PARITY CONSERVATION AND SYMMETRY LAWS IN STRONG INTERACTIONS

N. Dallaporta

University of Padua, Padua, Italy

These are some results of a work in collaboration with L. K. Pandit of the Tata Institute of Fundamental Research, Bombay.

1. One of the problems which has been considered with most attention in these last two years in the field of strong interactions is the possible connection which appears to exist, when all strong interactions are assumed to be of non-derivative type and *CP* invariance of universal validity, between exact parity conservation and charge state symmetries. Well known results, obtained by different authors [1] are that: for π^0 interactions, parity conservation follows directly from hermiticity of the Lagrangian and *CP* invariance; in order to obtain the same result for charged pion-nucleon and pion-Ξ interactions, it is necessary to impose further at least charge symmetry; and for Σ-pion interactions, at least charge independence. These cases strongly suggest that increasingly high symmetries may progressively compel parity conservation for a wider and wider class of interactions, so that one may hope to arrive at complete correlation between these two properties for all strong interactions if only sufficiently high symmetries are considered. This, in fact, was shown to be the case both by Sakurai [2] and by Feinberg and Gürsey [3], when one assumes the doublet approximation: that is, the 8 beryons arranged in terms of four similar doublets:

$$N_1 = \begin{pmatrix} p \\ n \end{pmatrix} \quad N_2 = \begin{pmatrix} \Sigma^+ \\ Y^0 \end{pmatrix} \quad N_3 = \begin{pmatrix} Z^0 \\ \Sigma^- \end{pmatrix} \quad N_4 = \begin{pmatrix} \Xi^0 \\ \Xi^- \end{pmatrix} \tag{1}$$

with

$$Y^0 = \frac{\Sigma^0 + \Lambda^0}{\sqrt{2}} \quad Z^0 = \frac{\Sigma^0 - \Lambda^0}{\sqrt{2}} \tag{2}$$

as the main scheme for the strong interactions, and imposes invariance not only with respect to charge symmetry for pion interactions, but also to corresponding analogous operations for neutral and charged *K* interactions, which could be termed " hypercharge symmetry operations " such as:

$$\begin{array}{llcll} N_1 \to N_2 & K^0 \to \bar{K}^0 & & N_1 \to N_3 & K^+ \to \bar{K}^- \\ & & \text{and} & & \\ N_3 \to N_4 & K^+ \to K^+ & & N_2 \to N_4 & K^0 \to K^0 \end{array} \tag{3}$$

or any combination of them.

A similar result was obtained by Dallaporta and Pandit [4] by applying a still higher symmerty to the *K* interactions in the doublet approximation case, namely " hypercharge independence " which will be formulated later.

However, we know today that the doublet approximation with equal masses for the baryons is quite unrealistic, because as was first shown by Pais [5], it leads to the prediction of reaction rates and selection rules which are quite in disagreement with the experimental evidence. Therefore, since the work of Pais, it has become customary not to present schemes of strong interactions without taking into account the perturbations responsible for spoiling the main symmetries acting on them. In order to present the situation in a somewhat schematic way, let us consider the Yukawa trilinear baryon-meson interactions:

$$\begin{aligned} & G_1 \bar{N} N \pi + G_2 \bar{\Sigma} \Sigma \pi + G_3 \bar{\Sigma} \Lambda \pi + G_4 \bar{\Xi} \Xi \pi \\ & + F_1 \bar{\Lambda} N K + F_2 \bar{\Sigma} N K + F_3 \bar{\Lambda} \Xi \bar{K} + G_4 \bar{\Sigma} \Xi \bar{K} \end{aligned} \tag{4}$$

with 8 independent constants; let us call $P(K)$, $P(\Sigma\Lambda)$, and $P(N\Xi)$ the relative parities of $K^0 K^+$, $\Sigma\Lambda$ and $N\Xi$. Then the condition of validity for the doublet approximation is:

$$P(\Sigma\Lambda) \text{ and } P(K) \text{ even ;} \tag{5}$$

and

$$G_2 = -G_3 \quad F_1 = -F_2 \quad F_3 = -F_4 \tag{6}$$

which is equivalent to the grouping in Eq. (1) of the doublets. Should we choose instead:

$$G_2 = +G_3 \quad F_1 = +F_2 \quad F_3 = +F_4 \tag{7}$$

this would lead us to group the doublets as:

$$N_1' = N_1 \quad N_2' = \begin{pmatrix}\Sigma^+\\ \Sigma^0\end{pmatrix} \quad N_3' = \begin{pmatrix}Y^0\\ \Sigma^-\end{pmatrix} \quad N_4' = N_4, \tag{8}$$

that is, to a trivial interchange of Y^0 with Z^0.

All the schemes which have been proposed in order to conform to the experimental situation may then be characterized by a definite way of breaking some of the symmetries expressed by Eqs. (5) and (6). The " intrinsic-perturbation " devices may preserve the conditions (6) for the constants and alter essentially the parity requirements; either by choosing odd relative parities for the baryon [$P(\Sigma\Lambda)$ and $P(N\Xi)$ both odd, (Behrends [6])] or leaving the baryon parities even and taking $P(K)$ odd (Pais [7]), while the " extrinsic perturbation " ones may preserve both Eqs. (5) and (6), but essentially consider some additional interactions (as $KK\pi$, $KK\pi\pi$, or $NNKK$) [7, 8] violating the unwanted invariances of the doublet approximation, or add some possible new baryon and meson states to the scheme, which thus becomes asymmetric [8a]. Generally the main stress in these attempts is put on the phenomenological situation requiring the violation of the symmetries, and the means used to achieve this are " overabundant ", in the sense that they often destroy more than is effectively required by the data (even charge independence for K interactions in some cases), so that one could hardly recognize that anything important is left of the original symmetrical interactions. One may wonder, however, if so complete a destruction of symmetries is really necessary, and in fact a third group of attempts, which for brevity can be termed as " quasi-symmetric " [2, 3, 4, 9] has shown that the amount of perturbation necessary to reproduce the experimental situation may, in fact, be introduced in such a way as to disturb as little as possible the symmetries of the main interactions, by leaving untouched the parity assumptions Eq. (5), (and taking further also $P(N\Xi)$ even) and altering instead some of the conditions Eq. (6) in a more or less regular manner.

