

Are the $\Theta^+(1540)$, $\Xi^{--}(1860)$ and $D^{*-}p(3100)$ Pentaquarks or Heptaquarks?

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We study the $\Theta^+(1540)$ discovered at SPring-8. We apply Quark Model techniques, that explain with success the repulsive hard core of nucleon-nucleon and kaon-nucleon exotic scattering, and the short range attraction present in pion-nucleon and pion-pion non-exotic scattering. We find a K-N repulsion which excludes the Θ^+ as a $K - N$ s-wave pentaquark. We explore the Θ^+ as a crypto-heptaquark, equivalent to a $K - \pi - N$ borromean boundstate, with positive parity and total isospin $I=0$. The attraction is provided by the pion-nucleon and pion-kaon interactions. The other candidates to pentaquarks $\Xi^{--}(1860)$, observed at NA49, and $D^{*-}p(3100)$, observed at H1, are also studied as linear molecular heptaquarks.

1 Introduction

The $uudd\bar{s}$ pentaquark $\Theta^+(1540)$ was discovered at LEPS¹ and DIANA². After the Jefferson Lab confirmation³, it was observed in several different experiences, with a mass of 1540 ± 10 MeV and a decay width of 15 ± 15 MeV. Recently the $ddss\bar{u}$ pentaquark $\Xi^{--}(1860)$ was observed at NA49⁴ and the $uudd\bar{c}$ pentaquark $D^{*-}p(3100)$ was observed at H1⁵. These are extremely exciting states because they may be the first exotic hadron to be discovered, with quantum numbers that cannot be interpreted as a quark and an anti-quark meson or as a three quark baryon. Exotic multiquarks are expected since the early works of Jaffe^{6,7}, and some years ago Diakonov, Petrov and Polyakov⁸ applied skyrmions to a precise prediction of the Θ^+ . The $\Xi^{--}(1860)$ and $D^{*-}p(3100)$ belong probably to the same family of exotic flavour pentaquarks.

We start in this talk by reviewing the Quark Model (QM) and the Resonating Group Method (RGM)⁹, which are adequate to study states where several quarks overlap. First we apply the RGM to show^{10,11,12} that the exotic $N - K$ hard core s-wave interaction is repulsive, excluding the Θ^+ as a bare s-wave pentaquark $uudd\bar{s}$ state or a tightly bound s-wave $N - K$ narrow resonance. However a π could also be present in this system, in which case the binding energy would be of the order of 30 MeV. Moreover this state of seven quarks would have a positive

Table 1: This table summarises the parameters μ, v, α, β (in Fm^{-1}) and scattering lengths a (in Fm).

channel	μ	v_{th}	α	β	a_{th}	a_{exp}
$K - N_{I=0}$	1.65	0.50	3.2	3.2	-0.14	-0.13 ± 0.04 ¹²
$K - N_{I=1}$	1.65	1.75	3.2	3.2	-0.30	-0.31 ± 0.01 ¹²
$\pi - N_{I=\frac{1}{2}}$	0.61	-0.73	3.2	11.4	0.25	0.246 ± 0.007 ¹⁷
$\pi - N_{I=\frac{3}{2}}$	0.61	0.36	3.2	3.2	-0.05	-0.127 ± 0.006 ¹⁷
$\pi - K_{I=\frac{1}{2}}$	0.55	-0.97	3.2	10.3	0.35	0.27 ± 0.08 ¹⁸
$\pi - K_{I=\frac{3}{2}}$	0.55	0.49	3.2	3.2	-0.06	-0.13 ± 0.06 ¹⁸

parity, and would have to decay to a p-wave $N - K$ system, which is suppressed by angular momentum, thus explaining the narrow width of the Θ^+ . We then put together the $\pi - N$, $\pi - K$ and $N - K$ interactions to show that the Θ^+ is possibly a borromean¹³ three body s-wave bound state of a π , a N and a K , with positive parity and total isospin $I = 0$. Finally we extend the crypto-heptaquark picture to flavour $SU(4)$ and study the Xi^{--} and $D^{*-}p$ multiquarks.

