Chapter 4
Bruno Touschek and Statistical Mechanics

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Abstract In this talk I will describe the history of the birth of the "Meccanica Statistica" book, that Bruno Touschek and I wrote in 1970. The book was conceived and brought to conclusion in the broad context of the stimulating atmosphere that the Touschek scientific and teaching activity had created in the Physics Department of the University of Rome "La Sapienza". I will present a recollection of my memories of the years from 1965 to 1970 during which the book was imagined, written, rewritten, corrected and polished till its final version. I will also briefly describe the content of the book underlining the unmistakable footprint of Touschek unconventional way of thinking.

4.1 Introduction

Just like many of the people participating in this Memorial symposium, celebrating the 100th anniversary of Bruno Touschek's birth, I was one of his students. I graduated in 1966 under his supervision defending the thesis "$e^+e^- \to \mu^+\mu^- + \gamma$ annihilation and the Bloch–Nordsieck method". The calculations confirmed Touschek's conjecture [1] that soft-photon emission could be elegantly described within the coherent photon state formalism. The results presented in the thesis appeared in my first paper, written in collaboration with Greco [2], in which employing the formalism developed in [1] (and subsequently extended in [3]), we rigorously proved that, as expected, the resummation of soft photon emission results in an annihilation cross section proportional to the factor $(\Delta\omega/E)^\beta$, where $\Delta\omega$ is the energy resolution of the experimental apparatus, E is the center of mass energy of the beams and β is

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the famous Bond-factor. Touschek named it this way because in the typical ADONE's kinematical conditions its value was just 0.07.

The coherent state formalism was subsequently extended to encompass the non-abelian case with applications to soft gluon emission corrections in parton processes in [4–6]. Although not explicitly acknowledged in the literature, after a moment of thought one recognizes that the kinematical kernel appearing in the description of the Maximally Helicity Violating amplitudes [7], today currently expressed in terms of spinor variables [8], is nothing else but the same basic kernel that describes soft photon emissions. It is amazing to realize how far Touschek's intuition has evolved.

4.1.1 Touschek's Teaching Activity

Many talks in this Conference are focused on the remarkable and at the time unexpected developments of the Touschek's simple and brilliant idea of having electrons and positrons running head-on in a ring, held on the same circular trajectory "by the CPT theorem". The construction of the "Anello di Accumulazione" (AdA) built in 1962 in the Frascati National Laboratories (LNF) proved the feasibility of workable e^+e^- colliders. The success of AdA prompted the construction of ADONE (big AdA, but also Adonis in English) which in the future years was followed by a number of e^+e^- machines all around the world with increasing center of mass energy and luminosity, culminating in the construction of the Large Electron-Positron Collider (LEP) at CERN.

In this contribution I'm not going to talk about these extraordinary developments, others in this Conference will do it. Instead, I want to focus on Touschek's large and varied teaching activity, a somewhat less known side of his scientific personality. In particular I shall describe the birth of the book "Meccanica Statistica" to which I had the privilege to contribute.

As a teacher Touschek carried out a wide, dedicated and highly valued activity. The list of courses below is just what I could reconstruct from the information I was able to collect from colleagues and former students, but it is certainly not complete

1. "Meccanica Statistica" (IV year)
2. "Metodi Matematici della Fisica" (III year)
3. "Renormalization" (Scuola di Perfezionamento)
4. "Quantum Electrodynamics" (Scuola di Perfezionamento)
5. "Sull'insegnamento della teoria dei quanti" (Lincei)
6. "The LASER effect" (private notes[1])
7. . . . and maybe more

[1] These notes are in my possession. In Touschek's view they should have been the basis of my "Tesi di laurea" which originally was supposed to be about the "LASER effect". I must confess, however, that the argument looked too difficult to me. A bit desperate, I asked Touschek to allow me to change the subject of my "Tesi", which he kindly did by associating me to the group of people (P. Di Vecchia, F. Drago, E. Etim, M. Greco, L. Pancheri, Y. Srivastava, . . .) already intensely working on the many theoretical aspects of relevance for the forthcoming of e^+e^- machines.

