Basics of inflationary cosmology

George Lazarides

Physics Division, School of Technology, Aristotle University of Thessaloniki, Thessaloniki 54124, Greece

E-mail: `lazaride@eng.auth.gr`

Abstract.   The early stages of the universe evolution are discussed according to the hot big bang model and the grand unified theories. The shortcomings of big bang are summarized and their resolution by inflationary cosmology is sketched. Cosmological inflation, the subsequent oscillation and decay of the inflaton field, and the resulting reheating of the universe are studied in some detail. The density perturbations produced by inflation and the temperature fluctuations of the cosmic microwave background radiation are introduced.   The hybrid inflationary model is described. Two natural variants of the supersymmetric version of this model which avoid the disaster encountered in its standard realization from the overproduction of magnetic monopoles are presented.

1. Introduction

The discovery of the cosmic microwave background radiation (CMBR) together with the observed Hubble expansion of the universe had established hot big bang as a viable model of the universe (for a textbook treatment of this model, see e.g. Ref. [1]). The success of nucleosynthesis (see e.g. Ref. [2]) in reproducing the observed abundance of light elements in the universe and the proof of the black body character of the CMBR then imposed hot big bang as the standard cosmological model. This model combined with the grand unified theories (GUTs) [3] of strong, weak, and electromagnetic interactions provides the scientific framework for discussing the early stages of the universe evolution.

The standard big bang (SBB) cosmological model, despite its great successes, had some long-standing shortcomings. One of them is the so-called horizon problem. The CMBR received now has been emitted from regions of the universe which, according to this model, had never communicated before sending this radiation to us. The question then arises how come the temperature of the black body radiation from these regions is so finely tuned as the measurements of the cosmic background explorer (COBE) [4] and the Wilkinson microwave anisotropy probe (WMAP) [5, 6] satellites show. Another important issue is the so-called flatness problem. It is a fact [6] that the present universe appears to be almost flat. This means that, in its early stages, the universe must have been flat with a great accuracy, which requires an extreme fine tuning of its initial conditions. Also, combined with GUTs which predict the existence of superheavy magnetic monopoles [7], the SBB model leads [8] to a catastrophe caused by the overproduction of these monopoles. Finally, the model has no explanation for the small density perturbations which are required for explaining the structure formation in the universe (for a pedagogical discussion, see e.g. Ref. [9]) and the generation of the observed [4, 5, 6] temperature fluctuations in the CMBR.

It is clear that cosmological inflation [10] offers an elegant solution to all these problems of the SBB model (for a textbook introduction or previous reviews on inflation, see e.g. Refs. [11, 12]). The idea is that, in the early universe, a real scalar field (the inflaton) was displaced from its vacuum value. If the potential energy density of this field happens to be quite flat, the roll-over of the field towards the vacuum can be very slow for a period of time. During this period, the energy density is dominated by the almost constant potential energy density of the inflaton. As a consequence, the universe undergoes a period of quasi-exponential expansion, which can readily solve the horizon and flatness problems by stretching the distance over which causal contact is established and reducing any pre-existing curvature in the universe. It can also adequately dilute the GUT magnetic monopoles. Moreover, it provides us with the primordial density perturbations which are needed for explaining the large scale structure in the universe [9] as well as the temperature fluctuations observed in the CMBR. Inflation can be easily incorporated in GUTs. It can occur during the GUT phase transition at which the GUT gauge symmetry breaks by the vacuum expectation value (VEV) of a Higgs field, which also plays the role of the inflaton.

After the termination of inflation, the inflaton field starts performing oscillations about the vacuum. These oscillations are damped because of the dilution of the field energy density by the cosmological expansion and the decay of the inflaton into light particles. The resulting radiation energy density eventually dominates over the field energy density and the universe returns to a normal big bang type evolution. The temperature at which this occurs is historically called 'reheat' temperature although there is neither supercooling nor reheating of the universe [13] (see also Ref. [14]).

The early realizations of inflation share the following important disadvantage. They require tiny parameters in order to reproduce the COBE or WMAP measurements on the CMBR. In order to solve this naturalness problem, hybrid inflation has been introduced [15]. The basic idea was to use two real scalar fields instead of one that was customarily used. One field may be a gauge non-singlet and provides the 'vacuum' energy density which drives inflation, while the other is the slowly varying field during inflation. This splitting of roles between two fields allows us to reproduce the temperature fluctuations of the CMBR with natural (not too small) values of the relevant parameters. Hybrid inflation, although it was initially introduced in the context of non-supersymmetric GUTs, can be naturally incorporated [16, 17] in supersymmetric (SUSY) GUTs.

It is unfortunate that the magnetic monopole problem reappears in hybrid inflation. The termination of inflation, in this case, is abrupt and is followed by a 'waterfall' regime during which the system falls towards the vacuum manifold and performs damped oscillations about it. If the vacuum manifold happens to be homotopically non-trivial, topological defects will be copiously formed [18] by the Kibble mechanism [19] since the system can end up at any point of this manifold with equal probability. Therefore, a cosmological disaster is encountered in the hybrid inflationary models which are based on a gauge symmetry breaking predicting magnetic monopoles.

One way [18, 20, 21, 22] to solve the magnetic monopole problem of SUSY hybrid inflation is to include into the standard superpotential for hybrid inflation the leading non-renormalizable term. This term cannot be excluded by any symmetries and, if its dimensionless coefficient is of order unity, can be comparable with the trilinear coupling of the standard superpotential (whose coefficient is typically $\sim 10^{-3}$). Actually, we have two options. We can either keep [20] both these terms or remove [18, 21] the trilinear term by imposing a discrete symmetry and keep only the leading non-renormalizable term. The pictures emerging in the two cases are quite different. However, they share an important common feature. The GUT gauge group is spontaneously broken already during inflation and, thus, no topological defects can form at the end of inflation. So, the magnetic monopole problem is solved.

2. The Big Bang Model

We will start with an introduction to the salient features of the SBB model [1] and a summary of the history of the early universe in accordance to GUTs.

2.1. Hubble Expansion

At cosmic times $t \gtrsim t_P \equiv M_P^{-1} \sim 10^{-44}$ sec ($M_P = 1.22 \times 10^{19}$ GeV is the Planck scale) after the big bang, the quantum fluctuations of gravity are suppressed and classical general relativity yields an adequate description of gravity. Strong, weak, and electromagnetic interactions, however, require quantum field theoretic treatment.

We assume that the universe is homogeneous and isotropic. The strongest evidence for this *cosmological principle* is the observed [4, 5, 6] isotropy of the CMBR. The space-time metric then takes the Robertson-Walker form

$$ds^2 = -dt^2 + a^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2(d\vartheta^2 + \sin^2 \vartheta \ d\varphi^2) \right], \tag{1}$$

where r, φ, and ϑ are 'comoving' polar coordinates, which remain fixed for objects that just follow the general cosmological expansion. The parameter k is the scalar curvature of the 3-space and $k = 0$, > 0, or < 0 corresponds to flat, closed, or open universe. The dimensionless parameter $a(t)$ is the scale factor of the universe. We take $a_0 \equiv a(t_0) = 1$, where t_0 is the present value of the cosmic time.

The instantaneous radial physical distance is given by

$$R = a(t) \int_0^r \frac{dr}{(1 - kr^2)^{\frac{1}{2}}}. \tag{2}$$

For flat universe ($k = 0$), $\bar{R} = a(t)\bar{r}$ ($\bar{r}$ is a comoving and $\bar{R}$ a physical radial vector in 3-space) and the velocity of an object is

$$\bar{V} = \frac{d\bar{R}}{dt} = \frac{\dot{a}}{a}\bar{R} + a\frac{d\bar{r}}{dt}, \tag{3}$$

where overdots denote derivation with respect to t. The second term in the right hand side (RHS) of this equation is the peculiar velocity, $\bar{v} = a(t)\dot{\bar{r}}$, of the object i.e. its velocity with respect to the comoving coordinate system. For $\bar{v} = 0$, Eq. (3) becomes

$$\bar{V} = \frac{\dot{a}}{a}\bar{R} \equiv H(t)\bar{R}, \tag{4}$$

where $H(t) \equiv \dot{a}(t)/a(t)$ is the Hubble parameter. This is the well-known Hubble law asserting that all objects run away from each other with velocities proportional to their distances and is the first success of SBB cosmology.

2.2. Friedmann Equation

In a homogeneous and isotropic universe, the energy-momentum tensor takes the form $(T_\mu^{\ \nu}) = \text{diag}(-\rho, p, p, p)$, where ρ is the energy density, p the pressure, and the indices μ, $\nu = 0, 1, 2, 3$ correspond to the space-time coordinates. Energy-momentum conservation then yields the continuity equation

$$\frac{d\rho}{dt} = -3H(t)(\rho + p), \tag{5}$$

where the first term in the RHS describes the dilution of the energy due to the Hubble expansion and the second term the work done by pressure.

For a universe described by the metric in Eq. (1), Einstein's equations

$$R_\mu{}^\nu - \frac{1}{2}\,\delta_\mu{}^\nu R = 8\pi G\, T_\mu{}^\nu, \tag{6}$$

where $R_\mu{}^\nu$ and R are the Ricci tensor and scalar curvature respectively, $\delta_\mu{}^\nu$ is the Kronecker delta, and $G \equiv M_{\mathrm{P}}^{-2}$ is the Newton's constant, lead to the Friedmann equation

$$H^2 \equiv \left(\frac{\dot{a}(t)}{a(t)}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}. \tag{7}$$

Averaging the pressure p, we write $\rho + p = (1 + w)\rho \equiv \gamma\rho$ and Eq. (5) gives $\rho \propto a^{-3\gamma}$. For a universe which is dominated by pressureless matter, $\gamma = 1$ and, thus, $\rho \propto a^{-3}$. This is interpreted as mere dilution of a fixed number of particles in a comoving volume due to the Hubble expansion of the universe. For a radiation dominated universe, $p = \rho/3$ and, thus, $\gamma = 4/3$, which gives $\rho \propto a^{-4}$. The extra factor of $a(t)$ is due to the red-shifting of all wave lengths by the cosmological expansion. Substituting $\rho \propto a^{-3\gamma}$ in Eq. (7) with $k = 0$, we get $a(t) \propto t^{2/3\gamma}$ which, for $a(t_0) = 1$, gives

$$a(t) = \left(\frac{t}{t_0}\right)^{\frac{2}{3\gamma}}. \tag{8}$$

For matter or radiation, we obtain $a(t) = (t/t_0)^{2/3}$ or $a(t) = (t/t_0)^{1/2}$ respectively. So, we see that a matter dominated universe expands faster than a radiation dominated one.

