Modified gravity and its reconstruction from the universe expansion history

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Abstract. We develop the reconstruction program for the number of modified gravities: scalar-tensor theory, $f(R)$, $F(G)$ and string-inspired, scalar-Gauss-Bonnet gravity. The known (classical) universe expansion history is used for the explicit and successful reconstruction of some versions (of special form or with specific potentials) from all above modified gravities. It is demonstrated that cosmological sequence of matter dominance, deceleration-acceleration transition and acceleration era may always emerge as cosmological solutions of such theory. Moreover, the late-time dark energy FRW universe may have the approximate or exact ΛCDM form consistent with three years WMAP data. The principal possibility to extend this reconstruction scheme to include the radiation dominated era and inflation is briefly mentioned. Finally, it is indicated how even modified gravity which does not describe the matter-dominated epoch may have such a solution before acceleration era at the price of the introduction of compensating dark energy.

1. Introduction

The explanation of the current universe speed-up which is due to mysterious dark energy remains to be one of the most fundamental problems of modern cosmology and theoretical physics. The number of theoretical models have been developed aiming to describe the dark energy universe (for recent review, see [1, 2]). The obtained observational data indicate that current universe has the effective equation of state parameter w being very close to -1 (for the review of observational data from the theoretical point of view, see [3] and for description of observable cosmological parameters, see [4]). Hence, it is still not quite clear if the universe lives in the phantom era (w less than -1), in the (most probably) cosmological constant epoch ($w = -1$) or in the quintessence phase (w more than -1). In such a situation, even phantom cosmological models (see [5, 6, 7] and refs. therein) which show quite strange properties are not ruled out because even being not phantomic one, the universe may currently enter to phantom dominated epoch.

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The dark energy problem may have the gravitational origin, calling for possible late-time modification of General Relativity. The number of modified gravity models (for a review, see [8]) starting from the simplest $1/R$ theory [9, 10] have been proposed as gravitational alternative for dark energy. Such models may be inspired by string/M-theory considerations [11, 12]. They may lead to (phantom, cosmological constant or quintessence) effective cosmology at late times without the need to introduce the scalar fields with strange properties (like negative kinetic energy) or with complicated potentials.

Generally speaking, modified gravity looks very attractive as it gives the qualitative answers to the number of fundamental questions about dark energy. Indeed, the origin of dark energy may be explained by some sub-leading gravitational terms which become relevant with the decrease of the curvature (at late times). Moreover, there are many proposals to consider the gravitational terms relevant at high curvature (perhaps, due to quantum gravity effects) as the source of the early-time inflation. Hence, there appears the possibility to unify and to explain both: the inflation and late-time acceleration as the modified gravity effects. Meanwhile, at the intermediate epoch the gravity may be approximated by General Relativity. Similarly, the coincidence problem may be explained. It is remarkable that modified gravity may pretend to describe also dark matter.

There are various ways to describe the modified gravity theory. Not only $f(R)$ but also modified Gauss-Bonnet theory [13] may be presented in the form of scalar-tensor gravity (perhaps, with higher derivative terms). Note that scalar-tensor theory itself may be understood as some kind of modified gravity. Moreover, one can present modified gravity as General Relativity with quite complicated ideal fluid [14] having non-standard equation of state. As it has been explained in ref.[15], this mathematical equivalence of different representations does not mean the physical equivalence (due to transformed form of the physical metric tensor). Nevertheless, it is remarkable that modified $f(R)$ gravity has some close analogy with non-trivial $f(\rho)$ equation of state ideal fluid [16, 7]. In general, modified gravity may be considered as General Relativity (GR) with inhomogeneous equation of state ideal fluid [17]. (The particular example of this sort is given by ideal fluid with time-dependent bulk viscosity [18]).

Of course, modified gravity should be fitted against the observational data (as is done in refs.[19, 20], mainly, for $f(R)$ theory). Similarly, it should pass the Solar System tests (for $f(R)$ models this is widely discussed in refs.[21, 22, 23, 20] while SdS metrics are studied in [24]). In principle, it may pass the requested tests as some cancellation of the contributions between different terms may occur as in the model of ref.[23]. Moreover, the corresponding initial (boundary) conditions of the form of the action at some (current) time of the universe expansion history may be used to bring the action to the approximate GR form (with possible earlier and (or) future deviations from it).

In such a situation, any realistic classical gravitation should describe the known sequence of the cosmological epochs (perhaps, without the inflationary era where quantum gravity effects may be quite strong). The purpose of this work is to review several versions of modified gravity and to show how it may be reconstructed from the known universe expansion history (for recent discussion of reconstruction program for usual gravity with matter, see [25]). In the next section this problem is solved for two scalar-tensor theory. The reconstruction method is developed. Several examples of realistic cosmology are considered: oscillating universe, ΛCDM cosmology and asymptotically de Sitter space. The corresponding scalar potentials admitting the sequence of matter dominated phase, decceleration-acceleration transition and acceleration are constructed in all above cases. Section three is devoted to the reconstruction of $f(R)$ gravity following ref.[15, 26]. The modified gravity admitting the unification of matter dominated and acceleration era is formulated. Its asymptotic behaviour at early and late times is found. It is shown that it does not conflict with three years WMAP data.

In section four the alternative scenario for the model of ref.[23] is developed. Adding the

compensating dark energy which role is negligible at the acceleration epoch it is shown that modified gravity [23] may be viable (the matter dominated phase occurs before the cosmic acceleration). In section five the reconstruction of string-inspired scalar-Gauss-Bonnet gravity and of modified Gauss-Bonnet gravity is made. Such theories have been suggested as dark energy models in refs.[12, 13], respectively. Its versions where dark energy universe emerges after matter dominance are given. Some discussion and outlook are presented in the Discussion section.

2. The reconstruction of scalar-tensor theory from the expansion history of the universe

2.1. General formulation

In this subsection, we show how *any* cosmology can be reproduced in the scalar-tensor theory (or scalar-tensor theory may be reconstructed). Our consideration is based on the method developed in ref.[27] (for recent study of scalar-tensor cosmology applied for dark energy description see [28, 7] and for earlier attempts, see [29]). Note that studying the transition from non-phantom phase to phantom phase, the instability of the one scalar model becomes infinite at the transition point [27]. That is why we only consider two scalar-tensor theory, whose instability could be always finite. Moreover, with two scalars it is easier to fit with universe history expansion.

We now consider two scalar-tensor gravity

$$S = \int d^4x \sqrt{-g} \left\{ \frac{1}{2\kappa^2} R - \frac{1}{2}\omega(\phi)\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}\eta(\chi)\partial_\mu\chi\partial^\mu\chi - V(\phi,\chi) \right\} \ . \tag{1}$$

Here $\omega(\phi)$ is a function of the scalar field ϕ and $\eta(\chi)$ is a function of the another scalar field χ. It is assumed the spatially-flat FRW metric $ds^2 = -dt^2 + a(t)^2 \sum_{i=1}^{3} \left(dx^i\right)^2$ and that ϕ and χ only depend on the time coordinate t. The FRW equations lead to

$$\omega(\phi)\dot{\phi}^2 + \eta(\chi)\dot{\chi}^2 = -\frac{2}{\kappa^2}\dot{H} \ , \quad V(\phi,\chi) = \frac{1}{\kappa^2}\left(3H^2 + \dot{H}\right) \ . \tag{2}$$

Then if

$$\omega(t) + \eta(t) = -\frac{2}{\kappa^2}f'(t) \ , \quad V(t,t) = \frac{1}{\kappa^2}\left(3f(t)^2 + f'(t)\right) \ , \tag{3}$$

the explicit solution follows

$$\phi = \chi = t \ , \quad H = f(t) \ . \tag{4}$$

One may choose that ω should be always positive and η be always negative, for example

$$\omega(\phi) = -\frac{2}{\kappa^2}\left\{ f'(\phi) - \sqrt{\alpha^2 + f'(\phi)^2} \right\} > 0 \ ,$$

$$\eta(\chi) = -\frac{2}{\kappa^2}\sqrt{\alpha^2 + f'(\chi)^2} < 0 \ . \tag{5}$$

Here α is a constant. Define a new function $\tilde{f}(\phi,\chi)$ by

$$\tilde{f}(\phi,\chi) \equiv -\frac{\kappa^2}{2}\left(\int d\phi\, \omega(\phi) + \int d\chi\, \eta(\chi) \right) \ . \tag{6}$$

The constant of the integration could be fixed to require

$$\tilde{f}(t,t) = f(t) \ . \tag{7}$$

If $V(\phi, \chi)$ is given by using $\tilde{f}(\phi, \chi)$ as

$$V(\phi, \chi) = \frac{1}{\kappa^2} \left(3\tilde{f}(\phi, \chi)^2 + \frac{\partial \tilde{f}(\phi, \chi)}{\partial \phi} + \frac{\partial \tilde{f}(\phi, \chi)}{\partial \chi} \right) , \tag{8}$$

the FRW and the scalar field equations are also satisfied:

$$
\begin{aligned}
0 &= \omega(\phi)\ddot{\phi} + \frac{1}{2}\omega'(\phi)\dot{\phi}^2 + 3H\omega(\phi)\dot{\phi} + \frac{\partial \tilde{V}(\phi, \chi)}{\partial \phi} , \\
0 &= \eta(\chi)\ddot{\chi} + \frac{1}{2}\eta'(\chi)\dot{\chi}^2 + 3H\eta(\chi)\dot{\chi} + \frac{\partial \tilde{V}(\phi, \chi)}{\partial \chi} .
\end{aligned}
\tag{9}
$$

