A NEEDLE IN A HAYSTACK (*)

J. Bernstein

Brookhaven National Laboratory, Upton, L. I., N.Y.

G. Feinberg

Columbia University, New York, N. Y.

(presented by J. Bernstein)

In reactions involving, for example, $\pi^- + p$ or $p - \bar{p}$ in the initial states, there are no selection rules which favour the production of ρ's rather than ω's or *vice versa*. Hence as they have about the same mass one may expect to produce them in comparable amounts in most experiments. The ρ is presumably a $J = 1^-$, $I = 1$ state and thus can decay into 2π's via a strong coupling. The ω is probably $J = 1^-$, $I = 0$. However, it can also decay into 2π's via the electromagnetic coupling, as was first emphasized by Glashow [1]. Since the ω is a 1^- object its decay into two pions will necessarily proceed into the $I = 1$ 2π state. Thus the G-forbidden ω decay and the allowed ρ decay lead into the same isotopic channel, which is to say, the two decays must be added coherently in the description of any process in which 2π's are observed coming from a source which is a mixture of ρ's and ω's.

We have made a theoretical description of this situation in terms of a set of coupled time dependent Schrödinger equations which mix ρ and ω states that we assume to be isotopically pure at the time of production. The resulting mathematical problem is classical and the details are given in several places [2]. One is led to a mass matrix, $\Gamma + iM$, whose eigenvalues and eigen-vectors express the exponentially decaying states, $\bar{\rho}$ and $\bar{\omega}$, in terms of the isotopically pure states ρ and ω; i.e., one solves the equation,

$$(\Gamma + iM)\begin{pmatrix}\rho\\ \omega\end{pmatrix} = \gamma\begin{pmatrix}\rho\\ \omega\end{pmatrix} \tag{1}$$

where

$$\Gamma + iM = \begin{pmatrix} im_\rho + \Gamma_\rho/2 & im + \Gamma \\ im^* + \Gamma^* & im_\omega + \Gamma_\omega/2 \end{pmatrix} \tag{2}$$

This matrix is not, *a priori*, symmetric and, as far as we can tell, it is only symmetric if one is willing to treat the unstable states ρ and ω as eigenstates of T, the time reversal operator. The magnitudes of the complex parameters which give the mixing of ρ with ω $\lambda = im + \Gamma$ and $\lambda' = im^* + \Gamma^*$, are not easy to estimate precisely but may be as large as $\alpha m_\rho \sim 5$ MeV [1]. In the numerical work that follows we have taken the real and imaginary parts of λ to be about 1 MeV; probably a conservative choice.

The most interesting quantities accessible to this formalism are the energy amplitudes $\psi_\rho(E)$ and $\psi_\omega(E)$ that generate the observed 2π and 3π decay spectra. These can be written very simply in terms of the isotopically pure states $|\rho\rangle$ and $|\omega\rangle$ as follows:

$$\psi_\rho(E) = \frac{|\rho\rangle(i(E - m_\omega) - \Gamma_\omega/2) + |\omega\rangle\lambda'}{(i(E - m_{\bar{\rho}}) - \Gamma_{\bar{\rho}}/2)(i(E - m_{\bar{\omega}}) - \Gamma_{\bar{\omega}}/2)} \tag{3}$$

$$\psi_\omega(E) = \frac{|\omega\rangle(i(E - m_\rho) - \Gamma_\rho/2) + |\rho\rangle\lambda}{(i(E - m_{\bar{\rho}}) - \Gamma_{\bar{\rho}}/2)(i(E - m_{\bar{\omega}}) - \Gamma_{\bar{\omega}}/2)} \tag{4}$$

In Eqs. (3) and (4) m_ρ, m_ω, Γ_ρ, Γ_ω refer to the parameters of the isotopically pure ρ and ω states. In the case of the ρ they correspond to the observed parameters of the charged ρ; $m_\rho \approx 750$ MeV, $\Gamma_\rho \approx 100$ MeV. The parameters $m_{\bar{\rho}}$, $m_{\bar{\omega}}$, $\Gamma_{\bar{\rho}}$, $\Gamma_{\bar{\omega}}$

(*) Work performed under the auspices of the U.S. Atomic Energy Commission.

characterize the exponentially decaying states $\bar{\rho}$ and $\bar{\omega}$. These parameters, presumably, differ somewhat from the ρ, ω parameters. However, in what follows, we shall ignore such differences which we are not able to estimate reliably in any case, and which appear to be small.

