Uncertainty propagation in efficiency calculation of HPGe detector using Monte Carlo method

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Introduction

Mankind have achieved great success in science by observing the physical phenomenon around us. To develop a technology from the gained knowledge requires the experimental data and its uncertainties. In nuclear physics, we mostly require values of physical quantities which cannot be directly measured and they have to be calculated from variables that can be directly measured by using their functional dependence on each other. It is pity that no act of measurement can give true value of a physical quantity and the outcomes of a measurement are always associated with some uncertainties [1]. We have to propagate the uncertainties of known variables to find the uncertainties of unknown variables based on the function relationship between them. We will discuss two methodologies for this task [2], one is deterministic approach (Sandwich formula of error propagation) and other is stochastic approach (Monte Carlo method).

First order error analysis by using Sandwich formula

The Sandwich formula for error propagation is traditional first order sensitivity analysis method. We will briefly discuss this method first. Consider an independent variable vector x of order n, and dependent variable vector y of order m. Let $y{=}f(x)$ then the mean value of y is given as $\overline{y} \approx f(\overline{x})$. The covariance matrix for Sandwich formula is given as [2]

$$C_y \approx H_x C_x H_x^T. \quad (1)$$

Here C_x is $n \times n$ covariance matrix of x, C_y is $m \times m$ covariance matrix of y and H_x is the sensitivity matrix with elements $H_{xij} = \left(\frac{\partial f_i}{\partial x_j}\right)$ $(i = 1, 2, \ldots, m; j = 1, 2, \ldots, n)$. This method works quite good for functions with small nonlinearity and small uncertainties. As the nonlinearity increases it produces unsatisfactory results. Higher order terms of Taylor expansion can be involved in calculations to have more accurate results. This can be achieved by using stochastic method (Monte Carlo method). In this method an $m \times m$ covariance matrix can be approximated by a sum of matrices, each corresponding to a distinct uncertainty attribute [3].

Uncertainty propagation by Monte Carlo method

In Monte Carlo method individual histories of the derived variables are simulated using random variables from the probability distributions of the primary variables. All events are considered independent to each other, as the number of histories become large, the average of the histories approach the true solution [3]. Let x be the n-dimensional vector of primary variables and V_x be covariance matrix representing covariance of the elements of x. Consider a m-dimensional vector of derived variables y, derived from m functional relationships. This method involves producing a large number of x_k, $k = 1, 2, \ldots K$, vectors by randomly varying each component x_i of x in accordance with the probability function $P(x_i)$ governing them. For each vector (x_k) produced the m values of elements of vector y are calculated. Hence we get large collection of vectors of derived variables from which sample means are calculated as [2, 3]

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$\langle y_i \rangle_K = \frac{\left(\sum_{K=1}^{K} y_{ik}\right)}{K}$, $i = 1, 2, \ldots m$. Sample variance and covariance are given as

$$(v_{yik})_K = \frac{\left(\sum_{K=1}^{K} y_{ik} y_{jk}\right)}{K} - \langle y_i \rangle_K \langle y_j \rangle_K \quad (2)$$

where $i, j = 1, 2, \ldots nm$.

Uncertainty propagation in efficiency calculation of HPGe detector

In this experiment the efficiency of the detector is determined at six different energies of the calibration source ^{152}Eu [4]. The number of counts (C) and gamma abundances (a) are taken from [4] (Table I). The activity (A_0) of the source at the time of its manufacturing

TABLE I: Efficiency (ϵ) and their standard deviations ($\Delta\epsilon$) by using Sandwich and Monte Carlo method.

Energy (keV)	S A Method $\epsilon(\Delta\epsilon)(10^{-2})$	MC Method $\epsilon(\Delta\epsilon)(10^{-2})$
244.675	3.3262(0.0903)	3.3274(0.0904)
411.116	1.9954(0.1236)	1.9963(0.1236)
867.378	0.9042(0.0563)	0.9045(0.0562)
964.079	0.8563(0.0236)	0.8567(0.0237)
1112.074	0.7817(0.0220)	0.7820(0.0221)
1299.140	0.7459(0.0676)	0.7462(0.0677)

was 7767.67± 155.35. The time elapsed (t) between manufacturing and the experiment date was 9.893 years, half-life (T) of ^{152}Eu is 13.537±0.006 years. The efficiency (ϵ) is given as [4]:

$$\epsilon = \frac{C}{\alpha A_0 e^{\left(\frac{-0.693}{T} t\right)}} \quad (3)$$

In first order sensitivity analysis method four attributes C, a, A_0 and T were considered to calculate their partial uncertainties. The micro correlation assigned were equal to identity matrix for counts and gamma abundances attributes, and one matrix, having all its entries equal to one, for half-life and activity attributes. To calculate efficiencies and covariance matrix using Monte Carlo method, all the input parameters were considered to follow normal probability distribution. The 10000 random variables having normal distribution, with mean and standard deviation given in input data set were generated using MATLAB command and then efficiency was calculated for each gamma energy using their corresponding random input variables.

Result and Discussion

Efficiencies calculated using Monte Carlo method are free from errors due to linearization and will be more accurate than calculated using sandwich formula. Since errors due to linearization are very small in this example, so the values calculated using both methods are close to each other.

TABLE II: Calculated covariance matrix using Sandwich method and Monte Carlo method($\times 10^{-7}$)

Covariance	8.145
matrix	2.655 15.266
using	1.203 0.722 3.165
Sandwich	1.139 0.684 0.31 0.559
method	1.04 0.624 0.283 0.268 0.486
	0.992 0.595 0.27 0.256 0.233 4.567
Covariance	8.172
matrix	2.646 15.263
using	1.206 0.710 3.161
Monte	1.146 0.686 0.309 0.560
Carlo	1.047 0.624 0.284 0.269 0.487
method	0.997 0.586 0.267 0.256 0.234 4.589

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References

[1] Wolf Mannhart, IAEA Report, INDC(NDS)-0588 (2011).
[2] Harshvardhan Kadvekar, Nuclear Science and Engineering, **183** (2016).
[3] D.L.Smith, N. Otuka, Nuclear Data Sheets **113**, 3006 (2012).
[4] B. S. Shivashankar, S. Ganesan et.al, Nuclear Science and Engineering, **179 (2015).**