quadrupoles and bending magnets are to be considered then the two error specifications

```
        RANDOM,   QUAD,   B,   VARI = .001  ;
        RANDOM,   BEND,   N,   VARI = .0005  ;
```

could be used.

The procedure described has the disadvantage that the sequence of random numbers used for any given type of error will be changed if the beam line is changed in any way. This will be true even if the change is to an element for which no random errors are specified. An example might be the splitting up of a drift space to make two shorter drift spaces. If this is done, the beam line is considered changed. The error specification given above, where a random error is given to the magnetic fields of the quadrupoles in the system will then give a different result. The correspondence of random numbers to magnetic field values will be altered due to the generation of an additional random number for the additional drift space. It will also be possible that different versions of TURTLE will give different results if

additional physical parameters can be given random errors. The special parameters element does not use random numbers, since it simply gives default values for the parameters of other elements.

In most cases the mathematical procedure of giving a certain error to a physical parameter is the same as the physical procedure. By the mathematical procedure, we mean simply changing the physical parameter and considering the beam line to be whatever results. By the physical procedure, we mean imposing certain constraints whereby the change in one parameter will cause a change in others resulting in a more physically realistic situation.

There are two cases where the two procedures are significantly different. The first is in the imposition of an error on the length of a magnet. An error may be specified as

```
RANDOM,  BEND,  L,  VARI = 0.02 ;
```

The mathematical error is the procedure currently in TRANSPORT, and therefore TURTLE. Here the length of a beam line will increase when the length of magnet is increased. The physical error will correspondingly shorten the adjacent drifts so that the length of the total beam line is unchanged. It is planned to change TRANSPORT and TURTLE in the future so that the physical error is calculated.

The second exception is the case of an error on the field of a bending magnet. An error may be specified by

```
RANDOM,  BEND,  B,  VARI = 0.005  ;
```

The angle of bend is calculated from the magnet length and field and the reference momentum. If the field is changed, the remainder of the beam line is moved. However, in this case, there is provision for introducing an excess field without altering the remainder of the beam line. The excess field is specified using the keyword RMPS either in the BEND element or with a special parameter. The parameter RMPS is the fractional error in the magnetic field. It may be considered as a mispowering of the magnet without affecting the beam layout. An error on this parameter may then by given by

```
RANDOM,  BEND,  RMPS,  VARI = 0.0001 ;
```

The initial value of the random number generator may be reset using the special parameter element SPECIAL with the keyword RANNO. If the appropriate SPECIAL element is included in the data, but the space for the initial value of the random number generator is left blank, then the initial value will be reset by the computer clock. Detailed description is given under the SPECIAL element.

Description of the Beam

BEAM — Input Beam

The phase space and the reference momentum of the input beam for a TURTLE calculation are specified by this element. Usually the `BEAM` element is on the fourth line of the data. It will be immediately preceded by the title, the indicator, and the unit set specification. If any individual units changes are to be made, the units elements (`UNIT`) must also precede the `BEAM` element Any `ORDER` or `RANDOM` elements also precede the `BEAM` element.

The mnemonic used in specifying the input phase space and reference momentum is `BEAM`. The input phase space is given by specifying its half widths. The units associated with the entries depend on the unit set specified.

Sigma Matrix Notation

The input is given in terms of the semi-axes of a six-dimensional erect[*] beam ellipsoid. The coordinates in the six-dimensional space represent the phase space variables x, x', y, y', ℓ, and δ. Each of these six parameters is entered as a positive quantity, but should be thought of as $\pm x$, $\pm x'$, etc.; hence, the total beam width is $2x$, the total horizontal beam divergence is $2x'$ and so forth.

In keyword notation the `BEAM` element might look like:

```
BEAM,  X=.1,  XP=1.,  Y=.1,  YP=1.,  DL=0.,  DEL=0.2,  P0 = 100.   ;
```

Any of these parameters except `P0` may be omitted and the default value of zero will be used.

---

[*]For a rotated (non-erect) phase ellipsoid input, see the `CORR` element description on page 69.

The keywords which can be used are:

Symbol	Keyword	Description
x	X	One-half the horizontal beam extent (millimeters with UMM, meters with UMETER).
x'	XP	One-half the horizontal beam divergence (milliradians with UMM, radians with UMETER).
y	Y	One-half the vertical beam extent (mm or meters).
y'	YP	One-half the vertical beam divergence (mr or radians).
ℓ	DL	One-half the longitudinal beam extent (mm or meters).
δ	DEL	One-half the momentum spread (permil with UMM, fraction with UMETER).
p_o	P0	The momentum of the central trajectory (GeV/c).
	GAUSS	Gaussian distribution for beam phase space
	SHELL	Surface of six-dimensional shell
	READ	Read trajectory coordinates from input file.

All quantities not specified on the BEAM element will be taken to be zero. The same set of keywords may be used for an rms addition to the beam phase space. Global unit set specifications other than UMM or UMETER may also be selected and are described on page 34. Individual units changes are described on page 38.

The default phase space distribution is described below, in the subsection entitled "Usage." The keywords GAUSS, BEAM, and READ are used to indicate alternate distributions.

If the keyword GAUSS is present, the trajectory coordinates are given a Gaussian distribution. The parameters in the BEAM element are taken to be the standard deviations of the distribution in each of the six coordinates.

If the keyword SHELL is present, the trajectory coordinates are projected onto the surface of a shell. The shell is specified as the six-dimensional ellipsoid whose dimensions are given by the parameters in the BEAM element.

The keyword READ indicates that the coordinates for each new trajectory are read from an input file. The trajectory coordinates are read with the format 6E12.4, from fortran file number 2. The file of trajectory coordinates might be something generated by another computer program.

Usage

Strictly speaking x' and y' are not angles and are therefore not measurable in either milliradians or radians. They are the tangents of the horizontal and vertical angles θ and ϕ between the given trajectory and the reference trajectory. However, up to and including second order, they are equal to the angles themselves.

In third order, the tangent of an angle is different from the angle. However, the principal effects in third order arise from aberrations which are nonlinear terms in the differential equations of motion. The difference between an angle and its tangent as a measure of beam divergence is still inconsequential. Milliradians as a measure of x' and y' may still be interpretated as parts in a thousand.

For the input beam, each ray is chosen at random with its coordinates constrained to lie within the limits specified in the `BEAM` element. In addition, if the default distribution is used, then the x and y coordinates are constrained to lie within an upright ellipse whose semi-axes are the quantities given with the `BEAM` element. The angular coordinates x' and y' are also constrained to lie within a similar ellipse. This prevents the implicit representation of a preferred direction in space by the choice of coordinates. In other words, targets may be circular and scattering is isotropic. Further restrictions on the phase space, such as the use of a multi-dimensional ellipsoid were felt not to be physically realistic. This is illustrated by the fact that the target is the same size for particles of different momentum.

If we let x_0, x'_0, y_0, y'_0, ℓ_0, and δ_0 represent the maximum extents of the beam envelope in each coordinate, then the additional restriction imposed by TURTLE is equivalent to the two inequalities.

$$\left(\frac{x}{x_0} \right)^2 + \left(\frac{y}{y_0} \right)^2 \leq 1, \tag{24}$$

$$\left(\frac{x'}{x'_0} \right)^2 + \left(\frac{y'}{y'_0} \right)^2 \leq 1,$$

The hypervolume of this four-dimensional region is equal to $\pi^2 x_0 x'_0 y_0 y'_0$. By contrast, the hypervolume of the region specified by the single ellipsoid condition

$$\left(\frac{x}{x_0} \right)^2 + \left(\frac{y}{y_0} \right)^2 + \left(\frac{x'}{x'_0} \right)^2 + \left(\frac{y'}{y'_0} \right)^2 \leq 1, \tag{25}$$

is equal to $(\pi^2/2)x_0 x'_0 y_0 y'_0$, or exactly half that of the region specified by the first set of conditions. This additional volume occurs near the boundaries of the region specified in the two-ellipse condition. When the multi-dimensional distribution is projected onto one or two dimensions, the ellipsoidal condition yields a distribution which is very sparse near the edges. The two-ellipse condition yields a projection where the edges of the distrubition are much more sharply defined and is thus much better suited to beam line studies.

User-written Subroutine for Input Beam

A different initial phase-space distribution from the above may be imposed via a user-written subroutine. For example, it may be desired to populate each coordinate independently and uniformly between the limits specified by the BEAM element. The result is what we would usually refer to as a "rectangular" distribution.

To do this, a user writes his or her own subroutine called RAYGEN, and substitutes it for the version presently in TURTLE. The valculated values of the six ray coordinates are to be placed in the array RAY(6) in the common block

```
COMMON /RAY/ RAY(6)
```

For the convenience of the user, the data in the beam element are contained in the common block

```
COMMON /RAYMAX/ RAYMAX(6)

COMMON /P0/ P0
```

The array RAYMAX and the variable P0 contain the seven numbers on the beam element, giving the maximum extent of the input beam in each coordinate and the central design momentum of the beam line. With a user-written subroutine, RAYMAX need not represent the maximum extent of the input beam, but can be employed by the user in any manner desired.

The default version of RAYGEN is listed here as an example. It distributes the initial trajectories as described earlier. Obviously, users may do anything they desire.

Version of Subroutine RAYGEN in TURTLE

```
      SUBROUTINE RAYGEN
      COMMON /RAY/    RAY(6)
      COMMON /RAYGEN/ RAYGEN(6)
      DIMENSION       VALX(4)
   10 DO 20 IR = 1, 4
   20 VALX(IR) = 2.0*RAYF(-1) - 1.0
      RN1 = VALX(1)**2 + VALX(3)**2
      IF (RN1 .GT. 1.0) GO TO 10
      RN2 = VALX(2)**2 + VALX(4)**2
      IF (RN2 .GT. 1.0) GO TO 10
      DO 30 IR = 1, 4
   30 RAY(IR) = RAYMAX(IR)*VALX(IR)
      RAY(5) = RAYMAX(5)*(2.0*RANF(-1) - 1.0)
      RAY(6) = RAYMAX(6)*(2.0*RANF(-1) - 1.0)
      RETURN
      END
```

RMS Addition to the Beam

For an RMS addition to the beam, the coordinates are chosen in the same manner as for the input beam. They are then added to the coordinates of the ray at the point where the RMS addition is made. Because of the random selection of both the original ray coordinates and the additions to them, the effective widths of the distributions in any coordinate will add in an RMS fashion. It should be remembered that the parameters on the element specify the maximum amplitude of the scattering. If the element immediately preceding an RMS addition is a SLIT, the RMS addition will be made only if the ray hits the slit. If the ray passes through the slit, its coordinates will be unchanged and it will continue unperturbed.

In DECAY TURTLE, it is necessary to specify which particle is experiencing the RMS addition. The keyword which states the particle is disturbed is the name of the particle. The default names are PARENT, DAUGHTER1, and DAUGHTER2. The default names can be changed, using the SPEC element to anything desired. One often used possibility is PION, MUON, and NEUTRINO. The scattering of muons in a beam line may then be represented by the element

```
BEAM,   ...   RMS,  MUON,  ;
```

User-Written Subroutine for RMS Addition to the Beam

A user-determined scattering distribution may also be imposed on the RMS addition to the beam. For example, it may be desired that the parameters on the BEAM element represent single standard deviations of a multi-dimensional Gaussian distribution.

A user would then write his or her own subroutine and call it SCATR, substituting it for the version in the TURTLE. The values of the trajectory coordinates will be received by SCATR in the array RAY(6), in the common block

```
COMMON /RAY/ RAY(6)
```

The user then modifies these values in a manner which represents the effect of the scattering, and returns them to TURTLE. The parameters on the element representing the RMS addition are contained in the common block

```
COMMON /RAYADD/ RAYADD(6), PGAIN
```

in the same order as on the BEAM element. The units are those used internally to TURTLE, namely meters and radians.

Version of Subroutine SCATR in TURTLE

.

```
      SUBROUTINE SCATR
      COMMON /RAY/     RAY(6)
      COMMON /RAYADD/ RAYADD(6), PGAIN
      DIMENSION        VALX(6)
C
   10 DO 15 IR = 1, 4
   15 VALX(IR) = 2.0*RANF(-1) - 1.0
      RN1 = VALX(1)**2 + VALX(3)**2
      IF (RN1 .GT. 1.0) GO TO 10
      RN2 = VALX(2)**2 + VALX(4)**2
      IF (RN2 .GT. 1.0) GO TO 10
      DO 20 IR = 1, 4
   20 RAY(IR) = RAY(IR) + RAYADD(IR)*VALX(IR)
      VALX(6) = 2.0*RANF(-1) - 1.0
      RAY(6) = RAY(6) + RAYADD(6)*VALX(6) + PGAIN/P0
      RETURN
      END
```

CORR(ELATION) — Beam (Rotated Ellipse)

The ellipse or ellipsoid, describing the beam input phase space, is not always upright. It can be tilted by virtue of having passed through another optical system before arriving at the one under study. The beam distributions described in the previous section all work as before, but with reference to a tilted ellipse, rather than an upright ellipse or ellipsoid.

CORR is not a MAD element. However, it can be expressed in keyword notation.

Notation

The keywords which can be used in specifying the beam matrix correlations are

C21, C31, C32, C41, C42, C43, C51, C52, C53, C54, C61, C62, C63, C64, C65.

The keywords Cij correspond in an obvious manner to the 15 correlations r_{ij} among the 6 beam components. The value assigned to Cij is just r_{ij}. The correlations are all dimensionless. In terms of the elements of the σ matrix, the correlations are given by:

$$r_{ij} = \frac{\sigma_{ij}}{\sqrt{\sigma_{ii}}\sqrt{\sigma_{jj}}}$$

Only the nonzero correlations need be typed. It is often the case that the only nonzero correlations are r_{21} and r_{43}. To specify the values of these correlations, the CORR element might look like

CORR,   C21 = 0.6,   C43 = -0.8   ;

The correlation element often requires several lines. For compatability with the MAD program an implicit semicolon is now assumed to be in column 81 at the end of each line. A line must now be explicitly marked to be continued. This is done by placing an ampersand (&) after the data on a given line.

Usage

Since this element is solely an extension of the beam input, the CORR element immediately follows the BEAM element.

CENTROID — Shift in the Beam Centroid

Sometimes it is useful to redefine the beam centroid such that it does not coincide with the reference trajectory. Provision has been made for this possibility via the CENTROID element.

CENTROID is not a MAD element, but it may be expressed in keyword notation.

Notation

The mnemonic used in specifying a shift in the beam centroid is CENTROID. The keywords which can be used are

Symbol	Keyword	Description
x	X	The shift in the x coordinate
x'	XP	The shift in x'
y	Y	The shift in y
y'	YP	The shift in y'
ℓ	DL	The shift in ℓ.
δ	DEL	The shift in δ.

The units are the same as that used for the beam ellipse. (See page 62.) In the UMM unit set, the units are mm, mr, and permil (parts in a thousand). In the UMETER set, they are meters, radians, and fractional measure. Other global unit sets are described on page 34.

An example of an increase in y' by 1.0 milliradians might be

        CENTROID, YP = 1.0 ;

Here the global unit set is assumed to be UMM.

Usage

The CENTROID element is intended as a means of directing a particle beam into an optical system so that the centroid is not coincident with the design axis of that optical system. It normally occurs only at the beginning of a beam line. The CENTROID element therefore immediately follows the BEAM element. If there are correlations in the beam ellipse, then the CORR element will immediately follow the BEAM element. The CENTROID element will, in turn, immediately follow the CORR element.

The centroid shift is considered as being due to an unspecified cause within a fixed optical system. A shift in the centroid coordinates can also occur without a physical centroid deflection. This latter coordinate shift occurs when there is a change in the coordinate system to which the centroid is referred. A change in coordinate systems often occurs when a particle beam is transferred from one optical system to another. Examples are the transference of a beam from an accelerator lattice to a transfer line or from a transfer line to another

accelerator lattice. The `SHIFT` element should be used for such purposes. In TRANSPORT, the `SHIFT` element affects the floor coordinates. The `CENTROID` element does not. Since floor coordinates are not used in TURTLE, the effects of the two elements `CENTROID` and `SHIFT` are the same.

Physical Elements

Symbolic Constants

TRANSPORT, and therefore TURTLE, recognize the mathematical and physical constants listed in the table below. The use of recognizably named constants is included for compatability with the MAD program. The list of constants is essentially the same as is found in the MAD program. Users should avoid using the names of such symbolic constants for any other purpose. The use of a symbolic constant name for a user-defined label will cause the user definition to supersede the symbolic constant meaning. Further use as a symbolic constant may then be inconsistent with the new meaning and cause unpredictable results. However, if a symbolic constant is redefined in the data as a parameter with the same value (next page), there will be no problem. Sets of data based on versions of the program before the inclusion of symbolic constants may have such defined parameters.

The symbolic constants recognized are:

Symbol	Name	Value Used	Unit
π	PI	3.14159265358979323	
2π	TWOPI	$2.0 * \pi$	
$180/\pi$	DEGRAD	$180.0/\pi$	
$\pi/180$	RADDEG	$\pi/180.0$	
e	E	2.718281828459045	
m_e	EMASS	$.510099906 * 10^{-3}$	GeV
m_p	PMASS	.93827231	GeV
c	CLIGHT	$2.99792458 * 10^8$	m/s
	MTOIN	.0254	
	INTOM	1.0/MTOIN	

Further constants can be defined using the parameter statement defined below.

PARAMETERS

The `PARAMETER` element is taken from the MAD program.

A numerical value to be used many times over in the definition of elements can be specified a single time by the use of a parameter statement. The parameter is given a proper name via the parameter element. It is then referred to by that proper name on all other elements. For example, a drift length `LD1` which is to be .5 meters can be defined by

```
LD1:  = .5 ;
```

or alternatively

```
PARAM, LD1 = 0.5 ;
```

In defining a drift space `D1` whose length is `LD1`, one can write

```
D1:  DRIFT, L = LD1 ;
```

It is also possible to define a parameter by its use on a given element. When used on a later element it is represented by the label of the first element followed by the keyword of the parameter in brackets. The length of the drift space `D1` may now be given directly on the drift specification as:

```
D1:  DRIFT, L = 0.5 ;
```

A second drift space, whose length is always to be the same as that of the first can be given as

```
D2:  DRIFT, L = D1[L] ;
```

The symbol `D1` after the equals sign indicates that the length of element `D2` is to be taken from element `D1`. The `L` in the brackets indicates that the parameter taken from `D1` is the length.

Algebraic Expressions

Any algebraic expression made up from previously defined parameters can also serve as a parameter. This algebraic expression can be evaluated either on another `PARAMETER` element, or as one of the physical parameters of a physical element. In the example above, a second drift space can be defined in terms of the parameter `LD1`

```
D2:  DRIFT, L = 10.0 - LD1 ;
```

The operations which may be used in the algebraic expression are:

`+`	Addition
`-`	Subtraction
`*`	Multiplication
`/`	Division
`SQRT`	Square root
`ALOG`	Natural logarithm (written Ln in engineering notation).
`EXP`	Exponential function
`SIN`	Sine function
`COS`	Cosine function
`SINH`	Hyperbolic sine function
`COSH`	Hyperbolic cosine function
`ASIN`	Inverse sine function
`ACOS`	Inverse cosine function

For example, the lengths of three drift spaces in a system can be made to increase in geometric progression. The first two drift spaces `D1` and `D2` can be made free parameters and given as:

```
D1:  DRIFT, L = LD1 ;
D2:  DRIFT, L = LD2 ;
```

A third drift space can be defined as

```
D3:  DRIFT, L = LD2*LD2/LD1
```

DRIFT

A drift space is a field-free region through which the beam passes. It is specified by a single parameter, which is its length. The drift space is a MAD compatible element and is expressed in MAD notation.

MAD Notation

A single keyword is used in specifying a drift space. It is L which indicates the length of the drift space. A specification for a drift space D1 of length 6.0 meters is written as

```
D1:  DRIFT,  L = 6.  ;
```

Usage

The dimensions of L are those chosen for longitudinal length. In any of the global units specifications (UMM, UMETER, UMAD, etc.) this unit is meters. An alternate longitudinal unit may be chosen via the UNIT element. For example, a change of the longitudinal unit to feet would be effected by placing the element

```
UNIT,  L,  FT  ;
```

preceding the BEAM element. Details about unit changes can be found on page 38.

For a the transverse coordinates of a normal drift space, the first-order transformation is the <u>exact</u> solution of the equations of motion. There are no higher-order terms. For the longitudinal coordinates (ℓ and δ), the transformation is exact only for massless particles.

For the effect of momentum on time of flight to be nonzero, the particle must be given a nonzero mass. The particle mass is specified via the Special Parameter element as explained on page 47.

A normal drift space is one which begins and ends at a plane which is perpendicular to the reference trajectory. It is sometimes desirable to have a drift space end at a plane which makes a different angle to the reference trajectory. One example might be the focal plane of a dispersive focusing system. The foci at the different momenta are spread out transversely because of the dispersion and longitudinally because of the chromatic aberration of the focusing devices (quadrupoles or their optical equivalents). A drift space can be specified to end at such a tilted focal plane. For details of implementation, see page 47.

BEND – Sector Bending Magnet

A sector bending magnet implies that the central trajectory of the beam line enters and exists perpendicularly to the pole-face boundaries. The `BEND` element is used to represent the interior field of a bending magnet. No fringe field effects are included in the `BEND` element. The `ROTATE` element (see page 87) represents the fringe field itself. The `SPECIAL` element (see page 44) may be used to define certain parameters used to describe the fringe field.

First-, second, and third-order geometric and magnetic field aberrations can be calculated by TURTLE for the `BEND` element. DECAY TURTLE is limited to second order for the `BEND` element. The order actually calculated will depend on the `ORDER` command as described on page 57. With no `ORDER` element the first-order transformation will be calculated. The number of parameters necessary to specify a `BEND` element depends on the order of calculation requested.

The element `BEND` is not found in MAD, but can be described by the same set of parameters as is used for the elements `RBEND` and `SBEND` in MAD. It can also be described by several other sets of parameters, including those used in the original TRANSPORT numerical notation.

To represent a bending magnet complete with fringing fields as a single element one should use the `RBEND` or `SBEND` elements described on page 91. A more complete description of the set of parameters used to represent the magnetic field is given earlier in this manual. Some of this description is repeated here for the user's convenience.

MAD-like Notation

The `BEND` element, without fringing fields, is not found in the MAD program. It is, however, a TRANSPORT (and TURTLE) element which can be described in keyword notation. The parameterization of it, described here as MAD-like, has been implemented for consistency with the `RBEND` and `SBEND` elements. To first order, a `BEND` element is specified by three real attributes.

        label: BEND, L = value , ANGLE = value , K1 = value;

A sample `BEND` element might look like:

        BR: BEND, L = 10., ANGLE = 0.010, K1 = 0.5E-4 ;

The bend angle shown is 0.010 radians or 0.574 degrees. The value in radians is shown since radians are the standard MAD units for bending angle. The use of MAD units is indicated by the choice of the global unit set `UMAD`. The same element, but with the bending angle expressed in degrees would be:

        BR: BEND, L = 10., ANGLE = 0.574, K1 = 0.5E-4 ;

The bend angle is expressed in milliradians in the UMM data set. Other global unit sets are described on page 34

The MAD-like keywords which can be used in specifying a sector bending magnet are

Symbol	Keyword	Description
L	L	The magnet length (normally meters)
α	ANGLE	The angle through which the reference trajectory is bent (milliradians with unit set UMM, radians with UMETER, default 0.0).
K_1	K1	The quadrupole coefficient (normally meters^{-2}, default 0.0), defined by

$$K_1 = \frac{1}{B\rho} \frac{\partial B_y}{\partial x}$$

K_2	K2	The sextupole coefficient (normally meters^{-3}, default 0.0), defined as

$$K_2 = \frac{1}{2B\rho} \frac{\partial^2 B_y}{\partial x^2}$$

K_3	K3	The octupole coefficient (normally meters^{-4}, default 0.0), defined as

$$K_3 = \frac{1}{6B\rho} \frac{\partial^3 B_y}{\partial x^3}$$

In terms of these parameters, the field expansion on the magnetic midplane is given by

$$B_y = (B_o\rho) \sum K_n x^n$$

$$B_x = 0$$

The definition of the multipole components α, K_1, K_2, and K_3 allows the orbit of a particle to be computed without knowledge of its momentum. The normalization of K_1, K_2, and K_3 is such that a multipole of unit strength will give unit angular deflection per unit magnet length to a trajectory one transverse unit from the reference trajectory. In the MAD program itself the values of K_1, K_2, and K_3 are defined as the x derivatives of the field divided by $B_0\rho$. The value of K_2 in MAD is then a factor of 2 greater than the value defined above. The MAD value of K_3 is a factor of 6 (= 3!) greater than that used here. When the global unit set UMAD is used, the MAD definition of the multipoles is also used.