2 A Quark Model Criterion for Repulsion/attraction

We use a standard Quark Model Hamiltonian. The Resonating Group Method is a convenient method to compute the energy of multiquarks and to study hadronic coupled channels. The RGM was first used by Ribeiro¹⁴ to explain the $N - N$ hard-core repulsion.

We compute the matrix element of the Hamiltonian in an antisymmetrised basis of hadrons.

$$V_{\text{meson } A \text{ baryon } B} = \frac{\langle \phi_B \phi_A | (1 + P_{AB}) [-(V_{13} + V_{23} + V_{14} + V_{24})P_{13} + A_{23} + A_{14}] | \phi_A \phi_B \rangle}{\langle \phi_B \phi_A | (1 + P_{AB})(1 - P_{13}) | \phi_A \phi_B \rangle}, \quad (1)$$

where the exchange operator P_{14} produces the states colour-octet x colour-octet, expected in multiquarks, and where A_{23} is the quark-antiquark annihilation potential, crucial to the chiral symmetry of the interaction^{15,16}.

The exchange overlap results in a separable potential, and we arrive at the criterion for the interaction of ground-state hadrons:

- whenever the two interacting hadrons have a common flavour, the repulsion is increased,
- when the two interacting hadrons have a matching quark and antiquark the attraction is enhanced.

3 Why the Θ^+ cannot be a simple $uudd\bar{s}$ or $K - N$ state

Applying the criterion to the $S=1, I=0$ pentaquark, arranged in the color singlet clusters $uud - d\bar{s}$ or $ddu - u\bar{s}$ we find repulsion! Indeed we are able to reproduce the repulsive $K - N$ exotic s-wave phase shifts, which have been understood long ago^{10,11,12}. Moreover all other $uudd\bar{s}$ systems are even more repulsive or unstable. Because we checked all our only approximations, say using a variational method, and neglecting the meson exchange interactions, we estimate that something even more exotic is probably occurring!

Suppose that a $q - \bar{q}$ pair is added to the system. Then the new system may bind. Moreover the heptaquark had a different parity and therefore it is an independent system (a chiral partner). Here we propose that the Θ^+ is in fact a heptaquark with the strong overlap of a $K - \pi - N$, where the π is bound by the $I=1/2$ $\pi - K$ and $\pi - N$ attractive interactions.

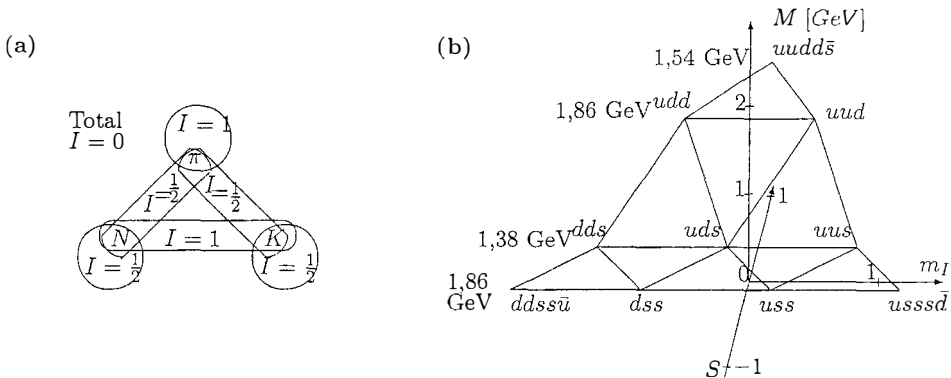


Figure 1: We show in (a) the isospin couplings in the Θ^+ . In (b) we exhibit, in a three dimensional strangeness-flavour-mass plot, the expected masses of the exotic anti-decuplet.