In a typical experiment a linear combination of Eqs. (3) and (4)

$$\alpha\psi_{\rho}(E)+\beta\psi_{\omega}(E)$$

is produced. The ratio, $f=\beta/\alpha$, depends on the incident pion energy, is complex and may be of order 1 in magnitude [3]. We use the relations

$$0=\langle 2\pi|\omega\rangle=\langle 3\pi|\rho\rangle$$

where we assume that T is conserved in the decay of the pure ω and ρ. This amounts to neglecting electromagnetism except in the $\rho-\omega$ mixing. From this, the 2π spectrum generated by the linear combination above, in the approximation $m_{\omega}=m_{\bar{\omega}}$; $\Gamma_{\omega}\lambda=\Gamma_{\bar{\omega}}$ is given by

$$N(E)_{2\pi}=\frac{1}{[(E-m_{\bar{\rho}})^2+\Gamma_{\bar{\rho}}^2/4]} \times\left[1+\frac{|\lambda f|^2-\Gamma_{\bar{\omega}}/2\ 2\,\mathrm{Re}(\lambda f)-2\,\mathrm{Im}(\lambda f)(E-m_{\bar{\omega}})}{[(E-m_{\bar{\omega}})^2+\Gamma_{\bar{\omega}}^2/4]}\right] \quad (5)$$

In Figs. 1 and 2 the function $N(E)_{2\pi}$ is plotted for a few choices of the parameters. The characteristic feature of all of these plots is the superposition of a sharp needle-like ω peak on top of the broad haystack-like ρ. The interference between the two resonances has the effect of extending the 2π spectrum toward the ω mass. The details of the shape of the spectra are very sensitive to the choice of parameters.

In actual experiments the function given by Eq. (5) must be folded into an experimental energy resolution function which in present experiments is about 10 MeV wide. Hence one should not expect to find the sharp spectra as given in Figs. 1 and 2. However, there can be observable effects such as an apparent shifting upward of the mass of the neutral ρ compared to the charged ρ or even some splitting in the neutral ρ peak. There may be hints of such effects in some of the experimental data on neutral ρ production [4] and it will be interesting to see if these effects persist as the statistics improve [5]. Since the ratio of ω to ρ production may vary appreciably with the incident energy, the interference effect could also vary appreciably as the energy is changed.

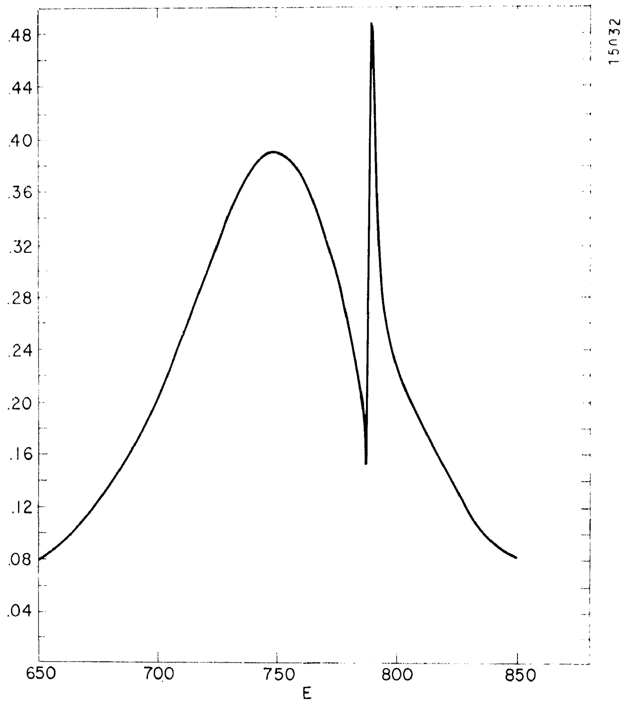

Fig. 1 This is a plot of $N(E)_{2\pi}$ with the parameters $m_{\bar{\omega}}=790$ MeV, $\Gamma_{\bar{\varrho}}=100$ MeV, $\Gamma_{\bar{\omega}}=1$ MeV, $f=1$, Re $\lambda=1$, and Im $\lambda=\frac{1}{2}$. The scale in both figures is arbitrary.

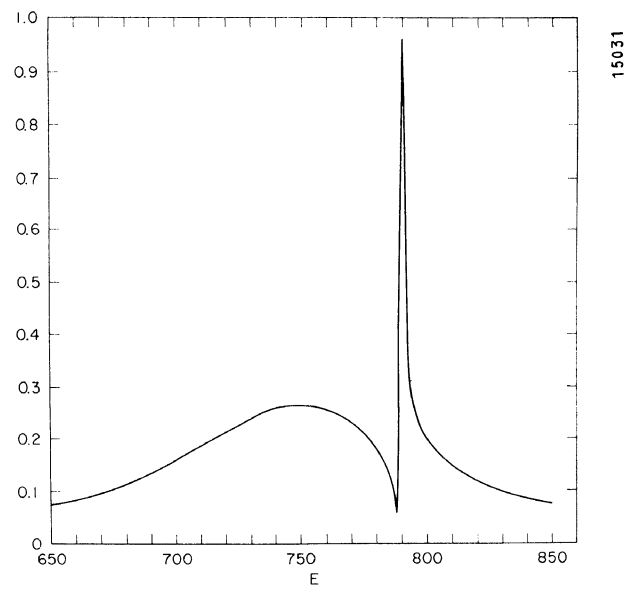

Fig. 2 This is a plot of $N(E)_{2\pi}$ with the parameters $m_{\bar{\omega}}=790$ MeV, $m_{\bar{\varrho}}=750$ MeV, $\Gamma_{\bar{\varrho}}=120$ MeV, $\Gamma_{\bar{\omega}}=1$ MeV, $f=1$, Re $\lambda=1$, and Im $\lambda=1$.