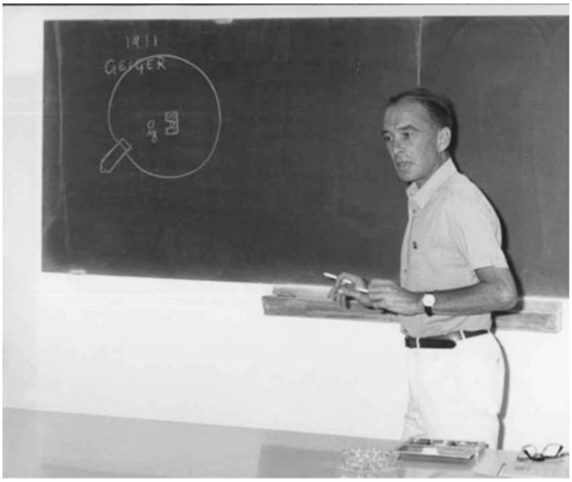

Fig. 4.1 Touschek at the blackboard with cigarette on the left hand and chalk on the right (photo from [9])

This list better that anything else may give an idea of the wide range of subjects on which Touschek had been lecturing in the years he spent in the Department of Physics of "La Sapienza" and of the amazingly large diversity of interests his scientific activity was covering (Fig. 4.1).

Between 1959 and 1968 Touschek was particularly keen to lecture on Statistical Mechanics. One of the reasons for this was that he thought it was necessary to update the program of the course and fill a number of holes in the curriculum followed by Physics students. One aspect of this was the fact that the teaching of Statistical Mechanics was commonly mainly focused on equilibrium physics and the theory of thermodynamic *ensembles*. In the standard courses there was nothing or very little about stationary but open systems and the many dynamical problems that the "Master Equation" could deal with. In other words, not much was usually taught about "statistical dynamics". The enlargement of the scope of the course with the inclusion of statistical dynamics as well as some unconventional and rarely discussed applications was highly appreciated by the students. Despite the difficulty of following Touschek lectures, "il corso di Meccanica Statistica" quickly became a "must" for many students, including myself.

4.2 The Birth of the Book "Meccanica Statistica"

The book "Meccanica Statistica" was published by Boringhieri in 1970 in the Series "Programma di Matematica Fisica Elettronica" [10]. It was the result of a rather long and elaborated journey. The first draft of the manuscript dates back to the winter of 1967. It was based on the notes that, as a fourth year student in Physics, I had taken two years before (i.e. in the academic year 1964–65), when I followed Touschek's course on Statistical Mechanics. This manuscript, carefully revised by Touschek himself, was then published by a local Editor, “La Goliardica”, as “dispense” for the students in 1969 [11].[2]

Almost contemporarily, with the idea of publishing a text-book on Statistical Mechanics, in the academic year 1967–68, Touschek started to put on paper with the help of his beloved Olivetti Lettera 22 (no PC’s were available at that time!) an English version of the lectures he was delivering week by week.

The final text of the published book resulted from the intersection of my translation of the English notes written by Touschek with the text I had drafted in Italian while attending (again) his lectures. As mentioned before, the manuscript was published in Italian by Boringhieri in 1970 in the Series “Programma di Matematica Fisica Elettronica” with the title “Meccanica Statistica” [10]. Touschek’ hope was that the book could also appear in English. Unfortunately, despite some initial interest from Wiley and Academic Press to his great regret this project never materialized.

In the long process of deciding the content of the book and the style of the text, Touschek’s guiding principle and his main concern was always clarity, as the book was supposed to be addressed to undergraduate students. For this reason we chose to use as plain and simple language as possible, and decided to end each chapter with a summary of the results and the main ideas that would be developed in the following chapter. Simplicity did not mean that all the subtleties inherent in the conceptual construction of the methods of Statistical Mechanics methods were ignored or overlooked. Quite the contrary! In the book not only standard subjects, like the construction of the various statistical *ensembles*, the proof of their equivalence and the derivation of the “Master Equation”, were inserted and discussed in a somewhat original, yet elementary way. A few unconventional problems were also addressed.

4.3 The Content of the Book

It is illuminating to look at the content of the book because it shows how modern and original was Touschek’s point of view even when covering standard topics in Statistical Mechanics. More than many words, Fig. 4.2, which was used to illustrate in an intuitive way the mathematics behind the saddle point method, is emblematic of the unmistakable footprint that Touschek had left in the book.

Here is the Table of content of the book.

- PARTE PRIMA: STATICA STATISTICA

1. Meccanica statistica e termodinamica dell’oscillatore armonico
2. Teoria dell’*ensemble* di Gibbs
3. Termodinamica covariante
4. Termodinamica di un gas ideale di particelle identiche

[2] I must add that a little earlier notes collected by another student, M. Gambarelli, where available to the students.

