ERGODICITY OF SOME SIMPLE MODEL SYSTEMS OF INFINITELY MANY PARTICLES

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1. Introduction

The aim of our talk is to propose some simple model system of infinitely many classical particles and to investigate its ergodic properties.

By many authors such as Sinai and Lanford dynamical systems with infinitely many degrees of freedom have been constructed [3],[4]. But their time evolutions are so complicated. It is hardly possible to study their ergodic properties except the cases of ideal gases in arbitrary dimension and the system of hard rods in one-dimensional space [1],[2].

Recently Hardy et al have proposed some simple 2-dimensional system of classical particles [5]. Simplicity of its dynamics allows to study some of its ergodic properties. Nevertheless it is still very difficult problem. They have studied it by "linearizing" the time evolution and could get some results.

Here we study the ergodic properties of its time evolution without "linearizing" but we assume the domain where collisions do occur is bounded.

We prove that the system so obtained is Bernoulli, so is mixing, and the time correlation functions are decreasing exponentially.

Unfortunately our system has no interactions between particles except those of which are in some bounded domain, so the system is to be considered as "perturbed ideal gas". It is also possible to consider it as a finite system surrounded by ideal gas.

2. Description of the model.

Now we describe the system more precisely [5]. Let Z^2 be 2-dimensional lattice.

The phase space $\mathfrak{X}$ of allowed configurations of particles is

$$\mathfrak{X} = \{ X ; X : Z^2 \times P \rightarrow \{ 0, 1 \} \}$$

where $P = \{ V = (v_x, v_y) \in Z^2 ; |v_x| + |v_y| = 1 \}$ is a momentum space.

Let $\mathfrak{X}_a$ be the space of configurations of particles on a site $a \in Z^2$:

$$\mathfrak{X}_a = \{ X_a ; X_a : \{ a \} \times P \rightarrow \{ 0, 1 \} \} .$$

Then naturally

$$\mathfrak{X} = \prod_{a \in Z^2} \otimes \mathfrak{X}_a .$$

The time evolution of the system is defined as follows.

Let T_0 be a translation of $\mathfrak{X}$, that is,

$$T_0 X (a, v) = X(a - v, v) \quad \text{for } \forall X \in \mathfrak{X} , \ \forall a \in Z^2 , \text{ and } \forall v \in P.$$

Collision C is defined by the interaction operator φ_a on each lattice site a, that is a bijection mapping of $\mathfrak{X}_a$ and satisfies the following conditions.

1) Preservation of the total momentum:

$$\sum_{v \,:\, \varphi_a X_a(v) = 1} v = \sum_{v \,:\, X_a(v) = 1} v ,$$

2) Preservation of the total number of particles:

$$\sum_{v \in P} \varphi_a X_a(v) = \sum_{v \in P} X_a(v).$$

Under these conditions φ_a is trivial, that is,

$$X_a = \varphi_a X_a \quad \text{for } \forall X_a \in \mathfrak{X}_a$$

or given by

$$(\varphi_a X_a)(v) = X_a(v) + (-1)^{v_x}[X_a(v^1)X_a(v^3)\{1 - X_a(v^2)\}\{1 - X_a(v^4)\} - \{1 - X_a(v^1)\}\{1 - X_a(v^3)\} X_a(v^2) X_a(v^4)]$$

for $\forall X_a \in \mathcal{X}_a$, $\forall v \in P$, where $v^1 = (1, 0)$, $v^2 = (0, 1)$, $v^3 = (-1, 0)$ and $v^4 = (0, -1)$.

This is the one considered in [5].

C is defined by

$$CX = \Pi\otimes_{a \in Z^2} \varphi_a X_a \quad \text{for } \forall X = \Pi\otimes_{a \in Z^2} X_a \in \mathcal{X} = \Pi\otimes_{a \in Z^2} \mathcal{X}_a.$$

The time evolution T of $\mathcal{X}$ is defined by

$$T = CT_o . \qquad (\text{or } T = T_o C T_o).$$

Under some conditions there exists a unique equilibrium state ρ on $\mathcal{X}$, that is, T-invariant probability measure, which has no correlations between particles with different velocities and lattice sites [5].
Henceforth we fix such state ρ .

Now we assume that the interaction operators φ_a on $\mathcal{X}_a$ are trivial if $a \notin V$ for some bounded domain V of Z^2 .

3. Results and some remarks.

We investigate the ergodic properties of the dynamical system $(\mathcal{X}, T, \rho)$.
We get

__Theorem__. $(\mathcal{X}, T, \rho)$ is a Bernoulli System, therefore, has a mixing property, and the time correlation functions are decreasing exponentially, that is, for any cylinder sets A and B (i.e. subsets of $\mathcal{X}$ defined on the sites that are in some bounded domain of Z^2) we have

$$|\rho(T^n(A) \cap B) - \rho(A)\rho(B)| \leq \text{const.} \; r^n \quad \text{for } \forall n \geq 1$$

where const. and r are constants depending only on the supports of A and B and $0 < r < 1$.

__Remark__ These results can be extended to more general one.
Firstly dimension of the lattice is not necessary to have two.
Secondly interactions between different sites can be permitted, in this case "dissipative" character of the interactions is enough.

References

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