Now, if the general derivation of parity conservation from CP invariance is connected with the symmetries, it is rather natural to presume that mainly by following the line of these " quasi-symmetric " schemes, one may succeed in reconciling together the two following apparently opposite requisites: that is, to introduce enough symmetries in order to deduce from them parity conservation from CP invariance for all strong interactions, and at the same time, enough asymmetries in order to violate the unwanted selection rules, and satisfy the experimental evidence. That this may be possible has already been shown, at least in particular cases, in some of the previously quoted papers [2, 3, 4].

What we propose to do in the present work is to discuss on a somewhat more general basis the compatibility of these two requisites, and precisely to show that all possible asymmetries necessary to meet the experimental situation may be made compatible with $CP \rightarrow P$ provided that: a) all baryon states may be considered as components of a single spinor; b) all operators acting on the different charge or hypercharge components of this unique spinor should be hermitian. We shall only show that these are sufficient conditions for the compatibility of the two requirements we are discussing; it is rather likely that they are not necessary.

The two conditions are in a sense an obvious extension of the familiar procedure expressing charge independence for pion-nucleon interactions. Here the gathering of the two nucleon states into a single spinor and the symmetries contained in the hermitian τ matrices permit the reduction of the $CP \rightarrow P$ derivation to the same line as for the π_0 interactions; and the same will now be achieved by extending this procedure to the eight component case.

2. Let us first illustrate our procedure in the case of the doublet approximation, which is the common basis of most of the " quasi-symmetric " schemes. We write for the Lagrangian:

$$L = G\sum_{j=1}^{3} \bar{\psi}\Gamma T_j \pi_j \psi + F\sum_{k=1}^{4} \bar{\psi}\Gamma \omega_k \phi_k \psi \tag{9}$$

Here ψ is the 8-component spinor represented by:

$$\psi = \left|\begin{array}{c} p \\ n \\ \Xi^0 \\ \Xi^- \\ -\Sigma^+ \\ Y^0 \\ Z^0 \\ \Sigma^- \end{array}\right| \tag{9a}$$

The T_j matrices are the usual three dimensional isospin matrices for the four doublets:

$$T_j = \left|\begin{array}{cc|cc} \tau_j & & & \\ & \tau_j & & \\ \hline & & \tau_j & \\ & & & \tau_j \end{array}\right| \qquad \tau_j \ 2\times 2 \text{ Pauli matrices}$$

and the three-dimensional isovector π_j is related in the usual manner to the π mesons by:

$$\pi = \pi_1 - i\pi_2 \quad \pi^* = \pi_1 + i\pi_2 \quad \pi^0 = \pi_3$$

the ω_k are four anticommuting Dirac type matrices first introduced by Tiomno [9], operating in the four dimensional isospin (or hypercharge) K space and given by:

$$\omega_1 = i\left|\begin{array}{cc|cc} & & & \mathrm{I} \\ & & \mathrm{I} & \\ \hline & -\mathrm{I} & & \\ -\mathrm{I} & & & \end{array}\right| \qquad \omega_2 = \left|\begin{array}{cc|cc} & & & \mathrm{I} \\ & & -\mathrm{I} & \\ \hline & -\mathrm{I} & & \\ \mathrm{I} & & & \end{array}\right|$$

$$\omega_3 = i\left|\begin{array}{cc|cc} & & \mathrm{I} & \\ & & & -\mathrm{I} \\ \hline -\mathrm{I} & & & \\ & \mathrm{I} & & \end{array}\right| \qquad \omega_4 = \left|\begin{array}{cc|cc} & & \mathrm{I} & \\ & & & \mathrm{I} \\ \hline \mathrm{I} & & & \\ & \mathrm{I} & & \end{array}\right| \tag{10}$$

$$\omega_5 = \omega_1\omega_2\omega_3\omega_4 = \left|\begin{array}{cc|cc} -1 & & & \\ & -\mathrm{I} & & \\ \hline & & \mathrm{I} & \\ & & & \mathrm{I} \end{array}\right| \qquad \mathrm{I} = \left|\begin{array}{c|c} 1 & 0 \\ \hline 0 & 1 \end{array}\right| ;$$

ϕ is a four-dimensional isovector in hypercharge space related to the K meson by the relations:

$$K^+ = \phi_1 - i\phi_2 \qquad -K^0 = \phi_3 - i\phi_4$$

$$\overline{K}^- = \phi_1 + i\phi_2 \qquad \overline{K}^0 = \phi_3 + i\phi_4$$

Γ is any kind of special coupling matrix, G is a unique pion field constant and F a unique K field constant. In this form the pion interactions express charge independence for each of the four baryon doublets in a three dimensional isospace and the K interactions "hypercharge independence" for rotations of the four doublets in a four dimensional hypercharge space. These rotational invariances provide the following independent constants of the motion; 1) for pions interactions: a total pion interaction isospin T, (baryon+pion contributions), the three baryon components being the T_j previously defined. 2) For K interactions: if one builds from the ω_k matrices (defined in Eq. (10))the momenta:

$$M_{lk} = -\frac{i}{4}[\omega_l\omega_k]$$

and from them two three-vectors [10]:

$$Y_l^{\pm} = \frac{M_{jk} \pm M_{ey}}{2} (j, k, l \text{ cyclic})$$

$$Y_1^+ = \tfrac{1}{2}\left|\begin{array}{cc|c} & \mathrm{I} & \\ \mathrm{I} & & 0 \\ \hline \multicolumn{2}{c|}{0} & 0 \end{array}\right| Y_2^+ = \tfrac{1}{2}\left|\begin{array}{cc|c} & -i\mathrm{I} & \\ i\mathrm{I} & & 0 \\ \hline \multicolumn{2}{c|}{0} & 0 \end{array}\right| Y_3^+ = \tfrac{1}{2}\left|\begin{array}{cc|c} \mathrm{I} & & \\ & -\mathrm{I} & 0 \\ \hline \multicolumn{2}{c|}{0} & 0 \end{array}\right| \tag{10a}$$