The units for any of the parameters may be changed as desired. The units of the parameters K_1, K_2, and K_3 are in inverse powers of the unit of longitudinal length. Global unit sets are described on page 34. Individual units changes are described on page 38.

Other Keyword Options

To allow keyword representation of data in the original TRANSPORT variables and to permit other options for specifying a bending magnet, the following keywords are also recognized. They may be used instead of or combined with the MAD-like keywords.

Symbol	Keyword	Description
B	B	The magnetic field (normally kilogauss)
ρ	RADIUS	The radius of curvature of the reference trajectory (normally meters).
n	N	The normalized field derivative (dimensionless).
ϵ	EPS	The quadratic field dependence (in inverse squared units of horizontal beam width – mm in UMM, meters in UMETER.
ϵ_3	EPS3	The cubic field dependence (in inverse cubed units of horizontal beam width).

The complete set of variables available for defining a bending magnet contains a great deal of redundacy. The bending magnet, used as an example earlier, may be described to first order in any of ten ways. We assume the beam momentum to be 600 GeV/c. The following five lines are then all equivalent.

```
        BEND,  L = 10.,  B = 20.,  N = 0.5 ;

        BEND,  L = 10.,  ANGLE = 10.,  N = 0.5 ;

        BEND,  L = 10.,  RADIUS = 1000.,  N = 0.5 ;

        BEND,  B = 20.,  ANGLE = 10.,  N = 0.5 ;

        BEND,  RADIUS = 1000.,  ANGLE = 10.,  N = 0.5 ;
```

Here the length is assumed to be in meters, the angle in milliradians, the field in kilo-Gauss, and the bend radius in meters. The angular unit specified by the global unit set UMM is milliradians. The angular unit specified by UMETER, is radians. Other units may be specified by use of an alternate global units specification (UMAD, UTRANS, etc.) or a UNITS element.

In addition to the above five options, the field gradient may be specified by K_1 instead of n. The two are related by

$$K_1 = - \frac{n}{\rho^2} = \frac{1}{B\rho} \frac{\partial B_y}{\partial x}$$

The expansion of the magnetic field on the magnetic midplane is given by

$$B_y = B_o(1 + r_s)(1 - nhx + \beta \, h^2 x^2 + \gamma \, h^3 x^3 + \ldots)$$

$$B_x = 0$$

The sextupole component β is related to the coefficients ϵ and K_2 by

$$K_2 = \frac{\epsilon}{\rho} = \frac{\beta}{\rho^3} = \frac{1}{2B\rho} \frac{\partial^2 B_y}{\partial x^2}$$

Only one of the two quantities ϵ and K_2 should be specified. If neither is specified, the sextupole component is set equal to the default value given by the special parameters (SPEC element). If the default value is also unspecified, it is taken to be zero.

The octupole component γ is related to the cubic variation of the magnetic field by.

$$K_3 = \frac{\epsilon_3}{\rho} = \frac{\gamma}{\rho^4} = \frac{1}{6B\rho} \frac{\partial^3 B_y}{\partial x^3}$$

The default value for the octupole component ϵ_3 is also set by the special parameters (SPEC element). Otherwise it is taken to be zero.

Mispowering and Violations of Midplane Symmetry

The vertical field in a bending magnet along the reference trajectory may be mispowered so that the reference particle will be bent away from the reference trajectory. This can happen simply by increasing the actual field in the magnet without changing the reference field. A physical bending magnet may also have a horizontal field component on what is taken as the magnetic midplane. A set of parameters to describe both the effects of excess bend field and violation of midplane symmetry follows. The multipole components which violate midplane symmetry are also known as skew multipoles.

Some of these parameters are MAD-like, and should be used if the midplane-symmetric field is given a MAD-like parameterization. Specifically the parameters K_1' and K_2' are MAD-like. The parameters n' and ϵ' are original-TRANSPORT-like. The two scaling parameters r_s and r_a can be used with either representation.

Symbol	Keyword	Description
r_s	RMPS	The fractional excess magnetic field (unitless).
r_a	RNMS	Scaling factor for the skew field components (unitless).
v_r	VR	The normalized skew dipole magnetic field (unitless).

K_1'	K1P	The skew quadrupole coefficient (normally meters^{-2}).
K_2'	K2P	The skew sextupole coefficient (normally meters^{-3}).
n'	NP	The midplane-symmetry-violating normalized field derivative (dimensionless).
ϵ'	EPSP	The midplane-symmetry-violating quadratic field dependence (same units as EPS).

In terms of these new parameters, the field expansion on the nominal magnetic midplane is given by

$$B_y = (B_o\rho)(1 + r_s) \sum K_n x^n$$

$$B_x = (B_o\rho)r_a \sum K_n' x^n$$

where

$$B_0 v_r = (B_0\rho)K_0'$$

or

$$B_y = B_o(1 + r_s)(1 - nhx + \beta \ h^2 x^2 + \gamma \ h^3 x^3 + \ldots)$$

$$B_x = B_o r_a(v_r - n'hx + \beta' h^2 x^2 + \ldots)$$

The quantity r_s represents the fractional excess field. The reference trajectory is defined as if r_s equalled zero. The field of the magnet may then be increased or decreased to steer the beam about the reference trajectory. The quantity r_a exists only to facilitate simultaneous scaling of all midplane symmetry violating multipoles.

If r_s and r_a are not specified for a particular BEND element, they are set equal to the default values given on the special parameter element (SPEC element – see page 54). If the default values are also unspecified, they are taken to be zero.

If the magnet in the example just above is mispowered so that the field is excessively strong by 1%, it might be specified as:

```
BR:  BEND,  L = 10.,  ANGLE = 10.,  K1 = 0.5E-4,  RMPS = 0.01  ;
```

If there is a skew quadrupole component which is whose strength is 1% that of the midplane symmetric quadrupole component, the bend magnet could be described as:

```
BR: BEND, L = 10., ANGLE = 10., K1 = 0.5E-4, RNMS = 0.01, K1P = 0.5E-4 ;
```

or as:

```
BR: BEND, L = 10., ANGLE = 10., K1 = 0.5E-4, RNMS = 1.0, K1P = 0.5E-6   ;
```

Any of the parameters, r_s, r_a, v_r, K_1', K_2', n', and β' may be set on the individual bend element, or left to equal the default value. The default values may be set on the special parameter element. Otherwise they are all zero. The parameters K_1' and K_2' are related to n', β', and ϵ' respectively in the same way that K_1 and K_2 are related to n, β, and ϵ. There are no third-order midplane-symmetry-violating transfer matrix elements in TRANSPORT or TURTLE.

Fringing Fields

If fringing field effects are to be included, a `ROTAT` element must immediately precede and follow the pertinent `BEND` element (even if there are no pole-face rotations). Thus a typical TRANSPORT (or TURTLE) input for a bending magnet including fringing fields might be:

```
ROTAT,  ANGLE = 0.   ;
BEND,  L = 10.0,  B = 20.,  N = 0.5  ;
ROTAT,  ANGLE = 0.   ;
```

For non-zero pole-face rotations a typical data input might be

```
ROTAT,  ANGLE = 10.   ;
BEND,  L = 10.0,  B = 20.0,  N = 0.5  ;
ROTAT,  ANGLE = 20.   ;
```

Transformation Through a Bending Magnet

If a second- or higher-order calculation is not specified, then a ray with the beam central momentum is transformed through the magnet using the first-order transfer matrix R so that $X(1) = RX(0)$. If the ray does not have the central momentum, we use an off-momentum R matrix which is obtained from the ordinary first-order matrix R^0 and the chromatic elements of the second-order matrix T via the equation

$$R_{ij} = R_{ij}^0 + T_{ijk}\epsilon \tag{26}$$

where

$$\epsilon = \Delta p/p = (p - p_0)/p. \tag{27}$$

The use of the ϵ parameter will be described below.

If a second-order specification is in the data, second-order geometric effects will also be included in the calculations. These effects correspond to the matrix elements T_{ijk}, where j

and k are both equal to or less than four. If a third-order specification is in the data, then third-order geometric effects will be included in the calculations.

In DECAY TURTLE, a complete second-order calculation is automatically invoked when a `DECAY` element is inserted calling for decay calculations to be made. The transformation through a bend is limited to second order for a non-uniform-field bending magnet.

To derive the matrix expressions used to calculate the chromatic terms, we first consider the equations of motion as given on page 24 of Ref. 1. If we separate the three components of the vector equation, and write the x and y terms, we have:

$$x'' - h(1 + hx) - \frac{x'}{(T')^2} \left[x'x'' + y'y'' + (1 + hx)(hx' + h'x) \right] \tag{28}$$

$$= \frac{e}{p} \left[y'B_t - (1 + hx)B_y \right]$$

$$y'' - \frac{y'}{(T')^2} \left[x'x'' + y'y'' + (1 + hx)(hx' + h'x) \right]$$

$$= \frac{e}{p} T'[(1 + hx)B_x - x'B_t]$$

At this point, there is no truncation by order. The equation of motion is still valid to all orders in the variables x and y and their derivatives.

To terminate the equations at a certain order, it is first necessary to expand the variables in the equations. We expand the variable T' and the magnetic field to the appropriate order. Since the individual momentum p of a given particle occurs in the denominator of the equations of motion, it is better if we use an expansion which terminates after two terms. We then write

$$\frac{p_0}{p} = \frac{p_0}{p_0(1 + \delta)} = 1 - \frac{\delta}{1 + \delta} \tag{29}$$

where

$$\delta = \left(\frac{p - p_0}{p_0} \right) \tag{30}$$

from Ref. 1. We can define a new quantity ϵ as

$$\epsilon = \frac{p - p_o}{p} = \frac{\delta}{1 + \delta} \tag{31}$$

The momentum dependence is given by a finite series, so that there is no dependence on ϵ^2 or higher powers. The second-order matrix elements are the same as in Ref. 1, except for T_{166} and T_{266}. The third-order matrix elements are the same as before, except for U_{1166}, U_{1266}, U_{1666}, U_{2166}, U_{2266}, U_{2666}, U_{3366}, U_{3466}, U_{4366}, and U_{4466}

For uniform field bending magnets, the trajectory through any portion is a circle. The following <u>exact</u> formula set is used to describe the trajectories.

$$x_2 + \rho_0 = (x_1 + \rho_0)\cos\alpha + \rho[\cos\theta_2 - cos(\theta_1 + \alpha)] \tag{32}$$

$$\rho \sin\theta_2 \; = \; \rho \sin(\theta_1 + \alpha) - (x_1 + \rho_0)\sin\alpha$$

$$y_2 \; = \; y_1 + y_1' \rho(\alpha + \theta_1 - \theta_2)\cos\theta_1$$

$$y_2' \; = \; y_1' \frac{\cos\theta_1}{\cos\theta_2}$$

$$\rho \; = \; \rho_0 \frac{1+\delta}{1+r_s} \frac{\sqrt{1+x_1'^2}}{\sqrt{1+x_1'^2 + y_1'^2}} \tag{33}$$

where:

ρ_0 is the radius of curvature of the central trajectory having momentum p_0.

ρ is the radius of curvature of a particular trajectory having momentum p.

α is the total bending angle of the central trajectory.

θ_1 is the entrance angle made between the central trajectory and the projection of the particular trajectory onto the bend (mid-) plane of the magnet.

θ_2 is the exit angle made between the central trajectory and the projection of a particular trajectory onto the bend (mid-) plane of the magnet.

y_1' is the tangent of the angle ϕ_1, made between the central trajectory and the projection of the particular trajectory onto a plane perpendicular to the midplane, at the entrance face of the magnet.

y_2' is the tangent of the angle ϕ_2 made between the central trajectory and the projection of a particular trajectory onto a plane perpendicular to the midplane, at the exit face of the magnet.

x_1, y_1, x_2, y_2 are the x and y coordinates of a particular trajectory at the entrance and exit planes of the magnet, relative to the central trajectory.

r_s is the fractional excess magnetic field of a mispowered bending magnet.

Direction of Bend

The sign conventions for bending magnet entries are illustrated in the following figure. For TRANSPORT a positive bend is to the right looking in the direction of particle travel. To represent a bend in another sense, the coordinate rotation matrix (SROT element) should be used as follows:

A bend up is represented by a rotation of the (x,y) coordinates by -90.0 degrees about the (z) beam axis as follows:

```
SROT,   ANGLE = -90.    ;
ROTATE,   ANGLE = 10.0   ;
BEND,  L = 10.0,  B = 20.0,  N = 0.5  ;
ROTATE,   ANGLE = 20.0   ;
SROT,   ANGLE = +90.    ; (returns coordinates to their initial orientation)
```

A bend down is accomplished via:

```
SROT,   ANGLE = +90.    ;
ROTATE
BEND
ROTATE
SROT,   ANGLE = -90.    ;
```

A bend to the left (looking in the direction of beam travel) is accomplished by rotating the x,y coordinates by 180 degrees, e.g.

```
SROT,   ANGLE = +180.    ;
ROTATE
BEND
ROTATE
SROT,   ANGLE = -180.    ;
```

The SROT need be used only with the BEND element. For the RBEND and SBEND elements the coordinate rotation may be specified on the element itself as the ROLL parameter.
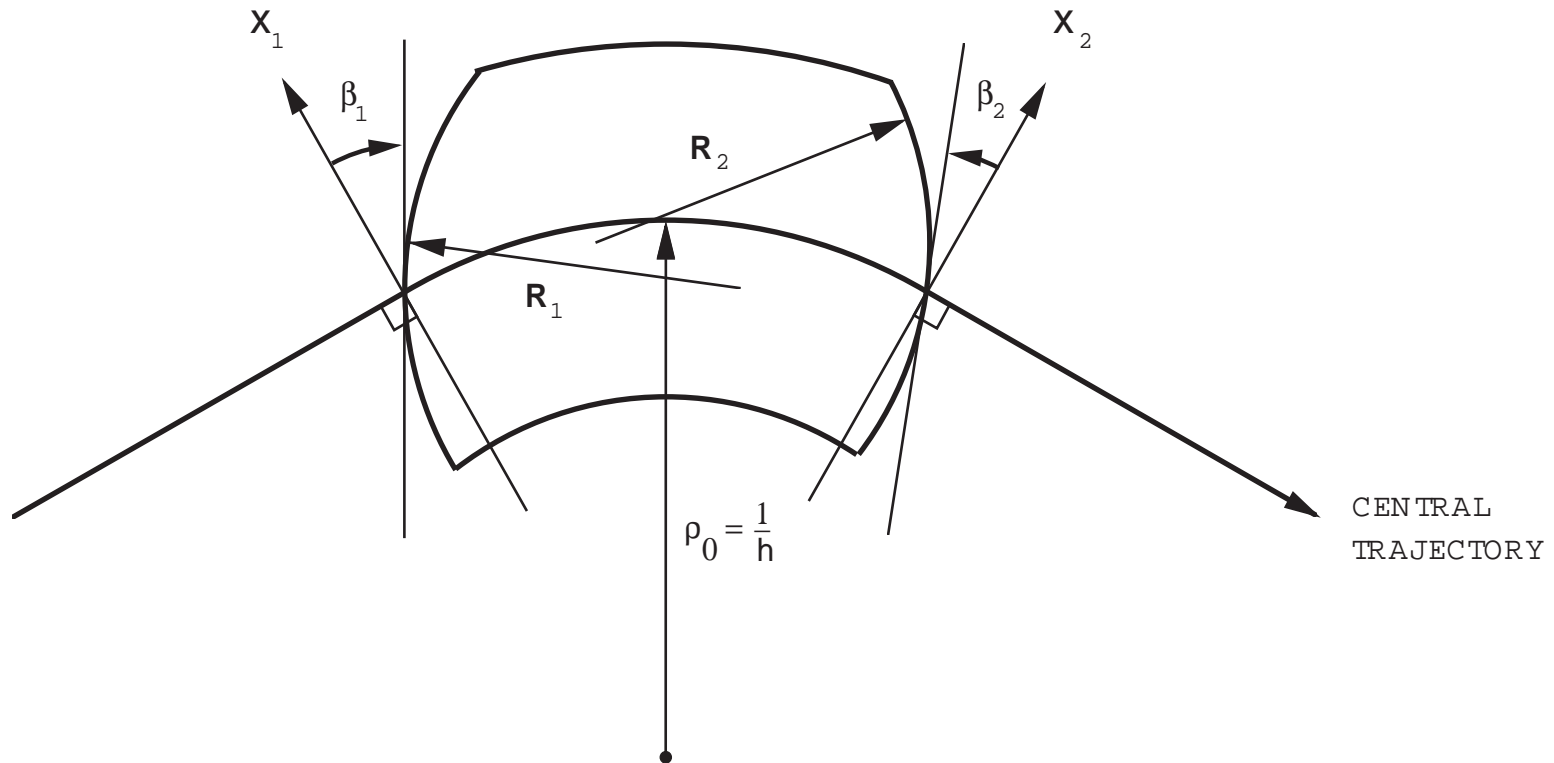
Field Boundaries for Bending Magnets

The TRANSPORT sign conventions for x, β, R and h are all positive as shown in the figure. The positive y direction is out of the paper. Positive β's imply transverse focusing. Positive R's (convex curvatures) represent negative sextupole components of strength $S = (-h/2R)\ sec^3\beta$. (See SLAC-75 [4], page 71 or SLAC-PUB-3381 [11], page 22.)
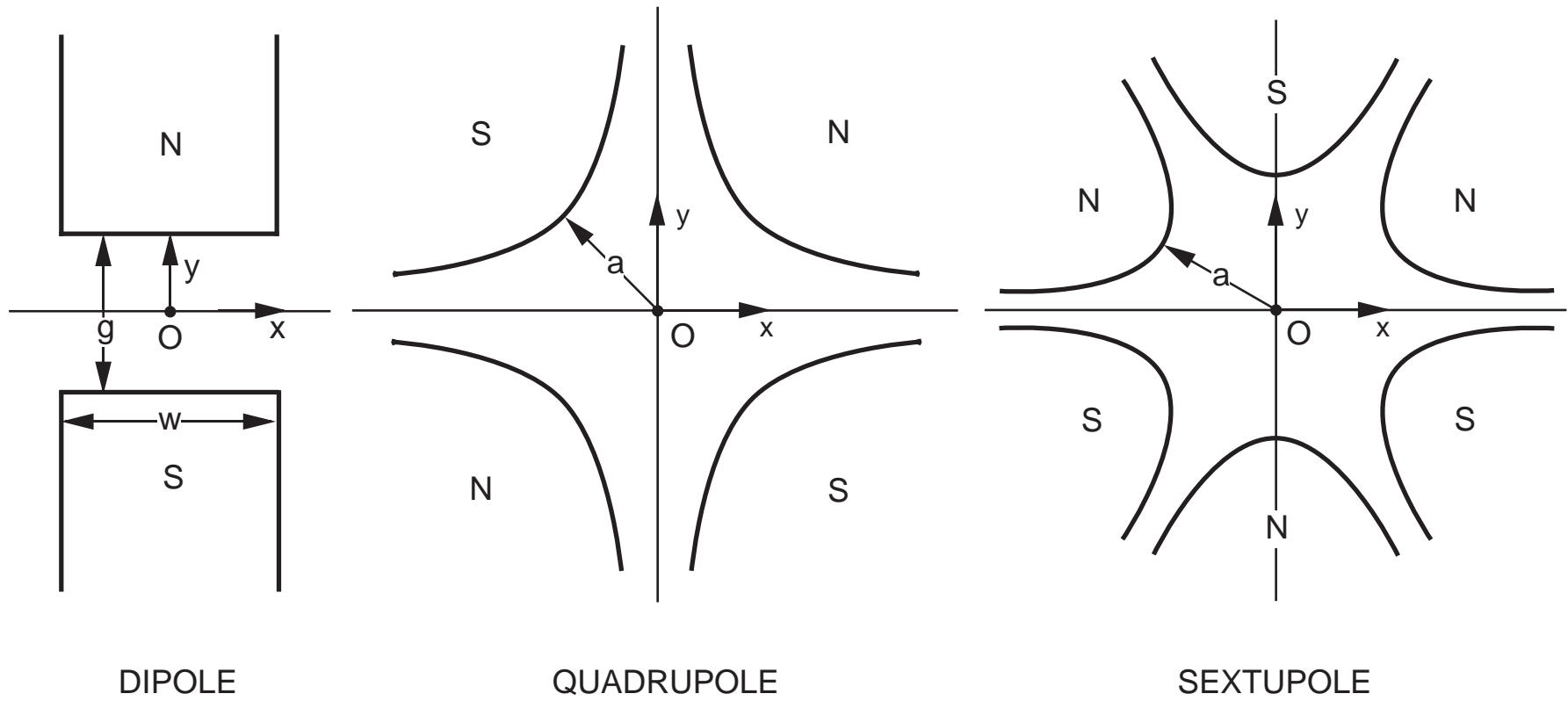
Illustration of the magnetic midplane (x axis) for dipole, quadrupole and sextupole elements. The magnet polarities indicate multipole elements that are positive with respect to each other. The coordinate system is right-handed, so that the beam particles will be emerging from the paper and moving into the face of the reader.

ROTATION — Fringing Fields and Pole-Face Rotations for Bending Magnets

To provide for fringing fields and/or pole-face rotations on bending magnets specified by the BEND element, the ROTATION element is used. The combined use of the BEND and the ROTATION elements allows the user to separate the effects of the fringing fields from those of the central body of a bending magnet.

First-, second-, and third-order matrix elements are included in TURTLE for the ROTATION element. In a third-order calculation, an extended fringe field (not an abrupt beginning or end) is required for the matrix elements to be finite (not infinite). For the fringe field to be extended, it is necessary to give a nonzero value to the magnet half gap (explained below).

Like the BEND element, the ROTATION element is not found in MAD. The combination of BEND and ROTATION elements may be replaced in MAD with the single RBEND or SBEND element. The ROTATION element is expressed in keyword notation, similar to MAD input.

Notation

A first-order specification of a ROTATION element will typically have a value for just the rotation angle.

    label: ROTAT,   ANGLE = value ;

A pole face rotation of 5 mr might be represented as:

    ROTAT,   ANGLE = 5.0   ;

The units for the pole face rotation angle will be milliradians in the global unit set UMM and radians with UMETER. Other global unit sets are described on page 34. Other individual unit changes are listed on page 38.

To represent the entrance face rotation the ROTATION element should be placed immediately <u>before</u> the BEND element. To represent the exit face rotation the ROTATION element should be placed immediately <u>after</u> the BEND element. Additional parameters (see below) are available for higher-order calculations or in situations where greater precision is required is describing the configuration of the fringing field.

In the above example we omit the optional label. ROTATION is not a MAD element and is typically not used when a beam line is assembled from previously defined elements. Labels used for other purposes, such as simple identification, are generally attached to the accompanying BEND element.

The mnemonic used in specifying a pole face rotation is ROTATION. The MAD-like keywords which can be used are

Symbol	Keyword	Description
β	ANGLE	Angle of pole-face rotation (mr in UMM, radians with UMETER).
$g/2$	HGAP	Half gap (mm with UMM, meters with UMETER, default 0.0).
κ_1	FINT	Fringe field integral (unitless, default 0.5).
$1/R_{1,2}$	H	Pole face curvature (normally meters^{-1}, default 0.0).

None of these parameters need be specified. The default value of the pole-face rotation angle is zero. The keywords HGAP, FINT, and H allow the user to override temporarily the default values. The values specified by these three keywords on any ROTAT element will apply only to that element. The default values of these three parameters may be changed by the special parameter element described on page 44.

A symmetrically oriented rectangular bending magnet whose total bend is 10 mr would be represented by the three entries:

```
ROTAT,   ANGLE = 5.    ;
BEND,    - - -   ;
ROTAT,   ANGLE = 5.   ;
```

In original TRANSPORT notation, only the pole-face rotation angle is given in the ROTAT element. The other three parameters are given by the special parameter element. The keywords HGAP, FINT, and H now allow the user to override temporarily the default values given by the special parameter element. The values specified by these three keywords on any ROTAT element will apply only to that element. The default values are given on page 44 and following.

The use of keywords to override the defaults for special parameters allows the user to specify the characteristics of a pole face rotation for a particular bending magnet. A complete bending magnet with fringe fields and pole face rotations and curvatures might be written as

```
ROTAT,   ANGLE = 5.0,   H = 0.01  ;
BM:  BEND,  L = 10.,   B = 20.   ;
ROTAT,   ANGLE = 5.0,   H = -0.1  ;
```

Here a bending magnet 10 meters long with a field of 20 kG has the entrance and exit pole face rotations specified also. Both pole face rotation angles are 5.0 mr. The entrance pole face is convex and has a curvature of 0.01 inverse meters, or a radius of curvature of 100 meters. The exit pole face is concave, and the radius of curvature is 10 meters. The units are based on the assumption that the global unit set UMM is employed.