The early universe is radiation dominated and its energy density is

$$\rho = \frac{\pi^2}{30}\left(N_b + \frac{7}{8}N_f\right) T^4 \equiv c\, T^4, \tag{9}$$

where T is the cosmic temperature and $N_{b(f)}$ the number of massless bosonic (fermionic) degrees of freedom. The quantity $g_* = N_b + (7/8)N_f$ is called effective number of massless degrees of freedom. The entropy density is

$$s = \frac{2\pi^2}{45}\, g_*\, T^3. \tag{10}$$

Assuming adiabatic universe evolution i.e. constant entropy in a comoving volume ($sa^3 =$ constant), we obtain $aT =$ constant. The temperature-time relation during radiation dominance is then derived from Eq. (7) (with $k = 0$):

$$T^2 = \frac{M_{\mathrm{P}}}{2(8\pi c/3)^{\frac{1}{2}}t}. \tag{11}$$

Classically, the expansion starts at $t = 0$ with $T = \infty$ and $a = 0$. This initial singularity is, however, not physical since general relativity fails for $t \lesssim t_{\mathrm{P}}$ (the Planck time). The only meaningful statement is that the universe, after a yet unknown initial stage, emerges at $t \sim t_{\mathrm{P}}$ with $T \sim M_{\mathrm{P}}$.

2.3. Important Cosmological Parameters

The most important parameters which describe the expanding universe are the following:

  i. The present value of the Hubble parameter (known as Hubble constant) $H_0 \equiv H(t_0) = 100\, h$ km sec^{-1} Mpc^{-1} ($h = 0.72 \pm 0.07$ from the Hubble space telescope [23]).

ii. The fraction $\Omega = \rho/\rho_c$, where ρ_c is the critical energy density corresponding to a flat universe. From Eq. (7), $\rho_c = 3H^2/8\pi G$ and $\Omega = 1 + k/a^2 H^2$. $\Omega = 1$, > 1, or < 1 corresponds to flat, closed or open universe. Assuming inflation (see below), the present value of Ω must be $\Omega_0 = 1$ in accord with the DASI observations which yield [24] $\Omega_0 = 1.04 \pm 0.08$. The low deuterium abundance measurements [25] in view of nucleosynthesis (see e.g. Ref. [2]) give $\Omega_B h^2 = 0.020 \pm 0.001$, where Ω_B is the baryonic contribution to Ω_0. This result implies that $\Omega_B \approx 0.039$. The total contribution Ω_m of matter to Ω_0 can then be determined from the measurements [26] of the baryon-to-matter ratio in clusters. It is found that, roughly, $\Omega_m \approx 1/3$, which shows that most of the matter in the universe is non-baryonic i.e. dark matter. Moreover, we see that about 2/3 of the energy density of the universe is not even in the form of matter and we call it dark energy. All these results are now confirmed and refined by the WMAP three year measurements [6] which, combined with the 2dF galaxy redshift survey [27], yield $\Omega_B h^2 = 0.02223^{+0.00066}_{-0.00083}$, $\Omega_m h^2 = 0.1262^{+0.0045}_{-0.0062}$, $\Omega_m = 0.236^{+0.016}_{-0.024}$, and, assuming that the dark energy is due to a non-zero cosmological constant, $\Omega_0 = 0.985^{+0.020}_{-0.016}$.

iii. The deceleration parameter

$$q = -\frac{(\ddot{a}/\dot{a})}{(\dot{a}/a)} = \frac{\rho + 3p}{2\rho_c}. \tag{12}$$

Measurements of type Ia supernovae [28] indicate that the universe is speeding up ($q_0 < 0$). This requires that, at present, $p < 0$ as can be seen from Eq. (12). Negative pressure can only be attributed to the dark energy since matter is pressureless. Equation (12) gives $q_0 = (\Omega_0 + 3w_X\Omega_X)/2$, where $\Omega_X = \rho_X/\rho_c$ and $w_X = p_X/\rho_X$ with ρ_X and p_X being the dark energy density and pressure. Observations prefer $w_X = -1$. Actually, the 95% confidence level limit $w_X < -0.6$ from the Ia supernovae data combined with constraints from large-scale structure (see Ref. [29]) is now improved, after the WMAP three year results [6] combined with the supernova legacy survey data [30], to $w_X < -0.83$ for a flat universe. Thus, dark energy can be interpreted as something very close to a non-zero cosmological constant (see below).

2.4. Particle Horizon

Light travels only a finite distance from the time of big bang ($t = 0$) until some cosmic time t. From Eq. (1), we find that the propagation of light along the radial direction is described by $a(t)dr = dt$. The particle horizon, which is the instantaneous distance at t travelled by light since $t = 0$, is then

$$d_H(t) = a(t) \int_0^t \frac{dt'}{a(t')}. \tag{13}$$

The particle horizon is an important notion since it coincides with the size of the universe already seen at time t or, equivalently, with the distance over which causal contact has been established at t. Equations (8) and (13) give

$$d_H(t) = \frac{3\gamma}{3\gamma - 2}t, \ \gamma \neq \frac{2}{3}. \tag{14}$$

Also,

$$H(t) = \frac{2}{3\gamma}t^{-1}, \ d_H(t) = \frac{2}{3\gamma - 2}H^{-1}(t). \tag{15}$$

For matter (radiation), these formulae become $d_H(t) = 2H^{-1}(t) = 3t$ ($d_H(t) = H^{-1}(t) = 2t$). Our universe was matter dominated until fairly recently. So assuming matter dominance, we

can obtain a crude estimate of the present particle horizon (cosmic time), which is $d_H(t_0) = 2H_0^{-1} \approx 6,000\ h^{-1}$ Mpc ($t_0 = 2H_0^{-1}/3 \approx 6.5 \times 10^9\ h^{-1}$ years $\approx 9 \times 10^9$ years). This is certainly an underestimate and had become a bit of a problem as independent estimates had suggested longer lifetimes for some old objects in our universe. After the WMAP three year measurements [6], the present age of our universe is estimated to be $t_0 = 13.73^{+0.13}_{-0.17} \times 10^9$ years. The present $\rho_c = 3H_0^2/8\pi G \approx 1.9 \times 10^{-29}\ h^2$ gm/cm^3.

2.5. Brief History of the Early Universe

We will now briefly summarize the early universe evolution according to GUTs [3]. We will consider a GUT gauge group G (= SU(5), SO(10), SU(3)^3, ...) with or without SUSY. At a scale $M_X \sim 10^{16}$ GeV (the GUT mass scale), G breaks to the standard model gauge group $G_S = $ SU(3)$_c \times$ SU(2)$_L \times$ U(1)$_Y$ by the VEV of an appropriate Higgs field ϕ. (For simplicity, we take this breaking occurring in one step.) G_S is, subsequently, broken to SU(3)$_c \times$ U(1)$_{em}$ at the electroweak scale M_W (SU(3)$_c$ and U(1)$_{em}$ are, respectively, the gauge groups of strong and electromagnetic interactions).

GUTs together with SBB provide a suitable framework for discussing the early universe for $t \gtrsim t_P \approx 10^{-44}$ sec. They predict that the universe, as it expands and cools, undergoes [31] a series of phase transitions during which the gauge symmetry is gradually reduced and important phenomena take place.

After the big bang, G was unbroken and the universe was filled with a hot 'soup' of massless particles which included photons, quarks, leptons, gluons, the weak gauge bosons $W^\pm$, Z^0, the GUT gauge bosons X, Y, ..., and several Higgs bosons. In the SUSY case, the SUSY partners were also present. At $t \sim 10^{-37}$ sec ($T \sim 10^{16}$ GeV), G broke down to G_S and the X, Y,..., and some Higgs bosons acquired masses $\sim M_X$. Their out-of-equilibrium decay could, in principle, produce [32, 33] the observed baryon asymmetry of the universe. Important ingredients are the violation of baryon number, which is inherent in GUTs (as well as in string inspired models [34]), and C and CP violation. This is the second (potential) success of SBB.

During the GUT phase transition, topologically stable extended objects [19] such as magnetic monopoles [7], cosmic strings [35], or domain walls [36] can also be produced. Monopoles, which exist in most GUTs, can lead into problems [8] which are, however, avoided by inflation [10, 11, 12] (see Secs. 3.3 and 4.3). This is a period of exponentially fast expansion of the universe which can occur during some GUT phase transition and can totally remove the monopoles from the scene. Alternatively, a more moderate inflation such as thermal inflation [37], which is associated with a phase transition occurring at a temperature of the order of the electroweak scale, can dilute them to an acceptable, but possibly measurable level. Cosmic strings, on the other hand, which are generically present in many GUT models [38, 39], would contribute [40] to the cosmological perturbations which are needed for structure formation [9] in the universe leading [41] to extra restrictions on the parameters of the model. Finally, domain walls are [36] catastrophic and GUTs should be constructed so that they avoid them (see e.g. Ref. [42]) or inflation should extinguish them. Note that, in some cases, more complex extended objects (which are topologically unstable) such as domain walls bounded by cosmic strings [43] or open cosmic strings connecting magnetic monopoles [44] can be (temporarily) produced.