We now investigate the (in)stability of the model. By introducing the new quantities, X_ϕ, X_χ, and Y as

$$X_\phi \equiv \dot{\phi} , \quad X_\chi \equiv \dot{\chi} , \quad Y \equiv \frac{\tilde{f}(\phi, \chi)}{H} , \tag{10}$$

the FRW equations and the scalar field equations (9) are:

$$
\begin{aligned}
\frac{dX_\phi}{dN} &= -\frac{\omega'(\phi)}{2H\omega(\phi)}\left(X_\phi^2 - 1\right) - 3(X_\phi - Y) , \\
\frac{dX_\chi}{dN} &= -\frac{\eta'(\chi)}{2H\eta(\chi)}\left(X_\chi^2 - 1\right) - 3(X_\chi - Y) , \\
\frac{dY}{dN} &= \frac{1}{2\kappa^2 H^2}\left\{ X_\phi\left(X_\phi Y - 1\right) + X_\chi\left(X_\chi Y - 1\right) \right\} .
\end{aligned}
\tag{11}
$$

Here $d/dN \equiv H^{-1}d/dt$. In the solution (4), $X_\phi = X_\chi = Y = 1$. The following perturbation may be considered

$$X_\phi = 1 + \delta X_\phi , \quad X_\chi = 1 + \delta X_\chi , \quad Y = 1 + \delta Y . \tag{12}$$

Hence

$$
\frac{d}{dN}\begin{pmatrix} \delta X_\phi \\ \delta X_\chi \\ \delta Y \end{pmatrix} = M \begin{pmatrix} \delta X_\phi \\ \delta X_\chi \\ \delta Y \end{pmatrix} ,
$$

$$
M \equiv \begin{pmatrix} -\frac{\omega'(\phi)}{H\omega(\phi)} - 3 & 0 & 3 \\ 0 & -\frac{\eta'(\chi)}{H\eta(\chi)} - 3 & 3 \\ \frac{1}{2\kappa^2 H^2} & \frac{1}{2\kappa^2 H^2} & \frac{1}{\kappa^2 H^2} \end{pmatrix} .
\tag{13}
$$

The eigenvalues of the matrix M are given by solving the following eigenvalue equation

$$
\begin{aligned}
0 &= \left(\lambda + \frac{\omega'(\phi)}{H\omega(\phi)} + 3\right)\left(\lambda + \frac{\eta'(\chi)}{H\eta(\chi)} + 3\right)\left(\lambda - \frac{1}{\kappa^2 H^2}\right) \\
&\quad + \frac{3}{2\kappa^2 H^2}\left(\lambda + \frac{\omega'(\phi)}{H\omega(\phi)} + 3\right) + \frac{3}{2\kappa^2 H^2}\left(\lambda + \frac{\eta'(\chi)}{H\eta(\chi)} + 3\right) .
\end{aligned}
\tag{14}
$$

The eigenvalues (14) for the two scalar model are clearly finite. Hence, the instability could be finite. In fact, right on the transition point where $\dot{H} = f'(t) = 0$ and therefore $f'(\phi) = f'(\chi) = 0$, for the choice in (5), we find

$$\omega(\phi) = -\eta(\chi) = \frac{2\alpha}{\kappa^2} , \quad \omega'(\phi) = -\frac{2\ddot{H}}{\kappa^2} , \quad \eta'(\chi) = 0 . \tag{15}$$

Then the eigenvalue equation (14) reduces to

$$
0 = \lambda^3 + (-A - B + 6)\lambda^2 + (AB - 3A - 3B + 9)\lambda - \frac{3}{2}AB + 9B \ ,
$$

$$
A \equiv \frac{\ddot{H}}{\alpha} \ , \quad B \equiv \frac{1}{\kappa^2 H^2} \ . \tag{16}
$$

Here we have chosen $\alpha > 0$. Then the eigenvalues are surely finite, which shows that even if the solution (4) is not stable, the solution has non-vanishing measure and therefore the transition from non-phantom phase to phantom one can occur. We should also note that the solution (4) can be in fact stable. For example, we consider the case $A, B \to 0$. Eq.(16) further reduces to

$$
0 = \lambda\left(\lambda + 3\right)^2 \ . \tag{17}
$$

Then the eigenvalues are given by 0 and -3. Since there is no positive eigenvalue, the solution (4) is stable in the case.

It is not difficult to extend the above formulation to the multi-scalars model, whose action is given by

$$
S = \int d^4x\sqrt{-g}\left\{\frac{1}{2\kappa^2}R - \frac{1}{2}\sum_i \omega_i(\phi_i)\partial_\mu\phi_i\partial^\mu\phi_i - V(\phi_i)\right\} \ . \tag{18}
$$

Here $\omega_i(\phi_i)$ is a function of the scalar field ϕ_i. We now choose ω_i to satisfy

$$
\sum_i(t) = -\frac{2}{\kappa^2}f'(t) \tag{19}
$$

by a proper function. and we also choose the potential $V(\phi_i)$ by

$$
V(\phi_i) = \frac{1}{\kappa^2}\left(3\tilde{f}(\phi_i) + \sum_i \frac{\partial \tilde{f}}{\partial \phi_i}\right) \ . \tag{20}
$$

Here

$$
\tilde{f}(\phi_i) \equiv -\frac{\kappa^2}{2}\sum_i \int d\phi_i \omega_i(\phi_i) \ . \tag{21}
$$

The constant of the integration in (21) is determined to satisfy

$$
\left.\tilde{f}(\phi_i)\right|_{\text{all }\phi_i = t} = f(t) \ . \tag{22}
$$

Then a solution of the FRW equations and the scalar field equations are given by

$$
\phi_i = t \ , \quad H = f(t) \ . \tag{23}
$$

The rest consideration coincides with the one given for two-scalars model.

2.2. The examples of scalar-tensor theory reconstruction
We now consider some explicit examples in order to demonstrate how the scalar-tensor theory can be reconstructed from known expansion history of the universe using the above developed formulation.

As a first example, we consider the model given by

$$
f(\phi) = h_0 + h_1 \sin(\nu\phi) \ . \tag{24}
$$

Here it is assumed h_0, h_1, and ν are positive. By choosing $\alpha = h_1 \nu$ in (5), one finds

$$\omega(\phi) = -\frac{2h_1\nu}{\kappa^2}\left\{\cos(\nu\phi) - \sqrt{1 + \cos^2(\nu\phi)}\right\} ,$$

$$\eta(\chi) = -\frac{2h_1}{\nu}\kappa^2\sqrt{1 + \cos^2(\nu\chi)} ,$$

$$\tilde{f}(\phi,\chi) = h_0 + h_1\sin(\nu\phi) - \frac{\sqrt{2}}{\nu}\left\{E\left(1/\sqrt{2}, \nu\phi\right) - E\left(1/\sqrt{2}, \nu\chi\right)\right\} . \tag{25}$$

Here $E(k, x)$ is the second kind elliptic integral defined by

$$E(k, x) = \int_0^x dx\sqrt{1 - k^2\sin^2 x} . \tag{26}$$

Note that similar reconstruction in case of the single scalar-tensor theory was presented in [27]. Eq.(24) shows that the Hubble rate H is given by

$$H = h_0 + h_1\sin(\nu t) , \tag{27}$$

which is oscillating. When $h_0 > h_1 > 0$, H is always positive and the universe is expanding. Since

$$\dot{H} = h_1\nu\cos(\nu t) , \tag{28}$$

when $h_1\nu > 0$, w_{eff}, which is defined by

$$w_{\text{eff}} = \frac{p}{\rho} = -1 - \frac{2\dot{H}}{3H^2} , \tag{29}$$

is greater than -1 (non-phantom phase) when

$$\left(2n - \frac{1}{2}\right)\pi < \nu t < \left(2n + \frac{1}{2}\right)\pi , \tag{30}$$

and less than -1 (phantom phase) when

$$\left(2n + \frac{1}{2}\right)\pi < \nu t < \left(2n + \frac{3}{2}\right)\pi . \tag{31}$$

In (30) and (31), n is an integer. Hence, in the model (24), there occur multiply oscillations between phantom and non-phantom phases. It could be that our universe currently corresponds to late-time acceleration phase in such oscillatory regime (for recent reconstruction examples for oscillatory universe, see [30] and references therein).

The present universe is expanding with acceleration. On the other hand, there occured the earlier matter-dominated period, where the scale factor a behaves as $a \sim t^{2/3}$. Such behavior could be generated by dust in the Einstein gravity. The baryons are dust and (cold) dark matter could be a dust. The ratio of the baryons and the dark matter in the present universe could be $1 : 5$ or $1 : 6$, which should not be changed even in matter dominant era. It is not clear what the dark matter is. For instance, the dark matter might not be the real matter but some (effective) artifact which appears in the modified/scalar-tensor gravity. In the present section, it is assumed that not only dark energy but also the dark matter may originate from the scalar field ϕ.

We now investigate that the transition from the matter dominant period to the acceleration period could be realized in the present formulation (for single scalar-tensor theory, see [31]). In

the following, the contribution from matter could be neglected since the ratio of the matter with the (effective) dark matter could be small.

First example is

$$H = f(t) = g'(t) = g_0 + \frac{g_1}{t} \ . \tag{32}$$

When t is large, the first term in (32) dominates and the Hubble rate H becomes a constant. Therefore, the universe is asymptotically deSitter space, which is an accelerating universe (for recent examples of late-time accelerating cosmology in scalar-tensor theory, see [28, 7]). On the other hand, when t is small, the second term in (32) dominates and the scale factor behaves as $a \sim t^{g_1}$. Therefore if $g_1 = 2/3$, the matter-dominated period could be realized. Here, one of scalars may be considered as usual matter.