Usage

The ROTATION element must either immediately precede a bending magnet (BEND ele-

ment) element (in which case it indicates an entrance fringing field and pole-face rotation) or <u>immediately</u> follow a `BEND` element (exit fringing field and pole-face rotation) with no other data entries between.[*] A positive sign of the angle on either entrance or exit pole-faces corresponds to a non-bend plane focusing action and bend plane defocusing action.

Even if the pole-face rotation angle is zero, (`ROTAT,  ANGLE = 0.   ;`) entries must be included in the data set before and after a `BEND` element entry if fringing-field effects are to be calculated.

A single `ROTAT` element entry that follows one bending magnet and precedes another will be associated with the <u>latter</u>.

Should it be desired to misalign such a magnet, an update element or alignment marker must be inserted immediately before the first `ROTATION` element. Then the convention appropriate to misalignment of a <u>set</u> of elements should be applied, since, indeed, three separate transformations are involved. See page 139 for a discussion of misalignment calculations and page 155 element for a discussion of updates or page 157 for the alignment marker.

Pole-Face Rotation Matrix

The first-order R matrix for a pole-face rotation used in a TURTLE calculation is as follows:

$$
R = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
\dfrac{tan\beta}{\rho_0} & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & -\dfrac{tan(\beta - \psi)}{\rho_0} & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
$$

where [†] [‡]

$$
\psi = \kappa_1 \left(\frac{g}{\rho_0}\right) \left(\frac{1 + sin2\beta}{cos\ \beta}\right) \left[1 - \kappa_1\kappa_2 \left(\frac{g}{\rho_0}\right)\ tan\ \beta\right]
$$

Definitions: β = Angle of rotation of pole-face (see figure in the preceding section for sign convention of β).
ρ_0 = Bending radius of central trajectory.
g = Total gap of magnet.

---

[*]*It is extremely important that no data entries be made between a* `ROTAT` *element and a* `BEND` *element* entry. If this occurs, it may result in an incorrect matrix multiplication in the program and hence an incorrect physical answer. If this rule is violated, an error message will be printed.

[†]See `SPECIAL` element on page 44 for input formats for g, κ_1, κ_2 parameters.

[‡]See SLAC-75 [3] (page 74) for a discussion of ψ.

$$\psi \quad = \quad \text{Correction term resulting from spatial extent of fringing fields.}$$

The transverse shift in the reference trajectory is given by

$$\Delta x = \frac{1}{cos^2\beta} \quad \frac{g^2}{\rho} \quad \kappa_0$$

RBEND and SBEND – Bending Magnets with Fringing Fields

The elements `RBEND` and `SBEND` indicate bending magnets including the fringing fields. The mnemonic `RBEND` indicates a rectangular bending magnet. The rectangular shape pertains to the entire magnet, as seen from above (plan view). <u>The default value for both the entrance and exit pole face rotation angles is half the bend angle.</u>

The mnemonic `SBEND` indicates a sector bending magnet. Here the the magnet pole faces are perpendicular to the reference trajectory as it enters and exits from the magnet. Illustrations of both the `RBEND` and the `SBEND` magnets can be found at the end of this section. These mnemonics are the ones used to describe a bending magnet in the MAD program.

For both `RBEND` and `SBEND` the default values for both the entrance and exit pole face curvatures are zero. The default values for the quadrupole, sextupole, and octupole components of the field are also zero. Any of these parameters can be set to whatever value is desired. A more complete description of the set of parameters used to represent the magnetic field is given earlier in this manual.

First-, second-, and third-order geometric effects and magnetic field nonlinearities are included in TURTLE for the `RBEND` and `SBEND` elements. DECAY TURTLE is limited to second order for the `RBEND` and `SBEND` elements. In a third-order calculation, an extended fringe field (not an abrupt beginning or end) is required for the matrix elements to be finite (not infinite). For the fringe field to be extended, it is necessary to specify a nonzero value to the magnet half gap (explained below).

The transformation through the main body of the bending magnet is the same as for the `BEND` element as described on page 81. If the field is nonuniform, then a Taylor's series expansion about the reference trajectory is used. The explicit momentum dependence is treated using the parameter ϵ instead of δ as described in the `BEND` section. For a uniform field bending magnet, an exact treatment is used, also as described in the `BEND` section.

`RBEND` and `SBEND` are MAD elements. They may be represented in MAD notation or other keyword notation. Strict MAD notation requires the use of the `UMAD` unit set. In TRANSPORT and TURTLE other unit sets, such as `UMM` can also be used. In the immediately following example, we first use the `UMM` unit set but also mention the `UMETER` unit set. The unit sets are described on page 38.

MAD Notation

To first order, an `RBEND` or an `SBEND` element might be specified by three real attributes.

        label: RBEND, L = value , ANGLE = value , K1 = value;

or

        label: SBEND, L = value , ANGLE = value , K1 = value;

A sample `SBEND` element might then look like:

```
BR:  SBEND,  L = 10.,  ANGLE = 0.010,  K1 = 0.5E-4  ;
```

The bend angle shown is 0.010 radians or .574 degrees. The value in radians is shown since radians are the standard MAD units for bending angle. The same element, but with the bending angle expressed in degrees would be

```
BR:  SBEND,  L = 10.,  ANGLE = 0.574,  K1 = 0.5E-4  ;
```

The bend angle is expressed in degrees in the `UTRANS` unit set.

The keywords used in the MAD program for `RBEND` and `SBEND` are

Symbol	Keyword	Description
L	L	The magnet length (normally meters).
α	ANGLE	The angle through which the reference trajectory is bent (mr with the unit set `UMM`, radians with `UMETER`).
K_1	K1	The quadrupole coefficient (normally meters^{-2}, default 0.0), defined by $$K_1 = \frac{1}{B\rho} \frac{\partial B_y}{\partial x}$$
β_1	E1	The rotation angle for the entrance pole face (mr in `UMM`, radians in `UMETER`, default 0.0).
β_2	E2	The rotation angle for the exit pole face (mr in `UMM`, radians in `UMETER`, default 0.0).
K_2	K2	The sextupole coefficient (normally meters^{-3}, default 0.0), defined as $$K_2 = \frac{1}{2B\rho} \frac{\partial^2 B_y}{\partial x^2}$$
	H1	The curvature of the entrance pole face (normally meters^{-1}, default 0.0).
	H2	The curvature of the exit pole face (normally meters^{-1}, default 0.0).
K_3	K3	The octupole coefficient (normally meters^{-4}, default 0.0), defined as $$K_3 = \frac{1}{6B\rho} \frac{\partial^3 B_y}{\partial x^3}$$
$g/2$	HGAP	The half gap of the magnet (mm in `UMM`, meters in `UMETER`, default 0.0).
	FINT	The field integral (given the symbol κ_1).

TILT   The rotation angle about the entrance reference trajectory. (degrees in UMM and UMETER, default 0.0).

In terms of these parameters, the field expansion on the magnetic midplane is given by

$$B_y = (B_o\rho) \sum K_n x^n$$

$$B_x = 0$$

The definition of the multipole components α, K_1, K_2, and K_3 allows the orbit or a particle to be computed without knowledge of the reference momentum. The normalization of K_1, K_2, and K_3 is such that a multipole of unit strength will give unit angular deflection per unit magnet length to a trajectory one transverse unit from the reference trajectory. In the MAD program itself the values of K_1, K_2, and K_3 are defined as the x derivatives of the field divided by $B_0\rho$. The value of K_2 in MAD is then a factor of 2 greater than the value defined above. The MAD value of K_3 is a factor of 6 ($= 3!$) greater than that used here. When the global unit set UMAD is used, the MAD definition of the multipoles is also used.

The units of any of the parameters may be changed as described on page 38. The units of the parameters K_1, K_2, and K_3 are in inverse powers of the unit of longitudinal length.

The keyword TILT indicates that the bending magnet is rotated in a clockwise sense about the entrance reference trajectory. The positive sense appears clockwise looking downstream since the z axis also points downstream. The value given is the angle of rotation in degrees. If the word TILT appears alone without a value, the angle of rotation is taken to be 90 degrees, corresponding to a downward bend. A bend upward can be obtained simply by reversing the sign of the bend angle.

An example of the same bend as above, but vertically bending in the downward direction would be:

         BR: SBEND, L = 10., ANGLE = 0.01, K1 = 0.5E-4, TILT ;

An example of the same bend but rotated by only 10° to provide a slight downward bend would be:

         BR: BEND, L = 10., ANGLE = 0.01, K1 = 0.5E-4, TILT = 10.0 ;

Here both bend angle and rotation angle about the beam axis are assumed to be in degrees. The units for the rotation about the beam axis may be changed as described on page 38. Rotations about the longitudinal axis are in degrees for all globally specified units sets (UMM, UMETER, etc.) except UMAD, for which it is in radians.

Other Keyword Options

To allow keyword representation of data in original TRANSPORT variables and to permit

other options for specifying a bending magnet, the following keywords are also recognized. Some may be used instead of the MAD variables, others in addition to a MAD specification.

Symbol	Keyword	Description
B	B	The magnetic field (normally kilogauss)
ρ	RADIUS	The radius of curvature of the reference trajectory (normally meters).
n	N	The normalized field derivative (unitless).
ϵ	EPS	The quadratic field dependence (in inverse squared units of horizontal beam width – mm in UMM, meters in UMETER.
ϵ_3	EPS3	The cubic field dependence (in inverse cubed units of horizontal beam width).

This set of variables contains a great deal of redundacy. The bending magnet, used as an example earlier in this section, may be described to first order in any of ten ways. For a beam momentum of 600 GeV/c, the following five items are then all equivalent.

```
SBEND,  L = 10.,   B = 20.,   N = 0.5  ;

SBEND,  L = 10.,   ANGLE = 10.0,  N = 0.5  ;

SBEND,  L = 10.,   RADIUS = 1000.,  N = 0.5  ;

SBEND,  B = 20.,   ANGLE = 10.0,  N = 0.5  ;

SBEND,  RADIUS = 1000.,   ANGLE = 10.0,   N = 0.5  ;
```

Here the length is assumed to be in meters, the angle in milliradians, the field in kilo-Gauss, and the bend radius in meters.. The angular unit specified by the global unit set UMM is milliradians. The angular unit specified by UMETER is radians. Other units may be specified by use of an alternate global units specification (UMAD, UTRANS, etc.) or a UNITS element.

In addition to the above five options, the field gradient may be specified by K_1 instead of n. The two are related by

$$K_1 = -\frac{n}{\rho^2} = \frac{1}{B\rho}\frac{\partial B_y}{\partial x}$$

The expansion of the magnetic field on the magnetic midplane is given by

$$B_y = B_o(1 + r_s)(1 - nhx + \beta\ h^2x^2 + \gamma\ h^3x^3 + \ldots)$$

$$B_x = 0$$

The sextupole component β is related to the coefficients ϵ and K_2 by

$$K_2 = \frac{\epsilon}{\rho} = \frac{\beta}{\rho^3} = \frac{1}{2B\rho} \frac{\partial^2 B_y}{\partial x^2}$$

The octupole component γ is related to the cubic variation of the magnetic field by.

$$K_3 = \frac{\epsilon_3}{\rho} = \frac{\gamma}{\rho^4} = \frac{1}{6B\rho} \frac{\partial^3 B_y}{\partial x^3}$$

The default value for the octupole component ϵ_3 is also set by the special parameters (SPEC element). Otherwise it is taken to be zero.

The multipole structure of a magnet should be specified in a consistent manner. One should use either K_1, K_2, and K_3 and the mid-plane symmetry violating counterparts or use n, β, and ϵ and their counterparts. One should not attempt to mix the two forms of field expansion. The two normalization parameters r_s, and r_a can be used with either type of specification.

Mispowering and Violations of Midplane Symmetry

The vertical field in a bending magnet along the reference trajectory may be mispowered so that the reference particle will be bent away from the reference trajectory. This can happen simply by increasing the field in the magnet without redefining the reference momentum. A physical bending magnet may also have a horizontal field component on what is taken as the magnetic midplane. A set of parameters to describe both the effects of excess bend field and violation of midplane symmetry follows. The multipole components which violate midplane symmetry are also known as skew multipoles. A discussion of the magnetic field components is given earlier in this manual.

Some of these parameters are MAD-like, and should be used if the midplane-symmetric field is given a MAD-like parameterization. Specifically the parameters K_1' and K_2' are MAD-like. The parameters n' and ϵ' are original-TRANSPORT-like. The two scaling parameters r_s and r_a can be used with either representation.

Symbol	Keyword	Description
r_s	RMPS	The fractional excess magnetic field (unitless).
r_a	RNMS	Scaling factor for skew field components (unitless).
v_r	VR	The normalized skew dipole magnetic field (unitless).
K_1'	K1P	The skew quadrupole coefficient (normally meters^{-2}).
K_2'	K2P	The skew sextupole coefficient (normally meters^{-3}).
n'	NP	The skew normalized field derivative (unitless).

ϵ' EPSP   The skew quadratic field dependence (same units as EPS).

In terms of these new parameters, the field expansion on the nominal magnetic midplane is given by:

$$B_y = (B_o \rho)(1 + r_s) \sum K_n x^n$$

$$B_x = (B_o \rho) r_a \sum K'_n x^n$$

where

$$B_0 v_r = (B_0 \rho) K'_0$$

or

$$B_y = B_o(1 + r_s)(1 - nhx + \beta \ h^2 x^2 + \gamma \ h^3 x^3 + \ldots)$$

$$B_x = B_o r_a (v_r - n' h x + \beta' h^2 x^2 + \ldots)$$

The quantity r_s represents the fractional excess field. The reference trajectory is defined as if r_s equalled zero. The field of the magnet may then be increased or decreased to steer the beam about the reference trajectory. The quantity r_a exists only to facilitate simultaneous scaling of all midplane symmetry violating multipoles.

The four parameters, r_a, v_r, n', and β' may also be set on the individual bend element, or left to equal the default value. The default values may be set on the special parameter element. Otherwise they are all zero. The parameters K'_1 and K'_2 are related to n', β', and ϵ' respectively in the same way that K_1 and K_2 are related to n, β, and ϵ.

If any of this set of parameters is not specified it is set equal to the default value given on the special parameter element (SPEC element). If the default value is also unspecified, it is taken to be zero.

If the magnet in the example above is mispowered so that the field is excessively strong by 1%, it might be specified as:

```
BR:  SBEND,  L = 10.,  ANGLE = 10.0,  K1 = 0.5E-4,  RMPS = 0.01  ;
```

If there is a skew quadrupole component which is whose strength is 1% that of the midplane symmetric quadrupole component, the bend magnet could be described as:

```
BR:  SBEND,  L = 10.,  ANGLE = 10.0,  K1 = 0.5E-4,  RNMS = 0.01,  K1P = 0.4E-4  ;
```

or as:

```
BR:   SBEND,   L = 10.,   ANGLE = 10.0,   K1 = 0.5E-4,   RNMS = 1.0,   K1P = 0.4E-6   ;
```

In the use of these keywords, the elements `RBEND` and `SBEND` are identical. In this example,
the default unit set is taken as `UMM`. The unit of angle is then milliradians.
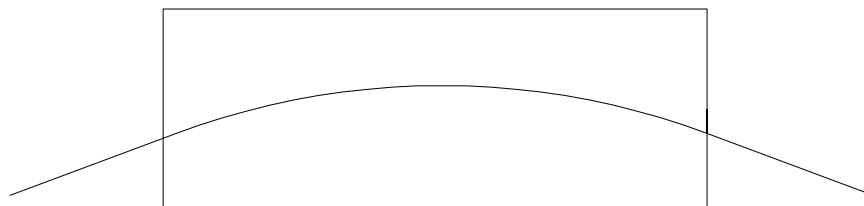
An RBEND Type of Bending Magnet
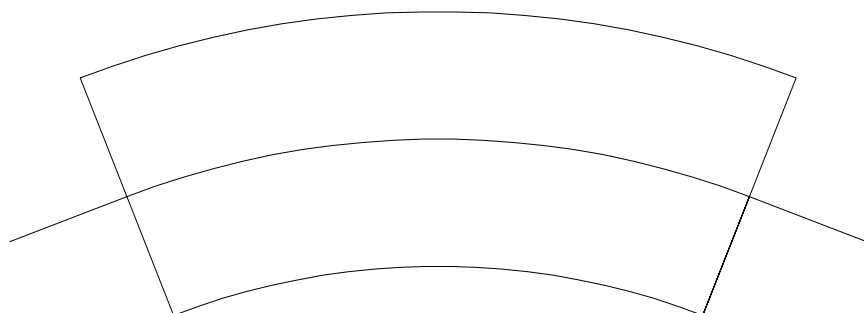
An SBEND Type of Bending Magnet

HKICK or VKICK – Horizontal and Vertical Steering Magnets

The difference between a steering magnet and a bending magnet is that the steering magnet does not affect the reference trajectory. If the beam line is perfectly aligned with no errors, and the beam centroid follows the reference trajectory, the field of the steering magnets is zero. The steering magnets are used to compensate for errors and misalignments in the beam line. If they are given a nonzero field, the beam line reference trajectory is not affected. However, the beam is steered either toward or away from the reference trajectory.

In TRANSPORT and TURTLE the elements `HKICK` and `VKICK` should always be used for steering magnets. The centroid shift element `CENTROID` should never be used for that purpose. The steering magnet elements contain a complete set of first-, second-, and third-order transfer matrix elements. In DECAY TURTLE the transfer matrix is limited to second order.

The higher-order terms for `HKICK` or `VKICK` include the effect of having the angle of deflection vary inversely with the momentum of the particle. The centroid shift element contains no such higher-order effects. It simply deflects the beam by a fixed distance or angle.

A positive angle bends to the right for the `HKICK` element or down for the `VKICK` element. This convention is consistent with that used for bending magnets. In accelerator usage, a steering magnet is often described by its "kick". In MAD notation, a "kick" is considered to be a change in x' or y'. A kick is considered to be positive if it increases p_x or p_y, or x' or y'. It is then opposite in sense but equal in magnitude to the change in angle.

There is no provision for non-uniform field in this element. The fringe fields are included automatically. The pole-face rotation angle is automatically set so that the magnet is rectangular. The entrance and exit pole faces are still both perpendicular to the reference trajectory, since the reference trajectory is considered to be undeflected by the kicking magnet. There is also no provision for pole face curvature.

Both `HKICK` and `VKICK` are MAD elements. They may also be expressed in other keyword notation.

MAD Notation

The `HKICK` and `VKICK` elements are specified by two real attributes:

```
        label:   HKICK,  L = value,  KICK = value  ;
or
        label:   VKICK,  L = value,  KICK = value  ;
```

An example of a horizontal steering magnet element would then be:

```
        HKICK,  L = 1.0,  KICK = -0.1  ;
```

Here the units for the kick are assumed to be in mr. Strict MAD compatability would require the use of `UMAD` units. The kick would then be measured in radians. The angle is the same as in subsequent examples in this section.

Symbol	Keyword	Description
L	L	The effective length of the magnet (meters).
$\Delta x'$ or $\Delta y'$	KICK	The kick angle (mr in `UMM`, radians in `UMETER`).

The units illustrated are those specified by one of the global unit sets `UMM` or `UMETER`. Other global unit sets are described on page 34.

Other Keyword Options

For consistency with bending magnet specifications, it is also possible to parameterize a `KICK` in terms of its length and bend angle. Recall that for a `BEND`, `RBEND`, or `SBEND` a positive bend is one which deflects the reference trajectory toward negative x. The bend angle is then equal in magnitude and opposite in sense from the kick value described above.

            label: HKICK, L = value, ANGLE = value ;

The same `KICK` element as above can then be expressed in terms of the `ANGLE` keyword as

            HKICK, L = 1.0, ANGLE = 0.1 ;

The keyword format can also be used to specify a steering magnet in terms of parameters which are more like the original TRANSPORT parameters for a bending magnet.

            label: HKICK, L = value, B = value ;

The keywords available are:

Symbol	Keyword	Description
L	L	The effective length of the magnet (meters).
α	ANGLE	The deflection angle (mr in `UMM`, radians in `UMETER`).
B	B	The magnetic field (kilogauss).

The units for either the length, the bend angle, or the magnetic field may be changed by use of the `UNIT` element.

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Assuming a 600 GeV beam, the following element is also equivalent to the previous examples.

```
HKICK,  L = 1.0,  B = 2.0  ;
```

KICKER – A Steering Magnet in Both Planes

The KICKER element is a steering magnet that steers in both planes simultaneously. The MAD standard does not have a TILT parameter on either HKICK or VKICK. Instead steering in both planes simultaneously is done using the KICKER element. All angle conventions applying to HKICK or VKICK apply to KICKER also.

The KICKER element exists only in MAD notation It also contains first-, second-, and third-order matrix elements. In DECAY TURTLE the matrix elements are limited to second order. The fringe fields are included automatically.

MAD Notation

The KICKER element is specified by three real attributes:

> label: KICKER, L = value , HKICK = value, VKICK = value ;

A simultaneous kick in the horizontal plane of $-.2$ mr and in the vertical plane of 0.12 mr could be represented as:

> KICKER, L = 1.0, HKICK = -0.2, VKICK = 0.12 ;

Symbol	Keyword	Description
L	L	The effective length of the magnet (meters).
$\Delta x'$	HKICK	The horizontal kick angle (radians in UMAD, degrees in UTRANS).
$\Delta y'$	VKICK	The vertical kick angle (radians in UMAD, degrees in UTRANS).

Here the kicks are assumed to be measured in mr, as in the unit set UMM. Strict MAD compatability would require the use of the unit set UMAD and therefore the angular unit of radians. Other unit sets are also allowable and are described on page 34.

QUADRUPOLE

A quadrupole provides focusing in one transverse plane and defocusing in the other. The transformation through a quadrupole is evaluated for each trajectory using the actual momentum of the trajectory. It is therefore good to all orders in chromatic effects.

A quadrupole has geometric aberrations of third and higher orders. When TURTLE performs a third-order calculation, the quadrupole fringe fields[*] are included automatically. The longitudinal field profile is assumed to be rectangular, meaning that the field begins and ends abruptly and is otherwise uniform. Fringe fields can be approximated by placing half-strength quadrupoles at the entrance and exit faces of the full-strength quadrupole.

The QUAD is a MAD element. It may be expressed in MAD notation or other keyword notation.

MAD Notation

For the MAD program, a magnetic quadrupole is specified by three real attributes.

        label: QUAD(RUPOLE),   L = value,   K1 = value,   TILT = value ;

A typical quadrupole specification might then look like:

        Q1:   QUAD,   L = 5.,   K1 = .01   ;

Symbol	Keyword	Description
L	L	The quadrupole length (normally in meters, default = 0.0)
K_1	K1	The quadrupole coefficient (defined below). A positive quadruole strength indicates horizontal focussing of positively charged particles. (The units are normally meters^{-2})
	TILT	The roll angle about the longitudinal axis (degrees with UMM and UMETER, radians with UMAD).

Strict MAD compatability requires the use of UMAD units, including radians for the TILT parameter. The units of any of the physical parameters can be changed to any desired. Other unit set specifications are described on page 34. Individual unit changes are described on page 38.

The keyword TILT indicates that the quadrupole is rotated in a clockwise sense about the entrance reference trajectory. The positive sense appears clockwise looking downstream since the z axis also points downstream. *If the word* TILT *appears alone without a value, the angle of rotation is taken to be 45 degrees.* This configuration is known to accelerator designers as a skew quadrupole.

---

[*]G.E.Lee-Whiting, Nuclear Instruments and Methods 83, 232 (1970).

Other Keyword Options

The same quadrupole can also be specified by other sets of variables. The first of the two following options corresponds to the original TRANSPORT variables expressed in keyword notation.

label: QUADRUPO,   L = value ,   B = value,   APER = value ;

label: QUADRUPO,   L = value ,   GRAD = value ;

The TILT parameter may also be used with either of these two options.

Symbol	Keyword	Description
L	L	The magnet length (normally meters).
B_o	B	The magnetic field (normally kilogauss).
a	APER(TURE)	The magnetic half aperture (mm with UMM, meters with UMETER).
g	GRAD(IENT)	The gradient of the magnetic field (kG/mm with UMM, kG/m with UMETER).

The UMM units are meters, kG, and mm respectively. The UMETER units for L, B and a are meters, kG, and meters, respectively. Global unit set specifications other than UMM or UMETER may also be selected and are described on page 34. Individual units changes are described on page 38.

The following three elements are all equivalent.