At $t \sim 10^{-10}$ sec or $T \sim 100$ GeV, the electroweak phase transition takes place and G_S breaks to SU(3)$_c \times$ U(1)$_{em}$. The weak gauge bosons $W^\pm$, Z^0, and the electroweak Higgs fields acquire masses $\sim M_W$. Subsequently, at $t \sim 10^{-4}$ sec or $T \sim 1$ GeV, color confinement sets in and the quarks get bounded forming hadrons.

The direct involvement of particle physics essentially ends here since most of the subsequent phenomena fall into the realm of other branches of physics. We will, however, sketch some of them since they are crucial for understanding the earlier stages of the universe evolution where their origin lies.

At $t \approx 180$ sec ($T \approx 1$ MeV), nucleosynthesis takes place i.e. protons and neutrons form nuclei. The abundance of light elements (D, ^{3}He, ^{4}He, ^{6}Li, and ^{7}Li) depends (see e.g. Ref. [45]) crucially on the number of light particles (with mass $\lesssim 1$ MeV) i.e. the number of light neutrinos, N_ν, and $\Omega_B h^2$. Agreement with observations [25] is achieved for $N_\nu = 3$ and $\Omega_B h^2 \approx 0.020$. This is the third success of SBB cosmology. Much later, at the so-called equidensity point, $t_{\rm eq} \approx 5 \times 10^4$ years, matter dominates over radiation.

At $t \approx 200,000 \ h^{-1}$years ($T \approx 3,000$ K), the decoupling of matter and radiation and the recombination of atoms occur. After this, radiation evolves as an independent component of the universe and is detected today as CMBR with temperature $T_0 \approx 2.73$ K. The existence of the CMBR is the fourth success of SBB. Finally, structure formation [9] starts at $t \approx 2 \times 10^8$ years.

3. Shortcomings of Big Bang

The SBB model has been successful in explaining, among other things, the Hubble expansion, the existence of the CMBR, and the abundance of the light elements formed during nucleosynthesis. Despite its successes, this model had a number of long-standing shortcomings which we will now summarize:

3.1. Horizon Problem

The CMBR which we receive now was emitted at the time of decoupling of matter and radiation when the cosmic temperature was $T_{\rm d} \approx 3,000$ K. The decoupling time, $t_{\rm d}$, can be estimated from

$$\frac{T_0}{T_{\rm d}} = \frac{2.73 \text{ K}}{3,000 \text{ K}} = \frac{a(t_{\rm d})}{a(t_0)} = \left(\frac{t_{\rm d}}{t_0}\right)^{\frac{2}{3}}. \tag{16}$$

It turns out that $t_{\rm d} \approx 200,000 \ h^{-1}$ years.

The distance over which the CMBR has travelled since its emission is

$$a(t_0) \int_{t_{\rm d}}^{t_0} \frac{dt'}{a(t')} = 3t_0 \left[1 - \left(\frac{t_{\rm d}}{t_0}\right)^{\frac{2}{3}}\right] \approx 3t_0 \approx 6,000 \ h^{-1} \text{ Mpc}, \tag{17}$$

which coincides with $d_H(t_0)$. A sphere of radius $d_H(t_0)$ around us is called the last scattering surface since the CMBR has been emitted from it. The particle horizon at decoupling, $3t_{\rm d} \approx 0.168 \ h^{-1}$ Mpc, expanded until now to become $0.168 \ h^{-1}(a(t_0)/a(t_{\rm d}))$ Mpc $\approx 184 \ h^{-1}$ Mpc. The angle subtended by this decoupling horizon now is $\vartheta_{\rm d} \approx 184/6,000 \approx 0.03$ rads. Thus, the sky splits into $4\pi/(0.03)^2 \approx 14,000$ patches which had never communicated before emitting the CMBR. The puzzle then is how can the temperature of the black body radiation from these patches be so finely tuned as COBE [4] and WMAP [5, 6] require.

3.2. Flatness Problem

The present energy density of the universe has been observed [24] to be very close to its critical value corresponding to a flat universe ($\Omega_0 = 1.04 \pm 0.08$). From Eq. (7), we obtain $(\rho - \rho_c)/\rho_c = 3(8\pi G \rho_c)^{-1}(k/a^2) \propto a$ for matter. Thus, in the early universe, $|(\rho - \rho_c)/\rho_c| \ll 1$ and the question is why the initial energy density of the universe was so finely tuned to its critical value.

3.3. Magnetic Monopole Problem

This problem arises only if we combine SBB with GUTs [3] which predict the existence of magnetic monopoles. According to GUTs, the universe underwent [31] a (second order) phase transition during which an appropriate Higgs field, ϕ, developed a non-zero VEV and the GUT gauge group, G, broke to $G_{\rm S}$.

The GUT phase transition produces magnetic monopoles [7]. They are localized deviations from the vacuum with radius $\sim M_X^{-1}$ and mass $m_M \sim M_X/\alpha_G$ ($\alpha_G = g_G^2/4\pi$, where g_G is the GUT gauge coupling constant). The value of ϕ on a sphere, S^2, of radius $\gg M_X^{-1}$ around the monopole lies on the vacuum manifold G/G_S and we, thus, obtain a mapping: $S^2 \to G/G_S$. If this mapping is homotopically non-trivial, the monopole is topologically stable.

The initial relative monopole number density must satisfy the causality bound [46] $r_{M,\text{in}} \equiv (n_M/T^3)_{\text{in}} \gtrsim 10^{-10}$ (n_M is the monopole number density), which comes from the requirement that, at monopole production, ϕ cannot be correlated at distances bigger than the particle horizon. The subsequent evolution of monopoles is studied in Ref. [8]. The result is that, if $r_{M,\text{in}} \gtrsim 10^{-9}$ ($\lesssim 10^{-9}$), the final relative monopole number density $r_{M,\text{fin}} \sim 10^{-9}$ ($\sim r_{M,\text{in}}$). This combined with the causality bound yields $r_{M,\text{fin}} \gtrsim 10^{-10}$. However, the requirement that monopoles do not dominate the energy density of the universe at nucleosynthesis gives

$$r_M(T \approx 1 \text{ MeV}) \lesssim 10^{-19} \tag{18}$$

and we obtain a clear discrepancy of about nine orders of magnitude.

3.4. Density Perturbations

For structure formation [9] in the universe, we need a primordial density perturbation, $\delta\rho/\rho$, at all length scales with a nearly flat spectrum [47]. We also need an explanation of the temperature fluctuations of the CMBR observed by COBE [4] and WMAP [5, 6] at angles $\vartheta \gtrsim \vartheta_{\text{d}} \approx 2^o$ which violate causality (see Sec. 3.1).

4. Inflation

The above four cosmological puzzles are solved by inflation [10, 11, 12]. Take a real scalar field ϕ (the inflaton) with (symmetric) potential energy density $V(\phi)$ which is quite flat near $\phi = 0$ and has minima at $\phi = \pm\langle\phi\rangle$ with $V(\pm\langle\phi\rangle) = 0$. At high cosmic temperatures, $\phi = 0$ due to the temperature corrections to $V(\phi)$. As the temperature drops, the effective potential tends to the zero-temperature potential but a small barrier separating the local minimum at $\phi = 0$ and the vacua at $\phi = \pm\langle\phi\rangle$ remains. At some point, ϕ tunnels out to $\phi_1 \ll \langle\phi\rangle$ and a bubble with $\phi = \phi_1$ is created in the universe. The field then rolls over to the minimum of $V(\phi)$ very slowly (due to the flatness of the potential $V(\phi)$) with the energy density $\rho \approx V(\phi = 0) \equiv V_0$ remaining practically constant for quite some time. The Lagrangian density

$$L = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) \tag{19}$$

gives the energy-momentum tensor

$$T_\mu{}^\nu = -\partial_\mu\phi\partial^\nu\phi + \delta_\mu{}^\nu\left(\frac{1}{2}\partial_\lambda\phi\partial^\lambda\phi - V(\phi)\right), \tag{20}$$

which during the slow roll-over becomes $T_\mu{}^\nu \approx -V_0\,\delta_\mu{}^\nu$ yielding $\rho \approx -p \approx V_0$. So, the pressure is opposite to the energy density in accord with Eq. (5). The scale factor $a(t)$ grows (see below) and the curvature term, k/a^2, in Eq. (7) diminishes. We thus get

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}V_0, \tag{21}$$

which gives $a(t) \propto e^{Ht}$, $H^2 = (8\pi G/3)V_0 = $ constant. So the bubble expands exponentially for some time and $a(t)$ grows by a factor

$$\frac{a(t_f)}{a(t_i)} = \exp H(t_f - t_i) \equiv \exp H\tau \tag{22}$$

between an initial (t_i) and a final (t_f) time.

The scenario just described is known as new [48] inflation. Alternatively, we can imagine, at t_{P}, a region of size $\ell_{\mathrm{P}} \sim M_{\mathrm{P}}^{-1}$ (the Planck length) where the inflaton is large and almost uniform carrying negligible kinetic energy. This region can inflate (exponentially expand) as ϕ slowly rolls down towards the vacuum. This scenario is called chaotic [49] inflation.

We will now show that, with an adequate number of e-foldings, $N = H\tau$, the first three cosmological puzzles are easily resolved (we leave the question of density perturbations for later).

4.1. Resolution of the Horizon Problem
The particle horizon during inflation

$$d_H(t) = e^{Ht} \int_{t_i}^{t} \frac{dt'}{e^{Ht'}} \approx H^{-1}\mathrm{exp}H(t - t_i), \tag{23}$$

for $t - t_i \gg H^{-1}$, grows as fast as $a(t)$. At t_f, $d_H(t_f) \approx H^{-1}\mathrm{exp}H\tau$ and ϕ starts oscillating about the vacuum. It then decays and reheats [13] the universe at a temperature $T_r \sim 10^9$ GeV [50] after which normal big bang cosmology is recovered. The particle horizon at the end of inflation $d_H(t_f)$ is stretched during the ϕ-oscillations by a factor $\sim 10^9$ and between T_r and the present time by a factor T_r/T_0. So, it finally becomes equal to $H^{-1}e^{H\tau}10^9(T_r/T_0)$, which must exceed $2H_0^{-1}$ if the horizon problem is to be solved. This readily holds for $V_0 \approx M_X^4$, $M_X \sim 10^{16}$ GeV, and $N \gtrsim 55$.