Since

$$f'(t) = -\frac{g_1}{t} < 0 \ , \tag{33}$$

instead of (5), by using a positive constant α, we may choose

$$\omega(\phi) = -\frac{2\,(1+\alpha)}{\kappa^2} f'(\phi) > 0 \ , \quad \eta(\chi) = \frac{2\alpha}{\kappa^2} f'(\phi) < 0 \ , \tag{34}$$

that is

$$\omega(\phi) = \frac{2(1+\alpha)}{\kappa^2} \frac{g_1}{\phi^2} \ , \quad \eta(\chi) = \frac{2\alpha}{\kappa^2} \frac{g_1}{\phi^2} \ . \tag{35}$$

One should note that in the limit $\alpha \to 0$, the scalar field χ decouples and single scalar model follows. Then one obtains

$$
\begin{aligned}
\tilde{f}(\phi, \chi) &= g_0 + \frac{(1+\alpha)g_1}{\phi} - \frac{\alpha g_1}{\chi} \ , \\
V(\phi.\chi) &= \frac{1}{\kappa^2} \left\{ 3 \left(g_0 + \frac{(1+\alpha)g_1}{\phi} - \frac{\alpha g_1}{\chi} \right)^2 - \frac{(1+\alpha)g_1}{\phi^2} + \frac{\alpha g_1}{\chi^2} \right\} \ .
\end{aligned}
\tag{36}
$$

Hence, for scalar-tensor theory with such potentials the matter dominated era occurs before the acceleration epoch.

Before going to the second example, we consider the Einstein gravity with cosmological constant and with matter characterized by the EOS parameter w. FRW equation has the following form:

$$\frac{3}{\kappa^2} H^2 = \rho_0 a^{-3(1+w)} + \frac{3}{\kappa^2 l^2} \ . \tag{37}$$

Here l^2 is the inverse cosmological constant. The solution of (37) is given by

$$a = a_0 e^{g(t)} \ , \quad g(t) = \frac{2}{3(1+w)} \ln \left(\alpha \sinh \left(\frac{3(1+w)}{2l} (t - t_0) \right) \right) \ . \tag{38}$$

Here t_0 is a constant of the integration and

$$\alpha^2 \equiv \frac{1}{3} \kappa^2 l^2 \rho_0 a_0^{-3(1+w)} \ . \tag{39}$$

As the second example, we consider metric(38) for scalar-tensor theory. Since

$$
\begin{aligned}
f(t) \equiv g'(t) &= \frac{1}{l} \coth \left(\frac{3(1+w)}{2l} (t - t_0) \right) \ , \\
f'(t) = g''(t) &= -\frac{3(1+w)}{2l^2} \sinh^{-2} \left(\frac{3(1+w)}{2l} (t - t_0) \right) < 0 \ ,
\end{aligned}
\tag{40}
$$

it is convenient to use (34) instead of (5). Then one arrives at

$$
\begin{aligned}
\omega(\phi) &= \frac{3(1+w)(1+\alpha)}{\kappa^2 l^2}\sinh^{-2}\left(\frac{3(1+w)}{2l}(\phi-t_0)\right) > 0 \ , \\
\eta(\chi) &= -\frac{3(1+w)\alpha}{\kappa^2 l^2}\sinh^{-2}\left(\frac{3(1+w)}{2l}(\chi-t_0)\right) < 0 \ , \\
\tilde{f}(\phi,\chi) &= \frac{1+\alpha}{l}\coth\left(\frac{3(1+w)}{2l}(\phi-t_0)\right) - \frac{\alpha}{l}\coth\left(\frac{3(1+w)}{2l}(\chi-t_0)\right) \ , \\
V(\phi,\chi) &= \frac{1}{\kappa^2}\left[\frac{3}{l^2}\left\{(1+\alpha)\coth\left(\frac{3(1+w)}{2l}(\phi-t_0)\right)\right.\right. \\
&\qquad \left.\left. -\alpha\coth\left(\frac{3(1+w)}{2l}(\chi-t_0)\right)\right\}^2\right. \\
&\qquad -\frac{3(1+w)(1+\alpha)}{l^2}\sinh^{-2}\left(\frac{3(1+w)}{2l}(\phi-t_0)\right) \\
&\qquad \left. +\frac{3(1+w)\alpha}{l^2}\sinh^{-2}\left(\frac{3(1+w)}{2l}(\chi-t_0)\right)\right] \ .
\end{aligned}
\tag{41}
$$

Thus, in both examples, (32) and (38), there occurs the matter dominated stage, the transition from matter dominated phase to acceleration phase and acceleration epoch. In the acceleration phase, in the above examples, the universe asymptotically approaches to deSitter space. This does not conflict with WMAP data. Indeed, three years WMAP data have been analyzed in Ref.[32]. The combined analysis of WMAP with supernova Legacy survey (SNLS) constrains the dark energy equation of state w_{DE} pushing it towards the cosmological constant. The marginalized best fit values of the equation of state parameter at 68% confidance level are given by $-1.14 \le w_{\mathrm{DE}} \le -0.93$. In case of a prior that universe is flat, the combined data gives $-1.06 \le w_{\mathrm{DE}} \le -0.90$. In the examples (32) and (38), the universe goes to asymptotically deSitter space, which gives $w_{\mathrm{DE}} \to -1$, which does not, of course, conflict with the above constraints. Note, however, one needs to fine-tune g_0 in (32) and $1/l$ in (38) to be $g_0 \sim 1/l \sim 10^{-33}$ eV, in order to reproduce the observed Hubble rate $H_0 \sim 70$ $\mathrm{km\,s}^{-1}\mathrm{Mpc}^{-1} \sim 10^{-33}$ eV.

The final remark is in order. There is no problem to include into the above reconstruction scenario usual matter (say, ideal fluid, dust, etc) and (or) to find the scalar potentials corresponding to realistic cosmology in multi-scalar case. Moreover, in the same way one may include the radiation dominated epoch where quantum effects may be still neglected in the above scenario. In the next section it is shown how similar reconstruction scheme may be developed for modified gravity.

3. Reconstruction of modified gravity with the unification of matter dominated and accelerated phases

3.1. General formulation

In the present section we develop the general reconstruction scheme for modified gravity with $f(R)$ action. It is shown how any cosmology may define the implicit form of the function f. The starting action of modified gravity is:

$$
S = \int d^4x \left\{ f(R) + \mathcal{L}_{\mathrm{matter}} \right\} \ .
\tag{42}
$$

First we consider the proper Hubble rate H, which describes the evolution of the universe with matter dominance era and accelerating expansion (for discussion of various accelerating cosmologies from above action, see [23, 33, 9, 10]). It turns out that one can find $f(R)$-theory

realizing such a cosmology (with or without matter). The construction is not explicit [15] and it is necessary to solve the second order differential equation and algebraic equation. It shows, however, that, at least, in principle, we could obtain any cosmology by properly reconstructing a function $f(R)$ on theoretical level.

The equivalent form of above action is

$$S = \int d^4x \sqrt{-g} \left\{ P(\phi)R + Q(\phi) + \mathcal{L}_{\text{matter}} \right\} . \tag{43}$$

Here P and Q are proper functions of the scalar field ϕ and $\mathcal{L}_{\text{matter}}$ is the matter Lagrangian density. Since the scalar field does not have a kinetic term, it may be regarded as an auxiliary field. In fact, by the variation over ϕ, it follows

$$0 = P'(\phi)R + Q'(\phi) , \tag{44}$$

which may be solved with respect to ϕ:

$$\phi = \phi(R) . \tag{45}$$

By substituting (45) into (43), one obtains $f(R)$-gravity:

$$S = \int d^4x \sqrt{-g} \left\{ f(R) + \mathcal{L}_{\text{matter}} \right\} , \quad f(R) \equiv P(\phi(R))R + Q(\phi(R)) . \tag{46}$$

By the variation of the action (43) with respect to the metric $g_{\mu\nu}$, we obtain

$$0 = -\frac{1}{2}g_{\mu\nu}\left\{P(\phi)R + Q(\phi)\right\} - R_{\mu\nu}P(\phi) + \nabla_\mu\nabla_\nu P(\phi) - g_{\mu\nu}\nabla^2 P(\phi) + \frac{1}{2}T_{\mu\nu} . \tag{47}$$

The equations corresponding to standard spatially-flat FRW universe are

$$0 = -6H^2 P(\phi) - Q(\phi) - 6H\frac{dP(\phi(t))}{dt} + \rho , \tag{48}$$

$$0 = \left(4\dot{H} + 6H^2\right)P(\phi) + Q(\phi) + 2\frac{d^2 P(\phi(t))}{dt} + 4H\frac{dP(\phi(t))}{dt} + p . \tag{49}$$

By combining (47) and (48) and deleting $Q(\phi)$, we find the following equation

$$0 = 2\frac{d^2 P(\phi(t))}{dt^2} - 2H\frac{dP(\phi(t))}{d\phi} + 4\dot{H}P(\phi) + p + \rho . \tag{50}$$

As one can redefine the scalar field ϕ properly, we may choose

$$\phi = t . \tag{51}$$

It is assumed that ρ and p are the sum from the contribution of the matters with a constant equation of state parameters w_i. Especially, when it is assumed a combination of the radiation and dust, one gets the standard expression

$$\rho = \rho_{r0}a^{-4} + \rho_{d0}a^{-3} , \quad p = \frac{\rho_{r0}}{3}a^{-4} , \tag{52}$$

with constants ρ_{r0} and ρ_{d0}. If the scale factor a is given by a proper function $g(t)$ as

$$a = a_0 e^{g(t)} , \tag{53}$$

with a constant a_0, Eq.(49) reduces to the second rank differential equation:

$$
\begin{aligned}
0 \;=\; & 2\frac{d^2 P(\phi)}{d\phi^2} - 2g'(\phi)\frac{dP(\phi))}{d\phi} + 4g''(\phi)P(\phi) \\
& + \sum_i (1+w_i)\,\rho_{i0} a_0^{-3(1+w_i)} \mathrm{e}^{-3(1+w_i)g(\phi)} \; .
\end{aligned}
\tag{54}
$$

In principle, by solving (54) we find the form of $P(\phi)$. Using (48) (or equivalently (49)), we also find the form of $Q(\phi)$ as

$$
Q(\phi) = -6\left(g'(\phi)\right)^2 P(\phi) - 6g'(\phi)\frac{dP(\phi)}{d\phi} + \sum_i \rho_{i0} a_0^{-3(1+w_i)} \mathrm{e}^{-3(1+w_i)g(\phi)} \; .
\tag{55}
$$

Hence, in principle, any cosmology expressed as (53) can be realized by some specific $f(R)$-gravity.

