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Hybrid high-energy factorization and evolution at NLO

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A scheme for NLO computations of generic observables in hadron collisions is presented within hybrid factorization, that is the framework of high-energy factorization with one off-shell and one on-shell initial-state parton. The ambiguity of projectile-target separation in the high-energy limit is governed by the Collins-Soper scale μ_Y. The NLO unintegrated PDF is constructed in terms of collinear PDFs, and its μ_Y-evolution reproduces the Collins-Soper-Sterman equation in the TMD limit of $|k_\perp| \ll \mu_Y$. BFKL-Collins-Ellis evolution of the Green's function in the UPDF takes care of the resummation of high-energy logarithms.

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1. Introduction

Partonic coefficient functions and anomalous dimensions of Collinear Factorization (CF) are enhanced by high-energy logarithms, and High-Energy Factorization (HEF) [1–4] was introduced to resum those. It concerns the logarithms of center-of-mass energy squared ($\hat{s}$) of the partons initiating the hard process with the hard scale μ in the regime $\Lambda_{\text{QCD}}^2 \ll \mu^2 \ll \hat{s}$. This resummation leads to the notion of "unintegrated PDF" (UPDF), which depends on the transverse momentum ($k_\perp$) of the parton initiating the hard process besides on the longitudinal momentum fraction (x). The UPDF is a process-independent quantity and should be accompanied by the corresponding "off-shell" x-and $k_\perp$-dependent matrix element of the hard process.

A proper definition of the $k_\perp$-dependent matrix element relies on gauge-invariant factorization of QCD matrix elements in the Regge limit, when the partonic energy $\hat{s}$ is much larger than any other scale. The auxiliary parton method [5] achieves such factorization for tree-level matrix elements. In [6], real-emission and virtual contributions were put together in the auxiliary-parton method, and the resulting expressions for the next-to-leading order (NLO) cross section in CF were separated into *target*, *Green's function* and *projectile* contributions. The *projectile* contribution is process-dependent and provides a precise prescription for future NLO computations in HEF with one off-shell parton, also referred to as "hybrid formalism" [7–9]. The *target* and *Green's function* contributions are process-independent and provide the expression for UPDF at NLO in α_{s} in terms of usual PDFs. In the following, we highlight the main elements of the approach and findings of [6].

2. Ingredients

The key ingredient is to embed the process aimed to be formulated in hybrid factorization into a process in collinear factorization. Consider the collision of two hadrons, with production of the final state of interest $\mathcal{H}$,

$$h(\lambda P) + h(\bar{P}) \to \mathcal{H} + \mathcal{X} \,, \tag{1}$$

where the final state $\mathcal{H}$ is defined by the jet-definition at LO in CF, which is correspondingly generalized to NLO. Eventually, we want to consider the high-energy limit regarding the first hadron, which we will achieve by making the parameter λ large. We introduce a rapidity Y_μ associated with $\mathcal{H}$, which separates events into "target" and "projectile" parts (see Fig. 1). Then we can, in an infra-red save manner, define the kinematic variables

$$x = \sum_j \theta\big(y_j < Y_\mu\big)\, \frac{p_j \cdot \bar{P}}{P \cdot \bar{P}} \quad , \quad k_\perp = -\sum_j \theta\big(y_j < Y_\mu\big)\, p_{j\perp} \,, \tag{2}$$

where the summation includes all momenta of particles p_j with rapidity $y_j < Y_\mu$, including those belonging to $\mathcal{H}$. Knowing x, one can associate rapidity scale μ_Y (or Collins-Soper scale) to the rapidity Y_μ as

$$\mu_Y = \nu x\, e^{-Y_\mu} \quad \Leftrightarrow \quad Y_\mu = \ln \frac{\nu x}{\mu_Y} \,, \tag{3}$$

where $\nu = \sqrt{2P\cdot\bar{P}}$. Due to IRC-safety of variables x and $k_\perp$, the corresponding hadronic differential

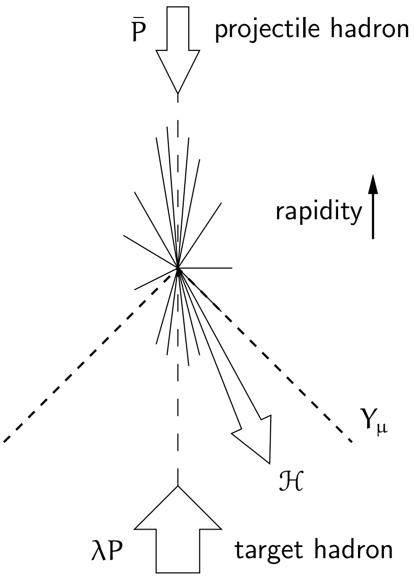

Figure 1: High-energy hadronic collision event with production of the system of interest $\mathcal{H}$.