4.2. Resolution of the Flatness Problem
The curvature term of the Friedmann equation, at present, is given by

$$\left(\frac{k}{a^2}\right)_0 \approx \left(\frac{k}{a^2}\right)_{\mathrm{bi}} e^{-2H\tau} \, 10^{-18} \left(\frac{10^{-13} \text{ GeV}}{10^9 \text{ GeV}}\right)^2, \tag{24}$$

where the terms in the RHS are the curvature term before inflation and its growth factors during inflation, ϕ-oscillations, and after reheating. Assuming $(k/a^2)_{\mathrm{bi}} \sim H^2$, we get $\Omega_0 - 1 = k/a_0^2 H_0^2 \sim 10^{48} \, e^{-2H\tau} \ll 1$ for $H\tau \gg 55$. Strong inflation implies that the present universe is flat with a great accuracy.

4.3. Resolution of the Monopole Problem
For $N \gtrsim 55$, the magnetic monopoles are diluted by at least 70 orders of magnitude and become irrelevant. Also, since $T_r \ll m_M$, there is no magnetic monopole production after reheating. Extinction of monopoles may also be achieved by non-inflationary mechanisms such as magnetic confinement [51]. For models leading to a possibly measurable magnetic monopole density see e.g. Refs. [52, 53].

5. Detailed Analysis of Inflation
The Hubble parameter during inflation depends on the value of ϕ:

$$H^2(\phi) = \frac{8\pi G}{3} V(\phi). \tag{25}$$

To find the evolution equation for ϕ during inflation, we vary the action

$$\int \sqrt{-\mathrm{det}(g)} \, d^4x \left(\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) + M(\phi)\right), \tag{26}$$

where g is the metric tensor and $M(\phi)$ the (trilinear) coupling of ϕ to light matter causing its decay. Assuming that this coupling is weak, one finds [54]

$$\ddot{\phi} + 3H\dot{\phi} + \Gamma_\phi\dot{\phi} + V'(\phi) = 0, \tag{27}$$

where the prime denotes derivation with respect to ϕ and Γ_ϕ is the decay width [55] of the inflaton. Assume, for the moment, that the decay time of ϕ, $t_d = \Gamma_\phi^{-1}$, is much greater than H^{-1}, the expansion time for inflation. Then the term $\Gamma_\phi\dot{\phi}$ can be ignored and Eq. (27) becomes

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0. \tag{28}$$

Inflation is by definition the situation where $\ddot{\phi}$ is subdominant to the 'friction term' $3H\dot{\phi}$ (and the kinetic energy density is subdominant to the potential one). Equation (28) then reduces to the inflationary equation [56]

$$3H\dot{\phi} = -V'(\phi), \tag{29}$$

which gives

$$\ddot{\phi} = -\frac{V''(\phi)\dot{\phi}}{3H(\phi)} + \frac{V'(\phi)}{3H^2(\phi)}H'(\phi)\dot{\phi}. \tag{30}$$

Comparing the two terms in the RHS of this equation with the friction term in Eq. (28), we get the conditions for inflation (slow roll conditions):

$$\epsilon, \; |\eta| \le 1 \;\; \text{with} \;\; \epsilon \equiv \frac{M_{\rm P}^2}{16\pi}\left(\frac{V'(\phi)}{V(\phi)}\right)^2, \; \eta \equiv \frac{M_{\rm P}^2}{8\pi}\frac{V''(\phi)}{V(\phi)}. \tag{31}$$

The end of the slow roll-over occurs when either of these inequalities is saturated. If ϕ_f is the value of ϕ at the end of inflation, then $t_f \sim H^{-1}(\phi_f)$.

The number of e-foldings during inflation can be calculated as follows:

$$N(\phi_i \to \phi_f) \equiv \ln\left(\frac{a(t_f)}{a(t_i)}\right) = \int_{t_i}^{t_f} H dt = \int_{\phi_i}^{\phi_f} \frac{H(\phi)}{\dot{\phi}}d\phi = -\int_{\phi_i}^{\phi_f} \frac{3H^2(\phi)d\phi}{V'(\phi)}, \tag{32}$$

where Eqs. (22), (29) were used. We shift ϕ so that the global minimum of $V(\phi)$ is displaced at $\phi=0$. Then, if $V(\phi) = \lambda\phi^\nu$ during inflation, we have

$$N(\phi_i \to \phi_f) = -\int_{\phi_i}^{\phi_f} \frac{3H^2(\phi)d\phi}{V'(\phi)} = -8\pi G\int_{\phi_i}^{\phi_f} \frac{V(\phi)d\phi}{V'(\phi)} = \frac{4\pi G}{\nu}(\phi_i^2 - \phi_f^2). \tag{33}$$

Assuming that $\phi_i \gg \phi_f$, this reduces to $N(\phi) \approx (4\pi G/\nu)\phi^2$.

6. Coherent Oscillations of the Inflaton

After the end of inflation at t_f, the term $\ddot{\phi}$ takes over in Eq. (28) and ϕ starts performing coherent damped oscillations about the global minimum of the potential. The rate of energy density loss, due to friction, is given by

$$\dot{\rho} = \frac{d}{dt}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right) = -3H\dot{\phi}^2 = -3H(\rho + p), \tag{34}$$

where $\rho = \dot{\phi}^2/2 + V(\phi)$ and $p = \dot{\phi}^2/2 - V(\phi)$. Averaging p over one oscillation of ϕ and writing $\rho + p = \gamma\rho$, we get $\rho \propto a^{-3\gamma}$ and $a(t) \propto t^{2/3\gamma}$ (see Sec. 2.2).

The number γ can be written as (assuming a symmetric potential)

$$\gamma = \frac{\int_0^T \dot{\phi}^2 dt}{\int_0^T \rho dt} = \frac{\int_0^{\phi_{\max}} \dot{\phi} d\phi}{\int_0^{\phi_{\max}} (\rho/\dot{\phi}) d\phi},$$

(35)

where T and $\phi_{\max}$ are the period and the amplitude of the oscillation. From $\rho = \dot{\phi}^2/2 + V(\phi) = V_{\max}$, where $V_{\max}$ is the maximal potential energy density, we obtain $\dot{\phi} = \sqrt{2(V_{\max} - V(\phi))}$. Substituting this in Eq. (35), we get [57]

$$\gamma = \frac{2 \int_0^{\phi_{\max}} (1 - V/V_{\max})^{\frac{1}{2}} d\phi}{\int_0^{\phi_{\max}} (1 - V/V_{\max})^{-\frac{1}{2}} d\phi}.$$

(36)

For $V(\phi) = \lambda \phi^\nu$, we find $\gamma = 2\nu/(\nu + 2)$ and, thus, $\rho \propto a^{-6\nu/(\nu+2)}$ and $a(t) \propto t^{(\nu+2)/3\nu}$. For $\nu = 2$, in particular, $\gamma = 1$, $\rho \propto a^{-3}$, $a(t) \propto t^{2/3}$ and ϕ behaves like pressureless matter. This is not unexpected since a coherent oscillating massive free field corresponds to a distribution of static massive particles. For $\nu=4$, we obtain $\gamma = 4/3$, $\rho \propto a^{-4}$, $a(t) \propto t^{1/2}$ and the system resembles radiation. For $\nu = 6$, one has $\gamma = 3/2$, $\rho \propto a^{-9/2}$, $a(t) \propto t^{4/9}$ and the expansion is slower (the pressure is higher) than in radiation.

7. Decay of the Inflaton

Reintroducing the decay term $\Gamma_\phi \dot{\phi}$, Eq. (27) can be written as

$$\dot{\rho} = \frac{d}{dt}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right) = -(3H + \Gamma_\phi)\dot{\phi}^2,$$

(37)

which is solved [13, 57] by

$$\rho(t) = \rho_f \left(\frac{a(t)}{a(t_f)}\right)^{-3\gamma} \exp[-\gamma \Gamma_\phi (t - t_f)],$$

(38)

where ρ_f is the energy density at t_f. The second and third factors in the RHS of this equation represent the dilution of the field energy due to the expansion of the universe and the decay of ϕ to light particles respectively.

All pre-existing radiation (known as old radiation) was diluted by inflation, so the only radiation present is the one produced by the decay of ϕ and is known as new radiation. Its energy density ρ_r satisfies [13, 57] the equation

$$\dot{\rho}_r = -4H\rho_r + \gamma \Gamma_\phi \rho,$$

(39)

where the first term in the RHS represents the dilution of radiation due to the cosmological expansion while the second one is the energy density transfer from ϕ to radiation. Taking $\rho_r(t_f)=0$, this equation gives [13, 57]

$$\rho_r(t) = \rho_f \left(\frac{a(t)}{a(t_f)}\right)^{-4} \int_{t_f}^t \left(\frac{a(t')}{a(t_f)}\right)^{4-3\gamma} e^{-\gamma \Gamma_\phi (t' - t_f)} \gamma \Gamma_\phi dt'.$$

(40)

For $t_f \ll t_d$ and $\nu = 2$, this expression is approximated by

$$\rho_r(t) = \rho_f \left(\frac{t}{t_f}\right)^{-\frac{8}{3}} \int_0^t \left(\frac{t'}{t_f}\right)^{\frac{2}{3}} e^{-\Gamma_\phi t'} dt',$$

(41)

which can be expanded as

$$\rho_r = \frac{3}{5} \rho \, \Gamma_\phi t \left[1 + \frac{3}{8} \, \Gamma_\phi t + \frac{9}{88} \, (\Gamma_\phi t)^2 + \cdots \right] \tag{42}$$

with $\rho = \rho_f (t/t_f)^{-2} \exp(-\Gamma_\phi t)$ being the energy density of the field ϕ.