3.2. $f(R)$ gravity: the transition of matter dominated phase to the acceleration phase
Let us consider realistic example where the total action contains also usual matter. The starting form of $g(\phi)$ is

$$
g(\phi) = h(\phi)\ln\left(\frac{\phi}{\phi_0}\right) \; ,
\tag{56}
$$

with a constant ϕ_0. It is assumed that $h(\phi)$ is a slowly changing function of ϕ. We use adiabatic approximation and neglect the derivatives of $h(\phi)$ ($h'(\phi) \sim h''(\phi) \sim 0$). Then the solution of Eq.(54) has the following form [15, 26]:

$$
P(\phi) = p_+ \phi^{n_+(\phi)} + p_- \phi^{n_-(\phi)} + \sum_i p_i(\phi)\phi^{-3(1+w_i)h(\phi)+2} \; .
\tag{57}
$$

Here $p_\pm$ are arbitrary constants and

$$
\begin{aligned}
n_\pm(\phi) &\equiv \frac{h(\phi)-1 \pm \sqrt{h(\phi)^2 + 6h(\phi)+1}}{2} \; , \\
p_i(\phi) &\equiv -\left\{(1+w)\rho_{i0} a_0^{-3(1+w_i)}\phi_0^{3(1+w)h(\phi)}\right\} \\
&\quad \times \left\{6(1+w)(4+3w)h(\phi)^2 - 2\left(13+9w\right)h(\phi) + 4\right\}^{-1} \; .
\end{aligned}
\tag{58}
$$

Especially for the radiation and dust, one has

$$
\begin{aligned}
p_r(\phi) &\equiv -\frac{4\rho_{r0}\phi_0^{4h(\phi)}}{3a_0^4\left(40h(\phi)^2 - 32h(\phi) + 4\right)} \; , \\
p_d(\phi) &\equiv -\frac{\rho_{d0}\phi_0^{3h(\phi)}}{a_0^3\left(24h(\phi)^2 - 26h(\phi) + 4\right)} \; .
\end{aligned}
\tag{59}
$$

We also find the form of $Q(\phi)$ as

$$
\begin{aligned}
Q(\phi) \;=\; & -6h(\phi)p_+\left(h(\phi)+n_+(\phi)\right)\phi^{n_+(\phi)-2} \\
& -6h(\phi)p_-\left(h(\phi)+n_-(\phi)\right)\phi^{n_-(\phi)-2} \\
& +\sum_i \left\{-6h(\phi)\left(-(2+3w)h(\phi)+2\right)p_i(\phi)\right. \\
& \left. +p_{i0}a_0^{-3(1+w)}\phi_0^{3(1+w)h(\phi)}\right\}\phi^{-3(1+w)h(\phi)} \; .
\end{aligned}
\tag{60}
$$

Eq.(56) tells that $H \sim h(t)/t$ and $R \sim 6\left(-h(t) + 2h(t)^2\right)/t^2$. Let assume $\lim_{\phi \to 0} h(\phi) = h_i$ and $\lim_{\phi \to \infty} h(\phi) = h_f$. Then if $0 < h_i < 1$, the early universe is in decceleration phase and if $h_f > 1$, the late universe is in acceleration phase. We may consider the case $h(\phi) \sim h_m$ is almost constant when $\phi \sim t_m$ ($0 \ll t_m \ll +\infty$). If h_1, $h_f > 1$ and $0 < h_m < 1$, the early universe is also accelerating, which could correspond to the inflation. After that the universe deccelerates, which corresponds to matter-dominated phase with $h(\phi) \sim 2/3$ there. Furthermore, after that, the universe could be in the acceleration phase.

The simplest example is

$$h(\phi) = \frac{h_i + h_f q \phi^2}{1 + q \phi^2} \; , \tag{61}$$

with constants h_i, h_f, and q. When $\phi \to 0$, $h(\phi) \to h_i$ and when $\phi \to \infty$, $h(\phi) \to h_f$. If q is small enough, $h(\phi)$ can be a slowly varying function of ϕ. Then we find [15, 26]

$$\phi^2 = \Phi_0(R) \;, \quad \Phi_0 \equiv \alpha_+^{1/3} + \alpha_-^{1/3} \;,$$

$$\alpha_\pm \equiv \frac{-\beta_0 \pm \sqrt{\beta_0^2 - \frac{4\beta_1^3}{27}}}{2} \;,$$

$$\beta_0 \equiv \frac{2\left(2R + 6h_f q - 12h_f^2 q\right)^3}{27 q^3 R^3}$$

$$-\frac{\left(2R + 6h_f q - 12h_f^2 q\right)\left(R + 6h_i q + 6h_f q - 4h_i h_f q\right)}{3qR}$$

$$+6h_i - 12h_i^2 \;,$$

$$\beta_1 \equiv -\frac{\left(2R + 6h_f q - 12h_f^2 q\right)^2}{3q^2 R^2} - \frac{R + 6h_i q + 6h_f q - 4h_i h_f q}{q^2 R} \;. \tag{62}$$

There are two branches besides Φ_0 but the asymptotic behaviour of R indicates that we should choose Φ_0 in (62). Then explicit form of $f(R)$ could be given by using the expressions of $P(\phi)$ (57) and $Q(\phi)$ (60) as

$$f(R) = P\left(\sqrt{\Phi_0(R)}\right) R + Q\left(\sqrt{\Phi_0(R)}\right) \;. \tag{63}$$

One may check the asymptotic behavior of $f(R)$ in (63). For simplicity, it is considered the case that the matter is only dust ($w = 0$) and that $p_- = 0$. Then

$$P(\phi) = p_+ \phi^{n_+(\phi)} + p_d(\phi) \phi^{-3h(\phi)+2} \;. \tag{64}$$

One may always get $n_+ - (-3h + 2) > 0$ in (64). Here n_+ is defined in (58). Then when ϕ is large, the first term in (64) dominates and when ϕ is small, the last term dominates. When ϕ is large, curvature is small and $\phi^2 \sim 6\left(-h_f + 2h_f\right)/R$ and $h(\phi) \to h(\infty) = h_f$. Hence, Eq.(64) shows that

$$P(\phi) \sim p_+ \left(\frac{6\left(-h_f + 2h_f\right)}{R}\right)^{\left(h_f - 1 + \sqrt{h_f^2 + 6h_f + 1}\right)/4} \;, \tag{65}$$

and therefore $f(R) \sim R^{-\left(h(\phi) - 5 + \sqrt{h_f^2 + 6h_f + 1}\right)/4}$. Especially when $h \gg 1$, we find $f(R) \sim R^{-h_f/2}$. Therefore there appears the negative power of R (for review of such theories, see [8]). As $H \sim h_f/t$, if $h_f > 1$, the universe is in acceleration phase.

On the other hand, when curvature is large, we find $\phi^2 \sim 6\left(-h_i + 2h_i\right)/R$ and $h(\phi) \to h(0) = h_i$. Then (57) shows $P(\phi) \sim p_d(0)\phi^{-3h_i+2}$. If the universe era corresponds to matter dominated

phase ($h_i = 2/3$), $P(\phi)$ becomes a constant and therefore $f(R) \sim R$, which reproduces the Einstein gravity.

Thus, in the above model, matter dominated phase evolves into acceleration phase and $f(R)$ behaves as $f(R) \sim R$ initially while $f(R) \sim R^{-\left(h(\phi)-5+\sqrt{h_f^2+6h_f+1}\right)/4}$ at late time. Moreover, for some parameter choice the asymptotic limit of above theory reproduces the model of ref.[23].

In our model, we can identify

$$w_{\mathrm{DE}} = -1 + \frac{2}{3h_f} \ , \quad \text{or} \quad h_f = \frac{2}{3\left(1+w_{\mathrm{DE}}\right)} \ , \tag{66}$$

which tells h_f should be large if h_f is positive. For example, if $w_{\mathrm{DE}} = -0.93$, $h_f \sim 9.51\cdots$ and if $w_{\mathrm{DE}} = -0.90$, $h_f \sim 6.67\cdots$. Thus, we presented the example of $f(R)$ gravity which describes the matter dominated stage, transition from decceleration to acceleration and acceleration epoch which is consistent with three years WMAP. Adding the radiation permits to realize the radiation dominated era before above cosmological sequence as is shown in [26].

4. Modified gravity and compensating dark energy

In this section we present another approach [15] to realistic cosmology in modified gravity. Specifically, we discuss the modified gravity which successfully describes the acceleration epoch but may be not viable in matter dominated stage. In this case, it is demonstrated that one can introduce the compensating dark energy (some ideal fluid) which helps to realize matter dominated and decceleration-acceleration transition phases. The role of such compensating dark energy is negligible in the acceleration epoch.

We now start with general $f(R)$-gravity action (42). In the FRW metric with flat spatial dimensions one gets

$$\begin{aligned}
\rho &= f(R) - 6\left(\dot{H} + H^2 - H\frac{d}{dt}\right) f'(R) \ , \\
p &= -f(R) - 2\left(-\dot{H} - 3H^2 + \frac{d^2}{dt^2} + 2H\frac{d}{dt}\right) f'(R) \ .
\end{aligned} \tag{67}$$

Here $R = 6\dot{H} + 12H^2$. If the Hubble rate is given (say, by observational data) as a function of t: $H = H(t)$, by substituting such expression into (67), we find the t-dependence of ρ and p as $\rho = \rho(t)$ and $p = p(t)$. If one can solve the first equation with respect to t as $t = t(\rho)$, by substituting it into the second equation, an equation of state (EOS) follows $p = p\left(t(\rho)\right)$. Of course, ρ and p could be a sum with the contribution of several kinds of fluids with simple EOS.

We now concentrate on the case that $f(R)$ is given by [23]

$$f(R) = -\frac{\alpha}{R^n} + \frac{R}{2\kappa^2} + \beta R^2 \ . \tag{68}$$

Furthermore, we write $H(t)$ as $H(t) = h(t)/t$ and assume $h(t)$ is slowly varying function of t and neglect the derivatives of $h(t)$ with respect to t.