Q1:   QUAD,   L = 5.,   B = 10.,   APER = 50.   ;

Q1:   QUAD,   L = 5.,   GRAD = 0.2 ;

Q1:   QUAD,   L = 5.,   K1 = .01   ;

Here we have assumed a beam momentum of 600 GeV/c. We have also assumed that the aperture is measured in the UMM unit of mm. The parameter K_1 is measured in the inverse square of the unit of longitudinal distance (usually meters). The description in terms of K_1 has the advantage that it does not require a separate specification of the beam momentum. The quantities K_1 and g are related to the pole tip field B_o and the aperture a by:

$$K_1 = \frac{g}{(B\rho)} = \frac{1}{(B\rho)} \frac{B_o}{a}$$

First-Order Quadrupole Matrix

For a perfect quadrupole, the first four components of the ray vector $x = (x, x', y, y', \ell, \delta)$ are carried through the quadrupole via the transformation matrix.

$$
\begin{bmatrix}
\cos k_q L & \dfrac{1}{k_q} \sin k_q L & 0 & 0 & 0 & 0 \\[2ex]
-k_q \sin k_q L & \cos k_q L & 0 & 0 & 0 & 0 \\[2ex]
0 & 0 & \cosh k_q L & \dfrac{1}{k_q} \sinh k_q L & 0 & 0 \\[2ex]
0 & 0 & k_q \sinh k_q L & \cosh k_q L & 0 & 0 \\[2ex]
0 & 0 & 0 & 0 & 1 & \dfrac{L}{\gamma^2} \\[2ex]
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
$$

These elements are for a quadrupole which focuses in the horizontal (x) plane (B positive). A vertically (y-plane) focusing quadrupole (B negative) has the first two diagonal submatrices interchanged.

Definitions:

L	=	the effective length of the quadrupole.
a	=	the radius of the aperture.
B_0	=	the field at the radius a.
k_q^2	=	$(B_0/a)(1/B_0\rho_0)$, where $(B\rho_0)$ = the magnetic rigidity (momentum) of the central trajectory.
γ	=	relativistic factor.

Higher-order multipole effects for a realistic quadrupole may also be introduced by the SPEC element. Multipoles up to and including a 40-pole may be included. In original TURTLE, the multipole aberrations are taken to be lumped at the longitudinal midpoint of the quadrupole. The ray is transformed half-way through the quadrupole, perturbed by the multipole aberrations, then transformed through the remainder of the quadrupole. If no multipole aberrations are included, the ray passes through the quadrupole in a single step. The multipole component strengths are normalized to unit gradient, and need be entered only once for similar but differently excited quadrupoles. The effect of the multipoles is also calculated from the actual momentum of the ray, making this effect corrrect to all orders chromatically.

In DECAY TURTLE, the treatment of the multipole aberrations is the same for all quadrupoles, except one in which a parent particle decays. In the case of such a quadrupole, the longitudinal location of the decay divides the quadrupole into two shorter virtual

quadrupoles. The trajectory is tracked to the longitudinal midpoint of the first of these quadrupoles. The effect of the aberrations is applied to the ray, and it is then tracked to the end of the first part. At that point, the decay in made to occur. Then the trajectory is tracked to the midpoint of the second part of the quadrupole. The aberrations for the second part of the quadrupole are imposed, and the trajectory is tracked through the second half of the second quadrupole. This procedure is complicated, but does not impose any new requirements on the user. The segmentation of the quadrupole by the decay, and the imposition of aberrations at the longitudinal midpoint of each segment are done automatically by TURTLE.
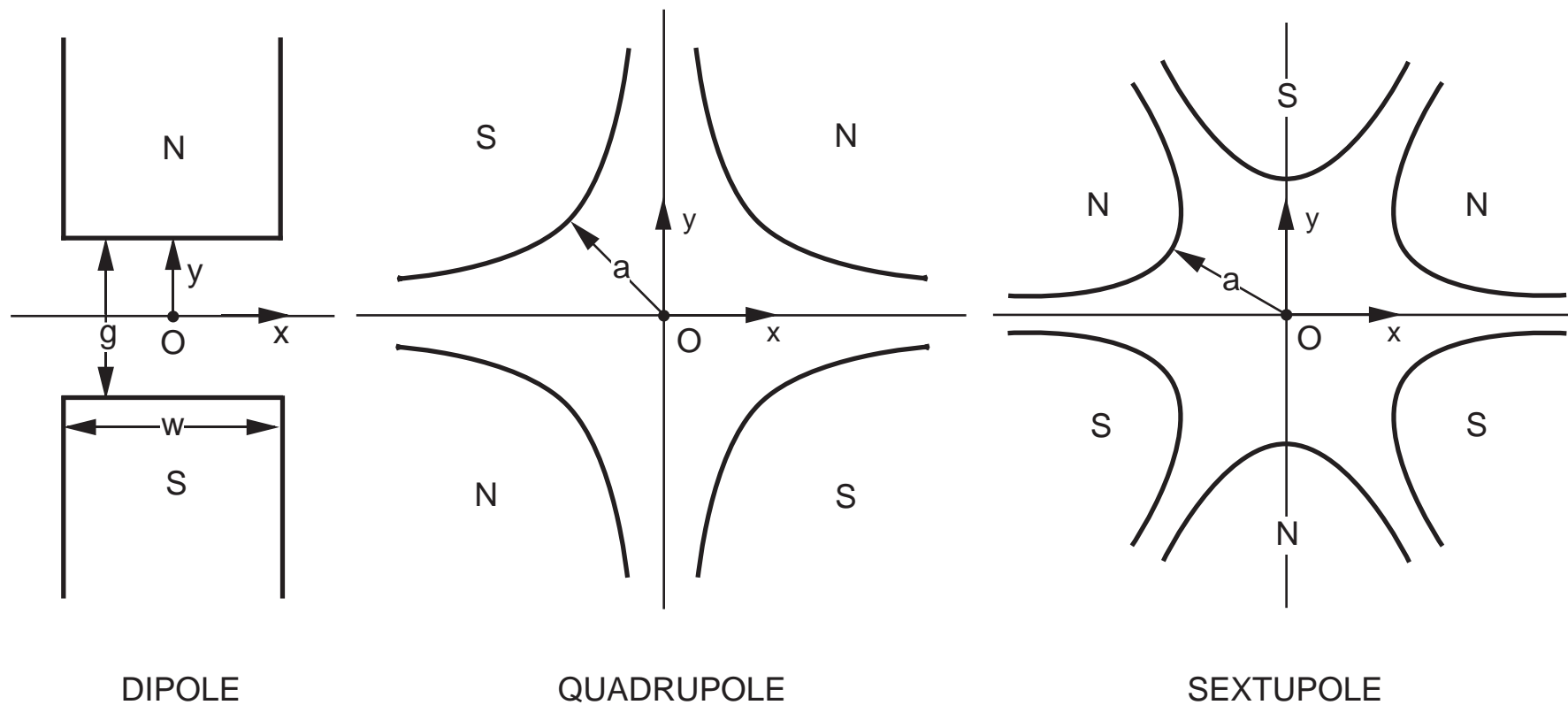
DIPOLE   QUADRUPOLE   SEXTUPOLE

Illustration of the magnetic midplane (x axis) for dipole, quadrupole and sextupole elements. The magnet polarities indicate multipole elements that are positive with respect to each other. The coordinate system is right-handed, so that the beam particles will be emerging from the paper and moving into the face of the reader.

SEXTUPOLE

Sextupole magnets are used to modify second-order aberrations in beam transport systems. The reference trajectory through a sextupole is a straight line. The action of a sextupole on beam particles is a second- and higher-order effect. In first order runs, a sextupole will act as a drift space. See SLAC-75 [3] for a tabulation of sextupole matrix elements.

First-, second-, and third-order effects in a sextupole are included in TURTLE. Since the transformation through a sextupole is evaluated using the actual momentum of each trajectory, no higher-order chromatic effects need be evaluated. Higher order geometric effects arise from the coupling between second-order terms at different longitudinal locations within the same sextupole. For the second- or third-order terms to have any effect, a higher-order calculation must be specified with the ORDER command described on page 57.

The SEXTUPOLE is a MAD element. It may be expressed in MAD notation or other keyword notation.

MAD Notation

For the MAD program, a magnetic sextupole is specified by three real attributes.

> label: SEXTUPOLE,   L = value,   K2 = value ,   TILT = value ;

A typical sextupole specification might be:

> S1:   SEXT,   L = 5.,   K2 = 0.2   ;

Symbol	Keyword	Description
L	L	The magnet length (normally meters).
K_2	K2	The normalized sextupole strength (in meters^{-3}).
	TILT	The angle of rotation about the optic axis (degrees with UMM and UMETER, radians with UMAD).

Strict MAD compatability requires the use of UMAD units, including radians for the TILT parameter. The units of any of the physical parameters can be changed to any desired. Other unit set specifications are described on page 34. Individual unit changes are described on page 38.

The quantity K_2 is related to the pole tip field B_o and the aperture a by:

$$K_2 = \frac{1}{(B\rho)} \frac{B_o}{a^2}$$

The normalization of K_2 is such that an sextupole of unit strength will give unit angular

deflection per unit magnet length to a trajectory one transverse unit from the reference trajectory. It should be noted that in the MAD program itself, the parameter K_2 is defined to be the third derivative of the field divided by $B_0\rho$. The MAD value of K_2 is then greater by a factor of 2 ($= 2!$) than the value defined here. Specification of the global unit set `UMAD` will cause TRANSPORT and TURTLE to use the MAD definition of multipole strength.

The keyword `TILT` indicates that the sextupole is rotated in a clockwise sense about the entrance reference trajectory. The positive sense appears clockwise looking downstream since the z axis also points downstream. The value given is the angle of rotation (degrees in `UTRANS`, radians in `UMAD`). *If the word* `TILT` *appears alone without a value, the angle of rotation is taken to be 30 degrees.*

Other Keyword Options

The same sextupole can be specified by another set of variables. The following parameterization corresponds to the original TRANSPORT variables expressed in keyword notation.

label: `SEXTUPOL`, `L` = value , `B` = value , `APER` = value ;

Symbol	Keyword	Description
L	`L`	The magnet length (normally meters).
B_o	`B`	The magnetic field (normally kilogauss).
a	`APER(TURE)`	The magnetic half-aperture or pole-tip radius (mm with `UMM`, meters with `UMETER`).

The `TILT` parameter may also be used with this option.

The `UMM` units are meters, kG, and mm respectively. The `UMETER` units for L, B and a are meters, kG, and meters, respectively. Global unit set specifications other than `UMAD` or `UTRANS` may also be selected and are described on page 34. Individual units changes are described on page 38.

For a beam momentum of 600 GeV/c, the following two elements are equivalent.

        S1:   SEXT, L = 5.,   B = 10.,   APER = 50.   ;

        S1:   SEXT, L = 5.,   K2 = 0.2 ;

Here we assume that the aperture is measured in mm. The parameter K_2 is measured in the inverse cube of the unit of longitudinal distance (usually meters). The description in terms of K_2 has the advantage that it does not require a separate specification of the beam momentum. The quantity K_2 is related to the pole tip field B_o and the half aperture a by:

$$K_2 = \frac{1}{(B\rho)} \frac{B_o}{a^2}$$

The normalization of K_2 is such that a sextupole of unit strength will give unit angular deflection per unit magnet length to a trajectory one transverse unit from the reference trajectory. It should be noted that in the MAD program itself, the parameter K_2 is defined to be the second derivative of the field divided by $B_0\rho$. The MAD value of K_2 is then greater by a factor of 2 than the value defined here.

The normal orientation of a sextupole is shown in the section describing the quadrupole. Other orientations of the sextupole may be obtained using the beam rotation element (SROT).

OCTUPOLE

Octupole magnets are used to affect third-order aberrations in charged particle optical systems. The effect of an octupole is limited to third and higher orders.

In TURTLE, the transformation through an octupole includes geometric effects of third order. The second-order terms for an octupole are all zero. The difference between an octupole and a drift space begins at third order. The chromatic effects are exact, since the transformation is calculated separately for each trajectory, using the actual momentum of the trajectory. In order to activate the third-order matrix elements for an octupole a third-order calculation must be specified via the `ORDER` element, described on page 57.

The `OCTUPOLE` is a MAD element. It may be expressed in MAD notation or other keyword notation.

MAD Notation

For the MAD program, a magnetic octupole is specified by three real attributes.

label: `OCTUPOLE,  L = value ,  K3 = value ,  TILT = value ;`

A typical octupole specification might then look like:

`O1:   OCTUPOLE,  L = 5.,  K1 = 4.0  ;`

Symbol	Keyword	Description
L	L	The magnet length (normally meters).
K_3	K3	The normalized octupole component (normally meters^{-4}, default 0.0).
	TILT	The angle of rotation about the optic axis (degrees with `UMM` and `UMETER`, radians in `UMAD`).

Strict MAD compatability requires the use of `UMAD` units, including radians for the `TILT` parameter. The units of any of the physical parameters can be changed to any desired. Other unit set specifications are described on page 34. Individual unit changes are described on page 38.

The quantity K_3 is related to the pole tip field B_o and the aperture a by:

$$K_3 = \frac{1}{(B\rho)} \frac{B_o}{a^3}$$

The normalization of K_3 is such that an octupole of unit strength will give unit angular deflection per unit magnet length to a trajectory one transverse unit from the reference trajectory.

It should be noted that in the MAD program itself, the parameter K_3 is defined to be the third derivative of the field divided by $B_0\rho$. The MAD value of K_3 is then greater by a factor of 6 ($= 3!$) than the value defined here. Specification of the global unit set UMAD will cause TRANSPORT and TURTLE to use the MAD definition of multipole strength. The description in terms of K_3 has the advantage that it does not require a separate specification of the beam momentum.

The keyword TILT indicates that the octupole is rotated in a clockwise sense about the entrance reference trajectory. The positive sense appears clockwise looking downstream since the z axis also points downstream. The value given is the angle of rotation in degrees. *If the word* TILT *appears alone without a value, the angle of rotation is taken to be 22.5 degrees.*

Other Keyword Options

The same octupole can also be specified by another set of variables. The following option exists for consistency with the original TRANSPORT set of variables

        label: OCTUPOLE, L = value ,   B = value ,   APER = value ;

Symbol	Keyword	Description
L	L	The magnet length (normally meters).
B_o	B	The magnetic field (normally kilogauss).
a	APER(TURE)	The magnetic aperture (mm in UMM, meters in UMETER).

The TILT parameter may also be used with this option.

The UMM units are meters, kG, and mm respectively. The UMETER units for L, B, and a are meters, kG, and meters respectively. Global unit set specifications other than UMAD or UTRANS may also be selected and are described on page 34. Individual units changes are described on page 38.

For a beam momentum of 600 GeV/c, the following two elements are equivalent.

        O1:   OCTUPOLE, L = 5.,   B = 10.,   APER = 50.   ;

        O1:   OCTUPOLE, L = 5.,   K1 = 4.0 ;

Here we assume that the aperture is specified in mm. The parameter K_3 is measured in the inverse fourth power of the longitudinal distance (usually meters). The description in terms of K_3 has the advantage that it does not require a separate specification of the beam momentum. The quantity K_3 is related to the pole tip field B_o and the half aperture a by:

$$K_3 = \frac{1}{(B\rho)} \frac{B_o}{a^3}$$

The normalization of K_3 is such that a octupole of unit strength will give unit angular deflection per unit magnet length to a trajectory one transverse unit from the reference trajectory. It should be noted that in the MAD program itself, the parameter K_3 is defined to be the second derivative of the field divided by $B_0\rho$. The MAD value of K_3 is then greater by a factor of 6 than the value defined here.

PLASMALENS – Plasma (Lithium) Lens

A plasma lens has focusing characteristics similar to those of a quadrupole, except that the plasma lens can focus in both transverse planes simultaneously. The specification for a plasma lens is the same as that for a quadrupole.

Since the transformation through a plasma lens is calculated separately for each trajectory, the chromatic terms are exact. Third-order geometric terms for a plasma lens are not in TRANSPORT or TURTLE at this time.

The plasma lens is not a MAD element. However, it can be described using the same parameters as a quadrupole. It therefore can be expressed in MAD-like notation or other keyword notation.

MAD-like Notation

The only reason that the notation is described as MAD-like is that this element is not in the MAD program. The keywords which can be used in specifying a plasma lens are the same as for a quadrupole. They are here repeated for convenience. A plasma lens is specified by two real attributes.

```
label: PLASMA,  L = value,  K1 = value  ;
```

A typical plasma lens specification might then look like:

```
P1:   PLAS,  L = 5.,  K1 = .01   ;
```

Symbol	Keyword	Description
L	L	The magnet length (normally meters).
K_1	K1	The normalized quadrupole strength (normally in meters^{-2}).

The units of any of the physical parameters can be changed to any desired. Other unit set specifications are described on page 34. Individual unit changes are described on page 38.

Other Keyword Options

The same plasma lens can also be specified by other sets of variables. The first of the two following options corresponds to the original TRANSPORT variables.

```
label: PLASMA,  L = value ,  B = value ,  APER = value ;
```

```
label: PLASMA,  L = value ,  GRAD = value ;
```

Symbol	Keyword	Description
L	L	The magnet length (normally meters).
B_o	B	The magnetic field (normally kilogauss).
a	APER(TURE)	The magnetic aperture (mm with UMM, meters with UMETER).
g	GRAD(IENT)	The gradient of the magnetic field (kg/mm with UMM, kg/m with UMETER).

If the unit set UMM is specified, the corresponding units are meters, kG, and mm respectively. If the global unit set UMETER is specified, the units for L, B and a are meters, kG, and meters, respectively. Global unit set specifications other than UMM or UMETER may also be selected and are described on page 34. Individual units changes are described on page 38.

For a beam momentum of 600 GeV/c, the following three elements are all equivalent.

```
P1:    PLAS,   L = 5.,   B = 10.,   APER = 50.    ;

P1:    PLAS,   L = 5.,   GRAD = 0.2   ;

P1:    PLAS,   L = 5.,   K1 = .01   ;
```

Here we assume that the aperture is measured in mm.

The description in terms of K_1 has the advantage that it does not require a separate specification of the beam momentum. The quantities K_1 and g are related to the pole tip field B_o and the aperture a by:

$$K_1 = \frac{g}{(B\rho)} = \frac{1}{(B\rho)} \frac{B_o}{a}$$

SOLENOID

The solenoid is most often used as a focusing element in systems passing low-energy particles. The solenoid transformation is also used in conjunction with particle detectors in interaction regions of particle colliders.

Particles in a solenoidal field travel along helical trajectories. The solenoid fringing field effects necessary to produce the focusing are included.

The first order transformation through a solenoid is included in TURTLE. Chromatic effects are included since the transformation is evaluated for each trajectory separately. Third-order matrix elements for a solenoid have yet to be included in TRANSPORT or TURTLE. The solenoid element is not done correctly, even to first order, in DECAY TURTLE. The order of the calculation is indicated by the `ORDER` element described on page 57.

The `SOLENOID` is a MAD element. It can be expressed in MAD notation or other keyword notation.

MAD Notation

For the MAD program, a solenoid is specified by two real attributes.

    label: SOLENOID, L = value , KS = value ;

A typical solenoid specification might then look like:

    SOL1:   SOLEN, L = 1.0, KS = .00025 ;

Symbol	Keyword	Description
L	L	Effective length of the solenoid (meters).
θ	KS	The solenoid strength in terms of the reference momentum rotation angle (degrees with `UMM` or `UMETER`, radians with `UMAD`).

Other units may be selected either by use of a different global unit specification (page 34) or individual unit changes (page 38).

Other Keyword Options

A solenoid may alse be specified by its length and field.

    label: SOLENOID, L = value , B = value ;

The same example as above would then be written as:

```
SOL1:   SOLEN,  L = 1.0,  B = 10.0  ;
```

Here we have assumed a beam momentum of 600 GeV/c. This is a higher value than than the momentum used in most solenoid applications. However, we continue to use it here for consistency with the other sections.

Symbol	Keyword	Description
L	L	Effective length of the solenoid (normally meters).
B	B	The field (normally kiloGauss).

The units are meters and kG in the three global unit sets `UMM`, `UMETER`, and `UMAD`. For strict compatability to the MAD program, the unit set `UMAD` is required.

If other units are desired they can be chosen via other global units specifications or the appropriate `UNIT` element preceding the `BEAM` element. The units of either physical parameter can be changed with a `UNIT` command as described on page 38.

The relation among its length and field and its rotation angle is

$$\frac{BL}{(B\rho)} = 2\theta$$

If the reference coordinate system is rotated by the angle θ after the solenoid, then the transfer matrix will take the block diagonalized form similar to that of a quadrupole.

First-Order Solenoid Matrix [*]

Definitions: L = effective length of solenoid.

$K = B_0/(2B\rho_0)$, where B_0 is the field inside the solenoid and $(B\rho_0)$ is the magnetic rigidity (momentum) of the central trajectory.

$C = \cos KL$

$S = \sin KL$

γ = relativistic factor

$$R(\text{Solenoid}) = \begin{pmatrix} C^2 & \frac{1}{K}SC & SC & \frac{1}{K}S^2 & 0 & 0 \\ -KSC & C^2 & -KS^2 & SC & 0 & 0 \\ -SC & -\frac{1}{K}S^2 & C^2 & \frac{1}{K}SC & 0 & 0 \\ KS^2 & -SC & -KSC & C^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \frac{L}{\gamma^2} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Alternate Forms of Matrix Representation of the Solenoid:

Rotating the transverse coordinates about the z axis by an angle $= -KL$ decouples the x and y first-order terms, i.e.

$$R(-KL) \cdot R(\text{Solenoid}) = \begin{pmatrix} C & \frac{1}{K}S & 0 & 0 & 0 & 0 \\ -KS & C & 0 & 0 & 0 & 0 \\ 0 & 0 & C & \frac{1}{K}S & 0 & 0 \\ 0 & 0 & -KS & C & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \frac{L}{\gamma^2} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

where $KS = \dfrac{1}{F}$, and F = focal length of solenoid.

---

[*] For a derivation of this transformation see report SLAC-4, [6].

ACCELERATION

An energy gain is reflected in both the divergence and the width of the beam. This element provides a simulation of a travelling wave linear accelerator energy gain over a field free drift length (i.e. no externally applied magnetic field).

The energy of the reference trajectory is assumed to increase linearly over the entire accelerator length. If this is not the case, an appropriate model may be constructed by combining separate ACCEL elements. An ACCEL element with a zero energy gain is identical to a drift length.

First-order matrix elements are included in TRANSPORT and TURTLE for the accelerator section. Second- or third-order matrix elements have not been incorporated in the program.

The ACCEL element is MAD compatible. It can be expressed in MAD notation or other keyword notation.

MAD Notation

For the MAD program, a travelling wave linear accelerator element is specified by four real attributes.

    label: ACCEL,   L = value ,   VOLT = value ,   LAG = value ,   FREQ = value ;

A typical accelerator element might then look like:

       ACCEL,   L = 10.0,   VOLT = 0.1,   LAG = 30.0,   FREQ = 30.   ;

Symbol	Keyword	Description
L	L	Accelerator length (meters).
	VOLT	Energy gain (GeV) at crest of wave.
ϕ	LAG	Phase lag (degrees).
ν	FREQ	The RF frequency in MHz (no default value).

The units of only the length and the energy gain may be changed. The different unit set specificiations are described on page 34. Individual unit changes are described on page 38.

Other Keyword Notation

The original TRANSPORT set of parameters can also be expressed in keyword notation by four attributes.

    label: `ACCEL, L` = value , `VOLT` = value , `LAG` = value , `WAVE` = value ;

The following two parameterizations of the accelerator element are equivalent.

    `ACCEL, L = 10.0, VOLT = 0.1, LAG = 30.0, FREQ = 30. ;`

    `ACCEL, L = 10.0, VOLT = 0.1, LAG = 30.0, WAVEL = 1000.0 ;`

Here we have taken the units of wavelength to be mm.

Symbol	Keyword	Description
L	`L`	Accelerator length (meters).
	`VOLT`	Energy gain (GeV) at crest of wave.
ϕ	`LAG`	Phase lag (degrees).
λ	`WAVELENGTH`	Wavelength (mm in `UMM`, meters in `UMETER`).

The units for the length, the energy gain, and the wavelength may be changed. The beam unit of longitudinal separation is used for the wavelength and may be changed by the `UNIT` command described on page 38.

The new beam energy is taken as the new energy of the reference particle and is printed as output. The energy gain of the reference particle is $\Delta E \cos \phi$.

SEPTUM

An electrostatic septum is used to split a particle beam into two portions differently directed. Wires are strung transversely on the element $(x - z)$ midplane. The electric field above the wires is in the positive y direction and deflects the particles upward. The electric field below the wires is either zero or in the negative y direction. There is therefore a sudden discontinuity in field direction at the plane of the wires.

The septum is assumed to be aligned along the incoming coordinate axes. If the field below the wires is not separately specified, it is taken to be equal and opposite to the field above the wires.

Above the wires, the septum element is treated like a kick. If the field below the wires is zero, the septum element acts as a drift space for particles below the wires. If the field below the wires is nonzero, the septum is treated like a kick for particles below the wires. If the field below the wires is in the opposite direction from the field above the wires, the kick induced by the field below the wires will be in the opposite direction from the kick induced by the field above the wires.

