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Scalar fields in Cosmology: dark matter and inflation

L. Arturo Ureña-López

Departamento de Física, DCI, Campus León, Universidad de Guanajuato, 37150, León, Guanajuato, México

E-mail: lurena@ugto.mx

Abstract. Scalar fields have been widely used in cosmology during the last three decades, but it is until now that we have been able to fully understand their role as possible major matter components for the evolution of the Universe. Here we briefly present recent studies on scalar fields that show how their intrinsic properties are translated into observables related to the process of structure formation. In the first case, we consider a massive scalar field as a dark matter component, and present the corresponding mass power spectrum. It is confirmed that one of the distinctive features of the model is the suppression of structure at small cosmological scales. In the second case, we describe a generic method to find inflationary solutions without the need to be specific about the scalar potential. The method shows that single-field models of inflation can be classified in two groups according to their predictions of inflationary observables.

1. Introduction
What we actually call the standard model of Cosmology is a phenomenological description of the physics that seems to be at play to reproduce the observed Universe. The predictions of such model are in strikingly good agreement with a variety of observational constraints, and it is because of this reason that it is the fiducial model used in present and future observational forecasts. The model is also known as the ΛCDM model, for the initials of its main matter components: the cosmological constant Λ and Cold Dark Matter (CDM). The core of the model is based on a spatially-flat expanding Universe whose gravitational dynamics is governed by the equations of motion of General Relativity; in addition, the seeds of structure formation are Gaussian distributed adiabatic fluctuations with an almost scale-invariant spectrum[1].

One major concern in the present literature is the understanding of the formation of cosmological structure, and one principal matter component in this process is the CDM. The latter is as simple as it gets: a pressureless fluid of particles that have no direct, weak at most, interaction with other matter fluids apart from the gravitational one. These two basic properties are enough to provide a complex picture of structure formation through hierarchical assembling that fits data at different scales[2, 3].

Apart from the CDM component, the ΛCDM model also needs to set up the appropriate initial conditions so that the formation of structure can start as required by present observations of the distribution of galaxies. Such initial conditions, which we refer above as the seeds of structure formation, must have been produced by some separate process in the early Universe. It is because of this that the idea of inflation, which refers to an accelerated stage in the very early Universe, remains as one of the cornerstone ideas of modern Cosmology[4, 5, 6].

In this paper we will present the case of a scalar field as a model for dark matter and inflation. This is possible because of the flexible behavior that scalar fields exhibit within

different cosmological settings (see for instance[7]). The results that will be presented in Secs. 2 and 3 below are part of recent studies on scalar fields[8, 9], with the aim to extract from them useful features that may help us to understand the subtleties behind the formation of cosmological structure.

2. Scalar field as dark matter

Here we present the equations of motion that must be solved in order to find the general behavior of cosmological scalar fields. For that, we shall consider the same features as that of the ΛCDM model: a spatially-flat Universe populated with standard matter fluids such as: radiation, neutrinos, baryons, and a cosmological constant Λ as the dark energy component. In addition, we consider a scalar field ϕ endowed with a generic potential $V(\phi)$, its Lagrangian density given by $\mathcal{L}_\phi = -(1/2)(\partial\phi)^2 - V(\phi)$.

If we use ρ_I and p_I to denote the energy density and pressure, respectively, for each matter fluid in the model apart from the scalar field, the full equations of motion for the background dynamics are:

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{\kappa^2}{3}\left[\sum_I \rho_I + \frac{1}{2}\dot{\phi}^2 + V(\phi)\right], \quad \dot{H} = -\frac{\kappa^2}{2}\left[\sum_I(\rho_I + p_I) + 2\dot{\phi}^2\right], \quad (1)$$

$$\dot{\rho}_I = -3H(\rho_I + p_I), \quad \ddot{\phi} = -3H\dot{\phi} - \partial_\phi V, \quad (2)$$

where $\kappa^2 = 8\pi G$, a dot denotes derivative with respect to cosmic time t, and $H = \dot{a}/a$ is the Hubble parameter, with a the scale factor of the Universe.