First we consider the case that the last term in (68) dominates $f(R) \sim \beta R^2$, which may correspond to the early (inflationary) epoch of the universe. It is not difficult to find

$$\rho \sim -\frac{36\beta\left(-1+2h(t)\right)h(t)^2}{t^{4n}} \ , \quad p \sim -\frac{36\beta\left(-1+2h(t)\right)h\left(3h(t)+1\right)}{t^{4n}} \ . \tag{69}$$

If h goes to infinity, which corresponds to deSitter universe, we find $\rho \sim p \sim h^3$ although $R \sim h^4$. Therefore $\rho, p \ll \beta R^2$ and contribution form the matter could be neglected. Then the inflation could be generated only by the contribution from the higher curvature term.

Second, we consider the case that the second term in (68) dominates $f(R) \sim R/2\kappa^2$, which may correspond to the matter dominated epoch after the inflation. In this case ρ and p behave as

$$\rho \sim \frac{12h(t) + 6h(t)^2}{\kappa^2 t^2} \ , \quad p \sim -\frac{4h(t) - 6h(t)^2}{\kappa^2 t^2} \ . \tag{70}$$

In the matter dominated epoch, we expect $h \sim 2/3 \left(a \sim t^{\frac{2}{3}}\right)$. Hence, one gets $\rho \sim 32/3\kappa^2 t^2$, $p \sim 0$. Therefore in the matter sector, dust with $w = 0$ ($p = 0$) should dominate, as usually expected.

Finally we consider the case that the first term in (68) dominates $f(R) \sim -\alpha/R^n$, which might describe the acceleration of the present universe. The behavior of ρ and p is given by

$$
\begin{aligned}
\rho \ &\sim \ \alpha \left\{ 6\,(n+1)\,(2n+1)\,h(t) + 6\,(n-2)\,h(t)^2 \right\} \\
&\times \left\{ -6h(t) + 12h(t)^2 \right\}^{-n-1} t^{2n} \ , \\
p \ &\sim \ \alpha \left\{ -4n\,(n+1)\,(2n+1) - 2\left(8n^2 + 5n + 3\right) h(t) - 6\,(n-2)\,h(t)^2 \right\} \\
&\times \left\{ -6h(t) + 12h(t)^2 \right\}^{-n-1} t^{2n} \ .
\end{aligned}
\tag{71}
$$

Thus, the effective EOS parameter $w_l \equiv p/\rho$ is given by

$$
\begin{aligned}
w_l \ \sim \ &\left\{ -4n\,(n+1)\,(2n+1) - 2\left(8n^2 + 5n + 3\right) h(t) \right. \\
&\left. -6\,(n-2)\,h(t)^2 \right\} \left\{ 6\,(n+1)\,(2n+1)\,h(t) + 6\,(n-2)\,h(t)^2 \right\}^{-1} \ .
\end{aligned}
\tag{72}
$$

In order that the acceleration of the universe could occur, we find $h > 1$. Let us now assume that $h(t) \to h_f$ when $t \to \infty$. Then one obtains $w_l \to w_f$. Here

$$
\begin{aligned}
w_f \ \equiv \ &\left\{ -4n\,(n+1)\,(2n+1) - 2\left(8n^2 + 5n + 3\right) h_f \right. \\
&\left. -6\,(n-2)\,h_f^2 \right\} \left\{ 6\,(n+1)\,(2n+1)\,h_f + 6\,(n-2)\,h_f^2 \right\}^{-1} \ ,
\end{aligned}
\tag{73}
$$

and $H(t) \to h_f/t$. Since the matter energy density ρ_{w_f} with the EoS parameter w_f behaves as

$$\rho_{w_f} \propto a^{-3(1+w_f)} \propto \exp\left(-3(1+w_f)\int dt\, H(t)\right) \ , \tag{74}$$

the energy density is

$$\rho_{w_f} \propto t^{-3(1+w_f)h_f} \ . \tag{75}$$

Comparing (75) with (71), we find $2n = -3(1 + w_f)h_f$, which can be confirmed directly from (73).

From the above consideration, we find ρ and p contain mainly contributions from dust with $w = 0$, $\rho_d(t)$, $p(t) = 0$ and "dark energy" with $w = w_l$ in (72), $\rho_l(t)$, $p_l(t)$. In the expressions of $\rho(t)$ and $p(t)$ in (67), there might be a remaining part:

$$\rho_R(t) \equiv \rho(t) - \rho_d(t) - \rho_l(t) \ , \quad p_R(t) \equiv p(t) - p_l(t) \ , \tag{76}$$

which may help the transition from the matter dominated epoch to the acceleration epoch. By deleting t in the expression of (76), we obtain the EOS for the remaining part:

$$p_R = p_R(\rho_R) \ , \tag{77}$$

which may be called the compensating dark energy. More concretely, one may have

$$\rho_d \sim \frac{32}{3\kappa^2 t_0^2}\mathrm{e}^{-3\int_{t_0}^t dt\frac{h(t)}{t}} \ , \tag{78}$$

and according to (71),

$$\begin{aligned}
\rho_l \ &\sim \ \alpha\left\{6\left(n+1\right)\left(2n+1\right)h_f + 6\left(n-2\right)h_f^2\right\} \\
&\times \left\{-6h_f + 12h_f^2\right\}^{-n-1} t_1^{2n}\mathrm{e}^{-3(1+w_f)\int_{t_1}^t dt\frac{h(t)}{t}} \ .
\end{aligned} \tag{79}$$

In (79), we choose t_1 to be large enough. When $t \sim t_0$, $\rho(t) \sim \rho_d$ and when $t \to \infty$, $\rho(t) \sim \rho_l$. Thus, ρ_R only dominates after $t = t_1$ but it becomes smaller in late times. Hence, the role of ρ_R (which perhaps may be identified partially with dark matter) is only to connect the matter dominated epoch to the acceleration epoch. The number of modified gravities can be made cosmologically viable via such scenario.

5. Reconstruction of scalar-Gauss-Bonnet theory and $F(G)$-gravity
5.1. Scalar-Gauss-Bonnet gravity
In this section we show how string-inspired, scalar-Gauss-Bonnet gravity may be reconstructed for any requested cosmology. The starting action is

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{2\kappa^2} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) - \xi_1(\phi)G\right] \ . \tag{80}$$

Here G is the Gauss-Bonnet invariant and the scalar field ϕ is canonical in (80). Note that scalar may be considered as matter component. We assume the FRW universe and the scalar field ϕ only depending on t. The FRW equations look like [12]:

$$0 \ = \ -\frac{3}{\kappa^2}H^2 + \frac{1}{2}\dot{\phi}^2 + V(\phi) + 24H^3\frac{d\xi_1(\phi(t))}{dt} \ , \tag{81}$$

$$\begin{aligned}
0 \ = \ &\frac{1}{\kappa^2}\left(2\dot{H} + 3H^2\right) + \frac{1}{2}\dot{\phi}^2 - V(\phi) - 8H^2\frac{d^2\xi_1(\phi(t))}{dt^2} \\
&-16H\dot{H}\frac{d\xi_1(\phi(t))}{dt} - 16H^3\frac{d\xi_1(\phi(t))}{dt} \ .
\end{aligned} \tag{82}$$

and scalar field equation:

$$0 = \ddot{\phi} + 3H\dot{\phi} + V'(\phi) + \xi_1'(\phi)G \ . \tag{83}$$

Now $G = 24\left(\dot{H}H^2 + H^4\right)$. Combining (81) and (82), one gets

$$\begin{aligned}
0 \ &= \ \frac{2}{\kappa^2}\dot{H} + \dot{\phi}^2 - 8H^2\frac{d^2\xi_1(\phi(t))}{dt^2} - 16H\dot{H}\frac{d\xi_1(\phi(t))}{dt} + 8H^3\frac{d\xi_1(\phi(t))}{dt} \\
&= \ \frac{2}{\kappa^2}\dot{H} + \dot{\phi}^2 - 8a\frac{d}{dt}\left(\frac{H^2}{a}\frac{d\xi_1(\phi(t))}{dt}\right) \ .
\end{aligned} \tag{84}$$

Eq.(84) can be solved with respect to $\xi_1(\phi(t))$ as

$$\begin{aligned}
\xi_1(\phi(t)) \ &= \ \frac{1}{8}\int^t dt_1\frac{a(t_1)}{H(t_1)^2}W(t_1) \ , \\
W(t) \ &\equiv \ \int^t \frac{dt_1}{a(t_1)}\left(\frac{2}{\kappa^2}\dot{H}(t_1) + \dot{\phi}(t_1)^2\right) \ .
\end{aligned} \tag{85}$$

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Combining (81) and (85), the scalar potential $V(\phi(t))$ is:

$$V(\phi(t)) = \frac{3}{\kappa^2}H(t)^2 - \frac{1}{2}\dot{\phi}(t)^2 - 3a(t)H(t)W(t) . \tag{86}$$

We now identify t with $f(\phi)$ and H with $g'(t)$ where f and g are some functions. Let us consider the model where $V(\phi)$ and $\xi_1(\phi)$ may be expressed in terms of two functions f and g as

$$
\begin{aligned}
V(\phi) &= \frac{3}{\kappa^2}g'\left(f(\phi)\right)^2 - \frac{1}{2f'(\phi)^2} - 3g'\left(f(\phi)\right)e^{g(f(\phi))}U(\phi) \\
\xi_1(\phi) &= \frac{1}{8}\int^\phi d\phi_1 \frac{f'(\phi_1)e^{g(f(\phi_1))}}{g'\left(f(\phi_1)\right)^2}U(\phi_1) , \\
U(\phi) &\equiv \int^\phi d\phi_1 f'(\phi_1)e^{-g(f(\phi_1))}\left(\frac{2}{\kappa^2}g''\left(f(\phi_1)\right) + \frac{1}{f'(\phi_1)^2}\right) .
\end{aligned}
\tag{87}
$$

By choosing $V(\phi)$ and $\xi_1(\phi)$ as (87), we can easily find the following solution for Eqs.(81) and (82):

$$\phi = f^{-1}(t) \quad (t = f(\phi)) , \quad a = a_0 e^{g(t)} \ \left(H = g'(t)\right) . \tag{88}$$

One can straightforwardly check the solution (88) satisfies the field equation (83).