The reference trajectory is defined as if the electric field were not present. It is a straight line continuation of the entrance reference trajectory. The beam centroid will be displaced from the reference trajectory in the direction of the electric field above the wires. The septum element will also produce a vertical dispersion. Unlike the case with magnetic elements, this dispersion will depend on the particle mass in addition to the reference momentum.

Multiple scattering in the wires is also simulated. The material of the wires is taken to be spread uniformly over the $(x - z)$ plane. Therefore any particle crossing the midplane will suffer some deflection.

The SEPTUM element is not a MAD element.

Example

Assuming the UMM set of units, a typical septum specification might appear as:

```
SEPTUM,  L = 3.0,  VOLT = 0.1,  APER = 10.0  ;
```

or

```
SEPTUM,  L = 3.0,  E = 0.01  ;
```

indicating a septum of length 3 meters and a voltage of 0.1 megavolts on a 10 mm aperture.

Keyword Notation

An electrostatic septum is specified by the following real attributes.

Symbol	Keyword	Description
L	L	The septum length (meters, default: 0.0 m)
V	VOLTAGE	The voltage across the septum gap (megavolts, default: 0.0 MV)
a	APERTURE	The septum gap (mm in UMM, meters in UMETER)
E	E	The electric field (Mv/mm in UMM, MV/m in UMETER)
E_O	EOTHER	The electric field below the septum wires and pointing downward.
Δy	OFFSET	The vertical offset of the wire plane (mm in UMM, meters in UMETER)
d	WIRE	The wire diameter (mm in UMM, meters in UMETER)
ΔL	SPACING	The wire spacing (normally in meters)
r	RADL	The radiation length in the wire (used for scattering calculations) (normally in meters)
	TILT	The roll angle about the longitudinal axis (degrees in UMM and UMETER). The septum is rotated in a clockwise sense about the entrance reference trajectory. The positive sense appears clockwise looking downstream since the z axis also points downstream. If the word TILT appears alone without a value, the angle of rotation is taken to be 90 degrees.

Global unit set specifications other than UMM or UMETER may also be selected and are described on page 34. Individual units changes are described on page 38.

Several of the parameters are redundant, and others are useful only for TURTLE. A partial description of an unrotated septum can be given, which does not include multiple scattering in the wires.

        label: SEPTUM,   L = value ,   VOLT = value ,   APER = value ;

or by

        label: SEPTUM,   L = value,   E = value ;

Multiple scattering in the wires is included by supplying values for the wire diameter and spacing and the radiation length of the material of the wire.

HORN — Neutrino Horn

A neutrino horn is a magnetic device which is used to focus a beam of mesons. The mesons decay to produce muons and neutrinos. Several horns are often used in a neutrino beam line. The effect of having more than one horn is that a set of horns can serve as a broad-band focusing device. Once the angular divergence of the parent mesons is reduced to be no greater than the angular divergence of the muons from the decay, then further focusing is of diminished value.

The representation of the horn in TURTLE is such as to be compatable with the representation in the neutrino flux program NUADA[*]. The specification of a single horn might be

```
HORN      .14
-1.1 -1.0 -0.2 0.1  2.0  2.5  2.7  3.0  3.1
0.15 .07   .007 .007 .09   .015 .015 .057 .15
.002 .002 .003 .002 .002 .002 .002 .002
```

The first line contains the keyword `HORN` and a numerical constant. The numerical constant is the current, in megamps, which is flowing through the horn. The current is positive if the flow is downstream, in which case positively charged mesons will be focused, or deflected toward the axis of the beam line. In NUADA, the values of the currents are not on the same line as the `HORN` keyword, but are listed under a separate keyword `CURRENT`.

The first line also indicates that the subsequent three lines of numerical data are a description of the horn. The format for the three lines of strictly numerical data is the same in TURTLE as it is in NUADA.

The horn is described as a sequence of connected line segments. The second line of horn data gives the z positions of the ends of these line segments. The third line gives the r positions The fourth line of data indicates the thickness of the horn material between successive vertices. A cross section of the horn in the $z - r$ plane is shown below.

The numbers are all in meters, as the program NUADA expresses all lengths in meters. The three lines of numerical data are all in format 12F5.0.

The horn itself is a cylindrically symmetric device. The surface of the horn is generated by taking the broken line segment and rotating it about the z axis of the beam line.

The absolute values of the z coordinate are used in NUADA to locate each horn. In TURTLE, only the relative values are important. TURTLE considers a neutrino horn to be a physical element like a quadrupole or a bending magnet. The beginning of the horn is placed at the end of the sequence of elements which precede it. The length of a horn is the

---

[*]NUADA, The Fermilab Neutrino Flux Program, D.C. Carey, in preparation. (A copy of an older version may be obtained from the author.)

difference between the z values for the end and the beginning.
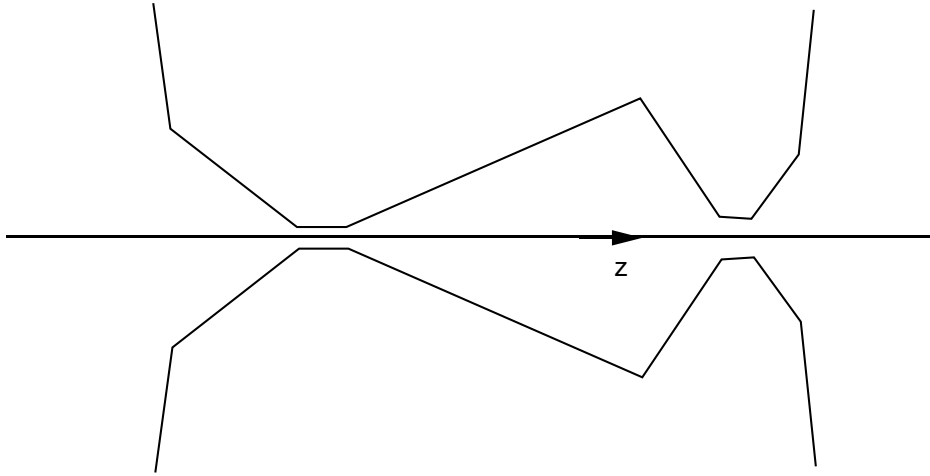
A cross section of a neutrino horn in the $z - r$ plane.

SLIT

The `SLIT` element cuts off the portion of the beam phase space which exceeds the dimensions of the slit. A slit element is considered to occupy no space longitudinally. A collimator can be constructed by separating two slits by a nonzero longitudinal distance. The slit can limit the dimensions of the beam in any of the six trajectory coordinates. Only for the transverse spatial coordinates x and y is this physically realizable in a direct manner. However, the other dimensions (x', y', ℓ, and δ) are also available since they make sense mathematically and might be useful.

The `SLIT` is not a MAD element, but may be expressed in keyword notation.

A slit element limiting the x coordinate to 1 cm half width might be expressed as:

    label: SLIT, X, HWIDTH = 10.0 ;

The value for the half width of 10.0 comes from the assumption that the unit set being used is `UMM`.

The following keywords are used in specifying a slit element.

Symbol	Keyword	Description
x	X	The x coordinate
x'	XP	The x' coordinate
y	Y	The y coordinate
y'	YP	The y' coordinate
ℓ	DL	The longitudinal separation ℓ.
δ	DEL	The relative momentum difference δ.
	NSLIT	Slit coordinate for user-written subroutine.
	particle name	Particle identification mnemonic (DECAY TURTLE only).
	HWIDTH	The half width of the slit.
	HHEIGHT	The half height of the slit (for an elliptical slit).

If a trajectory comes to a horizontal slit and the absolute value of its x coordinate exceeds the half aperture of the slit, the particle will be stopped and TURTLE will proceed to the next trajectory. If an RMS addition to the beam is specified immediately after the slit, the trajectory will not be stopped, but its coordinates will be changed by an amount within the range specified on the RMS addition. See the description of the `BEAM` element for further details.

If the absolute value of the x coordinate of the trajectory is within the half aperture of the slit, the particle will pass undeflected in all cases. This description applies similarly for a slit in any other coordinate.

An elliptic slit is also available. It may be specified by giving two coordinates and two half widths. For example, an elliptical slit in x and y might look like

```
SLIT,  X,  Y,  HWIDTH = 5.0,  HHEIGHT = 3.0  ;
```

This `SLIT` element describes an elliptical aperture with a horizontal semimajor axis of 5.0 mm and a vertical semimajor axis of 3.0 mm. The units are those specified by the global unit set `UMM`.

In keyword notation, the order of the parameters is not important. The keywords `HWIDTH` and `HHEIGHT` do <u>not</u> necessarily pertain to the two coordinates x and y in the order in which they appear on the `SLIT` element. They pertain to the two coordinates as they appear in the list of coordinates in the listing of keywords above. In other words, if `X` and `Y` were reversed in the keywords on the particular use of the `SLIT` element, the keyword `HWIDTH` would still apply to x, and `HHEIGHT` would apply to y.

In DECAY TURTLE the particle name indicates the particle for which the slit is active. The default particle names are `PARENT`, `DAUGHTER1`, and `DAUGHTER2`. The names may be changed via the `SPEC` element. A common choice is `PION`, `MUON`, and `NEUTRINO`. If there is no particle specification then the `SLIT` element is assumed to pertain to the `PARENT` particle or whatever is substituted for it. If the particle type is not the same as that in the slit, the slit will have no effect. For a random addition after a slit to have an effect, it is necessary that both the slit and the rms addition to the beam have the same particle type as the particle itself.

Example No. 1 A non-symmetric slit

An transversely displaced (non-symmetric) slit can be described using the `CENTROID` element. A slit which restricts the x coordinate so that $-60 \leq x \leq 100$ can be represented as

```
CENT,  X = 20.   ;
SLIT, X, HWIDTH = 80.    ;
CENT,  X = -20.   ;
```

The two coordinate shifts will cause all particles to be temporarily shifted in the positive x direction so that the beam centroid passes through through the slit with an x value of 20.

Example No. 2 A slit followed by a window.

```
SLIT,  X,  HWIDTH = 50.   ;
SLIT,  X,  MUON,  HWIDTH = 50.    ;
BEAM,  ...  RMS,  MUON  ;
BEAM,  ...  RMS,  PION  ;
```

Here we are dealing with two particle types, `PION` and `MUON`, so this is an example for

DECAY TURTLE. We assume that the parent particle has charge +1 and has been assigned the name PION. The first (charged) daughter particle is also positively charged and has the name MUON. The first SLIT restricts the phase space of the pions and produces no scattering. The second slit serves as an aperture restriction on the muons with scattering on the slit. The second RMS addition to the beam causes the pions still in the beam to scatter in the window. The second (neutral) daughter particles are not affected either by either slit or by the window.

Example No. 3 A typical neutrino decay channel

```
            .
            .            (Decay channel)
            .
SLIT,  PION,  X,  HWIDTH = 0.0  ;
BEAM,   ...   , MUON,  RMS  ;
            .
            .            (Muon channel)
            .
SLIT,  MUON,  X,  HWIDTH = 0.0  ;
DRIFT,   ...   , ; (Neutrino channel)
SLIT,  NEUTRINO,  X,  Y,  HWIDTH = 200.,  HHIGH = 150.    ;
```

This example deals with three types of particle. They are assumed to have been named PION, MUON, and NEUTRINO. The first slit stops all the pions. The first RMS addition to the beam, between the decay and muon channels, represents the scattering of muons in the beam stopper. The slit marked MUON is a beam stopper for all muons. At the end of the neutrino channel is an elliptical slit. Neutrinos are then counted only if they hit an elliptical detector having semimajor axes of 200 cm and 150 cm in the x and y directions respectively.

User-Written Subroutine for Slit Profiles

A slit whose profile is any function of any of the trajectory coordinates may be imposed via a user-written subroutine. A user programs a subroutine named SLIT, which replaces the version in the standard TURTLE program. This subroutine will be called for every place in the beam line where the SLIT element appears with a slit coordinate of 10 or greater. The slit coordinate is specified in the data by the keyword NSLIT. The element specifying a slit might then look like

```
SLIT,  NSLIT = 13,  HWIDTH = 0.5  ;
```

The values associated with NSLIT and HWIDTH keywords are then transmitted to the user-written subroutine.

There are two common blocks to be placed in the subroutine.

```
COMMON /RAY/ RAY(6)
COMMON /SLITT/ NSLIT, HWIDTH, HIT
LOGICAL HIT
```

The contents of the common block RAY will be the trajectory coordinates at the longitudinal location of the slit. The integer NSLIT is the coordinate in which the slit occurs. The parameter HWIDTH is the half width of the slit as specified in the SLIT element data. It may be used for any purpose desired.

The logical variable HIT is returned with a value indicating whether or not the particle has hit the slit. For example, if a circular slit of radius HWIDTH is to be imposed, the subroutine would contain the statement

```
HIT = SQRT(RAY(1)**2 + RAY(3)**2) .GT. HWIDTH
```

The presence of the integer NSLIT allows branching within the subroutine SLIT, and the representation of an unlimited variety of slit configurations.

MARKER – Location markers

In the MAD program, beam lines are defined by explicit commands which assemble the beam lines from previously defined components. Marker elements may be among those components. Their sole purpose is to demarcate a place in the beam line. Operations, which refer to those markers, can be defined after the beam line specification is complete. By the use of markers a beam line can be specified in an uninterrupted manner in MAD notation. The operations, not necessarily in MAD notation, will be all together and follow the beam line specification.

The `MARKER` is a MAD element. A marker has no additional keywords but is completely given by the word `MARKER` plus a label. Any histograms, storage of parameters, or other operations may refer to the markers placed in the beam line. When TURTLE runs through the beam line, it executes the operations when it encounters the corresponding marker.

A marker element with the label "`MAR1`" would have the appearance:

```
MAR1:    MARKER ;
```

If the MAD procedure for assembling beam lines is used, these markers may then be inserted into a beamline or subline defined by the `LINE` statements. This procedure is described on page 148.

If the beam line is defined by simply listing the elements, the `MARKER` element can be placed in order with the other physical elements. Reference to the markers can be made by any of the elements specifying operations. These operations include the `UPDATE` element, the storage of matrix elements (`STORE` element), and the alignment marker (`ALMARK`).

Other Transformations

MATRIX — Arbitrary Transformation Input

To allow for the use of empirically determined fringing fields and other specific (perhaps nonphase-space-conserving) transformations, provision has been made for reading in an arbitrary transformation matrix. First-, second-, and third-order matrix elements may be included. The order of calculation that will be used in the program is determined by the order parameters on the ORDER element described on page 57.

The first-order 6×6 matrix is read in row by row.

Notation

A typical MATRIX element setting the R_{12} matrix entry to 5.0 would look like:

        MATRIX, R12 = 5.0 ;

Any or all of the members of a given row of a matrix may be specified in a single element.

A complete matrix must be read and applied one row at a time. Rows that do not differ from the unit transformation need not be read. If one of the elements on the diagonal of the matrix is unspecified, it will be set equal to 1.0.

The units must be those used to print the transfer matrix; in other words, consistent with the BEAM input/output. The BEAM element and the units used for the specification of the input phase space are described on page 62.

Usage

MATRIX elements that immediately follow one another will all be used to form a single transformation matrix. If distinct matrices are desired, another element must be inserted to separate the MATRIX elements. Several do-nothing elements are available; for example, a zero length drift (DRIFT, L = 0.   ;) is a convenient one.

When the last of a sequence of MATRIX elements is read, the assembled transformation matrix will be printed in the output. Note that

$$\begin{pmatrix} 1 & 0 \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} a_{11} & a_{12} \\ 0 & 1 \end{pmatrix} \neq \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

Hence, a matrix formed by successive (MATRIX ; DRIFT, L = 0.   ; MATRIX ;) – elements is not always equal to the one formed by leaving out the (DRIFT, L = 0. ;) element.

Second Order

If it is desired to include some second-order matrix coefficients for the ith row, then the keywords used for such coefficients consist of the letter "T", followed by three integers. If, in the example given above, we also wished to set the T_{126} matrix element to 2.5E-2, then we would write the data element as

```
MATRIX,  R12 = 5.0,  T126 = 2.5E-2  ;
```

As in the first-order case, full rows not different from the identity matrix (i.e., $R_{ii} = 1$, all other $R_{ij} = 0$, and all $T_{ijk} = 0$) need not be read in.

Third Order

It is also possible to include any third-order matrix coefficients for the ith row by specifying any non-zero matrix elements.. If the U matrix element U_{1266} is to be set to -2.5E-4, then the example above would be modified to

```
MATRIX,  R12 = 5.0,  T126 = 2.5E-2,  U1266 = -2.5E-4  ;
```

All U matrix elements not specified otherwise will be assumed to be zero.

The `MATRIX` element must not be used with DECAY TURTLE to represent any element with nonzero physical length. The decay probability of a particle is determined using its lifetime, speed, and the length of the physical element through which it is passing. The `MATRIX` element contains no specification of a physical length.

SROT — Coordinate Rotation

The transverse coordinates x and y may be rotated through an angle α about the z axis (the axis tangent to the central trajectory at the point in question).* Thus a rotated bending magnet, quadrupole, or sextupole may be inserted into a beam transport system by preceding and following the element with the appropriate coordinate rotation. (See examples below.) The positive sense of rotation is clockwise looking in the direction of the positive z axis, i.e. the direction of beam travel.

The coordinate rotation is an <u>exact</u> first-order transformation. There are no higher order terms. However, the coordinate rotation will affect the higher-order terms from the elements that precede it.

The SROT element is used in MAD. Since there is only parameter to be specified, there is no other keyword notation.

Notation

For the MAD program, a rotation about the longitudinal axis is specified by a single physical attribute:

       label: SROT,   ANGLE=value ;

A rotation of 90° expressed in radians (using the global unit set UMAD) would then be:

       SROT,   ANGLE = 1.570796327   ;

and in degrees (using the global unit set UMM or UMETER) would be

       SROT,   ANGLE = 90.   ;

Symbol	Keyword	Description
α	ANGLE	The angle of rotation (degrees in UMM or UMETER, radians in UMAD, default: 0.0 rad)

The units of the rotation angle can be changed to any desired. Other unit set specifications are described on page 34. Individual unit changes are described on page 38.

---

*See SLAC-75 [3], page 45 and page 12, fig. 4, for definitions of x, y, and z coordinates.

Examples:

For a bending magnet, the beam rotation matrix may be used to specify that the magnet is rotated. With keyword notation, the rotation of certain magnetic elements can also be done by the use of the TILT parameter on the element specification. Bending magnets, including fringing fields, can be represented as a single RBEND or SBEND element. (See page 91) These two elements, along with the QUAD, SEXT, OCT, and SEPTUM elements, all can be rotated with a TILT parameter.

Example No. 1

A bend up is represented by rotating the x,y coordinates by -90.0 degrees as follows:

```
SROT,   ANGLE = -90.    ;
ROTAT,  -  ;
BEND,   -  -  -  ;
ROTAT,  -  ;
SROT,   ANGLE = +90.    ;    (returns coordinates to their initial orientation)
```

A bend down is accomplished via a +90 degree rotation.

```
SROT,   ANGLE = +90.    ;
ROTAT,  -  ;
BEND,   -  -  -  ;
ROTAT,  -  ;
SROT,   ANGLE = -90.    ;
```

A bend to the left (looking in the direction of beam travel) is accomplished by rotating the x,y coordinates by 180 degrees, e.g.

```
SROT,   ANGLE = +180.  ;
ROTAT,
BEND,
ROTAT,
SROT,   ANGLE = -180.   ;
```

Example No. 2

A quadrupole rotated clockwise by 60 degrees about the positive z axis would be specified as follows:

```
SROT,  ANGLE = +60.    ;
QUAD,   -  -  -  ;
SROT  ANGLE = -60.    ;
```

Beam rotation matrix

$$R = \begin{pmatrix}
C & 0 & S & 0 & 0 & 0 \\
0 & C & 0 & S & 0 & 0 \\
-S & 0 & C & 0 & 0 & 0 \\
0 & -S & 0 & C & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}$$

where $C = \cos\alpha$,
$S = \sin\alpha$,
α = angle of coordinate rotation about the beam axis.

e.g. for $\alpha = +90$ degrees, this matrix interchanges rows 1 and 2 with 3 and 4 of the accumulated R matrix as follows:

$$\begin{bmatrix}
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
-1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0
\end{bmatrix} \times \begin{bmatrix}
R(11) & R(12) & R(13) & R(14) \\
R(21) & R(22) & R(23) & R(24) \\
R(31) & R(32) & R(33) & R(34) \\
R(41) & R(42) & R(43) & R(44)
\end{bmatrix} = \begin{bmatrix}
R(31) & R(32) & R(33) & R(34) \\
R(41) & R(42) & R(43) & R(44) \\
-R(11) & -R(12) & -R(13) & -R(14) \\
-R(21) & -R(22) & -R(23) & -R(24)
\end{bmatrix}$$

(The rest of the matrix is unchanged.)

SHIFT — Shift in the Reference Coordinate System

If a particle beam is transferred from one optical system to another, there may be a discontinuity in the reference coordinate systems used. The centroid coordinates may change at a given point due to this discontinuity. The effect may be parameterized by this change in the centroid coordinates. The `SHIFT` element is specified in exactly the same manner as the `CENTROID` element, except that the keyword here is `SHIFT` instead of `CENTROID`.

In TRANSPORT the `SHIFT` element affects the floor coordinates. This property is the main difference between the `CENTROID` and the `SHIFT` elements. In TURTLE, there is no difference between the two elements.

The `SHIFT` element is not in MAD, but may be expressed in keyword notation.

Notation

The mnemonic used in specifying a shift in the reference coordinate system is `SHIFT`. The keywords which can be used are

Symbol	Keyword	Description
x	X	The shift in the x coordinate (mm with `UMM`, meters with `UMETER`)
x'	XP	The shift in x' (mr with `UMM`, radians with `UMETER`)
y	Y	The shift in y (mm or meters)
y'	YP	The shift in y' (mr or radians)
ℓ	L	The shift in ℓ. (mm or meters)
δ	DEL	The shift in δ (permil with `UMM`, fractional with `UMETER`).

The units are the same as for the `BEAM` element (page 62). The unit sets for which the units are given above are `UMM` and `UMETER`. Other unit sets in addition to those mentioned above can be used and are described on page 34. Individual units changes are described on page 38.

An example of a downward kink in the reference trajectory increasing the vertical angle by 1.0 milliradians might be

```
SHIFT,  YP = 1.0  ;
```

REALIGN — Realignment of the Reference Trajectory

(`REALIGN` ;): At the location of this element, the reference trajectory is shifted to line up with the <u>first-order image</u> of the initial beam centroid. Thus, the reference trajectory of the beam line, as followed by the program, will not be continuous. The beam centroid location will be continuous, but will seem to be discontinuous if expressed relative to the reference trajectory.

Part of a TRANSPORT or TURTLE set of data might then have the following appearance:

```
BEAM      .   .    .    .    .    .    ;
CENTROID  .   .    .    ;
DRIFT   .    .    .    ;
QUAD   .    .    .    ;
REALIGN   ;
DRIFT   .    .    .    ;
BEND   .    .    .    ;
DRIFT   .    .    .    ;
QUAD   .    .    .    ;
DRIFT   .    .    .    ;
```

The `CENTROID` element indicates an initial centroid displacement which is then propagated through the system. The centroid is focused by the quadrupoles. After traversing a quadrupole the beam will show dispersive effects, since the centroid does not follow the axis of the quadrupoles. A diagram of this simple system is shown on the following page.

When the `REALIGN` element is encountered, the centroid dispacement is set to zero. The reference trajectory is redefined to be a continuation of the previous centroid. The amount of shift will be that necessary to align the beam line components on the off-axis centroid.

An example of the use of the `REALIGN` command occurs when the first optical element in a beam line is a quadrupole, but because of geometric constraints, the optic axis of the quadrupole cannot be placed so that it is aligned with the center of the accepted phase space. The centroid may be placed at the center of the accepted phase space. The beam line can be laid out by allowing the centroid to be focused by the quadrupole and then redefining the reference trajectory to be lined up with the centroid as it exits the quadrupole.