In order to go beyond the background dynamics, we must take another step and calculate the linear perturbations to the background quantities; in this way, we will be able to study the influence of the scalar field on, for instance, the evolution of the CMB anisotropies. For that, we assume the synchronous gauge of metric perturbations with the line element $ds^2 = -dt^2 + a^2(t)(\delta_{ij} + h_{ij})dx^i dx^j$. The scalar field is given by $\phi(x,t) = \phi(t) + \varphi(x,t)$, where $\phi(t)$ is the background (homogeneous) field in Eqs. (1) to (2), and φ is its linear perturbation. Thus, the linearized KG equation for a Fourier mode $\varphi(k,t)$ reads[10, 11, 12, 13]:

$$\ddot{\varphi} = -3H\dot{\varphi} - (k^2/a^2 + \partial_\phi^2 V)\varphi - \frac{1}{2}\dot{\phi}\dot{h}, \quad (3)$$

where h is the trace of scalar metric perturbations (with $\dot{h}$ known as the metric continuity), and k is a comoving wavenumber.

We now apply the formalism above to the particular case of a scalar field as a dark matter component. For that, we need to consider a massive scalar field, and then the potential is:

$$V(\phi) = (1/2)m^2\phi^2, \quad (4)$$

where m is the mass scale of the scalar field. Other potentials have been considered in the literature, but their common feature is that they all can be approximated by the functional form (4), see for instance[14, 15, 16, 17].

The expected behavior at late times of the scalar field, under the influence of the quadratic potential (4), is that of rapid oscillations around the minimum of the potential located at $\phi = 0$. This is well known as a problematic regime which prevents an accurate solution of the scalar field equations of motion. However, as shown with more details in[8], such troublesome regime can be avoided by an appropriate change to polar variables for both the background and linear perturbations. Actually, it was in[8] that an accurate calculation of the full cosmological solutions was presented for the first time.
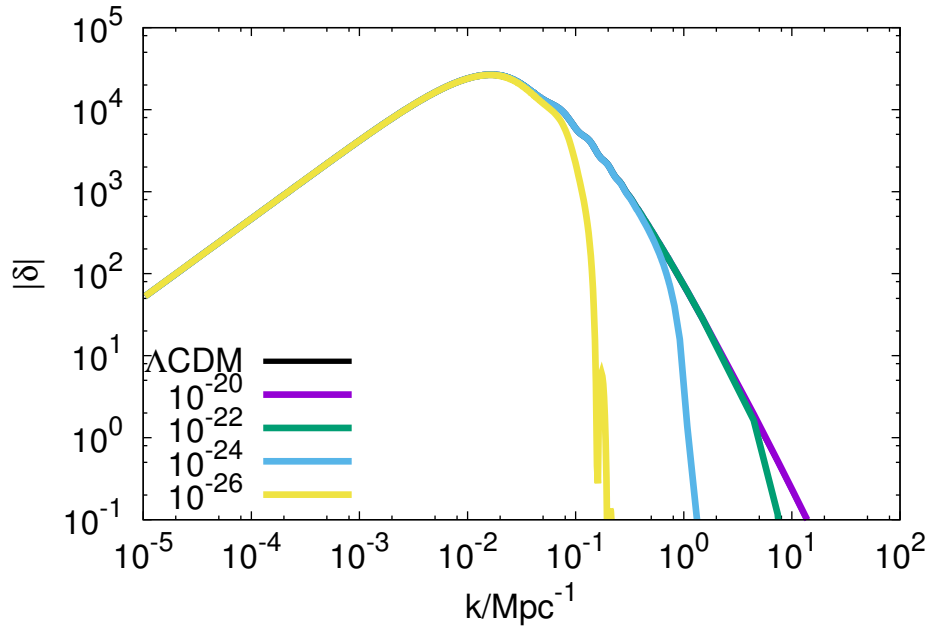
Figure 1. Mass power spectrum of scalar field density contrast calculated from the solution of Eq. (3) for the linear perturbation φ. The different colored lines denote the resultant power spectrum for different values, in units of eV, of the scalar field mass m (see Eq. (4)). We also show the case of the standard cosmological model ΛCDM (black solid line). The most prominent feature of the scalar field MPS is the sudden cut-off of the spectrum for large values of the wavenumber k, which means the suppression of cosmological structure at small scales.

As an example of the distinctive features of a scalar field as dark matter we present in figure 1 the mass power spectrum (MPS) of scalar field linear perturbations. The MPS was calculated with an amended version of the Boltzmann code CLASS[18] (the details of the amendments can be found in[8]). The most salient feature of the scalar field MPS is the sudden cut-off that appears for large values of the wavenumber k (here in units of Mpc^{-1}), which is interpreted as the suppression of cosmological structure at small scales $L < 1\,\mathrm{Mpc}$. The wavenumber at which the suppression starts is mainly determined by the scalar field mass m: the cosmological structure is suppressed at larger scales for smaller values of the scalar field mass, although the effect is only noticeable for masses $m < 10^{-20}\mathrm{eV}$.