An interesting cosmological example is

$$e^{g(t)} = \left(\frac{t}{t_0}\right)^{g_1} e^{g_0 t} , \quad \phi = f^{-1}(t) = \phi_0 \ln \frac{t}{t_0} . \tag{89}$$

Here t_0, g_0, g_1, and ϕ_0 are constants. We further choose

$$\phi_0^2 = \frac{2g_1}{\kappa^2} . \tag{90}$$

Then U is a constant: $U = U_0$ and

$$
\begin{aligned}
V(\phi) &= \frac{3}{\kappa^2}\left(g_0 + \frac{g_1}{t_0}e^{-\phi/\phi_0}\right)^2 - \frac{g_1}{\kappa^2 t_0^2}e^{-2\phi/\phi_0} \\
&\quad - U_0\left(g_0 + \frac{g_1}{\phi}\right)e^{-g_1\phi/\phi_0}\left(\frac{t}{t_0}\right)^{g_1}e^{g_0 t_0 e^{\phi/\phi_0}} , \\
\xi_1(\phi) &= \frac{U_0}{8}\int^{t_0 e^{\phi/\phi_0}}dt_1\left(g_0 + \frac{g_1}{t_1}\right)^{-2}\left(\frac{t}{t_0}\right)e^{g_0 t} .
\end{aligned}
\tag{91}
$$

Eq.(90) shows

$$H = g_0 + \frac{g_1}{t} . \tag{92}$$

Hence,, when t is small, the second term in (92) dominates and the scale factor behaves as $a \sim t^{g_1}$. Therefore if $g_1 = 2/3$, the matter-dominated period could be realized. On the other hand, when t is large, the first term in (92) dominates and the Hubble rate H becomes a constant. Therefore, the universe is asymptotically deSitter space, which is an accelerating universe. As in our model the universe goes to asymptotically deSitter space, we find $w_{DE} \to -1$. Therefore our model can easily accommodate three years WMAP values of w_{DE}. For example, if $g_0 \simeq 40$, one has $w_{DE} = -0.98$.

In the limit $U_0 \to 0$, the Gauss-Bonnet term in (80) vanishes and the action (80) reduces into that of the usual scalar tensor theory with potential

$$V(\phi) = \frac{3}{\kappa^2}\left(g_0 + \frac{g_1}{t_0}e^{-\phi/\phi_0}\right)^2 - \frac{g_1}{\kappa^2 t_0^2}e^{-2\phi/\phi_0} , \tag{93}$$

which reproduces the result [31].

Let us reconstruct the scalar-Gauss-Bonnet gravity from FRW cosmology (38). If a function $g(t)$ is given by (38) and $f(\phi)$ is given by

$$f(\phi) = t_0 - \frac{2l}{3(1+w)} \ln \tanh \left(-\frac{\kappa\sqrt{3(1+w)}}{4} \phi \right) \, , \tag{94}$$

$U(\phi)$ (87) becomes a constant again, $U = U(\phi)$. Then $V(\phi)$ and $\xi(\phi)$ are found to be

$$
\begin{aligned}
V(\phi) &= \frac{3}{\kappa^2 l^2} \cosh^2 \left(\frac{\kappa\sqrt{3(1+w)}}{2} \phi \right) \\
&\quad - \frac{3(1+w)}{2l^2\kappa^2} \sinh^2 \left(\frac{\kappa\sqrt{3(1+w)}}{2} \phi \right) \\
&\quad - \frac{3U_0}{l} \cosh \left(\frac{\kappa\sqrt{3(1+w)}}{2} \phi \right) \\
&\quad \times \left\{ -\frac{1}{\alpha} \sinh \left(\frac{\kappa\sqrt{3(1+w)}}{2} \phi \right) \right\}^{-2/(3(1+w))} \, , \\
\xi_1(\phi) &= -\frac{\alpha U_0 l^3 \kappa}{8\sqrt{3(1+w)}} \int^{\phi} d\phi_1 \cosh^{-2} \left(\frac{\kappa\sqrt{3(1+w)}}{2} \phi_1 \right) \\
&\quad \times \left\{ -\frac{1}{\alpha} \sinh \left(\frac{\kappa\sqrt{3(1+w)}}{2} \phi_1 \right) \right\}^{-2/(3(1+w))-1} \, ,
\end{aligned}
\tag{95}
$$

which again reproduces the result of ref.[31] in the limit of $U_0 \to 0$.

Eq.(38) shows that when $t \sim t_0$, the scale factor behaves as $a \sim (t - t_0)^{2/(3(1+w))}$. Therefore if $w = 0$, the matter-dominated period could be realized. On the other hand, when $t \to \infty$, a behaves as $a \sim \mathrm{e}^{t/l}$, which tells the universe goes to asymptotically deSitter with $w_{\mathrm{DE}} \to -1$. Hence, it could be consistent with WMAP data. The FRW cosmology of these and another versions of scalar-Gauss-Bonnet gravity may be studied in the same way as it was done in refs.[34, 35]. The investigation of cosmological perturbations can be also done [36] in the above model admitting the cosmological sequence of matter dominance, decceleration-acceleration transition and acceleration.

5.2. F(G)-gravity
The formulation of the previous section can be extended to so-called $F(G)$-gravity [13], whose action is given by

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{2\kappa^2} + F(G) \right] \tag{96}$$

The above action could be rewritten by introducing the auxilliary scalar field ϕ as

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{2\kappa^2} - V(\phi) - \xi_1(\phi)G \right] \, . \tag{97}$$

By the variation over ϕ, one obtains

$$0 = V'(\phi) + \xi_1'(\phi)G \, , \tag{98}$$

which could be solved with respect to ϕ as

$$\phi = \phi(G) \, . \tag{99}$$

By substituting the expression (99) into the action (97), we obtain the action of $F(G)$-gravity with

$$F(G) = -V(\phi(G)) + \xi_1(\phi(G)) G . \tag{100}$$

Note that the action (97) could be obtained also by dropping the kinetic term of ϕ in the action (80).

Assuming the spatially-flat FRW universe and the scalar field ϕ only depending on t, the FRW equations corresponding to (81) and (82) are:

$$0 = -\frac{3}{\kappa^2}H^2 + V(\phi) + 24H^3\frac{d\xi_1(\phi(t))}{dt} , \tag{101}$$

$$0 = \frac{1}{\kappa^2}\left(2\dot{H} + 3H^2\right) - V(\phi) - 8H^2\frac{d^2\xi_1(\phi(t))}{dt^2}$$
$$-16H\dot{H}\frac{d\xi_1(\phi(t))}{dt} - 16H^3\frac{d\xi_1(\phi(t))}{dt} . \tag{102}$$

Combining the above equations, one gets

$$0 = \frac{2}{\kappa^2}\dot{H} - 8H^2\frac{d^2\xi_1(\phi(t))}{dt^2} - 16H\dot{H}\frac{d\xi_1(\phi(t))}{dt} + 8H^3\frac{d\xi_1(\phi(t))}{dt}$$
$$= \frac{2}{\kappa^2}\dot{H} - 8a\frac{d}{dt}\left(\frac{H^2}{a}\frac{d\xi_1(\phi(t))}{dt}\right) , \tag{103}$$

which can be solved with respect to $\xi_1(\phi(t))$ as

$$\xi_1(\phi(t)) = \frac{1}{8}\int^t dt_1 \frac{a(t_1)}{H(t_1)^2}W(t_1) , \quad W(t) \equiv \frac{2}{\kappa^2}\int^t \frac{dt_1}{a(t_1)}\dot{H}(t_1) . \tag{104}$$

Combining (101) and (104), the following expression of $V(\phi(t))$ may be found:

$$V(\phi(t)) = \frac{3}{\kappa^2}H(t)^2 - 3a(t)H(t)W(t) . \tag{105}$$

Due to a freedom of the redefinition of the scalar field ϕ we may identify t with ϕ. Then one considers the model where $V(\phi)$ and $\xi_1(\phi)$ can be expressed in terms of a single function g as

$$V(\phi) = \frac{3}{\kappa^2}g'(\phi)^2 - 3g'(\phi)\,\mathrm{e}^{g(\phi)}U(\phi) \quad \xi_1(\phi) = \frac{1}{8}\int^\phi d\phi_1 \frac{\mathrm{e}^{g(\phi_1)}}{g'(\phi_1)^2}U(\phi_1) ,$$
$$U(\phi) \equiv \frac{2}{\kappa^2}\int^\phi d\phi_1 \mathrm{e}^{-g(\phi_1)}g''(\phi_1) . \tag{106}$$

By choosing $V(\phi)$ and $\xi_1(\phi)$ as (106), we can easily find the following solution for Eqs.(101) and (102):

$$a = a_0\mathrm{e}^{g(t)} \ \left(H = g'(t)\right) . \tag{107}$$

Then we can reconstruct $F(G)$-gravity in the way very similar [35, 37] to the scalar-Gauss-Bonnet theory in the previous sub-section.