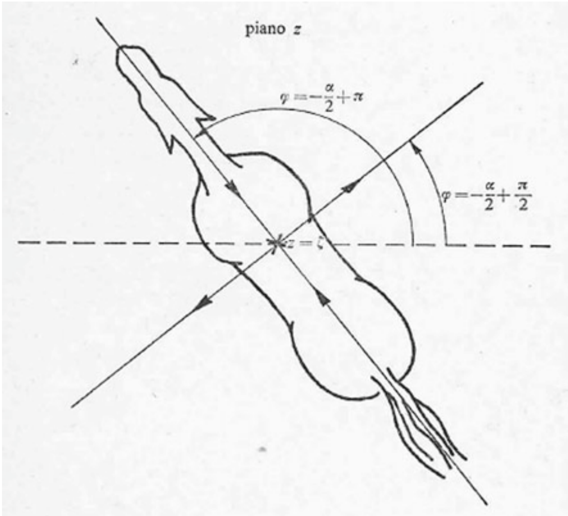

Fig. 4.2 The saddle point (method), from [10]

5. Gas degenere e imperfetto
6. Sistemi in cui il numero di particelle non è costante

- PARTE SECONDA: DINAMICA STATISTICA

1. Proprietà degli stati di non-equilibrio
2. I fondamenti microscopici della *master equation*
3. Applicazioni della *master equation*
4. Teoria del trasporto

- APPENDICI

1. A. Teoria quantistica del conteggio
2. B. Teorema adiabatico
3. C. Un esperimento statistico

Among the topics discussed in the book, it is worth to highlight three arguments that attracted the attention of the physics community as witnessed by the significant interest they stimulated in the specialized literature. The first topic is the solution of the problem posed by the definition of Temperature of a moving body in Special Relativity. The second is the application of the Master Equation to the study of the hourglass with an infinite or a finite number of grains. The third is a proposal to understand the apparent antinomy between microscopic reversibility and macroscopic irreversibility. Because of the general relevance of these questions in Physics and the originality of the arguments developed in the book, I would like to devote a few words to illustrate the physical and conceptual content of these problems and the proposed solutions. I will say something about each of these three items in the next three subsections.

4.3.1 Covariant Thermodynamics and the Lorentz Transformation Properties of Temperature

A rather non-standard topic which Touschek decided to include in the book (see chapter 3 PARTE PRIMA) was the discussion of the question of how to define the temperature of a moving body in Special Relativity. The problem, first analyzed in [12], is that it looks like we have a contradiction between two apparently equally acceptable physical definitions of temperature for a system in motion.

In fact, if one decides to make reference to the ideal gas law, $PV = RT$, in order to define the temperature of a moving system, one is led to conclude that the temperature should transform like a length under a Lorentz transformation, because P is a relativistic invariant (it is the trace of the energy-momentum tensor). As a result, the relation between the temperature of the system at rest and the temperature of the system in motion with (uniform) velocity v would be (we set $c = 1$)

$$T(v) = \frac{T(0)}{\gamma}, \qquad \gamma = \frac{1}{\sqrt{1-v^2}}. \tag{4.1}$$

On the other hand, if one decides to look at the second Law of Thermodynamics, $dS = \delta Q/T$, one is led to conclude that the temperature must transform like an energy, as entropy is just a number (it is the logarithm of the number of micro-states of the system) leading to the formula

$$T(v) = T(0)\gamma, \qquad \gamma = \frac{1}{\sqrt{1-v^2}}. \tag{4.2}$$

The way-out of this paradox lies in the observation that the usual operative definition of temperature actually refers to a measurement performed in the rest frame of the system (the one in which the thermometer is at rest with respect to the system). What one might call the temperature of a moving system is actually a matter of conventions, in the sense that different measurement procedures give raise to different definitions of the temperature of a body in motion, hence to apparently different Lorentz transformation properties.