The energy density of the new radiation grows relative to the energy density of the oscillating field and becomes essentially equal to it at a cosmic time $t_d = \Gamma_\phi^{-1}$ as one can deduce from Eq. (42). After this time, the universe enters into the radiation dominated era and the normal big bang cosmology is recovered. The temperature at t_d, $T_r(t_d)$, is historically called the reheat temperature although no supercooling and subsequent reheating of the universe actually takes place. Using Eq. (11), we find that

$$T_r = \left(\frac{45}{16\pi^3 g_*} \right)^{\frac{1}{4}} (\Gamma_\phi M_{\mathrm{P}})^{\frac{1}{2}}, \tag{43}$$

where g_* is the effective number of massless degrees of freedom. For $V(\phi) = \lambda \phi^\nu$, the total expansion of the universe during the damped field oscillations is

$$\frac{a(t_d)}{a(t_f)} = \left(\frac{t_d}{t_f} \right)^{\frac{\nu+2}{3\nu}}. \tag{44}$$

8. Density Perturbations from Inflation

Inflation not only homogenizes the universe but also generates the density perturbations needed for structure formation. To see this, we introduce the notion of event horizon at t. This includes all points with which we will eventually communicate sending signals at t. Its instantaneous radius is

$$d_e(t) = a(t) \int_t^\infty \frac{dt'}{a(t')}. \tag{45}$$

This yields an infinite event horizon for matter or radiation. For inflation, however, we obtain $d_e(t) = H^{-1} < \infty$, which varies slowly with t. Points in our event horizon at t with which we can communicate sending signals at t are eventually pulled away by the exponential expansion and we cease to be able to communicate with them emitting signals at later times. We say that these points (and the corresponding scales) crossed outside the event horizon. Actually, the exponentially expanding (de Sitter) space is like a black hole turned inside out. Then, exactly as in a black hole, there are quantum fluctuations of the thermal type governed by the Hawking temperature [58, 59] $T_H = H/2\pi$. It turns out [60, 61] that the quantum fluctuations of all massless fields (the inflaton is nearly massless due to the flatness of the potential) are $\delta\phi = T_H$. These fluctuations of ϕ lead to energy density perturbations $\delta\rho = V'(\phi)\delta\phi$. As the scale of this perturbations crosses outside the event horizon, they become [62] classical metric perturbations.

It has been shown [63] (for a review, see e.g. Ref. [64]) that the quantity $\zeta \approx \delta\rho/(\rho + p)$ remains constant outside the event horizon. Thus, the density perturbation at any present physical (comoving) scale ℓ, $(\delta\rho/\rho)_\ell$, when this scale crosses inside the post-inflationary particle horizon ($p{=}0$ at this instance) can be related to the value of ζ when the same scale crossed outside the inflationary event horizon (at $\ell \sim H^{-1}$). This latter value of ζ is found, using Eq. (29), to be

$$\zeta \mid_{\ell \sim H^{-1}} = \left(\frac{\delta\rho}{\dot{\phi}^2} \right)_{\ell \sim H^{-1}} = \left(\frac{V'(\phi)H(\phi)}{2\pi\dot{\phi}^2} \right)_{\ell \sim H^{-1}} = - \left(\frac{9H^3(\phi)}{2\pi V'(\phi)} \right)_{\ell \sim H^{-1}}. \tag{46}$$

Taking into account an extra 2/5 factor from the fact that the universe is matter dominated when the scale ℓ re-enters the horizon, we obtain

$$\left(\frac{\delta\rho}{\rho}\right)_\ell = \frac{16\sqrt{6\pi}}{5}\,\frac{V^{\frac{3}{2}}(\phi_\ell)}{M_{\mathrm{P}}^3 V'(\phi_\ell)}. \tag{47}$$

The calculation of ϕ_ℓ, the value of the field ϕ when the comoving scale ℓ crossed outside the event horizon, goes as follows. At the reheat temperature T_r, a comoving (present physical) scale ℓ was equal to $\ell(a(t_d)/a(t_0)) = \ell(T_0/T_r)$. Its magnitude at t_f was equal to $\ell(T_0/T_r)(a(t_f)/a(t_d)) = \ell(T_0/T_r)(t_f/t_d)^{(\nu+2)/3\nu} \equiv \ell_{\mathrm{phys}}(t_f)$, where the potential $V(\phi) = \lambda\phi^\nu$ was assumed. The scale ℓ, when it crossed outside the inflationary event horizon, was equal to $H^{-1}(\phi_\ell)$. We, thus, obtain

$$H^{-1}(\phi_\ell)e^{N(\phi_\ell)} = \ell_{\mathrm{phys}}(t_f), \tag{48}$$

which gives ϕ_ℓ and, thus, $N(\phi_\ell) \equiv N_\ell$, the number of e-foldings the comoving scale ℓ suffered during inflation. In particular, the number of e-foldings suffered by our present horizon $\ell = 2H_0^{-1} \sim 10^4$ Mpc turns out to be $N_Q \approx 50 - 60$.

Taking $V(\phi) = \lambda\phi^4$, Eqs. (33), (47), and (48) give

$$\left(\frac{\delta\rho}{\rho}\right)_\ell = \frac{4\sqrt{6\pi}}{5}\lambda^{\frac{1}{2}}\left(\frac{\phi_\ell}{M_{\mathrm{P}}}\right)^3 = \frac{4\sqrt{6\pi}}{5}\lambda^{\frac{1}{2}}\left(\frac{N_\ell}{\pi}\right)^{\frac{3}{2}}. \tag{49}$$

From the result of COBE [4], $(\delta\rho/\rho)_Q \approx 6 \times 10^{-5}$, one can then deduce that $\lambda \approx 6 \times 10^{-14}$ for $N_Q \approx 55$. We thus see that the inflaton must be a very weakly coupled field. In non-SUSY GUTs, the inflaton is necessarily gauge singlet since otherwise radiative corrections will make it strongly coupled. This is not so satisfactory since it forces us to introduce an otherwise unmotivated very weakly coupled gauge singlet. In SUSY GUTs, however, the inflaton could be identified [65] with a conjugate pair of gauge non-singlet fields $\bar{\phi}$, ϕ already present in the theory and causing the gauge symmetry breaking. Absence of strong radiative corrections from gauge interactions is guaranteed by the mutual cancellation of the D-terms of these fields.

The spectrum of density perturbations can be analyzed. For $V(\phi) = \lambda\phi^\nu$, we find $(\delta\rho/\rho)_\ell \propto \phi_\ell^{(\nu+2)/2}$ which, together with $N(\phi_\ell) \propto \phi_\ell^2$ (see Eq. (33)), gives

$$\left(\frac{\delta\rho}{\rho}\right)_\ell = \left(\frac{\delta\rho}{\rho}\right)_Q\left(\frac{N_\ell}{N_Q}\right)^{\frac{\nu+2}{4}}. \tag{50}$$

The scale ℓ divided by the size of our present horizon $(2H_0^{-1} \sim 10^4$ Mpc) should equal $\exp(N_\ell - N_Q)$. This gives $N_\ell/N_Q = 1 + \ln(\ell/2H_0^{-1})^{1/N_Q}$ which expanded around $\ell = 2H_0^{-1}$ and substituted in Eq. (50) yields

$$\left(\frac{\delta\rho}{\rho}\right)_\ell \approx \left(\frac{\delta\rho}{\rho}\right)_Q\left(\frac{\ell}{2H_0^{-1}}\right)^{\alpha_s} \tag{51}$$

with $\alpha_s = (\nu+2)/4N_Q$. For $\nu = 4$, $\alpha_s \approx 0.03$ and, thus, the density perturbations are essentially scale independent. The customarily used spectral index $n_s = 1 - 2\alpha_s$ is about 0.94 in this case.

9. Temperature Fluctuations

The density inhomogeneities produce temperature fluctuations in the CMBR. For angles $\vartheta \gtrsim 2^o$, the dominant effect is the scalar Sachs-Wolfe [66] effect. Density perturbations on the last scattering surface cause scalar gravitational potential fluctuations, which then produce

temperature fluctuations in the CMBR. The reason is that regions with a deep gravitational potential will cause the photons to lose energy as they climb up the potential well and, thus, these regions will appear cooler.

Analyzing the temperature fluctuations from the scalar Sachs-Wolfe effect in spherical harmonics, we obtain the corresponding quadrupole anisotropy:

$$\left(\frac{\delta T}{T}\right)_{Q-S} = \left(\frac{32\pi}{45}\right)^{\frac{1}{2}} \frac{V^{\frac{3}{2}}(\phi_\ell)}{M_{\rm P}^3 V'(\phi_\ell)}. \tag{52}$$

For $V(\phi) = \lambda\phi^\nu$, this becomes

$$\left(\frac{\delta T}{T}\right)_{Q-S} = \left(\frac{32\pi}{45}\right)^{\frac{1}{2}} \frac{\lambda^{\frac{1}{2}}\phi_\ell^{\frac{\nu+2}{2}}}{\nu M_{\rm P}^3} = \left(\frac{32\pi}{45}\right)^{\frac{1}{2}} \frac{\lambda^{\frac{1}{2}}}{\nu M_{\rm P}^3} \left(\frac{\nu M_{\rm P}^2}{4\pi}\right)^{\frac{\nu+2}{4}} N_\ell^{\frac{\nu+2}{4}}. \tag{53}$$

Comparing this with the COBE [4] result, $(\delta T/T)_Q \approx 6.6 \times 10^{-6}$, we obtain $\lambda \approx 6 \times 10^{-14}$ for $\nu = 4$ and number of e-foldings suffered by our present horizon scale during the inflationary phase $N_Q \approx 55$.