$$Y_1^- = \tfrac{1}{2}\left|\begin{array}{c|cc} 0 & \multicolumn{2}{c}{0} \\ \hline 0 & & \mathrm{I} \\ & \mathrm{I} & \end{array}\right| Y_2^- = \tfrac{1}{2}\left|\begin{array}{c|cc} 0 & \multicolumn{2}{c}{0} \\ \hline 0 & & -i\mathrm{I} \\ & i\mathrm{I} & \end{array}\right| Y_3^- = \tfrac{1}{2}\left|\begin{array}{c|cc} 0 & \multicolumn{2}{c}{0} \\ \hline 0 & \mathrm{I} & \\ & & -\mathrm{I} \end{array}\right|$$

generating rotations in two three-dimensional subspaces of the four-dimensional hypercharge space, then the moduli of total Y^+ and Y^- (baryon+K meson contributions) are also constants of the motion, and represent respectively the total hypercharge spin ($2Y_3^+$ being the hypercharge) and Y^- the total K^- interaction isospin; the total Gell-Mann-Nishijima isospin being:

$$J = Y^- + T ; \quad J_3 = Y_3^- + T_3 .$$

The values of these quantum numbers from all particles are given in Table I.

In case the interaction constants F and G are real and applying substitution (2), it is easily shown that (9) is completely equivalent to a d'Espagnat-Prentki Lagrangian [11]:

$$\begin{aligned} L = G\{&\overline{N}\Gamma\boldsymbol{\tau}\cdot\boldsymbol{\pi}N + \overline{\Xi}\Gamma\boldsymbol{\tau}\cdot\boldsymbol{\pi}\Xi + \overline{\boldsymbol{\Sigma}}\Gamma\times\boldsymbol{\Sigma}\cdot\boldsymbol{\pi} - (\overline{\boldsymbol{\Sigma}}\cdot\boldsymbol{\pi}\Gamma\Lambda + \\ &+\text{h.c.})\} - iF\{\overline{N}\Gamma\Lambda K + \text{h.c.} - (\overline{N}\Gamma\boldsymbol{\tau}\cdot\boldsymbol{\Sigma}K + \text{h.c.}) - \\ &-(\overline{\Xi}\Gamma\Lambda\overline{K} + \text{h.c.}) + (\overline{\Xi}\Gamma\boldsymbol{\tau}\cdot\boldsymbol{\Sigma}\overline{K} + \text{h.c.})\} \end{aligned} \tag{11}$$

Let us now consider the $CP \rightarrow P$ derivation; we start writing:

$$\begin{aligned} L = &\sum_{j=1}^{3} [\overline{\psi}(g_s + g_p\gamma_5)T_j\pi_j\psi + \overline{\psi}(g_s^* - g_p^*\gamma_5)T_j\pi_j\psi] + \\ &+ \sum_{k=1}^{4} [\overline{\psi}(f_s + f_p\gamma_5)\omega_k\phi_k\psi + \overline{\psi}(f_s^* - f_p^*\gamma_5)\omega_k\phi_k\psi] \end{aligned} \tag{9'}$$

Table I. Particle Quantum Numbers

Particle	T	T_3	Y^+	Y^-	Y_3^+	Y_3^-	$\mathcal{J}$	$\mathcal{J}_3 = Y_3^- + I + T_3$	$U = 2Y_3^+$	$Q = \mathcal{J}_3 + Y_3^+ + I$
p	½	½	½	0	½	0	½	½	1	1
n	½	−½	½	0	½	0	½	−½	1	0
Ξ^0	½	½	½	0	−½	0	½	½	−1	0
Ξ^-	½	−½	½	0	−½	0	½	−½	−1	−1
Σ^+	½	½	0	½	0	½	1	1	0	1
Y^0	½	−½	0	½	0	½	1,0	0	0	0
Z^0	½	½	0	½	0	−½		0	0	0
Σ^-	½	−½	0	½	0	−½	1	−1	0	−1
π^+	1	1	0	0	0	0	1	1	0	1
π^0	1	0	0	0	0	0	1	0	0	0
π^-	1	−1	0	0	0	0	1	−1	0	−1
K^+	0	0	½	½	½	½	½	½	1	1
K^0	0	0	½	½	½	−½	½	−½	1	0
$\bar{K}^0$	0	0	½	½	−½	½	½	½	−1	0
$\bar{K}^-$	0	0	½	½	−½	−½	½	−½	−1	−1

Then we transform Eq. (9′) according to Eq. (2); we get:

$$\begin{aligned}
L = {} & \bar{N}(g_s+\gamma_5 g_p)\boldsymbol{\tau}\cdot\boldsymbol{\pi}N+\bar{N}(g_s^*-g_p^*\gamma_5)\boldsymbol{\tau}\cdot\boldsymbol{\pi}N+\\
& +\bar{\Xi}(g_s+g_p\gamma_5)\boldsymbol{\tau}\cdot\boldsymbol{\pi}\Xi+\bar{\Xi}(g_s^*-g_p^*\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi})\Xi+\\
& +\bar{\boldsymbol{\Sigma}}(g_s+g_p\gamma_5)\times\boldsymbol{\Sigma}\cdot\boldsymbol{\pi}+\bar{\boldsymbol{\Sigma}}(g_s^*-g_p^*\gamma_5)\times\boldsymbol{\Sigma}\cdot\boldsymbol{\pi}-\\
& -\bar{\boldsymbol{\Sigma}}(g_s+g_p\gamma_5)\cdot\boldsymbol{\pi}\Lambda-\underline{\bar{\boldsymbol{\Sigma}}(g_s^*-g_p^*\gamma_5)\cdot\boldsymbol{\pi}\Lambda}-\\
& -\underline{\bar{\Lambda}\boldsymbol{\pi}\cdot(g_s+g_p\gamma_5)\boldsymbol{\Sigma}}-\bar{\Lambda}\boldsymbol{\pi}\cdot(g_s^*-g_p^*\gamma_5)\boldsymbol{\Sigma}-\\
& -i\bar{N}(f_s+f_p\gamma_5)\Lambda K-\underline{i\bar{N}(f_s^*-j_p^*\gamma_5)\Lambda K}+\\
& +\underline{i\bar{K}\bar{\Lambda}(f_s+f_p\gamma_5)N}+i\bar{K}\bar{\Lambda}(f_s^*-f_p^*\gamma_5)N+\\
& +i\bar{N}(f_s+f_p\gamma_5)\boldsymbol{\tau}\cdot\boldsymbol{\Sigma}K+\underline{i\bar{N}(f_s^*-f_p^*\gamma_5)\boldsymbol{\tau}\cdot\boldsymbol{\Sigma}K}-\\
& -\underline{i\bar{K}\bar{\boldsymbol{\Sigma}}\cdot\boldsymbol{\tau}(f_s+f_p\gamma_5)N}-i\bar{K}\bar{\boldsymbol{\Sigma}}\cdot\boldsymbol{\tau}(f_s^*-f_p^*\gamma_5)N+\\
& +i\bar{\Xi}(f_s+f_p\gamma_5)\Lambda\bar{K}+\underline{i\bar{\Xi}(f_s^*-f_p^*\gamma_5)\Lambda\bar{K}}-\\
& -\underline{iK\bar{\Lambda}(f_s+f_p\gamma_5)\Xi}-iK\bar{\Lambda}(f_s^*-f_p^*\gamma_5)\Xi-\\
& -i\bar{\Xi}(f_s+f_p\gamma_5)\boldsymbol{\tau}\cdot\boldsymbol{\Sigma}\bar{K}-\underline{i\bar{\Xi}(f_s^*-f_p^*\gamma_5)\boldsymbol{\tau}\cdot\boldsymbol{\Sigma}\bar{K}}+\\
& +\underline{iK\bar{\boldsymbol{\Sigma}}\cdot\boldsymbol{\tau}(f_s+f_p\gamma_5)\Xi}+iK\bar{\boldsymbol{\Sigma}}\cdot\boldsymbol{\tau}(f_s^*-f_p^*\gamma_5)\Xi\,. \qquad (9'')
\end{aligned}$$