3. Scalar fields as inflation

In the case of inflationary models with a single scalar field, the equations of motion can be simplified because of the assumption that the scalar field is the only matter field present in the early Universe. Another simplification is that we do not need to solve the equations of motion for linear perturbations, as there already exist general results for the features of the primordial perturbations that only require input from the dynamics in the background[4, 5, 6, 19].

Thus, the only equations we have to solve are:

$$H^2 = \frac{\kappa^2}{3}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi)\right], \quad \dot{H} = -\kappa^2\dot{\phi}^2, \quad \ddot{\phi} = -3H\dot{\phi} - \partial_\phi V. \tag{5}$$

In order to have a more manageable system of differential equations, we now apply a *polar*

change of variables in the form:

$$\frac{\kappa\dot{\phi}}{\sqrt{6}H} = \sin(\theta/2)\,, \quad \frac{\kappa V^{1/2}}{\sqrt{3}H} = \cos(\theta/2)\,, \quad y_1 = -2\sqrt{2}\frac{\partial_\phi V^{1/2}}{H}\,, \quad y_2 = -\frac{4\sqrt{3}}{\kappa}\frac{\partial_\phi^2 V^{1/2}}{H}\,. \qquad (6)$$

Thus, after straightforward manipulations, Eqs. (5) finally become:

$$\theta' = -3\sin\theta + y_1\,, \qquad (7)$$

$$y_1' = \frac{3}{2}\left(1 - \cos\theta\right)y_1 + \sin(\theta/2)y_2\,, \qquad (8)$$

where a prime denotes a derivative with respect to the number of e-folds of expansion $N \equiv \ln(a)$. In principle, we would also require an equation of motion for the second potential variable y_2, but we will show that this is not strictly necessary for the description of the inflationary solutions.

In any given inflationary setting, we expect the scalar field to change slowly from a potential $(V(\phi) \gg \dot{\phi}^2)$ to a kinetic dominated phase $(V(\phi) \ll \dot{\phi}^2)$. This is also called de Sitter inflation, as the first part of the inflationary state is similar to that produced by a cosmological constant. Such change in the behavior of the scalar field can be parametrised more precisely in terms of the scalar field equation of state (EoS) w_ϕ, which is defined as:

$$w_\phi = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)} = -\cos\theta\,, \qquad (9)$$

where the last expression on the rhs of Eq. (9) appears from the polar change of variables (6).

Hence, in terms of the EoS, the inflationary phase is represented by $w_\phi : -1 \to 1$; actually, it is only required that $w_\phi : -1 \to -1/3$ to span an accelerating phase. The latter can be written in terms of the angular variable as $\theta : 0 \to \theta_{end}$, where $\theta_{end} = \arccos(1/3) = 2\arcsin(1/\sqrt{3}) \simeq 1.2309\ldots$. This suggests that we can try a series solution of Eq. (8) with the following ansatz for the potential variable y_1:

$$y_1 = \sum_{j=1} k_{1j}\theta^j\,, \qquad (10)$$

where k_{1j} are all constant coefficients. Once with the values of k_{1j} at hand we can make an expansion of Eq. (7) in the form:

$$\theta' = (k_{11} - 3)\theta + k_{12}\theta^2 + (k_{13} + 1/2)\theta^3 + \ldots\,. \qquad (11)$$

Eq. (11) can then be solved at any order to provide a solution: $\theta_N = \theta_N(k_{1j}, \theta_{end}, N)$, where the subscript N denotes the number of e-foldings before the end of inflation.

For the calculation of inflationary quantities, we choose the so-called Hubble slow-roll (HSR) variables[20], under which various inflationary observables can be written in terms of the so-called HSR parameters ϵ_H and η_H. For instance, the spectral index n_S and the tensor-to-scalar ratio r, are given at first order in the HSR parameters as:

$$1 - n_S = 4\epsilon_H - 2\eta_H = 12\sin^2(\theta/2) - 6 + y_1\cot(\theta/2)\,, \qquad (12)$$

$$r = 16\epsilon_H = 48\sin^2(\theta/2)\,, \qquad (13)$$

where the last expressions on the rhs appear from the change of polar variables (6). Taking into account the expansions (10), Eqs. (13) can be written alternatively as:

$$1 - n_S \simeq 2(k_{11} - 3) + 2k_{12}\theta_N + (2k_{13} - k_{11}/6 + 3)\theta_N^2\,, \quad r \simeq 12\theta_N^2\,. \qquad (14)$$

In principle, Eqs. (11) and (14) is all what we need to find the values of the inflationary observables n_S and r, once the values of the expansion coefficients k_{1j} have been determined by other means. There is indeed a systematic, though a bit cumbersome, procedure to find the values of k_{1j} for any scalar field potential $V(\phi)$, the details of which can be found in[9].