The effect of the **REALIGN** command in the data above can be shown by the following figure:

NEW REFERENCE
TRAJECTORY

REALIGN

CENTROID

ORIGINAL REFERENCE
TRAJECTORY

QUAD   BEND   QUAD
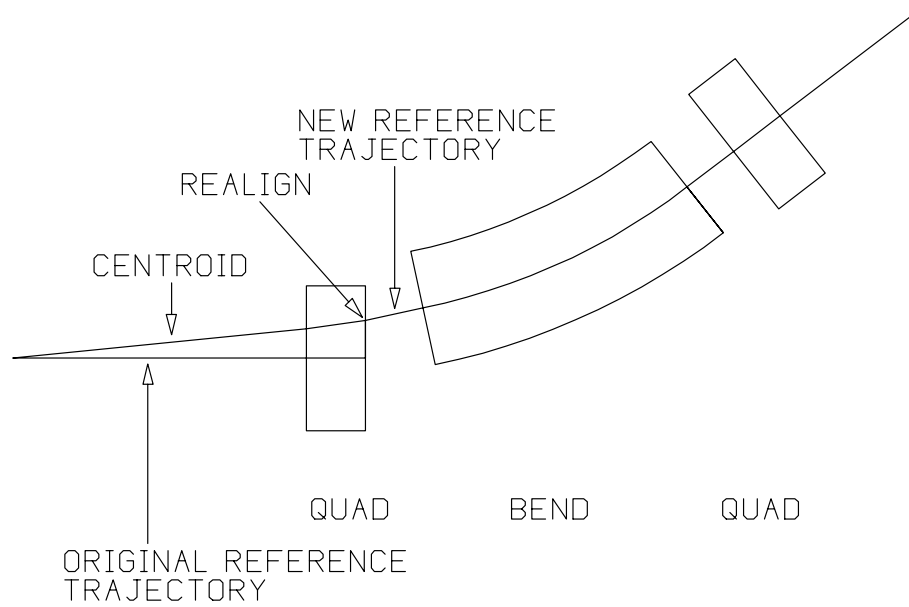
The Effect of the **REALIGN** element

ALIGN — Magnet Alignment Tolerances

The first-order effects of the misalignment of a magnet or group of magnets are a shift in the centroid of the beam and a change in the beam focusing characteristics. The transverse displacement of any focusing device, such as a quadrupole, will cause a deflection of the beam centroid, and therefore a downstream transverse shift in the position of the beam ellipse. A longitudinal shift of a quadrupole, or a rotation of a quadrupole about its optic axis, or a transverse displacement of a sextupole will cause a change in the beam focusing characteristics. These changes may affect histograms of any of the beam dimensions downstream from the point where the misalignment occurs.

An example of an element in keyword notation which specifies a roll of 2 mrad in the preceding magnet is

```
        MIS1:    ALIGN,   RZ = 2.0,   KNOWN   ;
```

A roll is a rotation about the longitudinal or z axis. Unless otherwise specified, a misalignment is taken as applying to the immediately preceding magnetic element. Other options are described below.

Types of Misalignment

In TRANSPORT, three varieties of misalignment simulation are commonly needed. In TURTLE, only numbers (2) and (3) make any sense, so number (1) will not be considered

-      (1) The physical position of the magnet is uncertain within a given tolerance.
- or   (2) The magnet is displaced and/or rotated by a known amount;
- or   (3) The magnet is displaced and/or rotated by a definite amount chosen from a user-specified range by computer-generated random numbers.

In TURTLE, the only visible consequence of a misalignment is in its effect on the histograms. The histogram output is most closely related to the beam matrix output in TRANSPORT.

The object(s) to be misaligned may be designated in three different ways:

-      (1) A single bending magnet, quadrupole, sextupole, or octupole
- or   (2) A section of a beam line
- or   (3) All bending magnets and/or quadrupoles)

In addition, there is some choice among coordinate systems to which the misalignment is to be referred. There are three possible positions for the origin or the coordinate system:

-      (1) The intersection of the reference trajectory with the entrance face of the magnet
- or   (2) The longitudinal midpoint of the magnet

or (3) The intersection of the reference trajectory with the exit face of the magnet

For each choice of origin, there are also two possibilities for the orientation of the coordinate misalignment coordinate system.

(1) For a magnet which changes the direction of the reference trajectory, the z axis of the misalignment coordinate system is along the chord of the magnet. If the longitudinal midpoint is chosen as the origin of the misalignment coordinate system, then the origin is taken to be on the midpoint of the chord. If the reference trajectory in the magnetic element is straight, the misalignment coordinate system is the same as the beam coordinate system.

or (2) The misalignment reference coordinates are the same as the beam coordinate system for bending magnets also. If the midpoint of the magnet is chosen as the origin of the misalignment coordinate systen, then that midpoint is the intersection of the projected entrance and exit reference trajectories. The z axis for the misalignments is parallel to the reference trajectory at the logitudinal midpoint.

The element `ALIGN` is a TRANSPORT or TURTLE element but is not a MAD element. However, in both TRANSPORT and TURTLE, it may be expressed either in keyword notation similar to the MAD format.

Notation

The mnemonic for magnet misalignments is `ALIGN`. The keywords which can be used in specifying the parameters describing the misalignments are given below. The parameters consist of three displacements and three rotations plus additional keywords indicating the type of misalignment. There is one parameter for a displacement along and one parameter for a rotation about each of the coordinate axes x, y, and z.

Here the global unit set specifications described are `UMM` and `UMETER` Other global unit set specifications may also be selected and are described on page 34. Individual units changes are described on page 38.

Symbol	Keyword	Description
Δx	X	The magnet displacement in the x direction (mm with `UMM`, meters with `UMETER`).
Δr_x	RX	A rotation about the x axis (mr with `UMM`, radians with `UMETER`).
Δy	Y	The magnet displacement in the y direction (mm or meters).
Δr_y	RY	A rotation about the y axis (mr or radians).
Δz	Z	The magnet displacement in the z (beam) direction (mm or meters).
Δr_z	RZ	A rotation about the z axis (mr or radians).

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Any parameters not specified are taken to equal zero. The meanings of other keywords used to describe the nature of the misalignment or the coordinate system to which it is referred are described below.

Additional Keywords

Additional keywords can be used to indicate whether the misalignment is a known displacement or a random error in position. They can also be used to specify the form in which the results are displayed, the magnet or beam line section to be misaligned, and both the origin and the orientation of the axes to which the misalignment is referred. Another option continuously restores the beam centroid to the reference trajectory. The meanings for the additional keywords are as follows:

A. The interpretations of the misalignment parameters and of the resulting beam dimensions are given by the following keyword options.

Keyword	Interpretation
KNOWN	The magnet or section of beam line is misaligned by a known amount. Both the beam centroid and phase space dimensions can be affected.
RANDOM	The other option is a random displacement. The effect of the misalignment will be calculated in the same manner as if the parameters describing the misalignment were known. However, those parameters will be determined by multiplying the parameters on the misalignment element by random numbers. The random numbers are distributed uniformly from -1 to 1. *The parameters on the misalignment element then represent the maximum magnitude of the misalignment.* The random numbers are drawn from the same sequence as those used for errors on the physical parameters of the elements. For a description of such errors, the user should see page 59.

B. Additional keywords define the mode of display of the results of the misalignment.

Keyword	Interpretation
BEAM	The effect of the misalignment is shown in whatever histograms are displayed. Histograms are printed wherever a HIST element or AHIST and DHIST elements are encountered. If no mode of display is specified, then the default option is BEAM

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C. A third set of keywords is used to specify the magnet(s) or section of the beam line to be misaligned.

Keyword	Interpretation

SINGLE   The single magnet (element) immediately preceding the align element is to be misaligned. This option is the default, so no specification of section to be misaligned is required to invoke this option. A bending magnet with fringe fields should be misaligned using one of the options described below.

R1   The last R1 matrix update (the start of the beam line or a (UPDATE R1 ;) element) or alignment marker marks the beginning of the section to be misaligned. The misalignment element itself marks the end. The section is treated as a unit and misaligned as a whole. The misalignments of quadrupole triplets and other combinations involving two or more quadrupoles may be studied using this keyword.

R2   The last R2 matrix update (see the UPDATE element for a list of elements which update R2) or alignment marker marks the beginning of the misaligned section. The misalignment element marks the end. This option makes use of the fact that R2 matrix updates do not affect the R1 matrix.
A bending magnet with fringing fields or pole face rotations (ROTAT element) should be misaligned using this option. See examples 1 and 2 below for an illustration of this. An array of quadrupoles provides another example of the use of this option. By successive application of align elements, the elements of a quadrupole triplet could be misaligned relative to each other, and then the triplet as a whole could be misaligned. See example 3 below for an illustration of this.

label   The label on an individual magnetic element may be used as a keyword. This usage indicates that the misalignment applies to that magnetic element. Misalignment specifications of this type are usually grouped together after the beam line specification.

MAGNETS   All subsequent bending magnets and quadrupoles are independently misaligned by the amount specified. This option is useful in conjunction with the tabular display of the misalignment results (see below). A ALIGN element with this keyword should be placed near the beginning of the data, before any magnets to which it applies. A bending magent, with fringing fields included, is treated as a single unit and misaligned accordingly.

BENDS   All subsequent bending magnets, including fringing fields, are independently misaligned by the amount specified. See the description for MAGNETS above for further comments.

QUADS   All subsequent quadrupoles are independently misaligned by the amount spec-
                 ified. See example 4 below for an illustration of this. See MAGNETS above for
                 further comments.

     Any combination of one keyword from each category (A, B, or C) may be used to define
the exact circumstances intended. However, only some of the combinations are useful. In the
above descriptions and in the following examples, we have tried to outline the more useful
combinations. A brief summary follows.

     The keywords KNOWN, BEAM, and SINGLE taken together indicate the effect on the beam
of a known shift in the position of a single magnet. The last two keywords mentioned
are the default, so the results would be the same if either of these keywords is omitted.
Their explicit presence simply makes the ALIGN element easier to read. The keyword BEAM is
typically included. The keyword SINGLE is typically omitted as it is a rather obvious default.

     D. A fourth set of keywords indicates the location of the origin of the coordinate system
to which the misalignment is referenced. The origin is the point which is unmoved when
any of the purely rotational misalignments are imposed. The default orientation for the
misalignment reference coordinate system depends on the location of its origin.

Keyword   Interpretation

ENTRANCE   The origin of the misalignment coordinate system is at the point where the
                 reference trajectory crosses the entrance face of the magnet. This option is
                 the default and need not be specified. For an element with a straight reference
                 trajectory, the default orientation for the misalignment coordinate system in
                 this case is that it coincide with the beam coordinate system. In other words,
                 the x axis is transverse and in the bend plane. The y axis is transverse and
                 perpendicular to the bend plane. The z axis is along the reference trajectory
                 at the entry point to the magnet.
                 For a bending magnet the position of the origin is the same, but the z axis
                 of the misalignment coordinate system points along the chord of the magnet.
                 The x axis is in the bend plane, perpendicular to the z axis. The y axis is
                 perpendicular to the bend plane and is the same as the y axis of the beam
                 coordinates.

MIDPOINT   The longitudinal midpoint of the magnet serves as the origin of the misalign-
                 ment reference coordinate system. The longitudinal midpoint is well defined if
                 the reference trajectory in the magnet is straight, as is the case for quadrupoles,
                 sextupoles, octupoles, and solenoids. If the magnet is curved, such as in a
                 BEND, RBEND, or SBEND the default position of the longitudinal midpoint is the
                 midpoint of the chord of the magnet.

EXIT   The point where the reference trajectory crosses the exit face of the magnet
                serves as the origin of the misalignment reference coordinate system. The de-
                fault orientation of the misalignment reference trajectory is such that the z
                axis points along a continuation of the chord of the magnet. The x axis is per-
                pendicular to the z axis and lies in the bend plane. The y axis is perpendicular
                to the bend plane and is the same as the y axis for the beam coordinates.

E. The fifth set of keywords indicates the orientation of the axes in the misalignment
reference coordinate system, if they are other than the default for the choice of origin.

Keyword	Interpretation

CHORD   For a magnet which changes the direction of the reference trajectory (BEND,
                RBEND, SBEND), the z axis or the misalignment reference coordinate system is
                along the chord of the magnet. The x axis is in the bend plane, perpendicular
                to the z axis. The y axis is perpendicular to the bend plane, as before. If
                the keyword MIDPOINT is used to indicate the location of the origin of the
                alignment coordinate system, then that origin is taken as being the midpoint
                of the chord.

TRAJECTORY   Here the misalignment coordinate system always always has the same orien-
                tation as the beam coordinate system. If the origin is specified as being at
                the entrance or exit face of the magnet, then the origin also coincides with the
                origin of the coordinate system for the beam. If the origin is specified as being
                at the MIDPOINT of the magnet, then it is at the intersection of the projected
                entrance and exit reference trajectories.

F. The sixth set of keywords allows the user to explore the effect of correction elements
to repair the damage to the beam resulting from the misalignments.

Keyword	Interpretation

FOCUS   The beam centroid is restored to the reference trajectory after each element.
                Only the focusing effect of the misalignment will be considered.

COMPLETE   The effect of this keyword is the default, and therefore it need never be included
                in the data. It does not insert hypothetical correction elements into the beam
                line. It does not limit the effect of the misalignment to the focusing effects,
                but instead includes the centroid displacement.

Examples

Example No. 1:   <u>A Bending Magnet with a Known Misalignment</u>

A bending magnet (including fringe fields) misaligned by a known amount might be represented as follows:

```
DRIFT,  L = 10.0  ;
UPDATE,  R2  ;
ROTATE,  ANGLE = 0.0  ;
BEND  L = 5.0,  B = 10.0,  N = 0.5  ;
ROTATE,  ANGLE = 0.0  ;
ALIGN,  RZ = 2.0,  KNOWN,  R2  ;
DRIFT,  L = 10.0  ;
```

This represents a known rotation of the bending magnet about the incoming beam direction (z axis) by 2.0 mr. The result of this misalignment will be a definite shift in the beam centroid, and a mixing of the horizontal and vertical coordinates. The use of the (`UPDATE R2 ;`) transform 2 update and the keywords `KNOWN` and `R2` is necessary because the magnetic array (bending magnet + fringing fields) consists of three elements instead of one.

The complete magnet including both central field and fringing fields can be represented by a single element using the `SBEND` element. This single element may then be misaligned using the keyword `KNOWN`. The `UPDATE` is now unnecessary. The above example would then look like:

```
DRIFT,  L = 10.0  ;
SBEND,  L = 5.0,  B = 10.0,  N = 0.5  ;
ALIGN,  RZ = 2.0,  KNOWN  ;
DRIFT,  L = 10.0  ;
```

An `RBEND` would be a rectangular bending magnet, but would be treated in the same manner.

Example No. 2:   A Randomly Aligned Quadrupole Triplet

The net effect of random errors in the placement of quadrupoles in a triplet may also be induced by a single misalignment element as follows:

```
ALIGN,  -   -   -   -   -   -   RANDOM, QUADS  ;
QUAD,  L = 1.,  B = -8.,  APER = 10.   ;
QUAD,  L = 2.,  B = 7.,  APER = 10.   ;
QUAD,  L = 1.,  B = -8.,  APER = 10.   ;
```

The quadrupoles in the triplet will be given displacements and rotations chosen randomly from the ranges specified on the **ALIGN** element. The qudrupoles will be misaligned independently of each other. The results of the misalignment can affect the beam centroid and phase space dimensions at any downstream point.

Different configurations can be created by changing the starting point of the random number generation. (See page 48.) The randomly aligned configurations can then be used to investigate different correction schemes for the effect of the misalignments.

Example No. 3:   Quadrupoles in a Triplet Individually Misaligned by Name

The quadrupoles in a triplet may be individually misaligned as follows;

```
Q1:   QUAD,  L = 1.,  B = -8.,  APER = 10.   ;
Q2:   QUAD,  L = 2.,  B = 7.,   APER = 10.   ;
Q3:   QUAD,  L = 1.,  B = -8.,  APER = 10.   ;
ALIGN, Q1,  -  -  -  -  -  -  KNOWN  ;
ALIGN, Q2,  -  -  -  -  -  -  KNOWN  ;
ALIGN, Q3,  -  -  -  -  -  -  KNOWN  ;
```

Here the label of the magnetic element to be misaligned is used as a keyword in the ALIGN element. The ALIGN elements may now all be grouped near the end of the data after the beam line specification. This arrangement makes it much easier to do a variety of misalignment studies on an otherwise fixed beam line.

Assembling Beam Lines

In TRANSPORT and TURTLE beam lines may be assembled from individual elements in two different ways.

1. The elements are all defined first, then assembled into beam lines and sublines by explicit instructions. This procedure is the one used in the MAD program.

2. The elements are listed in the order in which they occur in the beam line. An element is then defined as it is placed in the sequence of elements which make up the beam line.

Assembling Beam Lines with MAD Notation

A particle accelerator typically has a lattice structure which is highly repetitive. The MAD program was designed with particle accelerators in mind. Any of the elements is likely to be used many times.

The MAD method of assembling beam lines takes this repetitive structure into account. The elements are all defined first, then assembled into beam lines and sublines by explicit statements. Sublines can be treated as elements in assembling larger sublines or complete lines. The magnetic system over which the calculation is done is determined by the USE statement. It is this statement which signals the program that the MAD method of assembling beam lines is being used. If it is not present when the MAD procedure of assembling lines is being used, the user should not expect to get any meaningful results.

Simple Beam Lines

Using MAD notation, a simple beam line to be studied may be defined as a sequence of elements. As an example we define a simple beam line and give it the label 'SIMP'. It is composed from previously defined drifts `D1` and `D2`, quadrupoles `Q1` and `Q2`, and bending magnet `B1`.

```
SIMP:  LINE = (D1,Q1,D2,Q2,D2,B1,D2,B1,D2,B1,D2,Q2,D2,Q1,D1)
```

The alternative notation, used in earlier versions of MAD, will be equivalent

```
LINE  SIMP = (D1,Q1,D2,Q2,D2,B1,D2,B1,D2,B1,D2,Q2,D2,Q1,D1)
```

Sub-Lines

Lines themselves may serve as elements of other lines. Such a sub-line must be defined in a separate statement. Thus one can easily include a subline several times in a given beam line. Defining two such sublines, the above example may be written as

```
DOUB:  LINE = (D1,Q1,D2,Q2)
BPD:  LINE = (B1,D2)
SIMP:  LINE = (DOUB,D2,BPD,BPD,BPD,Q2,D2,Q1,D1)
```

Repetition and Reflection

A repeated element can be indicated by preceding it with an integer followed by an asterisk Thus our line SIMP becomes

```
SIMP:  LINE = (DOUB,D2,3*BPD,Q2,D2,Q1,D1)
```

Several consecutive elements may also be enclosed in parentheses and repeated. We could forgo the definition of BPD and write our beamline as

```
SIMP:  LINE = (DOUB,D2,3*(B1,D2),Q2,D2,Q1,D1)
```

A subline which is to be reflected (reverse order of elements) is indicated by preceding it with a minus sign. Thus our beam line SIMP may finally be shortened to

```
SIMP:  LINE = (DOUB,D2,3*(BPD),-DOUB)
```

In the TRANSPORT (and TURTLE) implementation of MAD notation, a repetition indication may not be preceded with a negative sign. Nothing is lost by this restriction since a reflected repeated sub-line is the same as a repeated subline with its elements reversed and in reverse order.

Replaceable Arguments

A beam line definition may include formal arguments. Our sub-line DOUB, for example, may not have Q2 included explicitly, but have a formal argument in its place. It is then defined as

```
DOUB:  LINE(X) = (D1,Q1,D2,X)              or

DOUB(X):  LINE = (D1,Q1,D2,X)
```

The formal arguments are included in the sequence of elements as if they were labels representing physical elements or sub-lines. They must also be included in parentheses before the equal sign.

When the sub-line DOUB is used, the formal argument must be replaced by the actual argument in parentheses following the label DOUB. Thus the line SIMP becomes

        SIMP: LINE = (DOUB(Q2),D2,3*(BPD),-DOUB(Q2))

Formal arguments may be single elements or sub-lines. When a subline has a single formal argument, it must be used with a single actual argument. When the subline has two formal arguments, it must be used with two actual arguments. In all cases, the number of formal arguments must be the same as the number of actual arguments.

The section DOUB may be defined with two formal arguments:

        DOUB(X,Y):   LINE = (D1,X,D2,Y)

The line SIMP would then be:

        SIMP: LINE = (DOUB(Q1,Q2),D2,3*(BPD),-DOUB(Q1,Q2))

USE Statement

The USE statement specifies which of the defined beam lines and sublines is to be used in the calculation. In our example, the most obvious candidate is the complete assembled beam line SIMP. The USE command would then read

        USE   SIMP ;

If, however, we wished to investigate the properties of the entrance doublet without creating a new data deck, then we could simply use the command

        USE   DOUB ;

TURTLE would then run through only the beamline named DOUB.

It is the USE statement which differentiates between the MAD procedure of defining beam lines explicitly and the original TRANSPORT procedure of just listing the elements in order. If no USE statement is included in the data, TRANSPORT or TURTLE considers the beam line to consist of the elements in the order listed.

Assembling Beam Lines by Listing the Elements

When the beam line is defined by simply listing the elements in order, the individual elements may be defined in any mannner described in this manual. Any complete set of parameters may be used to specify a given element. It is recommended for clarity that a user not mix types of parameters more than is necessary. For example, it would be awkward to have the quadrupole component of the field of some bending magnets specified by the `N` parameter, while that of others are given by `K1`.

The two commands `REPEAT` and `SECTION` are used only when a beam line is assembled by simply listing the elements. Ways to represent the same functions when MAD notation is used are described earlier in this section.

Repetition

Many systems include a set of elements that are repeated several times. To minimize the chore of input preparation, a 'repeat' facility is provided. A typical repeat element might look like:

```
REPEAT,   N = 5   ;
```

The only parameter is `N`. If `N` is nonzero, it states the number of repetitions desired from the point where it appears. If `N` is zero it marks the end of a repeating unit.

For example, a total bend of 12 degrees composed of four 3-degree bending magnets each separated by 0.5 metres could be represented by

```
REPEAT,   N = 4.    ;
BEND,   - - -  ;
DRIFT,   L = .5  ;
REPEAT,   N = 0.    ;
```

Those elements (in this case a bend and drift) between the (`REPEAT,   N = 4.    ;  `) and (`REPEAT,   N = 0.  ;`) would be used four times.

Repeat elements may be nested up to four deep. By "nesting" we mean a repeat within a repeat. An example is given below.

Example of Nesting

```
REPEAT,  N = 2.    ;
DRIFT,  L = 10.    ;
REPEAT,  N = 3.    ;
DRIFT,  L = 20.    ;
REPEAT,  N = 4.    ;
DRIFT,  L = 50.    ;
REPEAT,  N = 0.    ;
REPEAT,  N = 0.    ;
DRIFT,  L = 1.5    ;
REPEAT,  N = 0.    ;
```

Inner
Block

Next
Inner
Block

Outer
Block
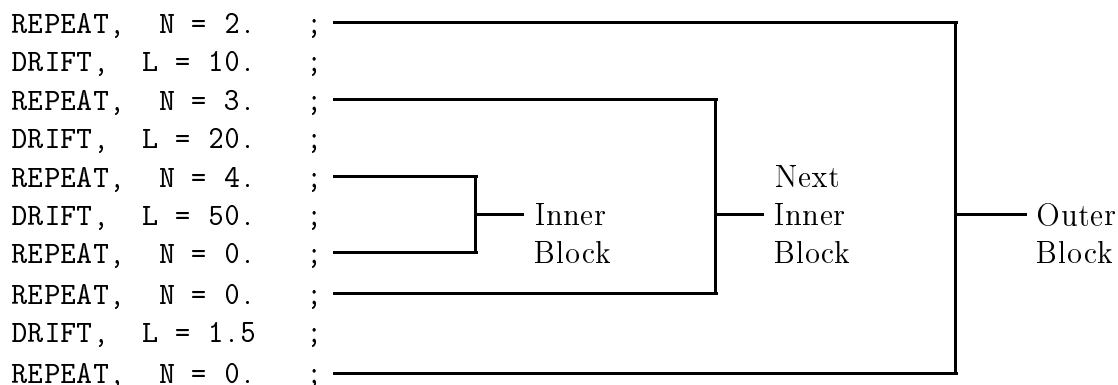
The total length of this sequence is:

$$2 * (10. + 3 * (20 + 4 * 50) + 1.5) = 1343 \quad .$$

Defined Section

A system may contain a section which is repeated at some later point. The repeated section may not occur immediately after the original section, so that the REPEAT element in not appropriate. It may also be useful to repeat a section, but with the elements listed in reverse order.

The keywords which can be used in defining a section of beam line are BEGIN, END, FORWARD, and BACKWARD. If a section of a beam line is to be identified to be used again at some later point, it is preceded by the command

```
        FRED:   SECTION, BEGIN   ;
```

and followed by the command

```
        FRED:   SECTION, END   ;
```

The label, in this case FRED:, identifies the section for later reference. If it is desired to repeat the same section at some later point in the beam line, one uses the command

```
        FRED:   SECTION, FORWARD   ;
```

If the section is to be repeated but in reverse order, use the command

```
        FRED: SECTION, BACKWARD   ;
```

A number of rules apply to the use of the SECTION element. Each defined section must have both its beginning and its end indicated, and the beginning must precede the end. The definition of a section must completely precede its use. A given name can be used only once to define a section, although it can be used many times to indicate a repeat of a section. The number of defined sections is limited to ten. The total z rotation (SROT element) within a defined section must equal zero. Finally, defined sections must nest properly with the REPEAT element. Defined sections, however, need not nest properly with each other.