There are also tensor fluctuations [67] in the temperature of the CMBR. The tensor quadrupole anisotropy is

$$\left(\frac{\delta T}{T}\right)_{Q-T} \approx 0.77 \frac{V^{\frac{1}{2}}(\phi_\ell)}{M_{\rm P}^2}. \tag{54}$$

The total quadrupole anisotropy is given by

$$\left(\frac{\delta T}{T}\right)_Q = \left[\left(\frac{\delta T}{T}\right)_{Q-S}^2 + \left(\frac{\delta T}{T}\right)_{Q-T}^2\right]^{\frac{1}{2}} \tag{55}$$

and the ratio

$$r = \frac{(\delta T/T)_{Q-T}^2}{(\delta T/T)_{Q-S}^2} \approx 0.27 \left(\frac{M_{\rm P}V'(\phi_\ell)}{V(\phi_\ell)}\right)^2. \tag{56}$$

For $V(\phi) = \lambda\phi^\nu$, we obtain $r \approx 3.4 \, \nu/N_Q \ll 1$, and the tensor contribution to the temperature fluctuations of the CMBR is negligible. Actually, the tensor fluctuations turn out to be negligible in all the cases considered here.

10. Hybrid Inflation
10.1. The non-Supersymmetric Version
The main disadvantage of inflationary scenarios such as the new [48] or the chaotic [49] scenario is that they require tiny parameters in order to reproduce the results of COBE [4]. This has led Linde [15] to propose, in the context of non-SUSY GUTs, hybrid inflation which uses two real scalar fields χ and σ instead of one. The field χ provides the vacuum energy density which drives inflation, while σ is the slowly varying field during inflation. This splitting of roles allows us to reproduce the COBE results with natural (not too small) values of the parameters.

The scalar potential utilized by Linde is

$$V(\chi,\sigma) = \kappa^2\left(M^2 - \frac{\chi^2}{4}\right)^2 + \frac{\lambda^2\chi^2\sigma^2}{4} + \frac{m^2\sigma^2}{2}, \tag{57}$$

where κ, $\lambda > 0$ are dimensionless constants and M, $m > 0$ mass parameters. The vacua lie at $\langle\chi\rangle = \pm 2M$, $\langle\sigma\rangle = 0$. For $m=0$, V has a flat direction at $\chi = 0$, where $V = \kappa^2 M^4$ and the

mass^2 of χ is $m_\chi^2 = -\kappa^2 M^2 + \lambda^2 \sigma^2/2$. So, for $\chi = 0$ and $|\sigma| > \sigma_c \equiv \sqrt{2}\kappa M/\lambda$, we obtain a flat valley of minima. For $m \neq 0$, the valley acquires a slope and the system can inflate as the field σ slowly rolls down this valley.

The ϵ and η criteria (see Eq. (31)) imply that inflation continues until σ reaches σ_c, where it terminates abruptly. It is followed by a waterfall i.e. a sudden entrance into an oscillatory phase about a global minimum. Since the system can fall into either of the two minima with equal probability, topological defects (magnetic monopoles, cosmic strings, or domain walls) are copiously produced [18] if they are predicted by the particular GUT model employed. So, if the underlying GUT gauge symmetry breaking (by $\langle \chi \rangle$) leads to the existence of magnetic monopoles or domain walls, we encounter a cosmological catastrophe.

The onset of hybrid inflation requires [68] that, at $t \sim H^{-1}$, H being the inflationary Hubble parameter, a region exists with size $\gtrsim H^{-1}$ where χ and σ are almost uniform with negligible kinetic energies and values close to the bottom of the valley of minima. Such a region, at t_P, would have been much larger than the Planck length ℓ_P and it is, thus, difficult to imagine how it could be so homogeneous. Moreover, as it has been argued [69], the initial values (at t_P) of the fields in this region must be strongly restricted in order to obtain adequate inflation. Several possible solutions to this problem of initial conditions for hybrid inflation have been proposed (see e.g. Refs. [70, 71, 72]).

The quadrupole anisotropy of the CMBR produced during hybrid inflation can be estimated, using Eq. (52), to be

$$\left(\frac{\delta T}{T}\right)_Q \approx \left(\frac{16\pi}{45}\right)^{\frac{1}{2}} \frac{\lambda \kappa^2 M^5}{M_P^3 m^2}. \tag{58}$$

The COBE [4] result, $(\delta T/T)_Q \approx 6.6 \times 10^{-6}$, can then be reproduced with $M \approx 2.86 \times 10^{16}$ GeV, the SUSY GUT VEV, and $m \approx 1.3\, \kappa\sqrt{\lambda} \times 10^{15}$ GeV. Note that $m \sim 10^{12}$ GeV for $\kappa, \lambda \sim 10^{-2}$.

10.2. The Supersymmetric Version

It has been observed [16] that hybrid inflation is tailor made for globally SUSY GUTs except that an intermediate scale mass for σ cannot be obtained. Actually, all scalar fields acquire masses $\sim m_{3/2} \sim 1$ TeV (the gravitino mass) from soft SUSY breaking.

Let us consider the renormalizable superpotential

$$W = \kappa S(-M^2 + \bar{\phi}\phi), \tag{59}$$

where $\bar{\phi}$, ϕ is a pair of G_S singlet left handed superfields belonging to conjugate representations of G and reducing its rank by their VEVs and S is a gauge singlet left handed superfield. The parameters κ and M ($\sim 10^{16}$ GeV) are made positive by field redefinitions. The vanishing of the F-term F_S gives $\langle \bar{\phi} \rangle \langle \phi \rangle = M^2$ and the D-terms vanish for $|\langle \bar{\phi} \rangle| = |\langle \phi \rangle|$. So, the SUSY vacua lie at $\langle \bar{\phi} \rangle^* = \langle \phi \rangle = \pm M$ (after rotating $\bar{\phi}$, ϕ on the real axis by a G transformation) and $\langle S \rangle = 0$ (from $F_{\bar{\phi}} = F_\phi = 0$). Thus, the superpotential W leads to the breaking of G.

The interesting observation [16] is that the same superpotential W also gives rise to hybrid inflation. The potential derived from W is

$$V(\bar{\phi}, \phi, S) = \kappa^2 |M^2 - \bar{\phi}\phi|^2 + \kappa^2 |S|^2 (|\bar{\phi}|^2 + |\phi|^2) + D - \text{terms}. \tag{60}$$

D-flatness implies that $\bar{\phi}^* = e^{i\theta}\phi$. We take $\theta = 0$, so that the SUSY vacua are contained. The superpotential W has a U(1)$_R$ R symmetry [73]: $\bar{\phi}\phi \to \bar{\phi}\phi$, $S \to e^{i\alpha}S$, $W \to e^{i\alpha}W$. Note, in passing, that global continuous symmetries such as this R symmetry can effectively arise [74] from the rich discrete symmetry groups encountered in many compactified string theories (see e.g. Ref. [75]). It is important to point out that W is the most general renormalizable superpotential allowed by G and U(1)$_R$. Bringing the fields $\bar{\phi}$, ϕ, and S on the real axis by

appropriate G and $U(1)_R$ transformations, we write $\bar{\phi} = \phi \equiv \chi/2$ and $S \equiv \sigma/\sqrt{2}$, where χ and σ are normalized real scalar fields. The potential V in Eq. (60) then takes the form in Eq. (57) with $\kappa = \lambda$ and $m = 0$. So, Linde's potential for hybrid inflation is almost obtainable from SUSY GUTs, but without the mass term of σ.

SUSY breaking by the vacuum energy density $\kappa^2 M^4$ on the inflationary valley ($\bar{\phi} = \phi = 0$, $|S| > S_c \equiv M$) causes a mass splitting in the supermultiplets $\bar{\phi}$, ϕ. We obtain a Dirac fermion with mass$^2 = \kappa^2|S|^2$ and two complex scalars with mass$^2 = \kappa^2|S|^2 \pm \kappa^2 M^2$. This leads [17] to one-loop radiative corrections to V on the valley which are calculated by using the Coleman-Weinberg formula [76]:

$$\Delta V = \frac{1}{64\pi^2} \sum_i (-)^{F_i} \, M_i^4 \ln \frac{M_i^2}{\Lambda^2}, \tag{61}$$

where the sum extends over all helicity states i with fermion number F_i and mass$^2 = M_i^2$ and Λ is a renormalization scale. We find that

$$\Delta V(|S|) = \kappa^2 M^4 \frac{\kappa^2 \mathcal{N}}{32\pi^2} \left(2 \ln \frac{\kappa^2|S|^2}{\Lambda^2} + (z+1)^2 \ln(1+z^{-1}) + (z-1)^2 \ln(1-z^{-1}) \right), \tag{62}$$

where $z \equiv x^2 \equiv |S|^2/M^2$ and $\mathcal{N}$ is the dimensionality of the representations to which $\bar{\phi}$, ϕ belong. These radiative corrections generate the necessary slope on the inflationary valley. Note that the slope is Λ-independent.

From Eqs. (33), (52), and (62), we find the quadrupole anisotropy of the CMBR:

$$\left(\frac{\delta T}{T} \right)_Q \approx \frac{8\pi}{\sqrt{\mathcal{N}}} \left(\frac{N_Q}{45} \right)^{\frac{1}{2}} \left(\frac{M}{M_P} \right)^2 x_Q^{-1} y_Q^{-1} \Lambda(x_Q^2)^{-1} \tag{63}$$

with

$$\Lambda(z) = (z+1)\ln(1+z^{-1}) + (z-1)\ln(1-z^{-1}), \tag{64}$$

$$y_Q^2 = \int_1^{x_Q^2} \frac{dz}{z} \Lambda(z)^{-1}, \; y_Q \geq 0. \tag{65}$$

Here, $x_Q \equiv |S_Q|/M$ with S_Q being the value of S when our present horizon crossed outside the inflationary horizon. Finally, from Eq. (62), one finds

$$\kappa \approx \frac{8\pi^{\frac{3}{2}}}{\sqrt{\mathcal{N} N_Q}} \, y_Q \, \frac{M}{M_P}. \tag{66}$$

The slow roll conditions for SUSY hybrid inflation are $\epsilon, |\eta| \leq 1$, where

$$\epsilon = \left(\frac{\kappa^2 M_P}{16\pi^2 M} \right)^2 \frac{\mathcal{N}^2 x^2}{8\pi} \Lambda(x^2)^2, \tag{67}$$

$$\eta = \left(\frac{\kappa M_P}{4\pi M} \right)^2 \frac{\mathcal{N}}{8\pi} \left((3z+1)\ln(1+z^{-1}) + (3z-1)\ln(1-z^{-1}) \right). \tag{68}$$

These conditions are violated only 'infinitesimally' close to the critical point ($x = 1$). So, inflation continues until this point, where the waterfall occurs.