In this case, as for the π^0 field, hermiticity requirement for the Lagrangian compels all scalar (s) constants to be real and all pseudoscalar (p) constants to be purely imaginary:

$$\begin{aligned}
G_s &= g_s+g_s^* & F_s &= f_s+f_s^*\\
iG_p &= g_p-g_p^* & iF_p &= f_p-f_p^*
\end{aligned}$$

It is then seen that by applying as usual the *CP* transform, we get *CP* invariance only if we have either:

$$\begin{pmatrix} F_s = 0 \\ G_s = 0 \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} F_p = 0 \\ G_p = 0 \end{pmatrix}$$

that is, we get parity conservation for all terms.

Owing to the identity of Lagrangian (9) with the d'Espagnat Prentki Lagrangian (11), one may wonder at first sight why complete parity conservation may be derived from (9) while, as it is well known, it cannot be derived from (11) in the case of the $\Sigma\Lambda\pi$ and all K interactions.

The reason may easily be seen if we write directly with complex constants the d'Espagnat Prentki Lagrangian [11] considered as formed by different baryons; we get:

$$\begin{aligned}
L = {} & \bar{N}(g_s+g_p\gamma_5)\boldsymbol{\tau}\cdot\boldsymbol{\pi}N+\bar{N}(g_s^*-g_p^*\gamma_5)\boldsymbol{\tau}\cdot\boldsymbol{\pi}W+\\
& +\bar{\Xi}(g_s+g_p\gamma_5)\boldsymbol{\tau}\cdot\boldsymbol{\pi}\Xi+\bar{\Xi}(g_s^*-g_p^*\gamma_5)\boldsymbol{\tau}\cdot\boldsymbol{\pi}\Xi+\\
& +\bar{\boldsymbol{\Sigma}}(g_s+g_p\gamma_5)\times\boldsymbol{\Sigma}\cdot\boldsymbol{\pi}+\bar{\boldsymbol{\Sigma}}(g_s^*-g_p^*\gamma_5)\times\boldsymbol{\Sigma}\cdot\boldsymbol{\pi}-\\
& -\bar{\boldsymbol{\Sigma}}(g_s+g_p\gamma_5)\cdot\boldsymbol{\pi}\Lambda-\bar{\Lambda}(g_s^*-g_p^*\gamma_5)\boldsymbol{\pi}\cdot\boldsymbol{\Sigma}-\\
& -i\bar{N}(f_s+f_p\gamma_5)\Lambda K+\bar{K}\bar{\Lambda}(f_s^*-f_p^*\gamma_5)N+
\end{aligned}$$

$$+i\overline{N}(f_s+f_p\gamma_5)\boldsymbol{\tau}\cdot\boldsymbol{\Sigma}K-i\overline{K}\overline{\boldsymbol{\Sigma}}\cdot\boldsymbol{\tau}(f_s{}^*-f_p{}^*\gamma_5)N+$$
$$+i\overline{\Xi}(f_s+f_p\gamma_5)\Lambda\overline{K}-iK\overline{\Lambda}(f_s-f_p\gamma_5)\Xi-$$
$$-i\overline{\Xi}(f_s+f_p\gamma_5)\boldsymbol{\tau}\cdot\boldsymbol{\Sigma}\overline{K}+iK\overline{\boldsymbol{\Sigma}}\cdot\boldsymbol{\tau}(f_s{}^*-f_p{}^*\gamma_5)\Xi\,. \qquad (11')$$

The underlined terms in (9″) do not exist in (11′) and it is their presence which compels parity conservation from CP for $\Sigma\Lambda\pi$ and all K interactions in (9″). These terms come essentially from the requirement of hermiticity for all operators ω_k and T_j expressing the hypercharge and charge independence of strong interactions. Therefore, although expressions (9) and (11) are equivalent, once parity is conserved and constants are real, their starting intrinsic symmetries are fundamentally different.

We may add that in this case the same results are obtained by using instead of hypercharge and charge independence the generalized hypercharge and charge symmetry operators (3) as done by Sakurai [2] and Feinberg and Gürsey [3].

3. We shall now discuss how, by starting from the doublet approximation, the necessary asymmetries to meet the experimental situation may be introduced in the "quasi-symmetric" theories according to our assumptions a) and b).