Example of a Defined Section

```
DOUB:   SECTION,   BEGIN   ; ─────────────── Begin defined section
QUAD,   L = 10.,   B = 5.,   APER = 2.    ;
DRIFT,  L = 8.    ;
QUAD,   L = 10.,   B = -5.,   APER = 2.    ;
DOUB:   SECTION,   END,   ; ─────────────── End defined section
          ⋮
DOUB:   SECTION,   FORWARD   ; ─────────────── Repeat defined section
          ⋮
DOUB:   SECTION,   BACKWARD,   ; ───────────── Repeat defined section in reverse order
```

The difference between the REPEAT command and the SECTION command is that the SECTION command does not require that the repetitions immediately follow the original. They may be separated by other portions of beam line. Thus we have that the sequence

```
        REPEAT,   N = 5.    ;
        (section of beam line)
        REPEAT,   N = 0.    ;
```

is identical to the sequence

```
        SAM:   SECTION,   BEGIN   ;
        (section of beam line)
        SAM:   SECTION,   END   ;
        SAM:   SECTION   FORWARD   ;
        SAM:   SECTION   FORWARD   ;
        SAM:   SECTION   FORWARD   ;
        SAM:   SECTION   FORWARD   ;
```

However, the example

```
SAM:   SECTION,   BEGIN   ;
(section of beam line)
SAM:   SECTION,   END   ;
(other elements)
SAM:   SECTION,   FORWARD   ;
```

cannot be done with the REPEAT element alone.

Operations

TRANSFORM 1 UPDATE [*]

In TRANSPORT, the `UPDATE` element is used to reinitialize the accumulated transfer matrix R1. TURTLE does not accumulate matrices, so the TRANSPORT usage of the `UPDATE` element does not occur. In TURTLE, the `UPDATE` element is occasionally used as a marker to specify a section of the beam line to be misaligned.

The `UPDATE` element represents an operation and is not MAD compatible. It is expressed in keyword notation.

Notation

To impose an R1 update, use the element.

        UPDATE, R1 ;

If the beam line is described simply by listing the elements in the order in which they occur physically, then the `UPDATE` instruction is placed at the location where the update is to be made.

If the beam line is assembled from elements and specified in a `LINE` statement, the `UPDATE` command may be placed after the beam line specification. It may refer to a marker which is included in the beam line specification. This is done with the `LOCA(TION)` keyword. Assume the marker

        MAR1:   MARKER ;

is included in the beam line description. Assume further that the user wishes to update the transfer matrix at the location of this marker. The command

        UPDATE, R1, LOCA = MAR1 ;

will perform the desired update operation.

---

[*]By "updating" we mean initiating a new starting point for the accumulation (multiplication) of the R1 matrix. At the point of update the previous accumulation is discontinued. When the next element possessing a transfer matrix is encountered, the accumulated transfer matrix R1 is set equal to the individual transfer matrix R for that element. Accumulation is resumed thereafter.

AUXILIARY TRANSFORMATION MATRIX (R2)

In TURTLE, the R2 update element is used only to mark the beginning of a section to be misaligned.

Notation

To update the auxilliary transfer matrix R2, one uses the element

```
UPDATE,   R2 ;
```

ALMARK – Alignment Marker

The beginnings of sections to be misaligned may also be indicated by the alignment marker element.

Notation for the alignment marker is exactly the same as for the `UPDATE` element. The keyword `ALMARK` simply replaces the keyword `UPDATE`. The keyword (`R1` or `R2`) on the `ALIGN` element will pertain to the most recent `ALMARK` element if it is more recent than the most recent `UPDATE` element of the same type.

Notation

The keyword `R1` on the misalignment element refers to an update of the R1 matrix. It can also refer to an `ALMARK` element if one occurs more recently than any R1 update. The `ALMARK` element should then read:

```
ALMARK,   R1   ;
```

If the beam line is assembled from elements and specified in a `LINE` statement, the `ALMARK` command may be placed after the beam line specification. It may refer to a marker which is included in the beam line specification. This is done with the `LOCA(TION)` keyword. Assume the marker

```
MAR1:   MARKER   ;
```

is included in the beam line description. Assume further that the user wishes to begin a misaligned section at the location of this marker. The command

```
ALMARK,   R1,   LOCA = MAR1   ;
```

will demarcate the location as a possible reference for subsequent `ALIGN` elements.

The keyword `R2` on a misalignment element refers to an update of the R2 matrix. Similarly, the command:

```
ALMARK,   R2 ;
```

can substitute for an update of the auxiliary R2 matrix.

Example

A bending magnet (including fringe fields) misaligned by a known amount was shown as
an example on page 145. Since the misalignment in that case was known, the example might
now be represented as follows:

```
DRIFT,   -  ;
ALMARK,   R2  ;
ROTATE,   ANGLE = 0.    ;
BEND,   -   -   -  ;
ROTATE,   ANGLE = 0.    ;
ALIGN,   RZ = 2.,   KNOWN,   R2  ;
DRIFT,   -  ;
```

This represents a known rotation of the bending magnet about the incoming beam di-
rection (z axis) by 2.0 mr. The result of this misalignment will be a definite shift in the
beam centroid, and a mixing of the horizontal and vertical coordinates. The use of the
(`ALMARK,  R2 ;`) alignment marker and the keyword `R2` is necessary because the magnetic
array (bending magnet + fringing fields) consists of three elements instead of one.

STORE – Storage of Matrix Elements

Any trajectory coordinate or algebraic combination of trajectory coordinates can be stored for use in a subsequent histogram. The stored quantity is given its own proper name on the STORE command. The quantity is referred to by this proper name at any later point. For example, the initial value of the x coordinate X may be stored under the name X0.

```
STORE,  X0 = X  ;
```

The value of X0 can be used in a later histogram by use of the proper name. An element indicating that the variable X0 is to serve as the horizontal coordinate of a two-dimensional histogram, would appear as:

```
AHIST,  NAME = X0,  LOWER = -1.0,  UPPER = 1.0  ,  STEP = 0.001 ;
```

The current value of x could be displayed as the vertical coordinate by use of the additional element:

```
DHIST,  X,  LOWER = -1.0,  UPPER = 1.0  ,  STEP = 0.001 ;
```

If the beam line is assembled from elements and specified in a LINE statement, the STORE instruction may be placed after the beam line specification. It may refer to a marker which is included in the beam line specification. This is done with the LOCA(TION) keyword. Assume the marker

```
MAR1:   MARKER  ;
```

is included in the beam line description. Assume further that the user wishes to store a value for later histogram entry at the location of this marker. The command

```
STORE,  XN = X,  LOCA = MAR1  ;
```

will store the horizontal coordinate x under the name XN at the location MAR1.

The STORE element should not be used in DECAY TURTLE as there is no provision for identification of the particle type to which the kinematic variables (x, x', etc.) apply.

HIST, AHIST, and DHIST – Histograms

A one- or two-dimensional histogram of beam parameters (listed below) may be requested at any position in the beam line. A one-dimensional histogram is initiated by a `HIST` element. The horizontally displayed coordinate of a two-dimensional histogram is indicated by an `AHIST` element. The vertically displayed coordinate is indicated by a `DHIST` element. The histogram element is placed at the point of the beam line where one wishes to histogram the trajectory coordinate specified.

The keywords used to specify any of these three element types are the same. The name of the coordinate (or algebraic combination of coordinates) to be histogrammed along with three real attributes are required.

```
label:  HIST,  coordinate,  LOWER  =  value,  UPPER  =  value,  STEP  =  value
;
```

A typical one-dimensional histogram specification might be:

```
H1:  HIST,  X,  LOWER = -10.0, UPPER = 10.0, STEP = 0.5 ;
```

A combination of `AHIST` and `DHIST` elements to produce a two-dimensional histogram might be:

```
HX: AHIST,  X,  LOWER = -10.0, UPPER = 10.0, STEP = 0.5 ;
HY: DHIST,  Y,  LOWER = -10.0, UPPER = 10.0, STEP = 0.5 ;
```

The keywords listed below can be used in defining the elements `HIST`, `AHIST`, and `DHIST`. The first few (ending with `NCOOR`) determine the coordinate to be histogrammed. The keyword `NCOOR` is the only coordinate specification requiring a value. For all the others, the name alone is sufficient. The particle name is used only in DECAY TURTLE, where there is a primary particle and two secondaries. The flag indicates a condition for entry in the histogram. The final three attributes are used in determining the structure of the histogram.

Symbol	Keyword	Description
x	X	The x coordinate
x'	XP	The x' coordinate
y	Y	The y coordinate
y'	YP	The y' coordinate
ℓ	DL	The longitudinal separation ℓ.
δ	DEL	The relative momentum difference δ.
z	L	The distance along the reference orbit. Such a histogram will show the <u>distribution</u> of particles which have been stopped by aperture limits.

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p	P	The momentum of the particle.
$\cos\theta$	COST	The cosine of the center-of-mass decay angle of the parent particle (DECAY TURTLE only)
	LD	The distance of the decay point along the reference orbit (DECAY TURTLE only).
r	R	The radius $\sqrt{x^2 + y^2}$
ξ	XI	The emittance angle $\sqrt{x'^2 + y'^2}$, the angle the ray makes with the central trajectory.
ψ	PSI	The polar angle $\arctan(y/x)$.
ζ	ZETA	The polar angle $\arctan(y'/x')$ of the velocity vector.
	variable name	Name defined on a previous STORE element.
	NAME	Indication of an algebraic combination of trajectory coordinates to be histogrammed (See below).
	NCOOR	Histogram coordinate number for user-written subroutine.
	particle name	Particle identification mnemonic (DECAY TURTLE only).
	FLAG	The name of the flag (if there is one).
	LOWER	The lower limit of the histogram
	UPPER	The upper limit of the histogram
	STEP	The bin size of the histogram

For DECAY TURTLE, it is necessary to specify the particle type for the histogram. If no keyword is place in the histogram element, then the default is the parent particle. The default names of the parent particle and two decay products are PARENT, DAUGHTER1, and DAUGHTER2. Other names may be specified by the SPEC element. A popular set of choices is PION, MUON, and NEUTRINO. A one-dimensional histogram for the x distribution of muons might be expressed as:

```
H1:  HIST,  X,  MUON,  LOWER = -10.0, UPPER = 10.0, STEP = 0.5 ;
```

A histogram may be flagged. This means that an entry will not be made in the histogram unless the trajectory reaches the location of the flag. For DECAY TURTLE, the particle type of the trajectory must match that of the flag. For more information, see the section on the FLAG element, following the sections on the histogram and mean value plot.

If either of the coordinates specified for a two-dimensional histogram is the distance z along the reference trajectory, the histogram will refer only to particles lost from the beam. Such a loss histogram cannot be flagged. A maximum of ten such histograms is allowed. A one-dimensional histogram will be created at any point in a beam line where a HIST element is inserted. The bins of the histogram are arranged vertically and the numerical range is given on the left side of the page. The beam intensity in a given interval is indicated by a horizontal row of the letter X.

A two-dimensional histogram is actually initiated by the DHIST element, while the AHIST element merely supplies a value for the horizontally displayed coordinate. Thus the AHIST

element and the `DHIST` element for a two-dimensional histogram need not occur at the same location in the beam line. However, for each `DHIST` element, there must be a `AHIST` element somewhere in the data preceding it. Several `DHIST` elements can refer to the same `AHIST` element to supply the horizontally displayed coordinate for the histogram. The `AHIST` element must simply occur at some point in the data earlier than the first `DHIST` element. Whenever a `DHIST` element appears, indicating a vertically displayed coordinate and initiating a histogram, the horizontally displayed coordinate is taken from the `AHIST` element most recently preceding it.

For both one- and two-dimensional histograms, the coordinates histogrammed with their units and position in the beam line are printed with the histogram. Examples of both one-and two-dimensional histograms are given on page 28 in the section on output.

For a two-dimensional histogram, the number of rays falling in a given bin is printed directly for 1-9 rays, represented by a letter `A-Z` for 10-35 rays, and represented by a $ for more than 35 rays. The $ was chosen because it is a fairly dark symbol. Sums of the rows and columns are given along the edges of the histogram. The numbers giving thesums of columns are themselves to be read vertically. Overflow in all four directions is given below the histogram.

A total of 100 histograms in a given beam line is allowed. The number of intervals allowed per histogram is limited directly only for the horizontal coordinate of a two-dimensional histogram, the limit being 100. If the limits and interval size specified are such that more than 100 intervals would result, TURTLE readjusts the upper limit so that the number of intervals equals 100. There is a limit of 10,000 total locations provided for histogram storage. If N is the number of intervals used in a one-dimensional histogram, the number of storage locations used is N+3. If NA and ND are the number of intervals used for the horizontal and vartical coordinates respectively of a two-dimensional histogram, the number of locations used is (NA+1)*(ND+1) + 5.

Algebraic Expressions

Any algebraic combination of trajectory coordinates may also be entered into a histogram. The algebraic combination is written after the keyword `NAME`. It is written just as a FORTRAN statement. The variables in the algebraic expression are any that might be histogrammed individually. If, for example, one wanted to histogram x^2, then an appropriate histogram request might be

```
        HIST,  NAME = X*X,  LOWER = 0.0,  UPPER = 20.,  STEP =  0.5  ;
```

A number of FORTRAN supplied intrinsic functions may also be used in an algebraic expression. The intrinsic functions now available are

`SQRT`   Square root

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```
ALOG        Natural logarithm (written Ln in engineering notation).
EXP         Exponential function
SIN         Sine function
COS         Cosine function
SINH        Hyperbolic sine function
COSH        Hyperbolic cosine function
ASIN        Inverse sine function
ACOS        Inverse cosine function
```

Additional functions can easily be added if there is a need for any of them.

User-Written Subroutine for Histogram Variables

Any variable which is any function of the trajectory coordinates may also be histogrammed. The user can write a subroutine with two arguments

```
SUBROUTINE HVAL(NCOO,VAL)
```

replacing the dummy version in the TURTLE program. This subroutine will be called at every place in the beam line where a histogram element has an integer associated with the keyword `NCOOR` and that integer is 10 or greater.

The integer argument NCOO is the coordinate number for the histogram. The user returns the value to be histogrammed in the floating point argument `VAL`.

The trajectory coordinates are contained in the common block

```
COMMON /RAY/ RAY(6)
```

From them, any variable to be histogrammed may be calculated. Branching within the user-written subroutine to obtain several alternate possibilities may be done with the argument `NCOO`.

User-computed histogrammed variables may be given user-selected names for identification in histogram captions. To name the variables, a user writes a subroutine

```
SUBROUTINE NAMIT(NUN,NAM,NAMM)
```

The integer `NUN` is the coordinate number of the histogram. The variables to be returned are `NAM` and `NAMM`. They represent literals (printed in A4) giving respectively the name of the variable and the name of the units for the coordinate numbered `NUN`. The names of the coordinates and units for coordinate numbers 1 through 6 are contained in the common block

```
COMMON /ELM15D/ NOM(13), NOM1(4)
```

The array `NOM` contains the names of the standard coordinates. The array `NOM1` contains the names `R`, `XI`, `PSI`, and `ZETA`.

PROFILE – Mean Value Plot

One creates a mean value plot by binning according to one kinematic variable and then calculating, within each bin, the average value of another kinematic variable. For example, if one wanted to plot the x value of the effective beam center as a function of relative momentum deviation δ, one might use the following sequence of two elements.

```
HP: AHIST,  DEL,  LOWER  =  -10.0,  UPPER  =  10.0,  STEP  =  0.5 ;
HX: PROFILE,  X,  LOWER  =  -10.0,  UPPER  =  10.0,  STEP  =  0.5 ;
```

The AHIST command first determines the bin in which the first kinematic variable δ falls. The PROFILE command then calculates the average value of x in the bin given by the AHIST command.

The keywords used are the same as for the two-dimensional histogram. Because it is necessary to store, for each bin, the number of trajectories and the mean and standard deviation of the second kinematic variable, the number of bins required is 3*NA + 7. An example of a mean value plot is given on page 30 in the section on output.

FLAG – Histogram Flag

If a flag is indicated in the data for a `HIST`, `AHIST`, or `DHIST` histogram element, then that histogram element is considered flagged. A `FLAG` element with the same proper name as indicated on the histogram element must exist somewhere in the data. Rays will not be entered in the histogram unless they reach the location of the flag. Unflagged histograms are not affected.

For original TURTLE, the two coordinates of a two-dimensional histogram can refer to only one kind of particle. Therefore only one flag can be associated with a two-dimensional histogram and must be specified on the `DHIST` element. Any flag on the `AHIST` element will be ignored.

In DECAY TURTLE, a flag is permitted on both the `AHIST` and `DHIST` elements, and both flags must be satisfied for the histogram to be entered.

The specification of the `FLAG` is similar to that of the `MARKER`. It consists of a label and the element mnemonic. From the histogram specification, the flag is referred to by its label. A flag element with the label "`F1`" would have the appearance.

```
F1:   FLAG  ;
```

In DECAY TURTLE, there may be three particles involved: the primary (parent) particle, and the two secondaries (the first and second daughters). The proper names (such as muon or proton) and charges of any of these particles can be set by the `SPECIAL` element. The default names of the particles are `PARENT`, `DAUGHTER1`, and `DAUGHTER2`. The default charges of the three particles are respectively +1, +1, and 0. Particles to which flags or histograms pertain may be indicated by the particle name.

If no particle is specified on a flag element, then the required particle type for the flag is taken from the histogram specification which refers to the flag. If two histogram specifications use the same flag, but refer to two different particle types, then the required particle for the flag will depend on the flagged histogram. The required particle for a flag to be satisfied can also be specified explicitly on the `FLAG` element. In this case, the particle types for the flagged histogram and the flag itself may be different. For example, one can histogram all pions which subsequently decay and produce muons passing a later point. An explicit muon flag, for example, might be

```
MU1:   FLAG, MUON  ;
```

In addition, any aperture constraint may be required for a flag to be satisfied. The aperture types are the same as for the `SLIT` element. As with the `SLIT` element, a third parameter gives the half width of the aperture. An elliptical aperture is also available as described under the `SLIT` element on page 126. The aperture constraint does not intercept the trajectory, but is required for the flag to be satisfied. Continuing the previous example,

a flag requiring a muon to pass with x less than 0.4 may be written as

```
MU1:    FLAG,   MUON,   X,   HWIDTH = 0.4 ;
```

Example No. 1

Histogram of the phase-space acceptance and emittance of a beam line. There is no particle specification or decay here, so this case can be run through regular TURTLE.

```
BEAM  ....  ;    Initial beam specification

AHIST,  X,  LOWER = -20., UPPER = 20., STEP = 1.0  ;
DHIST,  XP,  LOWER = -10., UPPER = 10., STEP = .5, FLAG = ACC ;
   .
   .
   .
ACC:  FLAG  ;
AHIST,  X,  LOWER = -20.,  UPPER = 20.,  STEP = 1.0  ;
DHIST,  XP,  LOWER = -10.,  UPPER = 10.,  STEP = .5  ;
SENTINEL
```

Example No. 2

Histogram for a typical decay channel. Here there are several types of particles. Even though a DECAY element is not shown explicitly, the presence of one is implied.

```
BEAM  ....  ;    Initial beam specification

AHIST,  DEL,  LOWER = -10.,  UPPER = 10.,  STEP = 0.5,  FLAG = F1  ;
.
.
.
F1:   FLAG  ;
.
.
.
F2:   FLAG  ;
.
.
.
DHIST,  P,  DAUG,  LOWER = 0.,  UPPER = 40.   STEP = 1.,  FLAG = F2  ;
```

The AHIST element at the beginning of this data set indicates that $\delta = \Delta p/p_0$ is to be the horizontal coordinate for the histogram. The histogram is displayed at the location of

the `DHIST` element, near the end of the data.

There is no particle specification on flag `F1` so the particle type is taken from any histogram which refers to the flag `F1`. Since there is also no particle specification on the `AHIST` element, we revert to the default particle type of `PARENT`. Only those parents that have passed through the position of `F1` are used for the histogram.

The flag `F2` is referred to by the `DHIST` element. Since the (neutral) daughter particle (probably a neutrino) is specified on the `DHIST` element, the flag is also understood as pertaining to the neutral daughter particle. Only neutral daughters created before and passing through the location of `F2` and which reach the `DHIST` element are entered into the histogram.

Finally, the `DHIST` element indicates that the momentum p of the neutral daughter is used as the vertical coordinate for the histogram displayed at that point.

Since an entry is made in the histogram only if both particles are present, the result is a histogram containing only those particles which decay between the two flagged positions `F1` and `F2`. No other `AHIST` element should appear between the `AHIST` and `DHIST` entries shown. Otherwise the flagging process will not function as described above.

DECAY – Particle Decay

Both original TURTLE and DECAY TURTLE will track single particles through a magnetic system. DECAY TURTLE will also include two-body decay and can track both of the decay products through the remainder of the system after the decay occurs.

The decay is activated by a `DECAY` element at the beginning of the data set. There are four other numbers which must be specified. A label is superfluous since the `DECAY` element is neither a representation of a physical object nor does it have a predetermined place in a beam line, like an operation. The form of the `DECAY` element would then be:

    `DECAY, MASS` = value, `MASSC` = value, `MASSN` = value, `LIFETIME` = value ;

The specification of the parent particle as a charged π meson and the two decay products as a muon and a neutrino would then be:

    `DECAY, MASS` = 0.1396, `MASSC` = 0.1055, `MASSN` = 0.0, `LIFETIME` = 26.0 ;

Symbol	Keyword	Description
m	MASS	The mass of the parent particle in GeV.
m_c	MASSC	The mass of the first (charged) daughter particle in GeV.
m_n	MASSN	The mass of the second (neutral) daughter particle in GeV.
τ	LIFETIME	The lifetime of the parent particle in nanoseconds.

The particle masses are given in the same units as the design momentum of the beam line , normally GeV (ignoring the factor c). The unit for mass can be changed by means of a

       `UNIT, P, ... ;`

element, preceding the `DECAY` element.

The charges of the primary (parent) and two secondary particles may be respecified via the (`SPEC, PARENT` ;), (`SPEC, DAUGHTER1` ;), and (`SPEC, DAUGHTER2` ;) elements. In the past, the two secondary particles have sometimes been referred to as the "charged daughter" and "neutral daughter." Now it is possible to respecify the "charged daughter" to be neutral, and the "neutral daughter" to be charged. Thus we use the terms "daughter1" and "daughter2" for the two decay products.

The standard version of DECAY TURTLE contains the decay kinematics for isotropic two-body decay. The particle masses must always be specified. At present, no default values are known to the program. A different decay process can be accommodated by changing one or more of the following subroutines:

SUBROUTINE DECAY

generates the two secondary rays from the parent ray:
RAYC for the first daughter particle,
RAYN for the second daughter particle.

SUBROUTINE DECAYP

decodes the `DECAY` element and prepares data for the subroutine DECAY.

SUBROUTINE DECAYZ(ZD)

returns the generated random decay position.

The initial phase space of the parent beam has been described in the section on the `BEAM` element. For decay problems, it is often required to enter a different initial distribution. In such a case, the user has to change the subroutine EMIT which generates a new parent ray at each call.

Users who wish to alter one of the indicated routines are referred to the comments in the main program which describe the significance of all COMMON variables in the program.

SENTINEL

A TURTLE data set must be terminated with the word `SENTINEL`. The word `SENTINEL` need not be on a separate line. For a description of the form of a TURTLE data set see page 13.

Original TRANSPORT Notation

Positional Format

Element Description

In the original TRANSPORT notation, the elements are written in positional notation. The input in the first version of TURTLE was made to be compatable with TRANSPORT. In positional notation, the element specification begins with a numerical type code. The physical parameters follow the numerical type code. They are given by a sequence of numbers, separated by spaces and terminated by a semicolon. The end of a line also serves as a semicolon.

The physical parameters are the quantities which describe the physical element represented. Their meaning depends on the numerical type code and their position in the sequence of numbers. There is no possibility of describing an element in an alternate set of parameters using positional notation. The label, if present, identifies the element. It may contain up to fifteen characters and is enclosed by single quotes.

In positional notation, a sector bending magnet would be specified as:

```
4.   10.   20.   0.5  'BM1'  ;
```

The first integer (4.) is a type code, indicating that this element is a bending magnet. The additional numbers here represent, respectively, the magnet length (10.) in meters, the magnetic field (20.) in kilogauss, and the dimensionless field index n (0.5). The sequence of characters BM1 is the label. A summary, indicating the proper order for the physical parameters is given in Table 3.

For any element the numerical type code is considered to be the first parameter. The second parameter is also known as the first physical parameter. It is the second entry in Table 3. In some cases the parameters of an element do not really refer to physical quantities, but will nevertheless be referred to as such in this manual.

In positional notation the label may be placed anywhere among the numbers. Labels are useful in problems with many elements and/or, in TRANSPORT, when sequential fitting is used.

If the physical parameters of an element are specified in positional notation, then a vary field forms an extension of the type code. The vary field is used by TRANSPORT to indicate which physical parameters of the element are to be adjusted if there is to be any fitting. It is placed immediately (no intervening blanks) to the right of the decimal point which follows the type code.

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```
        4.01  10.    20.    0.5  'BM1'  ;
```

The vary code is explained here since it may be found in a data set which has been run through TRANSPORT. There is no fitting in TURTLE.