Using COBE [4] and eliminating x_Q between Eqs. (63) and (66), we obtain M as a function of κ. The maximal M which can be achieved is $\approx 10^{16}$ GeV (for $\mathcal{N} = 8$, $N_Q \approx 55$) and, although somewhat smaller than the SUSY GUT VEV, is quite close to it. As an example, take $\kappa = 4 \times 10^{-3}$ which gives $M \approx 9.57 \times 10^{15}$ GeV, $x_Q \approx 2.633$, $y_Q \approx 2.42$. The slow roll conditions are violated at $x - 1 \approx 7.23 \times 10^{-5}$, where $\eta = -1$ ($\epsilon \approx 8.17 \times 10^{-8}$ at $x = 1$). The spectral index $n_s = 1 - 6\epsilon + 2\eta$ [77] is about 0.985.

SUSY hybrid inflation is considered natural for the following reasons:

i. There is no need of tiny coupling constants ($\kappa \sim 10^{-3}$).

ii. The superpotential W in Eq. (59) has the most general renormalizable form allowed by G and $U(1)_R$. The coexistence of the S and $S\bar{\phi}\phi$ terms implies that $\bar{\phi}\phi$ is neutral under all symmetries and, thus, all the non-renormalizable terms of the form $S(\bar{\phi}\phi)^n/M_S^{2(n-1)}$, $n \geq 2$, are also allowed [20] ($M_S \approx 5 \times 10^{17}$ GeV is the string scale). The leading term of this type $S(\bar{\phi}\phi)^2/M_S^2$, if its dimensionless coefficient is of order unity, can be comparable to $S\bar{\phi}\phi$ (recall that $\kappa \sim 10^{-3}$) and, thus, play a role in inflation (see Sec. 11). All higher order terms of this type with $n \geq 3$ give negligible contributions to the inflationary potential (provided that $|\bar{\phi}|$, $|\phi| \ll M_S$ during inflation). The symmetry $U(1)_R$ guarantees [78] the linearity of W in S to all orders excluding terms such as S^2 which could generate an inflaton mass $\gtrsim H$ and ruin inflation by violating the slow roll conditions.

iii. SUSY guarantees that the radiative corrections do not ruin [65] inflation, but rather provide [17] the necessary slope on the inflationary path.

iv. Supergravity (SUGRA) corrections can be brought under control leaving inflation intact. The scalar potential in SUGRA is given by

$$V = \exp\left(\frac{K}{m_P^2}\right)\left[\left(K^{-1}\right)_i^{\ j} F^i F_j - 3\frac{|W|^2}{m_P^2}\right], \tag{69}$$

where K is the Kähler potential, $m_P = M_P/\sqrt{8\pi} \approx 2.44 \times 10^{18}$ GeV is the reduced Planck mass scale, $F^i = W^i + K^i W/m_P^2$, and upper (lower) indices denote derivation with respect to the scalar field ϕ_i (ϕ^{j*}). The Kähler potential can be readily expanded as $K = |S|^2 + |\bar{\phi}|^2 + |\phi|^2 + \alpha|S|^4/m_P^2 + \cdots$, where the quadratic terms constitute the minimal Kähler potential. The term $|S|^2$, whose coefficient is normalized to unity, could generate a mass^2 $\sim \kappa^2 M^4/m_P^2 \sim H^2$ for S on the inflationary path from the expansion of the exponential prefactor in Eq. (69). This would ruin inflation. Fortunately, with this form of W (including all the higher order terms), this mass^2 is cancelled in V [16, 79]. The linearity of W in S, guaranteed to all orders by $U(1)_R$, is crucial for this cancellation. The $|S|^4$ term in K also generates a mass^2 for S via the factor $(\partial^2 K/\partial S \partial S^*)^{-1} = 1 - 4\alpha|S|^2/m_P^2 + \cdots$ in Eq. (69), which is however not cancelled (see e.g. Ref. [80]). In order to avoid ruining inflation, one has then to assume [71, 81] that $|\alpha| \lesssim 10^{-3}$. All other higher order terms in K give suppressed contributions on the inflationary path (since $|S| \ll m_P$). So, we see that a mild tuning of just one parameter is adequate for controlling SUGRA corrections. (In other models, tuning of infinitely many parameters is required.) Moreover, note that with special forms of K one can solve this problem even without a mild tuning. An example is given in Ref. [72], where the dangerous mass^2 term could be cancelled to all orders in the presence of fields without superpotential but with large VEVs generated via D-terms. Such a mechanism is necessary in variants of hybrid inflation (see Ref. [82]) where inflation takes place at large values of the inflaton field S ($|S| \sim m_P$) since the higher order terms are, in this case, unsuppressed and thus tuning of an infinite number of parameters would be otherwise required. All the above methods for controlling the SUGRA corrections also apply [82] to the extensions of the model that we will consider in Sec. 11.

In summary, for all these reasons, we consider SUSY hybrid inflation (with its extensions) as an extremely natural inflationary scenario.

11. Extensions of Supersymmetric Hybrid Inflation

Applying (SUSY) hybrid inflation to higher GUT gauge groups predicting magnetic monopoles, we encounter the following cosmological problem. Inflation is terminated abruptly as the system

reaches the critical point and is followed by the waterfall regime during which the scalar fields $\bar{\phi}$, ϕ develop their VEVs starting from zero and the spontaneous breaking of the GUT gauge symmetry takes place. The fields $\bar{\phi}$, ϕ can end up at any point of the vacuum manifold with equal probability and, thus, magnetic monopoles are copiously produced [18] via the Kibble mechanism [19] leading to a disaster.

One of the simplest GUTs predicting magnetic monopoles is the Pati-Salam (PS) model [83] with gauge group $G_{PS} = SU(4)_c \times SU(2)_L \times SU(2)_R$. These monopoles carry [84] two units of Dirac magnetic charge. We will present solutions [18, 20] of the monopole problem of hybrid inflation within the SUSY PS model, although our mechanisms can be extended to other gauge groups such as the trinification group $SU(3)_c \times SU(3)_L \times SU(3)_R$ (see e.g. Ref. [85]), which predicts [53] magnetic monopoles with triple Dirac charge.

11.1. Shifted Hybrid Inflation

One idea [20] for solving the monopole problem is to include into the standard superpotential for hybrid inflation (in Eq. (59)) the leading non-renormalizable term, which, as explained, cannot be excluded. If its dimensionless coefficient is of order unity, this term competes with the trilinear term of the standard superpotential (with coefficient $\sim 10^{-3}$). A totally new picture then emerges. There appears a non-trivial flat direction along which G_{PS} is broken with the appropriate Higgs fields acquiring constant values. This 'shifted' flat direction acquires a slope again from radiative corrections [17] and can be used as inflationary path. The end of inflation is again abrupt followed by a waterfall but no magnetic monopoles are formed since G_{PS} is already broken during inflation.

The spontaneous breaking of the gauge group G_{PS} to G_S is achieved via the VEVs of a conjugate pair of Higgs superfields

$$
\begin{aligned}
\bar{H}^c &= (4, 1, 2) \equiv \left(\begin{array}{cccc} \bar{u}_H^c & \bar{u}_H^c & \bar{u}_H^c & \bar{\nu}_H^c \\ \bar{d}_H^c & \bar{d}_H^c & \bar{d}_H^c & \bar{e}_H^c \end{array} \right), \\
H^c &= (\bar{4}, 1, 2) \equiv \left(\begin{array}{cccc} u_H^c & u_H^c & u_H^c & \nu_H^c \\ d_H^c & d_H^c & d_H^c & e_H^c \end{array} \right)
\end{aligned}
\tag{70}
$$

in the $\bar{\nu}_H^c$, ν_H^c directions. The relevant part of the superpotential, which includes the leading non-renormalizable term, is

$$
\delta W = \kappa S(-M^2 + \bar{H}^c H^c) - \beta \frac{S(\bar{H}^c H^c)^2}{M_S^2},
\tag{71}
$$

where β is taken to be real and positive for simplicity. D-flatness implies that $\bar{H}^{c*} = e^{i\theta} H^c$. We restrict ourselves to the direction with $\theta = 0$ ($\bar{H}^{c*} = H^c$) which contains the shifted inflationary trajectory (see below). The scalar potential derived from the superpotential δW in Eq. (71) then takes the form

$$
V = \left[\kappa(|H^c|^2 - M^2) - \beta \frac{|H^c|^4}{M_S^2} \right]^2 + 2\kappa^2 |S|^2 |H^c|^2 \left[1 - \frac{2\beta}{\kappa M_S^2} |H^c|^2 \right]^2.
\tag{72}
$$

Defining the dimensionless variables $w = |S|/M$, $y = |H^c|/M$, we obtain

$$
\tilde{V} = \frac{V}{\kappa^2 M^4} = (y^2 - 1 - \xi y^4)^2 + 2w^2 y^2 (1 - 2\xi y^2)^2,
\tag{73}
$$

where $\xi = \beta M^2 / \kappa M_S^2$. This potential is a simple extension of the standard potential for SUSY hybrid inflation (which corresponds to $\xi = 0$).