LIST OF REFERENCES

1. S. Glashow, Phys. Rev. Letters *7*, 469 (1961); see also J. C. Taylor, Phys. Rev. Letters *8*, 219 (1962).
2. For example, see T. D. Lee, R. Oehme, and C. N. Yang, Phys. Rev. *106*, 340 (1957); K. Aizu, Nuovo Cimento *6*, 1040 (1957); J. Bernstein and G. Feinberg, Nuovo Cimento (to be published) (1962).
3. A. Pevsner, E. Pickup, D. K. Robinson, and J. Steinberger, private communications.
4. We are especially grateful to Drs. E. Pickup and D. K. Robinson for discussions of their results on ϱ production in $\pi^- + p$ reactions.
5. There are also " mixing " effects on the 3π spectrum given by projecting Eqns. (3) and (4) on the state $|3\pi\rangle$. Because $\Gamma_\varrho \gg \Gamma_\omega$ these are probably too small to be observed.

DISCUSSION

FEINBERG: I would like to comment that most of these considerations have been made independently by Dr. J. C. Taylor.

JACKSON: Is there any obvious reason why the interference minimum occurs on the low side? It seems odd that the signs are such that it always happens.

BERNSTEIN: I do not think there is a general reason; I have not taken a case numerically where the dip did not occur on the low side.

LEVY: Maybe the 7090 knows the answer?

BERNSTEIN: Maybe Feinberg knows the answer?

FEINBERG: You can get the dip on the high side or no dip on the low side if you change the sign of the admixing parameter, for example, since the interference term is linear in the admixing parameter.

JACKSON: Consequently experimentalists should not be looking for that particular shape as opposed to some other shape.

BERNSTEIN: I would not suggest looking for that shape, but it is apparently a fact that the neutral ϱ looks different from the charged ϱ. It seems as if the neutral ϱ likes to push itself up towards a higher mass than the charged ϱ.

OPPENHEIMER: Is it a model that makes you choose a 1 MeV width for the ω?

BERNSTEIN: The 1 MeV is an under-estimate which was suggested by the calculation of Gell-Mann and Zachariasen who discussed the electromagnetic properties of the nucleon using these particles. If you take their model then the admixing parameter actually is something like α times the mass of the ϱ, which is of the order of 5 MeV. So, just to be on the modest side, we took 1 MeV.

MANDELSTAM: If I am right the experiments taken literally indicated a needle on the other side of the haystack but, of course, there might be some experimental error. Do you know whether it is appreciably easier to fit a curve, like the one you showed, to the experiments or to fit a single peak to the experiment, which after all is not completely out of the question?

BERNSTEIN: The statistics on these experiments are so far extremely bad and the conclusions you draw depend on which of the experimental groups you talk to. Pickup and Robinson have an experiment at Brookhaven to study both the charged and neutral ϱ and they have taken a number of our curves and have found a natural fit to their data with this needle-in-the-haystack effect, but it is not completely clear they could not get a fit without it. Steinberger on the other hand feels there is no evidence in his data on the neutral ϱ for this particular kind of structure and I think we must wait until there are more statistics.

FEINBERG: It might be worthwhile to comment that experimental curves may look very different from experiment to experiment because they depend on these parameters α and β which are the relative amount of ϱ and ω produced in the experiment and that, of course, can depend on all sorts of things.

MANDELSTAM: Whatever your parameters, you still have the needle on the right of the haystack, simply because the energy of the ω is greater than the energy of the ϱ.

PUPPI: There is one experimental feature which is stable, which differentiates clearly the neutral ϱ from the charged ϱ. This is not the shape of the peak, because the peak is very badly defined in all those experiments, but it is a strong forward-backward asymmetry in the angular distribution of the neutral ϱ, in contrast with the charged ϱ which at the resonance shows a perfect symmetry. If you take the result on the charged ϱ in which the interference changes sign at the resonance (as an indication that this is a resonance) then, unless you invent a very pathological model with waves of different parity you are not able to explain this strong forward-backward asymmetry. Is your model able to explain this strong forward-backward asymmetry at the peak?

BERNSTEIN: We certainly have not considered the angular distribution.

FEINBERG: In any case, in order to get a forward-backward asymmetry, you need an interference between a background, say, *S*-wave, and this *P*-wave (the 2 pions come out on a *P*-wave). Just having a $\varrho - \omega$ admixture by itself is not enough to explain it.