Each element must be terminated by a semicolon (;). For a single element per line, the end of a line serves as a semicolon. If there is more than one element per line of data, then each element must be terminated by a semicolon. Thus the **4.** above may be followed by a drift as follows

```
        4.   10.    20.    0.5  'BM1'  ;  3.    10.
```

Here the **4.** element must be followed by a semicolon. The end of the line serves as a semicolon for the drift element.

Spaces before and after the semicolon are allowed but not required. If the program encounters a semicolon before the expected number of parameters has been read in and if the indicator was a zero (0), the remaining parameters are set to zero. Thus, the element

```
        4.   10.    20.     'BM1'  ;
```

is the same as

```
        4.   10.    20.    0.0  'BM1'  ;
```

For a bend, three physical parameters are to be specified in original TRANSPORT notation. If only two physical parameters are specified, a single additional parameter is assumed with the value 0.0.

Type Codes

In original TRANSPORT, the element types are designated by numerical type codes. Not all TRANSPORT elements can be expressed as a sequence of numbers. For those that can, the numerical type code and the meaning are given in the table following.

Type Code	Mnemonic	Element
1.	BEAM	Input phase space dimensions and reference trajectory momentum.
2.	ROTATION	Fringing fields and pole-face rotations for bending magnets
3.	DRIFT	Drift space – a field free region
4.	BEND	Sector bending magnet (without fringing fields)

5.	QUADRUPOLE	Quadrupole magnet
6.	SLIT	Physical aperture
7.	CENTROID	Shift in the beam centroid
8.	ALIGN	Magnet alignment tolerances
9.	REPEAT	Repetition of a segment of a beam line
10.	FIT	Fitting constraint
11.	ACCELERATOR	Travelling wave linear accelerator
12.	CORRELATION	Beam rotated ellipse
13.	PRINT	Output print control instructions
14.	MATRIX	Arbitrary matrix input
15.	UNITS	Input-output units
16.	SPECIAL	Special input parameters
17.	ORDER	Zeroeth, first, second, or third order
18.	SEXTUPOLE	Sextupole magnet
19.	SOLENOID	Solenoid magnet
20.	SROT	Coordinate Rotation
24.	SECTION	Defined section of beam line
25.	OCTUPOLE	Octupole magnet
27.	ETA	Accelerator eta function
28.	RBEND	Rectangular bending magnet (with fringing fields)
29.	SBEND	Sector bending magnet (with fringing fields)
31.	MARKER	Position marker
34.	PLASMALENS	Plasma (lithium) lens
35.	HKICKER	Horizontal steering magnet
36.	VKICKER	Vertical steering magnet
37.	ALMARK	Alignment marker
41.	SEPTUM	Electrostatic septum
43.	SHIFT	A shift in the reference coordinate system
50.	HIST	A one-dimensional histogram.
51.	AHIST	The horizontally displayed coordinate of a two-dimensional histogram
52.	DHIST	The vertically displayed coordinate of a two-dimensional histogram
53.	FLAG	A histogram flag
54.	PROFILE	A mean-value plot
60.	DECAY	Two-body particle decay

The Use of Labels

Labels are available for identification of individual elements. Labels are used in TRANS-PORT to modify data from one step to the next. Labels may also be inserted for the user's convenience. In this case the placement of a label on a given element is optional.

In positional notation the label may be placed anywhere among the parameters of a given element. It must be enclosed in quotes. Blanks within a label are ignored. The maximum

length of a label is fifteen non-blank characters. It may contain letters, numerals, underscore, percent (%), and dollar sign ($).

As an example, the following all denote the same drift space:

```
'DRF' 3.  1.5        ;
3'DRF'15-1;
3.  .15E1 'D R F'  ;
```

On a unit specification 15 element, the label is the unit name. A unit specification indicating the longitudinal length (unit type 8) is to be measured in feet would then be:

```
15.   8.  'FT' ;
```

Table 3: Summary of Elements in Positional Notation

Preliminary Specifications in Original TRANSPORT Notation

Physical Element	Type	2nd Entry	3rd Entry	4th Entry
Apertures	13.	Code		
Units (Transport Dimensions)	15	Code	Unit Symbol	Scale Factor (if required)
Quadratic Term for Bend Field	16.0	1.0	$\epsilon(1) = \beta \left(\dfrac{1}{\rho_0} \right)^2$ ρ_0 in units of transverse length	
Fringe Field Correction Coefficient	16	2.0	κ_0	
Mass of Particles in beam	16	3.0	M/m m = mass of electron	
Half Aperture of Bending Magnet in x Plane	16	4.0	$w/2$	
Half Aperture of Bending Magnet in y plane	16	5.0	$g/2$	
Length of System	16	6.0	L	
Fringe Field Correction Coefficient	16	7.0	κ_1	
Reference Trajectory Momentum	16	11.0	p_0	
Curvature of Entrance Face of Bending Magnet	16.0	12.0	$(1/R_1)$	
Curvature of Exit Face of Bending Magnet	16.	13.0	$(1/R_2)$	
Random Number	16	14.0	Initial value of random number	
Focal Plane Rotation	16	15.0	Angle of focal plane rotation	

Physical Element	Type	2nd Entry	3rd Entry	4th Entry
Reference Momentum for Magnetic Fields	16	21.0	p_{ref}	
Fractional Excess bend	16	22.0	r_s	
Scaling Factor for Skew Field Components	16	23.0	r_a	
Skew Bend Field	16	24.0	v_r	
Skew Quadrupole Component for a Bending Magnet	16	25.0	n'	
Skew Sextupole Component for a Bending Magnet	16	26.0	ϵ'	
Octupole Component for a Bending Magnet	16	27.0	ϵ_3	
Quadrupole radius or horizontal semimajor axis	16	100	a	
Quadrupole hyperbolic aperture	16	101	a	
Quadrupole vertical semimajor axis	16	102	b	
Charge for parent particle	16	200	+1 or 0 or -1	
Charge for first daughter particle	16	201	+1 or 0 or -1	
Charge for second daughter particle	16	202	+1 or 0 or -1	
Order of Calculation	17	n_1	n_2	

Beam Description in Original TRANSPORT Notation

Physical Element	Type	2nd Entry	3rd Entry	4th Entry	5th Entry	6th Entry	7th Entry	8th Entry	9th Entry
Beam	1.0	x	x'	y	y'	ℓ	δ	p_0	
rms Addition to Beam Envelope	1.00	Δx	$\Delta x'$	Δy	$\Delta y'$	$\Delta \ell$	$\Delta \delta$	p_0	0
Beam (Rotated Ellipse)	12	The fifteen correlations among the six elements (This entry must be preceded by a 1 element.)							
Beam Centroid Shift	7.	Δx	$\Delta x'$	Δy	$\Delta y'$	$\Delta \ell$	$\Delta \delta$	p_0	

Physical Elements in Original TRANSPORT Notation

Physical Element	Type	2nd Entry	3rd Entry	4th Entry	5th Entry
Drift	3	Length			
Bending Magnet	4	Length	Field	Field gradient (n value)	
Pole Face Rotation	2	Angle of Rotation			
Rectangular Bending Magnet	28	Length	Field	Field gradient (n value)	
Sector Bending Magnet	29	Length	Field	Field gradient (n value)	
Horizontal Steering Magnet	35	Length	Field		
Vertical Steering Magnet	36	Length	Field		
Quadrupole	5	Length	Field	Half aperture	
Sextupole	18	Length	Field	Half aperture	
Octupole	25	Length	Field	Half aperture	
Plasma Lens	34	Length	Field	Half aperture	
Solenoid	19	Length	Field		
Accelerator	11	Length	E (energy gain)	ϕ (phase lag)	Wavelength
Electrostatic Septum	41	Length			
Slit	6	Coordinate	Half width		
Marker	31				

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Other Transformations in Original TRANSPORT Notation

Physical Element	Type	2nd Entry	3rd Entry	4th Entry	5th Entry	6th Entry	7th Entry	8th Entry
Arbitrary Matrix	14	R_{j1}	R_{j2}	R_{j3}	R_{j4}	R_{j5}	R_{j6}	J
		Continuation codes also allow the input of second- and third-order terms.						
Coordinate Rotation	20	Angle of rotation						
Shift	43	Δx	$\Delta x'$	Δy	$\Delta y'$	$\Delta \ell$	$\Delta \delta$	p_0
Alignment Tolerance	8	Δx	$\Delta \theta$	Δy	$\Delta \phi$	Δz	$\Delta \alpha$	Code Number

Assembling Beam Lines in Original TRANSPORT Notation

Physical Element	Type	2nd Entry
Repeat control	9.	Number of repetitions
Defined Section	24.	Code

Operations in Original TRANSPORT Notation

Physical Element	Type	2nd Entry	3rd Entry	4th Entry	5th Entry
Transform 1 Update	6.	0.	1.		
Transform 2 Update	6.	0.	2.		
Alignment Marker	37.	0.	1. or 2.		
One-dimensional histogram	50.n	Coordinate	Lower limit	Upper limit	Step size
Horizontal coordinate for two-dimensional histogram	51.n	Coordinate	Lower limit	Upper limit	Step size
Vertical coordinate for two-dimensional histogram	52.n	Coordinate	Lower limit	Upper limit	Step size
Histogram flag	53.n				
Mean value plot	54.n				

Elements with Code Digits

Several of the elements have parameters which are not a measure of any physical quantity, but rather a code indicating the action to be taken by the element. The code digits for such elements are explained in the following pages. The equivalent mnemonic is given for the numerical codes, along with a short explanation. Much more comprehesive explanations are given in the keyword descriptions of the various elements.

RMS Addition to the Beam – Type Code 1

To indicate an RMS addition to the beam, a code digit with value zero is appended to the element . For DECAY TURTLE, this digit also identifies the particle to which the random addition is to be made. The meaning of the code digit is as follows:

id = 0   adds a random addition to the parent particles only
id = 100   adds a random addition to the first (charged) daughter particles only
id = 200   adds a random addition to the second (neutral) daughters only

SLIT or UPDATE– Type Code 6

The second item on the slit element indicates the coordinate which is restricted by the slit. If this coordinate is a zero (0), then type code 6 indicates an `UPDATE` element The list of trajectory coordinates and the integers corresponding to them is.

1	-	x coordinate
2	-	x' coordinate
3	-	y coordinate
4	-	y' coordinate
5	-	ℓ – longitudinal separation
6	-	δ – fractional momentum deviation

In DECAY TURTLE, there is also a particle indentification indicating which particle is affected by the slit. The particle identification is added to the integer indicating the slit coordinate. The particle identification integer is:

id =	0	adds a random addition to the parent particles only
id =	100	adds a random addition to the first (charged) daughter particles only
id =	200	adds a random addition to the second (neutral) daughters only

In other words, it is the same as for the RMS addition to the beam.

Magnet Alignment Tolerances – Type Code 8

The last numerical parameter on the type code 8. element in original TRANSPORT format is a code indicating the nature of the misalignment. It determines whether the misalignment is uncertain or known and displayed in the beam matrix or a table. The code which indicates the beam matrix in TRANSPORT is used to indicate the histograms in TURTLE. It also specifies the element or section of the beam line to be misaligned.

It is much better formulated in keywords. The use of keywords also has allowed the expansion of the number of varieties of misalignment that can be represented. Code numbers that can be understood by TRANSPORT here are given, along with equivalent mnemonics and descriptions. The symbol "X", in a code digit, is not to be taken literally. It simply means that the position occupied by the "X" is irrelevant to the definition being given. It may take on any of the possibilities given when its position is being defined.

Code Digit	Corresponding Mnemonic	Description
0XX	UNCERTAIN	The magnet position is uncertain.
1XX	KNOWN	The magnet or section of beam line is misaligned by a known amount.
2XX.	RANDOM	Random numbers are used to calculate the displacement.
X0X	BEAM	The beam matrix contains the results of the misalignment.
X1X	TABLE	A table is used to store the results of misalignments.
XX0	SINGLE	The single magnet (element) immediately preceding the align element is to be misaligned.
XX1	R1	The last R1 matrix update or alignment marker marks the beginning of the section to be misaligned.
XX2	R2	The last R2 matrix update or alignment marker marks the beginning of the misaligned section.
XX3	MAGNETS	All subsequent bending magnets and quadrupoles are independently misaligned by the amount specified.
XX4	BENDS	All subsequent bending magnets, including fringing fields, are independently misaligned by the amount specified.
XX5	QUADS	All subsequent quadrupoles are independently misaligned by the amount specified.

Repeat – Type Code 9

The second parameter on the `REPEAT` element represents the number of repetitions through a section of beam line, provided the `REPEAT` element is demarcating the beginning of that section. This parameter is zero if the `REPEAT` element is demarcating the end of the section.

An example of the use of the `REPEAT` element might be:

```
REPEAT  4.   ;
BEND  - - -  ;
DRIFT  .5  ;
REPEAT  0.   ;
```

Those elements (in this case a bend and drift) between the (`REPEAT  4.   ;`) and (`REPEAT  0.   ;`) would be used four times.

More information about the repeat element can be found on page 151.

Apertures and Reference Trajectory – Type Code 13

To cause TURTLE to observe magnet apertures, we use the element

```
13.    10.    ;
```

For DECAY TURTLE, a code digit is added to the second number, indicating the particle to which the instruction applies. The meaningful instructions then become:

Aperture observed for parent particle:

```
13.    10.    ;
```

Aperture observed for first (charged) daughter particle

```
13.    110.    ;
```

Aperture observed for second (neutral) daughter particle

```
13.    210.    ;
```

The reference trajectory will be the path of a neutral particle following the instruction:

```
13.    109.    ;
```

It is restored to the path of a particle with unit positive charge by the element:

```
13.    108.    ;
```

All type code 13.0 entries normally used in TRANSPORT and not mentioned above will be ignored by TURTLE.

Matrix – Type Code 14

For the arbitrary matrix with positional input, all the elements up to and including the ones to be specified must be included in the input. This means that not only does each first-order row contain six coefficients, but each second-order row contains 21 coefficients, and each third-order row contains 56 coefficients. To avoid having to allocate space for higher-order matrix elements when only lower-order ones are present, a continuation code is used. There is one continuation code to indicate the presence of second-order terms, and another for third-order terms.

The first-order representation of a single row of a matrix might look like:

```
14.   -.1  .9  0.   0.   0.   0.   2.   ;
```

There are eight parameters. The first is the type code (14), which indicates a matrix element. The second through seventh give the values of the matrix elements. The eighth is the row number.

Second Order

If it is desired to include the second-order matrix coefficients for the ith row, then the following 22 additional numbers may be read in.

          9   continuation code `0`.
10 to 30   The 21 coefficients:

$$T_{i11} \quad T_{i12} \quad T_{i13} \quad T_{i14} \quad T_{i15} \quad T_{i16}$$
$$T_{i22} \quad T_{i23} \quad T_{i24} \quad T_{i25} \quad T_{i26} \quad T_{i33}$$
$$T_{i34} \quad T_{i35} \quad T_{i36} \quad T_{i44} \quad T_{i45} \quad T_{i46}$$
$$T_{i55} \quad T_{i56} \quad T_{i66}$$

in that order, where i is the row number.

Third Order

It is also possible to include the third-order matrix coefficients for the ith row by reading in the following 57 additional numbers.

      31   continuation code `0`.
32 to 87   The 56 coefficients:

$$U_{i111} \quad U_{i112} \quad U_{i113} \quad U_{i114} \quad U_{i115} \quad U_{i116}$$
$$U_{i122} \quad U_{i123} \quad U_{i124} \quad U_{i125} \quad U_{i126} \quad U_{i133}$$
$$U_{i134} \quad U_{i135} \quad U_{i136} \quad U_{i144} \quad U_{i145} \quad U_{i146}$$
$$U_{i155} \quad U_{i156} \quad U_{i166} \quad U_{i222} \quad U_{i223} \quad U_{i224}$$
$$U_{i225} \quad U_{i226} \quad U_{i233} \quad U_{i234} \quad U_{i235} \quad U_{i236}$$

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U_{i244} $\quad U_{i245}$ $\quad U_{i246}$ $\quad U_{i255}$ $\quad U_{i256}$ $\quad U_{i266}$
U_{i333} $\quad U_{i334}$ $\quad U_{i335}$ $\quad U_{i336}$ $\quad U_{i344}$ $\quad U_{i345}$
U_{i346} $\quad U_{i355}$ $\quad U_{i356}$ $\quad U_{i366}$ $\quad U_{i444}$ $\quad U_{i445}$
U_{i446} $\quad U_{i455}$ $\quad U_{i456}$ $\quad U_{i466}$ $\quad U_{i555}$ $\quad U_{i556}$
U_{i566} $\quad U_{i666}$

in that order, where i is the row number.

Units Changes – Type Code 15

To change length to feet, width to inches, and momentum to MeV/c, add to the front of the deck the elements

```
15.    8.    'FT'   0.3048;
15.    1.    'IN'   2.54;
15.    11.   'MEV'  0.001;
```

The name of the new unit is given by the label of the unit element (FT, IN, and MEV). The scale factors (0.3048, 2.54, and 0.001) give the size of the new unit (feet, inches, and MeV) in terms of the reference unit (meters, etc). The three code digits (8, 1, and 11) indicate which unit is to be altered. The code digit 8, for example, indicates the longitudinal distance along the reference trajectory.

Other code digits, their keyword equivalent, and a short description of the meaning of the unit type are given below. Users may not simply replace a code digit by a keyword and expect the unit element to be intelligible to TRANSPORT. An element must be either completely in keyword notation, or completely in positional notation.

The original TRANSPORT units and their meanings are:

Code Digit	Mnemonic	Quantity	Original TRANSPORT Unit
1.0	X	horizontal and vertical transverse dimensions, magnet apertures, and misalignment displacements	cm
2.0	XP	horizontal and vertical angles and misalignment rotation angles	mr
5.0	DL	pulsed beam length and wave length in accelerator	cm
6.0	DP	momentum spread	percent
7.0	ANGLE	bend, pole face rotation, and coordinate layout angles	degrees
8.0	L	length (longitudinal) of elements, layout coordinates, and bending magnet pole face curvatures	meters
9.0	B	magnetic fields	kG
10.0	MASS	mass	electron mass
11.0	P	momentum and energy gain in accelerator section	GeV/c GeV
12.0	PHASE	Betatron phase shift	degrees

| 13.0 | ROLL | Rotations about the reference trajectory | degrees |
| 14.0 | V | Electric fields and voltages | MV |

∗ These codes should not be used if the coordinate rotation (SROT) element is used anywhere in the system.

Special Parameters – Type Code 16

Keywords and Code Digits

The specification of the quadratic dependence of a bending magnet field might be done through a special parameter element. The element would apply to all subsequent bending magnets, until reset, and, in positional notation, would look like

```
16.   1.   .001   ;
```

The code digit (1) would specify which special parameter is being given a new value. The following decimal number (.001) would give that value.

The code digits which can be used in specifying the special input parameters are given below. The equivalent keywords and short descriptions are given also. For more complete descriptions, the user should consult the section on the keyword formulation of Special Input Parameters.

The code digits are used only in original TRANSPORT notation. A warning is that the user cannot obtain a valid element simply by replacing the code digit by the keyword. The element must be written completely either in keyword format or in positional format.

Code Digit	Symbol	Keyword	Description
1.	$\epsilon(1)$	EPS	A second-order measure of magnetic field inhomogeneity in bending magnets.
2.	κ_o	FINT0	An integral used to calculate the reference orbit displacement.
3.	(M/m)	PMASS	Mass of the particles comprising the beam.
4.	$W/2$	HWIDTH	Horizontal half-aperture of bending magnet.
5.	$g/2$	HGAP	Vertical half-aperture of bending magnet.
6.	L	LENGTH	Cumulative length of system.
7.	κ_1	FINT	An integral related to the extent of the fringing field of a bending magnet.
8.	κ_2	FINT2	A second integral related to the extent of the fringing field.
11.	p_0	P0	The momentum of the central trajectory.
12.	$1/R_1$	H1	Where R_1 is the radius of curvature of the entrance face of bending magnets.
13.	$1/R_2$	H2	Where R_2 is the radius of curvature of the exit face of bending magnets.
14.		RANNO	The input value of the random number used for error calculation.
15.	α	FOTILT	The tilt of the focal plane.

21.	p_{ref}	PREF	The reference momentum (normally GeV/c) to which the magnetic fields are normalized.
22.	r_s	RMPS	The fractional excess bend field on a bending magnet.
23.	r_a	RNMS	Overall scaling factor for the midplane-symmetry-violating field components of a bending magnet.
24.	v_r	VR	The vertical (midplane-symmetry-violating) bend field of a bending magnet.
25.	n'	NP	The midplane-symmetry-violating normalized gradient of a bending magnet.
26.	ϵ'	EPSP	The midplane-symmetry-violating sextupole component of a bending magnet.
27.	ϵ_3	EPS3	Cubic term for the field of a bending magnet. The quantity ϵ_3 is equal to γh^3. The units are the inverse cubed of the horizontal beam width unit.
100.	a	QAPA	The radius or horizontal semimajor axis of a quadrupole aperture
101.	a	QAPH	The diagonal distance to the pole tip for a quadrupole hyperbolic aperture.
102.	b	QAPB	The vertical semimajor axis of a quadrupole aperture.
200.		PARENT	The charge of the parent particle.
201.		DAUGHTER1	The charge of the first (charged) daughter particle
202.		DAUGHTER2	The charge of the second (neutral) daughter particle

Defined Section – Type Code 24.

In positional notation, the second parameter of the Defined Section element is a numerical code. It indicates whether the section is being defined or used. The beginning of a section to be identified can be preceded by the element

        24.   1.   'FRED'   ;

and followed by the element

        24.   1.   'FRED'   ;

To repeat the same section later in the beam line, one inserts the element

        24.   3.   'FRED'   ;

To insert the section, but in the reverse direction, use the element

        24.   4.   'FRED'   ;

For more information, see

Histograms – Type Codes 50, 51, 52, and 54

The type codes 50, 51, 52, and 54 correspond respectively to the mnemonics `HIST`, `AHIST`, `DHIST`, and `PROFILE`.

The variables which can be entered in the histogram are specified by the following numerical codes:

1	-	x coordinate
2	-	x' coordinate
3	-	y coordinate
4	-	y' coordinate
5	-	ℓ – longitudinal separation
6	-	δ – fractional momentum deviation
8	-	L, the distance along the reference orbit. Such a histogram will show the distribution of particles which have been stopped by aperture limits.
11	-	p, the momentum of the particle.
12	-	COST, the cosine of the center-of-mass decay angle of the parent particle.
18	-	LD, the distance of the decay point along the reference orbit.
21	-	$r = \sqrt{x^2 + y^2}$, the radius.
22	-	XI $= \sqrt{x'^2 + y'^2}$ the emittance angle, the angle the ray makes with the central trajectory.
23	-	PSI $= \arctan\left(y/x\right)$ - The polar angle.
24	-	ZETA $= \arctan\left(y'/x'\right)$ - The polar angle of the velocity vector

The codes involving decay are used only by DECAY TURTLE. Also for DECAY TURTLE, the codes given above all apply to the parent particles only. In order to specify the first (charged) daughter particles, add 100 to the relevant code. For the second (neutral) daughters, add 200 to the relevant code.

The flag on a histogram specification is given by an integer following the type code. Thus the type codes are 50.n, 51.n, and 52.n for a flagged one-dimensional histogram or either coordinate of a two-dimensional histogram. The flag number is similarly indicated on the 53. element.

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Histogram Flag – Type Code 53.n

The histogram flag is indicated by an integer placed after the decimal point on type code 53. Thus the element

        53.3   ;

indicates flag number 3. It then serves as a flag on any histogram or histogram coordinate which has the integer 3 after the decimal point on the type code.

In DECAY TURTLE, the flag, as described above, pertains to the parent particle. To indicate another particle, as second parameter is needed. If the first (charged) daughter particle is to be flagged, the second parameter is set to 100. If the second (neutral) daughter particle is to be flagged the second parameter is set to 200.

A flag for a muon would then be given by

        53.3   100.   ;

If the flag is to have an aperture condition associated, then the coordinate which is restricted by the aperture is indicated by an integer which is added to the second parameter. Thus a limit of 0.5 cm on the x coordinate of a secondary muon would be written as:

        53.3   101.   0.5 ;

Acknowledgements

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