One remarkable result of the above method is that de Sitter inflation in single-field models can be comprised in two general classes, which we called Class I and Class II, irrespective of the particular form of the scalar field potential $V(\phi)$. The key parameter for the new classification of inflationary models is the coefficient k_{12}. In the large N limit of the two classes we end up with only two typical behaviors in the solutions, namely[9]: $\theta_N \sim N^{-1/2}$ ($k_{12} = 0$: Class I), or $\theta_N \sim N^{-1}$ ($k_{12} \neq 0$: Class II), which in turn suggest that either $r \sim N^{-1}$, or $r \sim N^{-2}$, respectively, even though $1 - n_S \sim N^{-1}$ at the leading order in N for the two classes. Such classification of inflationary solutions was suggested before in[21, 22, 23], but the method in[9] is the first to give a general explanation of its existence.

To finish this section, we show in figures 2 and 3 the generic predictions for the observables n_S and r of the two classes of inflationary solutions, for $N = 60$, which is a typical number or e-folds before the end of inflation. It can be seen that it is Class I, rather than Class II, the one set of models that can provide a good fit to the observational data[9].
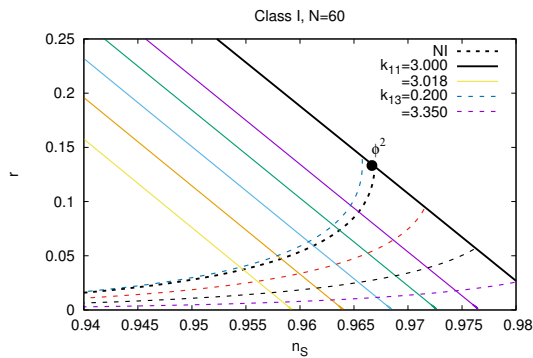

Figure 2. General results on the plane n_S vs r for the Class I of single field models of inflation, for different values of the coefficients k_{11} and k_{13}. The plots show the results for $N = 60$. Notice that Class I is not completely ruled out by observations, mostly because it can provide low enough values of the tensor-to-scalar ratio r. The predictions from the typical Large Field Inflation for the potential ϕ^2[24], and Natural Inflation (NI)[25], are indicated in the figure for comparison.
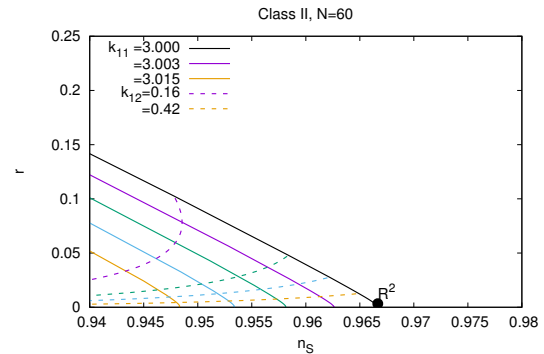
Figure 3. General results on the plane n_S vs r for Class II of single field models of inflation, for different values of the coefficients k_{11} and k_{12}. The plots show the results for $N = 60$. Notice that Class II covers a smaller area than Class I (see Fig. 2), and then it has less freedom to fit the observational data. The predictions from the typical Starobinsky's model (R^2)[26] model is indicated in the figure for comparison.

4. Conclusions and perspectives

We have presented two common instances in which scalar fields are used in Cosmology in order to provide models for unanswered riddles of the Universe.

In the first one, related to dark matter, we showed how the MPS, one of the most important quantities for the process of structure formation, is affected by the presence of a scalar field: the appearance of sudden cut-off at large wavenumbers, which is translated in the suppression of structure at small scales. This cut-off in the MPS has been the subject of study in many

recent works about models of dark matter with scalar fields: from constraints coming from CMB anisotropies and other astrophysical probes[27, 28, 29, 14], to the most recent topic of the non-linear process of structure formation[30, 31], and the analysis of objects of galactic size[32, 33, 34].

The second instance was about single-field models of inflation and their generic predictions. Although there is already a large tradition on the study of inflationary models (see[19] and references therein), the method briefly presented in Sec. 3 provides a brand new way to show the intrinsic properties of scalar field models of inflation. Interestingly enough, the method gives a more solid confirmation of the existence of two generic classes of inflationary models. However, the question for the correct model, i.e. the particular $V(\phi)$ that could be the ultimate responsible for inflation, still remains open.