For constant w (or $|S|$), $\tilde{V}$ in Eq. (73) has extrema at

$$y_1 = 0, \ y_2 = \frac{1}{\sqrt{2\xi}}, \ y_{3\pm} = \frac{1}{\sqrt{2\xi}}\sqrt{(1 - 6\xi w^2) \pm \sqrt{(1 - 6\xi w^2)^2 - 4\xi(1 - w^2)}}. \tag{74}$$

The first two extrema (at y_1, y_2) are $|S|$-independent and, thus, correspond to classically flat directions, the trivial one at $y_1 = 0$ with $\tilde{V}_1 = 1$ and the shifted one at $y_2 = 1/\sqrt{2\xi} = $ constant with $\tilde{V}_2 = (1/4\xi - 1)^2$, which we will use as inflationary trajectory. The trivial trajectory is a valley of minima for $w > 1$, while the shifted one for $w > w_0 = (1/8\xi - 1/2)^{1/2}$, which is its critical point. We take $\xi < 1/4$, so that $w_0 > 0$ and the shifted path is destabilized before w reaches zero. The extrema at $y_{3\pm}$, which are $|S|$-dependent and non-flat, do not exist for all values of w and ξ, since the expressions under the square roots in Eq. (74) are not always non-negative. These two extrema, at $w = 0$, become SUSY vacua. The relevant SUSY vacuum (see below) corresponds to $y_{3-}(w = 0)$ and, thus, the common VEV v_0 of $\bar{H}^c$, H^c is given by

$$\left(\frac{v_0}{M}\right)^2 = \frac{1}{2\xi}(1 - \sqrt{1 - 4\xi}). \tag{75}$$

We will now discuss the structure of $\tilde{V}$ and the inflationary history for $1/6 < \xi < 1/4$. For fixed $w > 1$, there exist two local minima at $y_1 = 0$ and $y_2 = 1/\sqrt{2\xi}$, which has lower potential energy density, and a local maximum at y_{3+} between the minima. As w becomes smaller than unity, the extremum at y_1 turns into a local maximum, while the extremum at y_{3+} disappears. The system then falls into the shifted path in case it had started at $y_1 = 0$. As we further decrease w below $(2 - \sqrt{36\xi - 5})^{1/2}/3\sqrt{2\xi}$, a pair of new extrema, a local minimum at y_{3-} and a local maximum at y_{3+}, are created between y_1 and y_2. As w crosses $(1/8\xi - 1/2)^{1/2}$, the local maximum at y_{3+} crosses y_2 becoming a local minimum. At the same time, the local minimum at y_2 turns into a local maximum and inflation ends with the system falling into the local minimum at y_{3-} which, at $w = 0$, becomes the SUSY vacuum.

We see that, no matter where the system starts from, it passes from the shifted path, where the relevant part of inflation takes place. So, G_{PS} is broken during inflation and no magnetic monopoles are produced at the waterfall.

After the termination of inflation, the system could fall into the minimum at y_{3+} instead of the one at y_{3-}. This, however, does not happen since in the last e-folding or so the barrier between the minima at y_{3-} and y_2 is considerably reduced and the decay of the 'false vacuum' at y_2 to the minimum at y_{3-} is completed within a fraction of an e-folding before the y_{3+} minimum even appears.

The only mass splitting within supermultiplets on the shifted path appears [20] between one Majorana fermion in the direction $(\bar{\nu}_H^c + \nu_H^c)/\sqrt{2}$ with $m^2 = 4\kappa^2|S|^2$ and two real scalar fields $\text{Re}(\delta\bar{\nu}_H^c + \delta\nu_H^c)$ and $\text{Im}(\delta\bar{\nu}_H^c + \delta\nu_H^c)$ with $m_\pm^2 = 4\kappa^2|S|^2 \mp 2\kappa^2 m^2$. Here, $m = M(1/4\xi - 1)^{1/2}$ and $\delta\bar{\nu}_H^c = \bar{\nu}_H^c - v$, $\delta\nu_H^c = \nu_H^c - v$, where $v = (\kappa M_S^2/2\beta)^{1/2}$ is the value of $\bar{H}^c$, H^c on the shifted inflationary path.

The radiative corrections on the shifted inflationary trajectory can be readily constructed and $(\delta T/T)_Q$ and κ can be evaluated. We find the same formulas as in Eqs. (63) and (66) with $\mathcal{N} = 2$ and $\mathcal{N} = 4$ respectively and M generally replaced by m. The COBE results [4] can be reproduced, for instance, with $\kappa \approx 4 \times 10^{-3}$, corresponding to $\xi = 1/5$, $v_0 \approx 1.7 \times 10^{16}$ GeV ($N_Q \approx 55$, $\beta = 1$). The scales $M \approx 1.45 \times 10^{16}$ GeV, $m \approx 7.23 \times 10^{15}$ GeV, the inflaton mass $m_{\text{infl}} \approx 4.1 \times 10^{13}$ GeV, and the inflationary scale, which characterizes the inflationary vacuum energy density, $v_{\text{infl}} = \kappa^{1/2}m \approx 4.57 \times 10^{14}$ GeV. The spectral index $n_s \approx 0.985$ [86]. It is interesting to note that this scenario can also be realized [82] with only renormalizable superpotential couplings.

11.2. Smooth Hybrid Inflation

An alternative solution to the magnetic monopole problem of hybrid inflation has been proposed in Ref. [18]. We will present it here within the SUSY PS model of Sec. 11.1, although it can be applied to other semi-simple gauge groups too. The idea is to impose an extra Z_2 symmetry under which $H^c \to -H^c$. The whole structure of the model remains unchanged except that now only even powers of the combination $\bar{H}^c H^c$ are allowed in the superpotential terms.

The inflationary superpotential in Eq. (71) becomes

$$\delta W = S \left(-\mu^2 + \frac{(\bar{H}^c H^c)^2}{M_S^2} \right), \tag{76}$$

where we absorbed the dimensionless parameters κ, β in μ, M_S. The resulting scalar potential V is then given by

$$\tilde{V} = \frac{V}{\mu^4} = (1 - \tilde{\chi}^4)^2 + 16\tilde{\sigma}^2 \tilde{\chi}^6, \tag{77}$$

where we used the dimensionless fields $\tilde{\chi} = \chi/2(\mu M_S)^{1/2}$, $\tilde{\sigma} = \sigma/2(\mu M_S)^{1/2}$ with χ, σ being normalized real scalar fields defined by $\bar{\nu}_H^c = \nu_H^c = \chi/2$, $S = \sigma/\sqrt{2}$ after rotating $\bar{\nu}_H^c$, ν_H^c, S to the real axis.

The emerging picture is completely different. The flat direction at $\tilde{\chi} = 0$ is now a local maximum with respect to $\tilde{\chi}$ for all values of $\tilde{\sigma}$ and two new symmetric valleys of minima appear [18, 21] at

$$\tilde{\chi} = \pm\sqrt{6}\tilde{\sigma} \left[\left(1 + \frac{1}{36\tilde{\sigma}^4} \right)^{\frac{1}{2}} - 1 \right]^{\frac{1}{2}}. \tag{78}$$

They contain the SUSY vacua lying at $\tilde{\chi} = \pm 1$, $\tilde{\sigma} = 0$ and possess a slope already at the classical level. So, in this case, there is no need of radiative corrections for driving the inflaton. The potential on these paths is [18, 21]

$$\tilde{V} = 48\tilde{\sigma}^4 \left[72\tilde{\sigma}^4 \left(1 + \frac{1}{36\tilde{\sigma}^4} \right) \left(\left(1 + \frac{1}{36\tilde{\sigma}^4} \right)^{\frac{1}{2}} - 1 \right) - 1 \right]. \tag{79}$$

The system follows a particular inflationary path and ends up at a particular point of the vacuum manifold leading to no production of monopoles.

The end of inflation is not abrupt since the inflationary path is stable with respect to $\tilde{\chi}$ for all $\tilde{\sigma}$'s. It is determined by using the ϵ and η criteria. Moreover, as it has been shown [87], the initial values (at t_P) of the fields which can lead to adequate inflation in this model are less restricted than in the standard SUSY hybrid inflationary scenario.

This model allows us to take the VEV $v_0 = (\mu M_S)^{1/2}$ of $\bar{H}^c$, H^c equal to the SUSY GUT VEV. COBE [4] then yields $M_S \approx 7.87 \times 10^{17}$ GeV and $\mu \approx 1.04 \times 10^{15}$ GeV for $N_Q \approx 57$. Inflation ends at $\sigma = \sigma_0 \approx 1.08 \times 10^{17}$ GeV, while our present horizon crosses outside the inflationary horizon at $\sigma = \sigma_Q \approx 2.72 \times 10^{17}$ GeV. Finally, $m_{\text{infl}} = 2\sqrt{2}\mu^2/v_0 \approx 1.07 \times 10^{14}$ GeV and the spectral index $n_s \approx 0.97$.

12. Conclusions

We summarized the shortcomings of the SBB cosmological model and their resolution by inflation, which suggests that the universe underwent a period of exponential expansion. This may have happened during the GUT phase transition at which the relevant Higgs field was displaced from the vacuum. This field (inflaton) could then, for some time, roll slowly towards the vacuum providing an almost constant vacuum energy density. Inflation generates the density perturbations needed for the large scale structure of the universe and the temperature

fluctuations of the CMBR. After the end of inflation, the inflaton performs damped oscillations about the vacuum, decays, and reheats the universe.

The early inflationary models required tiny parameters. This problem was solved by hybrid inflation which uses two real scalar fields. One of them provides the vacuum energy density for inflation while the other one is the slowly rolling field. Hybrid inflation arises naturally in many SUSY GUTs, but leads to a disastrous overproduction of magnetic monopoles. We constructed two extensions of SUSY hybrid inflation which do not suffer from this problem.

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