Although the above formulation is very similar to that in the scalar-Gauss-Bonnet theory, there could be some difference. In the scalar-Gauss-Bonnet gravity, since the scalar field has a kinetic term, the scalar field could propagate and there could be a possibility to generate extra force besides the Newton force. On the other hand, $F(G)$-gravity has no kinetic term for scalar field and extra force could not be generated. In fact, one can consider the perturbation around

the deSitter background, writing the metric as $g_{\mu\nu} = g_{(0)\mu\nu} + h_{\mu\nu}$. Here the Riemann tensor in the deSitter background is given by

$$R_{(0)\mu\nu\rho\sigma} = H_0^2 \left(g_{(0)\mu\rho} g_{(0)\nu\sigma} - g_{(0)\mu\sigma} g_{(0)\nu\rho} \right) \ . \tag{108}$$

The flat background corresponds to the limit of $H_0 \to 0$. For simplicity, if we choose the gauge conditions $g_{(0)}^{\mu\nu} h_{\mu\nu} = \nabla_{(0)}^\mu h_{\mu\nu} = 0$, we find from the equation of motion without energy-momentum tensor,

$$0 = \frac{1}{4\kappa^2} \left(\nabla^2 h_{\mu\nu} - 2H_0^2 h_{\mu\nu} \right) \ . \tag{109}$$

Since the contribution form the Gauss-Bonnet term does not appear except the length parameter $1/H_0$ of the deSitter space, the only propagating mode should be graviton in the $F(G)$-gravity. This also shows that Newton law is effectively the same as in GR.

Finally, let us note that it is not difficult to extend the above reconstruction scheme for modified Gauss-Bonnet gravity so that the (ideal fluid) matter may be naturally included into the formulation [37].

6. Discussion

In summary, the reconstruction program is developed for the number of modified gravities: scalar-tensor theory, $f(R)$, $F(G)$ and scalar-Gauss-Bonnet gravity. It is explicitly demonstrated which versions of above theories may be reconstructed from the known universe expansion history. Specifically, it is shown that the cosmological sequence of matter dominance, decceleration-acceleration transition and acceleration era may always emerge as the cosmological solutions of such modified gravities. Moreover, it is explained that the (exact or approximated) ΛCDM cosmology may be also the solution of such gravity theories. Several examples of corresponding reconstruction for it as well as for oscillating universe where ΛCDM dark energy describes the one of the oscillation periods are given. In principle, it is not difficult to include into this reconstruction scheme also radiation dominated era and, perhaps, the inflationary epoch. This will be studied elsewhere.

It is interesting that even if specific modified gravity which is well suited with acceleration epoch does not describe the matter dominance period, such a period may be included there as the solution at the price of the introduction of compensating dark energy. The corresponding example is worked out for $f(R)$ model of ref.[23]. It may be also extended for scalar-Gauss-Bonnet or $F(G)$ gravity.

It is quite remarkable that modified gravity may not only naturally describe dark energy (unlike to conventional GR) but also it can be reconstructed using the realistic universe expansion history. It is also promising that it may be fitted with observational data. As well it may be compatible with Solar System tests. Of course, some controversial results exist here what is not strange due to fact that modified gravity is seriously considered as the alternative cosmological theory only last several years while GR played this role almost one century. One should also bear in mind that most of cosmological data were obtained (at least, up to some extent) using GR foundation. Hence, very serious reconsideration (as well as more precise and complete observational data) are requested in order to have the answer to the fundamental question: what is the gravitation theory which governs the expansion of our universe?

References

[1] Copeland E, Sami M and Tsujikawa S 2006 *Preprint* hep-th/0603057
[2] Padmanabhan T 2003 *Phys. Rept.* **380** 235 (*Preprint* astro-ph/0603114)
[3] Jassal H, Bagla J and Padmanabhan T 2005 *Preprint* astro-ph/0506748
    Jassal H, Bagla J and Padmanabhan T 2006 *Preprint* astro-ph/0601389
    Nesseris S and Perivolaropoulos L 2006 *Preprint* astro-ph/0602053
    Nesseris S and Perivolaropoulos L 2006 *Preprint* astro-ph/0610092
[4] Tegmark M *et al.* 2004 *Phys. Rev.* D **69** 103504
[5] Caldwell R R, Kamionkowski M and Weinberg N N 2003 *Phys. Rev. Lett.* **91** 071301 (*Preprint* astro-ph/0302506)
    McInnes B 2002 *JHEP* **0208** 029 (*Preprint* hep-th/0112066)
    McInnes B 2005 *Preprint* hep-th/0502209
    Faraoni V 2002 *Int. J. Mod. Phys.* D **64** 043514
    Nojiri S and Odintsov S D 2003 *Phys. Lett.* B **565** 1 (*Preprint* hep-th/0304131)
    Nojiri S and Odintsov S D 2004 *Phys. Lett.* B **595** 1 (*Preprint* hep-th0405078)
    Gonzalez-Diaz P 2004 *Phys. Lett.* **B586** 1 (*Preprint* astro-ph/0312579)
    Gonzalez-Diaz P 2004 *Preprint* hep-th/0408225
    Sami M and Toporensky A 2004 *Mod. Phys. Lett.* A **19** 1509 (*Preprint* gr-qc/0312009)
    Stefancic H 2004 *Phys. Lett.* B **586** 5
    Chimento L P and Lazkoz R 2003 *Phys. Rev. Lett.* **91** 211301 (*Preprint* gr-qc/0307111)
    Chimento L P and Lazkoz R 2004 *Mod. Phys. Lett.* A **19** 2479 (*Preprint* gr-qc/0405020)
    Hao J and Li X 2005 *Phys. Lett.* B **606** 7 (*Preprint* astro-ph/0404154)
    Babichev E, Dokuchaev V and Eroshenko Yu 2005 *Class. Quantum Grav.* **22** 143 (*Preprint* astro-ph/0407190)
    Zhang X, Li H, Piao Y and Zhang X 2005 *Preprint* astro-ph/0501652
    Elizalde E, Nojiri S, Odintsov S D and Wang P 2005 *Phys. Rev.* D **71** 103504 (*Preprint* hep-th/0502082)
    Dabrowski M and Stachowiak T 2004 *Preprint* hep-th/0411199
    Lobo F 2005 *Preprint* gr-qc/0502099
    Cai R-G Zhang H-S and Wang A 2005 *Preprint* hep-th/0505186
    Arefeva I Ya, Koshelev A S and Vernov S Yu 2004 *Preprint* astro-ph/0412619
    Arefeva I Ya, Koshelev A S and Vernov S Yu 2005 *Preprint* astro-ph/0507067
    Lu H, Huang Z and Fang W 2005 *Preprint* hep-th/0504038
    Bauer F 2005 *Preprint* gr-qc/0501078
    Godlowski W and Szydlowski M 2005 *Phys. Lett.* B **623** 10
    Sola J and Stefancic H 2005 *Preprint* astro-ph/0505133
    Guberina B, Horvat R and Nicolic H 2005 *Preprint* astro-ph/0507666
    Dabrowski M, Kiefer C and Sandhofer B 2006 *Phys. Rev.* D **74** 044022
    Barboza E and Lemos N 2006 *Preprint* gr-qc/0606084
    Szydlowski M, Hrycyna O and Krawiec A 2006 *Preprint* hep-th/0608219
[6] Nojiri S and Odintsov S D 2003 *Phys. Lett.* B **562** 147 (*Preprint* hep-th/0303117)
    Nojiri S and Odintsov S D 2003 *Phys. Lett.* B **571** 1 (*Preprint* hep-th/0306212)
[7] Nojiri S, Odintsov S D and Tsujikawa S 2005 *Phys. Rev.* D **71** 063004 (*Preprint* hep-th/0501025)
[8] Nojiri S and Odintsov S D 2006 *Preprint* hep-th/0601213
[9] Capozziello S 2002 *Int. J. Mod. Phys.* D **11** 483
    Capozziello S, Carloni S and Troisi A 2003 *Preprint* astro-ph/0303041
[10] Carroll S M, Duvvuri V, Trodden M and Turner S 2004 *Phys. Rev.* D **70** 043528
[11] Nojiri S and Odintsov S D 2003 *Phys. Lett.* B **576** 5 (*Preprint* hep-th/0307071)
[12] Nojiri S, Odintsov S D and Sasaki M 2005 *Phys. Rev.* D **71** 123509 (*Preprint* hep-th/0504052)
[13] Nojiri S and Odintsov S D 2005 *Phys. Lett.* B **631** (2005) 1 (*Preprint* hep-th/0508049)
    Nojiri S, Odintsov S D and Gorbunova O 2005 *Preprint* hep-th/0510183
    Cognola G, Elizalde E, Nojiri S, Odintsov S D and Zerbini S 2006 *Phys. Rev.* D **73** 084007
    Brevik I and Quiroga J 2006 *Preprint* gr-qc/0610044
    Andrew K, Bolen B and Middleton C 2006 *Preprint* hep-th/0608127
    Elizalde E, Makarenko A, Obukhov V V, Osetrin K E and Filippov A 2006 *Preprint* hep-th/0611213
[14] Capozziello S, Nojiri S and Odintsov S D 2006 *Phys. Lett.* B **634** 93 (*Preprint* hep-th/0512118)