4 the $K - N$, $\pi - K$, $\pi - N$ and $K - \pi - N$ systems

We now investigate the borromean¹³ binding of the exotic Θ^+ constituted by a N , K and π triplet. We arrive at the separable potentials for the different two-body potentials^{11,16},

$$\begin{aligned}
 V_{K-N} &= \frac{2 - \frac{4}{3}\vec{\tau}_A \cdot \vec{\tau}_B (M_\Delta - M_N)}{\frac{5}{4} + \frac{1}{3}\vec{\tau}_A \cdot \vec{\tau}_B} \frac{(2\sqrt{\pi})^3}{3} e^{-\frac{p_\lambda^2}{2\beta^2}} \int \frac{d^3 p'_\lambda}{(2\pi)^3} e^{-\frac{p'^2_\lambda}{2\beta^2}} \\
 V_{\pi-N} &= \frac{2}{9} (2M_N - M_\Delta) \vec{\tau}_A \cdot \vec{\tau}_B \mathcal{N}_\alpha^{-2}, \\
 V_{\pi-K} &= \frac{8}{27} (2M_N - M_\Delta) \vec{\tau}_A \cdot \vec{\tau}_B \mathcal{N}_\alpha^{-2},
 \end{aligned} \tag{2}$$

where $\vec{\tau}$ are the isospin matrices.

Because the pion is quite light we start by computing the pion energy in an adiabatic $K-N$ system. Our parameter set, tested in 2-body channels, is presented in Table 1. The only favourable flavour combination is shown in Fig. 1 (a). Indeed we get quite a bound pion, but it only binds at very short $K - N$ distances. However when we remove the adiabaticity, by allowing the K and N to move in the pion field, we find that the pion attraction overcomes the $K - N$ repulsion but not yet the $K - N$ kinetic energy. Other effects may further increase attraction. We are planning to include full three-body Fadeev equations, the coupling to the $K - N$ p-wave channel and the short-range two-pion-exchange-interaction.

5 $SU(4)$ flavour: the $\bar{K} - N - \bar{K}$ and anti-charmed systems

Extending the pentaquark and the molecular heptaquark picture to the full $SU(3)$ anti-decuplet we arrive at the picture shown in Fig. 1 (b), where,

- The $\Xi^{--}(1860)$ cannot be a $dds\bar{u}$ pentaquark because it would suffer from repulsion.
- Adding a $q - \bar{q}$ pair we arrive at a $I = 1/2$ $\bar{K} - N - \bar{K}$ linear molecule where the $\bar{K} - N$ system has isospin $I=1$, and it is an attractive system. We find that the $\bar{K} - N - \bar{K}$ molecule is bound, although we are not yet able to arrive at a binding energy of - 60 MeV.
- Then the $I = 1/2$ elements of the exotic anti-decuplet are $K - \bar{K} - N$ molecules.
- Only the $I=1$ elements are pentaquarks, or equivalently overlapping $\bar{K} - N$ systems.

In what concerns anti-charmed pentaquarks like the very recently observed $D^{*-}p$, or anti-bottomed ones, this extends the anti-decuplet to flavour $SU(4)$ or $SU(5)$. Anti-charmed pentaquarks were predicted by many authors, replacing the s by a c . Again the pentaquark $uudd\bar{c}$ is unbound, and we are researching the possible $D(D^*) - \pi - N$ molecular heptaquarks.

6 Conclusion

We conclude that the $\Theta^+(1540)$, $\Xi^{--}(1860)$ and $D^{*-}p(3100)$ hadrons very recently discovered cannot really be s-wave pentaquarks.

- We also find that they may be a heptaquark states, with two repelled K and N clusters bound third π cluster.

- More effects need to be included, say exact Fadeev equations, the K-N p-wave coupled channel, and medium range interactions.

- This is a difficult subject with the interplay of many effects. The theoretical models should not just explain the pentaquarks, they should also comprehend other hadrons.

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