Up to now, our knowledge of the different reactions is still so incomplete and rough, that all that is presently required from a scheme is to account for the following qualitatively important disturbances which appear to be quite independent from each other even by their order of magnitude: 1) the $N\Xi$ mass difference, the effect of which is the breakdown of hypercharge independence for the K interactions, of which only the conservation of Y_3^+ (hypercharge) is left; and 2) the mixture of the two hyperon doublets to form the singlet Λ and triplet Σ, whose consequence will be the breakdown of the separate conservations for both T_3 and T on one hand and of both Y_3^- and Y^- on the other, and survival of only the total sum of both $T_3+Y_3^-$ and $T+Y^-$ as constants of motion. (See Table I.) Moreover, the alteration of some conditions (6) will allow reactions such as $K^++n\rightarrow K^0+p$ as required by the data, which are forbidden in the doublet approximation [5].

The introduction of these two asymmetries will generally require no more than two new independent constants, so that one will need altogether four interaction constants instead of the eight available; this implies that it is possible to attribute the observed asymmetries to a given kind of interactions only, and so the schemes divide into two main branches, according to whether the K or the pion interactions are) held responsible for the breakdown of the fundamental symmetries. This distinction is especially well marked for what concerns the $N-\Xi$ mass difference.

The global symmetric attempts, following the original idea of Gell-Mann [12], attribute the full symmetry to the pion interactions (charge independence and equal constants for all pion interactions, and the $N-\Xi$ mass difference to the K interactions. A possible way of expressing this fact in the actual notation is given in [4] by adding to (9) a second K interaction term:

$$F'\sum_{k=1}^{4}\overline{\psi}\Gamma\omega_k'\phi_k\psi \qquad (12)$$

with

$$\omega_1'=-i\omega_2\omega_5 \quad \omega_2'=i\omega_1\omega_5 \quad \omega_3'=-i\omega_4\omega_5$$
$$\omega_4'=i\omega_3\omega_5 \qquad (13)$$

the interference of the two K terms resulting in weighting all nucleon interactions by $F-F'$ and all Ξ interactions by $F+F'$.

The cosmic symmetric attempts, following the original idea of Schwinger's first paper [13] and essentially developed by Sakurai [2] attribute the full four-dimensional isosymmetry to the K interactions (either hypercharge symmetry or hypercharge independence) and the perturbation leading to the $N\Xi$ mass difference to the pion interactions which should be hypercharge dependent. The simplest expression for such an interaction is given by adding to Eq. (9) a term of type:

$$G'\sum_{j=1}^{3}\overline{\psi}\Gamma Y_3^+T_j\pi_j\psi \qquad (14)$$

It is presently not yet possible to decide if any of these two assumptions is likely to be true: global symmetry for some years has been the leading idea of most of the phenomenological research concerning strong interactions, perhaps just because it appeared in the beginning as the simplest perturbation picture in rough agreement with what was known when it was formulated. One must say, however, that the subsequent investigations and more detailed data have not improved the first impression of agreement. For about a year, moreover, there have been indications that global symmetry may be disproved in some K capture reaction [14, 15].

A possible breakdown of global symmetry (if it will in fact occur), should not of course prove by itself that cosmic symmetry is right, and direct confirmation of this possible symmetry actually seems rather difficult to reach, because essentially it would imply comparison between nucleon and Ξ reaction rates (ex. $\bar{K}+N\to\bar{K}+N$; $\bar{K}+N\to K+\Xi$) in an energy region (at least a few BeV) in which the $N\Xi$ mass difference could be considered as negligibly small. However, even if cosmic symmetry would allow us to consider the $N\Xi$ mass difference as a purely dynamical disturbance to be avoided by working at sufficiently high energy, the correctness of treating the K interactions independently from the assumed hypercharge dependent pion interactions which are of much greater range [6] appears *a priori* to be rather doubtful and, of course, the existence of a (possibly not weak) $KK\pi\pi$ interaction would make the situation still worse; as such a boson-boson interaction could be responsible at high energy for part of the K nucleon scattering, thus spoiling the possibility of observing the assumed hypercharge symmetry or independence of the pure K-baryon interactions. So we shall consider both kinds of perturbations (12) or (14) in our treatment.

If we turn now to the $\Lambda\Sigma$ splitting, the procedure used by most of the quoted authors in order to introduce it is essentially based on the following device: we have up to now considered doublets [1]; however as no physical difference is supposed to make us distinguish between the two neutral states Y^0 and Z^0, there is no reason why we should not consider also the grouping (8) so that in general one should consider a linear combination of these two possibilities in the Lagrangian of strong interactions. This may be expressed in the present formalism by writings:

$$L=a_1G\sum_{j=1}^{3}\bar{\psi}\Gamma T_j\pi_j\psi+a_2F\sum_{k=1}^{4}\bar{\psi}\Gamma\omega_k\phi_k\psi+$$

$$+b_1G\sum_{j=1}^{3}\bar{\psi}\Gamma JT_jJ\pi_j\psi+b_2F\sum_{k=1}^{4}\bar{\psi}\Gamma J\omega_kJ\phi_k\psi \qquad (15)$$

where

$$J=J^{\dagger}=J^{-1}=\left|\begin{array}{cccc|cccc} 1 &&&&&&& \\ & 1 &&&&&& \\ && 1 &&&&& \\ &&& 1 &&&& \\ \hline &&&& 1 &&& \\ &&&&& 0 & 1 & \\ &&&&& 1 & 0 & \\ &&&&&&& 1 \end{array}\right|$$

interchanges Y^0 with Z^0.

Now the different schemes differ from each other only in the different choices of the combination constants a and b. The simplest choice as first proposed by Pais [5] consists in taking either $a_2=b_1=0$ or $a_1=b_2=0$ [17], that is, one type of doublet grouping as characterizing pion interaction and the other grouping the K interactions. Another simple possibility is [17] $b_1=0$, $a_2=b_2$ or $b_2=0$, $a_1=b_1$.

A more general case has been considered in [4] by taking $a_1=a_2$, $b_1=b_2$. One may easily see that the use of transformation (2) leads always to a Lagrangian yielding to $\Lambda\Sigma$ splitting as required with different combinations of the $K\Sigma N$, $K\Lambda N$, $\pi\Sigma\Sigma$ and $\pi\Sigma\Lambda$ interaction constants according to the case.