Acknowledgments

References

[1] Ade P *et al.* (Planck Collaboration) 2015 (*Preprint* `1502.02114`)
[2] Vogelsberger M, Genel S, Springel V, Torrey P, Sijacki D, Xu D, Snyder G F, Nelson D and Hernquist L 2014 *Mon. Not. Roy. Astron. Soc.* **444** 1518–1547 (*Preprint* `1405.2921`)
[3] Genel S, Vogelsberger M, Springel V, Sijacki D, Nelson D, Snyder G, Rodriguez-Gomez V, Torrey P and Hernquist L 2014 *Mon. Not. Roy. Astron. Soc.* **445** 175–200 (*Preprint* `1405.3749`)
[4] Liddle A R and Lyth D 2000 *Cosmological inflation and large scale structure*
[5] Mukhanov V 2005 *Physical foundations of cosmology*
[6] Bassett B A, Tsujikawa S and Wands D 2006 *Rev. Mod. Phys.* **78** 537–589 (*Preprint* `astro-ph/0507632`)
[7] Copeland E J, Sami M and Tsujikawa S 2006 *Int. J. Mod. Phys.* **D15** 1753–1936 (*Preprint* `hep-th/0603057`)
[8] Urea-Lpez L A and Gonzalez-Morales A X 2016 *JCAP* **1607** 048 (*Preprint* `1511.08195`)
[9] Urena-Lopez L A 2015 (*Preprint* `1512.07142`)
[10] Ratra B 1991 *Phys.Rev.* **D44** 352–364
[11] Ferreira P G and Joyce M 1997 *Phys.Rev.Lett.* **79** 4740–4743 (*Preprint* `astro-ph/9707286`)
[12] Ferreira P G and Joyce M 1998 *Phys.Rev.* **D58** 023503 (*Preprint* `astro-ph/9711102`)
[13] Perrotta F and Baccigalupi C 1999 *Phys.Rev.* **D59** 123508 (*Preprint* `astro-ph/9811156`)
[14] Matos T and Urena-Lopez L A 2001 *Phys.Rev.* **D63** 063506 (*Preprint* `astro-ph/0006024`)
[15] Sahni V and Wang L M 2000 *Phys.Rev.* **D62** 103517 (*Preprint* `astro-ph/9910097`)
[16] Beyer J and Wetterich C 2014 (*Preprint* `1407.0141`)
[17] Marsh D J E and Silk J 2013 *Mon.Not.Roy.Astron.Soc.* **437** 2652 (*Preprint* `1307.1705`)
[18] Lesgourgues J 2011 (*Preprint* `1104.2932`)
[19] Vennin V, Koyama K and Wands D 2015 *JCAP* **1511** 008 (*Preprint* `1507.07575`)
[20] Liddle A R, Parsons P and Barrow J D 1994 *Phys.Rev.* **D50** 7222–7232 (*Preprint* `astro-ph/9408015`)
[21] Roest D 2014 *JCAP* **1401** 007 (*Preprint* `1309.1285`)
[22] Galante M, Kallosh R, Linde A and Roest D 2014 (*Preprint* `1412.3797`)
[23] Mukhanov V 2013 *Eur.Phys.J.* **C73** 2486 (*Preprint* `1303.3925`)
[24] Linde A D 1983 *Phys.Lett.* **B129** 177–181
[25] Freese K and Kinney W H 2014 (*Preprint* `1403.5277`)
[26] Starobinsky A A 1980 *Phys.Lett.* **B91** 99–102
[27] Hlozek R, Grin D, Marsh D J E and Ferreira P G 2014 (*Preprint* `1410.2896`)
[28] Marsh D J and Ferreira P G 2010 *Phys.Rev.* **D82** 103528 (*Preprint* `1009.3501`)
[29] Magana J, Matos T, Suarez A and Sanchez-Salcedo F 2012 *JCAP* **1210** 003 (*Preprint* `1204.5255`)
[30] Schive H Y, Chiueh T and Broadhurst T 2014 (*Preprint* `1406.6586`)
[31] Woo T P and Chiueh T 2009 *Astrophys.J.* **697** 850–861 (*Preprint* `0806.0232`)
[32] Du X, Behrens C and Niemeyer J C 2016 (*Preprint* `1608.02575`)
[33] Schwabe B, Niemeyer J C and Engels J F 2016 *Phys. Rev.* **D94** 043513 (*Preprint* `1606.05151`)
[34] Martinez-Medina L A, Bray H L and Matos T 2015 *JCAP* **1512** 025 (*Preprint* `1505.07154`)