cross section

$$\frac{d\sigma_\lambda^{\rm CF}}{dx d^2k_\perp}\,, \tag{4}$$

should be computable in CF, at least up to NLO, and in the limit

$$\lambda \to \infty \quad , \quad x, k_\perp\text{– fixed}\,. \tag{5}$$

The cross section can be differential further in variables associated with $\mathcal{H}$, and is equal to:

$$\frac{d\sigma_\lambda^{\rm CF}}{dx d^2k_\perp}(x, k_\perp, \dots) = \sum_{i,\bar{\imath}} \int_0^1 dX\, f_i(X) \int_0^1 d\bar{x}\, f_{\bar{\imath}}(\bar{x}) \times \frac{d\hat{\sigma}_{i\bar{\imath}}^{\rm CF}}{dx d^2k_\perp}(\lambda X, \bar{x}\,; x, k_\perp, \dots)\,, \tag{6}$$

where by "…" we denote final-state kinematic variables defining $\mathcal{H}$, which are independent of x and $k_\perp$. The parton with momentum $\lambda X P^\mu$ is the initial-state auxiliary parton, and the momentum difference $(\lambda X - x)P^\mu$ is carried by final-state auxiliary partons.

It turns out, that in order to consistently define the real radiation contribution and cancel all divergences at NLO, a hierarchy of rapidity limits

$$\lambda \succ \lambda_0 \succ \lambda_1 \to \infty \tag{7}$$

needs to be introduced rather than just one parameter $\lambda \to \infty$, and the cross section needs to be separated in a number of different contributions. First of all, there is the *non-k_T-factorizable* contribution, corresponding to values of X that are smaller than λ_0/λ and for which λX grows too slowly. The other, *k_T-factorizable* contributions, are separated as follows (Fig. 2):

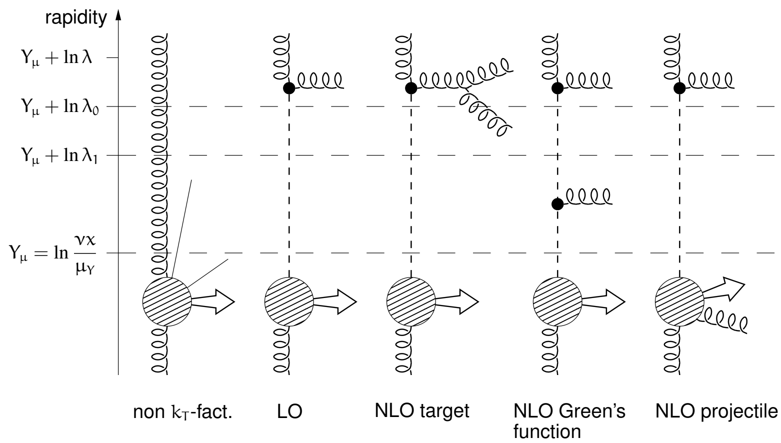

Figure 2: Contributions to the k_T-factorizable part of the cross section in the $\lambda \to \infty$ limit. The vertical dashed lines indicate the gauge-invariant factorization of projectile, target and the contribution of an emission in multi-Regge kinematics, which can be formulated in terms of the t-channel exchange of the off-shell gluon.

1. **The LO contribution:** the rapidity of auxiliary parton is $y_q > Y_\mu + \ln\lambda_0$
2. **The NLO target impact-factor (IF) contribution:** both the auxiliary parton and the radiative parton have rapidity $y_{q,r} > Y_\mu + \ln\lambda_1$.
3. **The Green's function contribution:** corresponds to $y_q > Y_\mu + \ln\lambda_1$ while $Y_\mu > y_r > Y_\mu + \ln\lambda_1$.
4. **The NLO projectile contribution:** the radiative parton has $y_r < Y_\mu$. This restriction is implemented by subtraction of the corresponding "collinear" asymptotics of the matrix element, with $y_r > Y_\mu$, from the "naive" real radiation contribution which is obtained by adding the radiation after $\lambda \to \infty$.

The parameter λ_1 separates the target IF and Green's function contributions, and is also used to remove energy logarithms from the IF contribution and resum them into the Green's function.