[15] Capozziello S, Nojiri S, Odintsov S D and Troisi A 2006 *Phys. Lett.* B **639** 135 (*Preprint* astro-ph/0604431)
     Nojiri S and Odintsov S D 2006 *Phys. Rev.* D **74** 086005 (*Preprint* hep-th/0608008)
[16] Nojiri S and Odintsov S D 2004 *Phys. Rev.* D **70** 103522 (*Preprint* hep-th/0408170)
     Stefancic H 2005 *Phys. Rev.* D **71** 084024 (*Preprint* astro-ph/0411630)
[17] Nojiri S and Odintsov S D 2005 *Phys. Rev.* D **72** 023003 (*Preprint* hep-th/0505215)
     Nojiri S and Odintsov S D 2006 *Phys. Lett.* B **639** 144 (*Preprint* hep-th/0606025)
[18] Barrow J 1988 *Nucl. Phys.* B **310** 743
     Brevik I and Gorbunova O 2005 *Preprint* gr-qc/0504001
     Giovannini M 2005 *Phys. Lett.* B **622** 349
     Ren J and Meng X 2005 *Preprint* astro-ph/0511163
     Meng X, Hu M and Ren J 2005 *Preprint* astro-ph/0509250
[19] Capozziello S, Cardone V F and Troisi A 2005 *Phys. Rev.* D **71** 043503
     Capozziello S, Cardone V F and Francaviglia M 2006 *Gen. Rel. Grav.* **38** 711
     Amarzguioui M, Elgaroy O, Mota D F and Multamaki T 2005 *Preprint* astro-ph/0510519
     Mena O, Santiago J and Weller J 2006 *Phys. Rev. Lett.* **96** 041103
     Koivisto T and Kurki-Suonio H 2006 *Class. Quantum Grav.* **23** 2355
     Capozziello S, Cardone V F, Elizalde E, Nojiri S and Odintsov S D 2006 *Phys. Rev.* D **73** 043512 (*Preprint* astro-ph/0508350)
     Poplawski N 2006 *Preprint* gr-qc 0607124
     Borowiec A, Godlowski W and Szydlowski M 2006 *Preprint* astro-ph/0602526
     Bludman S 2006 *Preprint* astro-ph/0605198
     Capozziello S, Stabile A and Troisi A 2006 *Preprint* gr-qc/0603071
     Allemandi G and Ruggiero M 2006 *Preprint* astro-ph/0610661
     Song Y, Hu W and Sawicki I 2006 *Preprint* astro-ph/0610532
     Li B, Chan K and Chu M 2006 *Preprint* astro-ph/0610794
     Mendoza S and Rosas-Guevara Y 2006 *Preprint* astro-ph/0610390
[20] Brookfield A W, van de Bruck C and Hall L 2006 *Preprint* hep-th/0608015
[21] Allemandi G, Francaviglia M, Ruggiero M and Tartaglia A 2005 *Preprint* gr-qc/0506123
     Capozziello S 2004 *Preprint* gr-qc/0412088
     Meng X and Wang P 2004 *Gen. Rel. Grav.* **36** 1947
     Domingues A and Barraco D 2004 *Phys. Rev.* D **70** 043505
     Koivisto T 2005 *Preprint* gr-qc/0505128
     Clifton T and Barrow J 2005 *Phys. Rev.* D **72** 103005
     Cembranos J 2006 *Phys. Rev.* D **73** 064029
     Sotiriou T 2005 *Preprint* gr-qc/0507027
     Navarro I and Van Acoleyen K 2005 *Phys. Lett.* B **622** 1
     Shao C, Cai R, Wang B and Su R 2005 *Preprint* gr-qc/0511034
     Soussa M E and Woodard R P 2004 *Gen. Rel. Grav.* **36** 855
     Capozziello S and Troisi A 2005 *Phys. Rev.* D **72** 044022
     Atazadeh K and Sepangi H 2006 *Preprint* gr-qc/0602028
     Woodard R 2006 *Preprint* astro-ph/0601672
     Jin X, Liu D and Li X 2006 *Preprint* astro-ph/0610854
[22] Dolgov A D and Kawasaki M 2003 *Phys. Lett.* B **573** 1
     Faraoni V 2006 *Preprint* astro-ph/0610734
     Faraoni V 2006 *Preprint* gr-qc/0607016
[23] Nojiri S and Odintsov S D 2003 *Phys. Rev.* D **68** 123512 (*Preprint* hep-th/0307288)
[24] Brevik I, Nojiri S, Odintsov S D and Vanzo L 2004 *Phys. Rev.* D**70** 043520 (*Preprint* hep-th/0401073)
     Cognola G, Elizalde E, Nojiri S, Odintsov S D and Zerbini S 2005 *JCAP* **0502** 010 (*Preprint* hep-th/0501096)
     Paul B and Paul D 2005 *Preprint* hep-th/0511003
     Sobouti Y 2006 *Preprint* astro-ph/0603302
     Multamaki T and Vilja I 2006 *Preprint* astro-ph/0606373
[25] Sahni V and Starobinsky A 2006 *Preprint* astro-ph/0610026
[26] Nojiri S and Odintsov S D 2006 *Preprint* hep-th/0610164
[27] Nojiri S and Odintsov S D 2006 *Gen. Rel. Grav.* **38** 1285 (*Preprint* hep-th/0506212)
     Capozziello S, Nojiri S and Odintsov S D 2006 *Phys. Lett.* B **632** 597 (*Preprint* hep-th/0507182)
[28] Faraoni V 2003 *Phys. Rev.* D **68** 063508
     Faraoni V 2004 *Phys. Rev.* D **69** 123520
     Faraoni V 2005 *Ann. Phys.* **317** 366
     Faraoni V 2005 *Class. Quantum Grav.* **22** 3235

Elizalde E, Nojiri S and Odintsov S D 2004 *Phys. Rev.* D **70** 043539 (*Preprint* hep-th/0405034)

Tsujikawa S 2006 *Phys. Rev.* D **73** 103504

Wei H and Cai R-G 2005 *Phys. Rev.* D **72** 123507

Barrow J and Clifton T 2006 *Phys. Rev.* D **73** 103520

Andrianov A, Cannata F and Kamenshchik A 2005 *Phys. Rev.* D **72** 043531

Perivolaropoulos L 2006 *JCAP* **0510** 001

Gumjudpai B, Naskar T, Sami M and Tsujikawa S 2005 *JCAP* **0506** 007

Nesseris S and Perivolaropoulos L 2004 *Phys. Rev.* D **70** 123529

Wei Y 2006 *Preprint* astro-ph/0607359

Wei Y 2005 *Mod. Phys. Lett.* A **20** 1147

Yurov A, Moruno P and Gonzalez-Diaz P 2006 *Preprint* astro-ph/0606529

Fang W, Lu H and Huang Z 2006 *Preprint* hep-th/0606032

Fang W, Lu H and Huang Z 2006 *Preprint* hep-th/0604160

Huang Z *et al.* 2006 *Preprint* hep-th/0610018

Huang Z *et al.* 2006 *Preprint* hep-th/0610019

Chimento L P and Lazkoz R 2006 *Phys. Lett.* B **639** 591

Gao C and Zhang S 2006 *Preprint* astro-ph/0605682

Srivastava S 2006 *Preprint* astro-ph/0602118

Tsujikawa S and Sami M 2004 *Phys. Lett.* B **603** 113

Guo Z, Piao Y, Zhang X and Zhang Y 2006 *Preprint* astro-ph/0608165

Alimohammadi M and Mohseni H 2006 *Phys. Rev.* D **73** 083527

Dzhunushaliev V, Folomeev V, Myrzakulov K and Myrzakulov R 2006 *Preprint* gr-qc/0608025

Setare M 2006 *Preprint* hep-th/ 0610190

[29] Barrow J 1993 *Phys. Rev.* D **47** 5359

Barrow J 1995 *Phys. Rev.* D **51** 2729

[30] Nojiri S and Odintsov S D 2006 *Phys. Lett.* B **637** 139 (*Preprint* hep-th/0603062)

[31] Nojiri S, Odintsov S D and Stefancic H 2006 *Phys. Rev.* D **74** 086009 (*Preprint* hep-th/0608168)

[32] Spergel D N *et al.* 2006 *Preprint* astro-ph/0603449

[33] Nojiri S and Odintsov S D 2004 *Gen. Rel. Grav.* **36** 1765 (*Preprint* hep-th/0308176)

Nojiri S and Odintsov S D 2004 *Mod. Phys. Lett.* A **19** 627 (*Preprint* hep-th/0310045)

Allemandi G, Borowiec A and Francaviglia M 2004 *Phys. Rev.* D **70** 043524

Meng X and Wang P 2005 *Class. Quantum Grav.* **22** 23

Meng X and Wang P 2004 *Class. Quantum Grav.* **21** 951

Easson D 2004 *Int. J. Mod. Phys.* A **19** 5343

Multamaki T and Vilja I 2006 *Phys. Rev.* D **73** 024018

Poplawski N 2005 *Preprint* gr-qc/0510007

Poplawski N 2006 *Preprint* gr-qc/0610133

Allemandi G, Borowiec A, Francaviglia M and Odintsov S D 2005 *Phys. Rev.* D **72** 063505 (*Preprint* gr-qc/0504057)

Faraoni V 2005 *Phys. Rev.* D**72** 124005

Abdalla M C B, Nojiri S and Odintsov S D 2005 *Class. Quantum Grav.* **22** L35 (*Preprint* hep-th/0409177)

Brevik I 2006 *Preprint* gr-qc/0603025

Brevik I 2006 *Preprint* gr-qc/0601100

Sotiriou T 2006 *Class. Quantum Grav.* **23** 5117

Sotiriou T 2005 *Preprint* gr-qc/0509029

Srivastava S 2006 *Preprint* hep-th/0605010

Srivastava S 2006 *Preprint* astro-ph/0608241

Samart D 2006 *Preprint* astro-ph/0606612

[34] Sami M, Toporensky A, Tretjakov P V and Tsujikawa S 2005 *Phys. Lett.* B **619** 193 (*Preprint* hep-th/0504154)

Calcagni G, Tsujikawa S and Sami M 2005 *Class. Quantum Grav.* **22** 3977 (*Preprint* hep-th/0505193)

Carter B and Neupane I 2005 *Preprint* hep-th/0512262

Neupane I 2006 *Preprint* hep-th/0602097

Tsujikawa S and Sami M 2006 *Preprint* hep-th/0608178

Sanyal A 2006 *Preprint* astro-ph/0608104

[35] Nojiri S, Odintsov S D and Sami M 2006 *Phys. Rev.* D **74** 046004 (*Preprint* hep-th/0605039)

[36] Koivisto T and Mota D 2006 *Preprint* hep-th/0609155

Koivisto T and Mota D 2006 *Preprint* astro-ph/0606078

Calcagni G, Carlos B and De Felice A 2006 *Nucl. Phys.* B **752** 404

Guo Z, Ohta N and Tsujikawa S 2006 *Preprint* hep-th/0610336

[37] Cognola G, Elizalde E, Nojiri S, Odintsov S D and Zerbini S 2006 *Preprint* hep-th/0611198