4. The preceding discussion shows that all the different asymmetry proposals may be taken into account by adding terms like (12), (14) and (15) to (9). This leads to the expression for the most general parity non-conserving Lagrangian:

$$L=\sum_{j=1}^{3}[\bar{\psi}(g_s+g_p\gamma_5)T_j\pi_j\psi+\text{h.c.}+\bar{\psi}(A_sg_s+A_pg_p\gamma_5)\times$$

$$\times JT_jJ\pi_j\psi+\text{h.c.}]+\sum_{k=1}^{4}[\bar{\psi}(f_s+f_p\gamma_5)\omega_k\phi_k\psi+\text{h.c.}+$$

$$+\bar{\psi}(f_s'+f_p'\gamma_5)\omega_k'\phi_k\psi+\text{h.c.}+\bar{\psi}(b_sf_s+b_pf_p\gamma_5)\times$$

$$\times J\omega_kJ\phi_k\psi+\text{h.c.}+\bar{\psi}(b_s'f_s'+b_p'f_p'\gamma_5)J\omega_k'J\phi_k\psi+$$

$$+\text{h.c.}]$$

where

$$f_s, f_s', b_s, b_s', g_s, \quad \text{and} \quad A_s$$

$$A_s=\left|\begin{array}{cccc} a_{Ns} &&& \\ & a_{\Xi s} && \\ && a_{Ys} & \\ &&& a_{Ys} \end{array}\right|$$

are the different 8 interaction constants for the scalar terms and f_p, f'_p, etc. a similar set for the *ps* terms. A_s may be considered as a shorthand for

$$A_s = p\mathrm{I} + q\omega_5 + 2Y_3^+ =$$

$$= p\left|\begin{matrix} \mathrm{I} &&& \\ & \mathrm{I} && \\ && \mathrm{I} & \\ &&& \mathrm{I} \end{matrix}\right| + q\left|\begin{matrix} -\mathrm{I} &&& \\ & -\mathrm{I} && \\ && \mathrm{I} & \\ &&& \mathrm{I} \end{matrix}\right| + 2\left|\begin{matrix} \mathrm{I} &&& \\ & -\mathrm{I} && \\ && 0 & \\ &&& 0 \end{matrix}\right|.$$

$$p-q+2 = a_{Ns} \quad p-q-2 = a_{\Xi s} \quad p+q = a_{Ys}$$

Now even for such a Lagrangian, (in fact more asymmetric than is strictly necessary as all asymmetries have been put together in it, while of course only some of them are simultaneously required), it is still possible to derive parity conservation from *CP* invariance exactly as was done in the case of (9), as all matrices in (18) are hermitian.

Transformation (2) applied to (18) gives a Lagrangian of d'Espagnat-Prentki type (which for brevity we omit writing); and the hermiticity requirement applied to it compels all constants to be either real or imaginary; *CP* invariance then requires that for each kind of mesons (charged pions, charged *K*'s and neutral *K*'s) either the scalar or the pseudoscalar constants are zero. Once parity conservation is imposed (let us choose all surviving terms to be pseudoscalars), we obtain for our Lagrangian the expression :

$$\begin{aligned} L = {} & i\bar{N}G_p(1+A_{Np})\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi}N + i\bar{\Xi}G_p(1+A_{\Xi p})\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi}\Xi \\ & + i\bar{\boldsymbol{\Sigma}}G_p(1+A_{yp})\gamma_5\times\boldsymbol{\Sigma}\cdot\boldsymbol{\pi} - i\{\bar{\boldsymbol{\Sigma}}\cdot\boldsymbol{\pi}G_p(1-A_{yp})\gamma_5\Lambda + \text{h.c.}\} \\ & + \{\bar{N}[F_p(1-B_p) - F'_p(1-B'_p)]\gamma_5\Lambda K + \text{h.c.}\} - \\ & - \{\bar{N}[F_p(1+B_p) - F'_p(1+B'_p)]\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\Sigma}K + \text{h.c.}\} - \\ & - \{\bar{\Xi}[F_p(1-B_p) + F'_p(1-B'_p)]\gamma_5\Lambda\bar{K} + \text{h.c.}\} + \\ & + \{\bar{\Xi}[F_p(1+B_p) + F'_p(1+B'_p)]\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\Sigma}\bar{K} + \text{h.c.}\} \end{aligned} \tag{19}$$

where

$$iG_p = g_p - g_p{}^*, \;\; iG_pA_{np} = g_pa_{Np} - g_p{}^*a_{Np}{}^*,$$

$$iF_pB_p = f_pb_p - f_pb_p \text{ etc.}$$

Thus even for a d'Espagnat-Prentki Lagrangian with 8 different interaction constants it is possible to derive parity conservation from *CP* invariance provided our conditions a) and b) are assumed.

5. Thus, the wide possibility of choices for the Lagrangian compatible with the *CP*→*P* derivation on one side does not exclude any of the particular assumptions made in the different schemes proposed; but from the other side it leaves open the possibility of using any other desirable criterion of discrimination among them. Particularly, and in the general spirit of the " quasi-symmetric " schemes, one might then try to look among all possible devices intended to obtain in a satisfactory way the $N\Xi$ and $\Lambda\Xi$ mass differences, for the one appearing as the most " symmetric ": intending by this expression in a rather qualitative and not precisely definite sense, the possibility of reaching the desired effects in a way most naturally related to the main symmetries of the fundamental interactions themselves. Among such " symmetric " possibilities, we should like to consider as a last point (although not necessarily connected with the preceding ones), a particularly " natural " assumption for explaining the $\Lambda\Sigma$ mass splitting, already outlined by Feinberg and Gürsey[3], which in the frame of the four-dimensional isobaric symmetry of *K* interactions may appear as rather suited to its aim.

A general feature and perhaps a common drawback of the preceding approaches is the introduction of the two different doublet groupings (1) and (8), necessary to obtain the $\Lambda\Sigma$ splitting, just on the same footing and with different arbitrary weights for each of them in the different interactions; this, even if it leads to satisfactory results from a practical point of view, cannot conceal a rather phenomenological aspect of the whole way of looking at the problem. The following proposal tries to avoid this arbitrariness by giving a well defined meaning to the two groupings (1) and (8).