A similar decomposition is applied to the virtual contribution. By applying the limit $\lambda \to \infty$ as in the Born case directly to the loop amplitude, one obtains an expression with two types of terms, that in [10] were designated as *familiar* $d\mathrm{V}^{\mathrm{fam}}_{\star\bar{\imath}}$ and *unfamiliar* $d\mathrm{V}^{\mathrm{unf}}_{i\bar{\imath}}$. The former are very similar to expressions with the off-shell gluon replaced by an on-shell gluon, and in particular all divergences are identical to the on-shell case. Also it has a smooth limit for $|k_\perp| \to 0$. The unfamiliar contribution is exactly proportional to the Born contribution $d\mathrm{B}_{\star\bar{\imath}}$, but contains "new" divergences compared to the on-shell case, and does not have a smooth limit for $|k_\perp| \to \infty$. The target, projectile, and Green's function partonic differential cross sections are obtained by reshuffling terms among them as

$$d\mathrm{V}^{\mathrm{target}}_{i\bar{\imath}}(\epsilon,\lambda,\lambda_1,\mu_Y) = d\mathrm{V}^{\mathrm{unf}}_{i\bar{\imath}}(\epsilon,\lambda) - d\mathrm{B}_{\star\bar{\imath}} \times a_\epsilon \left[\left(\frac{\mu^2}{\mu_Y^2}\right)^{\epsilon}\frac{N_\mathrm{c}}{\epsilon^2} + \frac{\gamma_g}{\epsilon} + \left(\frac{\mu^2}{|k_\perp|^2}\right)^{\epsilon}\frac{2N_\mathrm{c}\ln\lambda_1}{\epsilon}\right] \tag{8}$$

$$d\mathrm{V}^{\mathrm{Green}}_{\star\bar{\imath}}(\epsilon,\lambda_1) = d\mathrm{B}_{\star\bar{\imath}} \times a_\epsilon \left[\left(\frac{\mu^2}{|k_\perp|^2}\right)^{\epsilon}\frac{2N_\mathrm{c}\ln\lambda_1}{\epsilon}\right] \tag{9}$$

$$d\mathrm{V}^{\mathrm{projectile}}_{\star\bar{\imath}}(\epsilon,\mu_Y) = d\mathrm{V}^{\mathrm{fam}}_{\star\bar{\imath}}(\epsilon) + d\mathrm{B}_{\star\bar{\imath}} \times a_\epsilon \left[\left(\frac{\mu^2}{\mu_Y^2}\right)^{\epsilon}\frac{N_\mathrm{c}}{\epsilon^2} + \frac{\gamma_g}{\epsilon}\right], \tag{10}$$

where $a_\epsilon = \frac{\alpha_\mathrm{s}}{2\pi}\frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)}$, μ is the renormalization scale, N_c is the number of colors, n_f the number of active light quarks, $\gamma_g = \frac{11N_\mathrm{c}}{6} - \frac{n_f}{3}$, and ϵ the dimensional regulator as $D = 4 - 2\epsilon$.

3. Results

Including also both collinear counter terms associated with $f_i(X)$ and $f_{\bar{\imath}}(\bar{x})$, one ends up with individually finite target, Green's function, and projectile contributions. Demanding that the projectile contributions is independent of μ_Y leads to an evolution equation for the UPDF, or rather $\hat{F}(x,k_\perp;\mu_Y) = xF(x,k_\perp;\mu_Y)$, given by

$$\frac{d\hat{F}(x,k_\perp;\mu_Y)}{d\ln\mu_Y^2} = \frac{\alpha_\mathrm{s}N_\mathrm{c}}{2\pi}\int\frac{d^2r_\perp}{\pi|r_\perp|^2}\left\{\hat{F}\left(x\left[1+\frac{|r_\perp|}{\mu_Y}\right],k_\perp+r_\perp;\mu_Y\right)\theta\left(|r_\perp|<\mu_Y\,\frac{1-x}{x}\right) - \theta\big(\mu_Y-|r_\perp|\big)\,\hat{F}(x,k_\perp;\mu_Y)\right\}. \tag{11}$$

This equation is very similar to the equation found in [11] from the Born-Oppenheimer renormalization group. Considering the Fourier transform of $\hat{F}$ with respect to $k_\perp$, which then depends on $x_\perp$ instead, one sees that the equation reduces to the usual Collins-Soper-Sterman equation of Transverse-Momentum Dependent (TMD) factorization [12] in the limit $|k_\perp| \ll \mu_Y$. Also, at vanishing $x_\perp$, the equation becomes a DGLAP-type equation, essentially with the finite part of the splitting function missing.

The equation above comes from the projectile contribution and dictates the evolution in μ_Y, thereby resummming logarithms of $\mu_Y/|k_\perp|$. The sum of the finite target and Green's function contribution provide the initial condition at $\mu_Y = |k_\perp|$ to the UPDF, and is found to have the form

$$F\big(x, k_\perp, \mu_Y = |k_\perp|\big) = \sum_i \int_x^1 dX\, f_i(X, \mu_F) \int d^2k'_\perp I_i\big(k'_\perp, \mu_F\big)\, G\Big(k'_\perp, k_\perp, \frac{X}{x}, \mu_F\Big)\,, \qquad (12)$$

where I_i is exactly the LO impact factor plus its NLO corrections, and G is the Green's function containing the LO BFKL kernel. Consequently, the expression includes the resummation of the high-energy logarithms of $1/x$.

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