Let us assume always a unique well defined choice for the baryon doublets as represented by the general baryonic state (9a) or the grouping (1); let us take the *K* interactions as most highly symmetric and hypercharge independent. They are then given by a term such as :

$$F\sum_{k=1}^{4}\bar{\psi}\gamma_5\omega_k\phi_k\psi' \tag{20}$$

We shall now suppose that the pion interactions break this high symmetry essentially because there is an interference between a pion term sharing this four-dimensional rotational invariance in hypercharge space with another one which is only rotational invariant in the three-dimensional isospace. If further we consider the pion field as an antiself-dual antisymmetric isotensor satisfying the relations:

$$\pi_{12} = -\pi_{34} = \pi_2,\ \pi_{23} = -\pi_{14} = \pi_1,\quad \pi_{31} = -\pi_{24} = \pi_2 \tag{21}$$

the first four-dimensional invariant term may then be written as:

$$\frac{i}{2} G'' \sum_{ik} \overline{\psi} \gamma_5 M_{ik} \pi_{ik} \psi = i G'' \sum_{j=1}^{3} \overline{\psi} \gamma_5 Y_j^- \pi_j \psi \tag{22}$$

Then it is easily seen by referring to Eq. (10*a*) that this term leads to a pion coupling only for the hyperons with hypercharge zero (Y_3^- is zero for N and Ξ) in which these behave as if they were forming two doublets of type (8).

We may now choose for the second three-dimensional invariant term the usual pion-baryon interaction:

$$i G \sum_{j=1}^{3} \overline{\psi} \gamma_5 T_j \pi_j \psi \tag{23}$$

In this case the hyperon doublets are obviously grouped according to (1). Thus the combination of (22) and (23) gives us immediately the $\Lambda\Sigma$ splitting as an interference between two interactions acting respectively in the four-dimensional hyperspace and the three-dimensional isospace. Following the general line of cosmic symmetry, the $N-\Xi$ mass splitting can then be naturally introduced as an interference between (23) and a hypercharge dependent term of type (14). The breakdown of hypercharge independence thus obtained through pion interactions may be compared to the breakdown of charge independence through electromagnetic interactions, which in the present frame could be written as:

$$ie \sum_{\mu=1}^{4} \overline{\psi} \gamma_\mu (Y_3^+ + Y_3^- + T_3) A_\mu \psi = ie \sum_{\mu=1}^{4} \overline{\psi} \gamma_\mu (Y_3^+ + J_3) A_\mu \psi \tag{24}$$

The assumptions $G = G'$ or $G = G''$ would of course allow us to reduce the number of independent constants to 3 or even 2 and still present a picture qualitatively compatible with the data. We think that a Lagrangian of the form:

$$L = iF \sum_{\mu=1}^{4} \overline{\psi} \gamma_5 \omega_\mu \phi_\mu \psi + i \sum_{j=1}^{3} \overline{\psi} \gamma_5 [G'' Y_j^- + G T_j + G' Y_3^+ T_j] \pi_j \psi + ie \sum_{\mu=1}^{4} \overline{\psi} \gamma_\mu (Y_3^+ + J_3) A_\mu \psi \tag{25}$$

could represent a choice satisfying both the $CP \to P$ derivation for all interactions and expressing the symmetries in a rather homogeneous way of successive interferences between interactions invariant in isospaces of gradually decreasing dimensionality.

LIST OF REFERENCES

1. Feinberg, C. Phys. Rev. **108**, p. 878 (1957); Gupta, S. N. Can. J. Phys. **35**, p. 1309 (1957); Gell-Mann, H. and Rosenfeld, A. H. Ann. Rev. Nucl. Sci. **7**, p. 407 (1957).
2. Sakurai, J. J. Phys. Rev. **113**, p. 1679 (1959).
3. Feinberg, G. and Gürsey, F. Phys. Rev. **114**, p. 1153 (1959).
4. Dallaporta, N. and Pandit, L. K. Nuovo Cimento **16**, p. 135 (1960).
5. Pais, A. Phys. Rev. **110**, p. 574 (1958).
6. Behrends, R. E. Nuovo Cimento **11**, p. 424 (1959).
7. Pais, A. Phys. Rev. **112**, p. 624 (1958).
8. Barshay, S. Phys. Rev. **109**, p. 2160 (1958). Budini, P., Dallaporta, N. and Fonda, L. Nuovo Cimento **9**, p. 316 (1958).
9. Tiomno, J. Nuovo Cimento **6**, p. 69 (1957); Salam, A. and Polkinghorne, J. C. Nuovo Cimento **4**, p. 848 (1955); Peaslee, D. C. Phys. Rev. **117**, p. 873 (1960).
10. Pais, A. Proc. Nat. Acad. Sc. 40, **9**, p. 835 (1954).
11. D'Espagnat, B. and Prentki, J. Nuclear Phys. **1**, p. 33 (1956); Salam, A. Nuclear Phys. **2**, p. 173 (1956); d'Espagnat, B., Prentki, J. and Salam, A. Nuclear Phys. **3**, p. 466 (1957).
12. Gell-Mann, M. Phys. Rev. **106**, p. 1296 (1957).
13. Schwinger, J. Phys. Rev. **104**, p. 1164 (1956).
14. Salam, A. Kiev Conference Report.
15. D'Espagnat, B. and Prentki, J. Nuovo Cimento **15**, p. 130 (1960).
16. Sakurai, J. J. Phys. Rev. **115**, p. 1304 (1959).
17. Feynman, R. P. (unpublished).

DISCUSSION

Sudarshan : I want to comment that from this point of view of *deriving* parity conservation from *CP* invariance and charge symmetry, the observed degree of parity conservation in pion-nucleon interaction is the best test of charge symmetry since at the present time the technique of calculating electromagnetic corrections to charge symmetry are not as precise as the experimentally observed degree of parity conservation.

Feinberg : I would like to comment that if you proceed the way Gürsey and I did to try to see what kind of a Lagrangian will supply *CP* invariance, then there are two solutions in which the *K*-nucleon and the *K*-cascade interactions have equal sign or have opposite sign, and I think that in some sense one of your choices corresponds to adding those two possibilities together. One of the interactions you introduced to give the nucleon cascade mass difference involves getting a constant $f+f'$ for one and $f-f'$ for another. Now I think one can regard that as taking these two possible Lagrangians and just adding them.

Dallaporta : Yes.

Feinberg : The question I would like to ask, however, is : you have this principle that only hermitian operators should appear in the interaction and I wonder if this has any physical consequence that you can see?

Dallaporta : I must say that we are just beginning to wonder about this question. In our present research we have not yet considered what the deeper meaning of this condition could effectively be, as we have arrived at the present formulation by phenomenological steps. So, up to now, I think that we have not quite understood if there is a more physical meaning to this assumption.

Tiomno : I would like to make some comments which are related to Professor Dallaporta's talk. He has written the usual π and K interactions in the notation which I have proposed in 1957 and has, as in the D'Espagnat-Prentki interaction, 8 independent constants. Of course, everybody would prefer that the fundamental interactions (at least for the bare interaction) depend on a minimum number of constants. To take only one, as I have proposed, would be unsatisfactory, as at least π and K coupling constants are found to be very different. My first observation is that at least qualitatively we could have only two such constants for bare interactions. If we accept the idea that the larger the coupling constant the larger should be the symmetry, it is reasonable to accept global symmetry for the interaction :

$$ig\bar{\psi}\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi}\psi\ .$$

Now, in order to break the symmetry in the (weaker) K interaction without introducing more than one coupling constant we could remember that in the known examples of failure of higher symmetries, operators with different transformation properties are combined in equal amounts, say : $Q = I_3 + Y_2$ in electromagnetic interactions and $J_\mu^\pm = (\gamma_\mu(1+\gamma_5))\tau^\pm$ in weak (strangeness conserving) interactions. So we could form the K interactions using, instead of ω_i the combination $\omega_i+\omega_i'$ with the four ω_i' matrices conveniently chosen. As an example which does not disagree qualitatively with the experiment we might take $\omega_i' = p\omega_i p$ where $p = p^\dagger = p^{-1}$ is a matrix which applied on ψ changes the sign of the Ξ components as well as those of the $\boldsymbol{\Sigma}$ components :

$$P = \begin{vmatrix} 1 &&&&&&& \\ & 1 &&&&&& \\ && -1 &&&&& \\ &&& -1 &&&& \\ &&&& -1 &&& \\ &&&&& 0 & -1 & \\ &&&&& -1 & 0 & \\ &&&&&&& -1 \end{vmatrix}$$

Thus, we would have the bare interaction :

$$\frac{i}{2}G\bar{\psi}(\omega_i+\omega_i')\gamma_5\psi K^i = iG(\bar{N}K\gamma_5\Lambda+\bar{\Xi}\boldsymbol{\tau}\bar{K}\cdot\gamma_5\boldsymbol{\Sigma})$$

(of course, due to π interaction some amount of $\bar{N}\boldsymbol{\tau}K\cdot\gamma_5\boldsymbol{\Sigma}$ and $\bar{\Xi}\bar{K}\gamma_5\Lambda$ terms would appear in the effective interaction.) This simple interaction would give the correct kind of splitting of the baryon mass multiplet.

My second observation is that we could still keep the supersymmetric (cosmic symmetry) interaction for K-mesons: $iG\bar{\psi}\omega_i\gamma_5\psi K^i$. We use for breaking this symmetry the *scalar* K meson $K' = \begin{pmatrix} K^{+\prime} \\ K^{0\prime} \end{pmatrix}$. The indications for its existence come from the analysis of the Λ forward-backward asymmetry in π, Λp reactions. I have mentioned it after Professor Gell-Mann's talk on weak interactions. If there is a $K'K\pi$ interaction similar to the $KK\pi$ one proposed by Pais (but invariant under rotations in ordinary isotopic spin space) and a $N\Lambda K'$ interaction, agreement is found with experiment if the graph in which a K' is exchanged is dominant, and if the mass of the K' is of the order of the sum of the masses of π and K mesons. Now if the bare interaction of K' is only $G'(\bar{N}K'\Lambda + \text{h.c.})$ of course a $\bar{N}\tau K'\cdot\mathbf{\Sigma}$ is generated by the π interaction. We will have again the appropriate mass splitting of baryons and enough asymmetry to allow for the observed processes which are forbidden by the cosmic symmetric interaction of K.

MITRA: I would like to know whether the type of vertex that Professor Tiomno has put on the blackboard does not imply opposite parities for the two K mesons.

TIOMNO: It does, but they do not belong to the same doublet as in Pais' proposal. The K^+ and K^0 mesons which are observed are pseudoscalar. $K^{+\prime}$ and $K^{0\prime}$ are scalar and I am choosing this parity in order to get agreement with experience in the Λ production. I could choose the opposite assignment of parities because the parity of the K is not well established, but then this K particle should have a mass of the order of the π mass and this would lead to fast disintegration of K into $K'+\pi$ which is not observed.

REMARKS CONCERNING THE LOW ENERGY K-NUCLEON INTERACTION (*)

J. D. Bjorken

Institute of Theoretical Physics, Department of Physics, Stanford University, Stanford, California

We shall first discuss speculation regarding possible dynamics for the K-nucleon interaction, secondly the formalism used to calculate the scattering amplitudes, and finally some numerical results based upon these considerations. Our approach will be based upon the observation made years ago by Edwards and Matthews [1] that one reproduces the main qualitative features of pion-nucleon scattering by considering the Born approximation amplitudes as a potential and solving a Schrödinger equation. While this procedure appears to be an unjustifiable bypassing of field theory, it can actually be made quite acceptable by using the recent advances in calculational methods provided by the double dispersion relations. We shall discuss this later in more detail.

We begin by discussing possible interactions which predominate in the low energy scattering—i.e., in which Feynman graphs provide the major contribution to the K-nucleon potential. The K^--nucleon scattering has several spectacular features: the most prominent are the large low energy s-wave cross sections with extremely strong coupling into the π-Σ inelastic channels. Certainly graphs which couple $\bar{K}$-N to π-Σ *must* be important. The simplest graphs which do this and which presumably have large coupling constants associated with them are in two classes

---

(*) This work was supported in part by the United States Air Force through the Air Force Office of Scientific Research.