This argument can be made rigorous by constructing a covariant formulation of Thermodynamics and Statistical Mechanics which can be done by making fully covariant the construction of statistical *ensembles*. For instance, with reference to the Gibbs *ensemble*, in addition to including the constraint energy conservation, one needs to enforce the conservation of three-momentum. The occupation numbers will then satisfy the equations

$$\ln a_n + \lambda + \beta_\mu p_n^\mu = 0, \tag{4.3}$$

which represent the solution of the covariant extension of the standard variational problem of classical Statistical Mechanics. In (4.3) λ is the Lagrange multiplier associated with particle number conservation, $\sum_n a_n = N$, and β_μ the time-like four-vector associated with the four-momentum conservation, $\sum_n a_n p_n^\mu = P^\mu$. The covariant formulation of Statistical Mechanics will then be expressed by the equations

$$a_n = \frac{N}{Z} e^{-\beta_\mu p_n^\mu} \tag{4.4}$$

$$Z = \sum_n e^{-\beta_\mu p_n^\mu} \qquad P_\mu = -N \frac{\partial \ln Z}{\partial \beta^\mu} \,. \tag{4.5}$$

One can prove that the relation between β_μ and the temperature at rest, $T(0)$, is given by (k_B is the Boltzmann constant)[3]

$$\beta_\mu = \frac{u_\mu}{k_B T(0)} \,, \qquad u_\mu = \gamma(1, \mathbf{v}) \,. \tag{4.6}$$

This formula shows that the Lorentz transformation properties of the temperature depend on how the latter is measured, i.e. which component of β_μ is employed to define T for a body in motion with four-velocity, u_μ.

4.3.2 The Hourglass and the Periodic Statistical Clock

In this section I want to present an amusing application of the Master Equation to the time evolution of the hourglass (see Fig. 4.3) and the periodic statistical clock, discussed in chapter 3 of the PARTE SECONDA of the book.

4.3.2.1 The Hourglass

The statistical description of the time behaviour of the hourglass can be described by the equations

$$\dot{p}_0(t) = -\lambda p_0(t) \qquad\qquad p_0(0) = 1 \tag{4.7}$$

$$\dot{p}_{s+1}(t) = \lambda[p_s(t) - p_{s+1}(t)] \,, \quad p_{s+1}(0) = 0 \,, \quad s = 0, 1, \ldots \tag{4.8}$$

where $p_s(t)$ is the probability of having s grains in the lower part of the hourglass at time t and λ is the transition rate from the state s to the state $s+1$. Naturally the boundary conditions we are interest is are such that the initial time ($t_0 = 0$) state is the one where the lower part of the hourglass is empty as indicated in the right part of the Eqs. (4.7) and (4.8). Dots in (4.8) mean that we are considering the case is which N, the number of grains, is infinitely large. In this situation the system (4.8)

[3] Technically (4.6) holds under the rather mild assumption that the partition function is a relativistic invariant.

Fig. 4.3 The hourglass (figure by G. C. Rossi)

can be solved exactly, yielding

$$p_s(t) = \frac{(\lambda t)^s}{s!} e^{-\lambda t} \,, \quad s = 0, 1, \ldots \tag{4.9}$$

The probability distribution is therefore Poissonian with

$$\langle s \rangle = \lambda t \,, \qquad \frac{\sigma}{\langle s \rangle} = \frac{1}{\sqrt{\langle s \rangle}} \,. \tag{4.10}$$

From these equations we see that we can actually use the hourglass as a clock as the average number of grains in the lower part grows proportionally with time and the relative fluctuations in the number of grains dies out like $1/\sqrt{t}$.

4.3.2.2 The Periodic Statistical Clock

We now consider the case in which N is finite. In this case the hourglass Master Equations cannot be solved exactly. A related soluble problem is, however, the "periodic clock", i.e. a system in which, defining $p_s(t)$ the probability for the system to be in the state s at time t, we have

$$p_s(t) = p_{s+N}(t) \,, \quad s = 0, 1, \ldots, N-1 \,. \tag{4.11}$$

The Master Equation for the periodic statistical clock reads

$$\dot{p}_{s+1}(t) = \lambda[p_s(t) - p_{s+1}(t)]\,. \tag{4.12}$$

This set of linear first order differential equations can be solved by normal modes decomposition. If we start with the initial condition $p_s(0) = \delta_{s,0}$, one finds that the system visits it periodically, every $T = N/\lambda$. In principle for appropriate choices of N and λ this system can then be used as a clock.

4.3.3 Micro-reversibility Versus Macro-irreversibility

In this subsection I want to illustrate the proposal, discussed in the APPENDICE C of the book, for a solution (or an understanding) of the problem of reconciling the invariance under time reversal of the fundamental laws of Physics (both Newton's equations in classical physics and the Schrödinger equation in quantum physics), and the irreversibility we observe in macroscopic processes.

The question has not only drawn an enormous attention in the specialized literature, but it has also generated a large philosophical debate because of its epistemological implications for the very definition of the notion of time. The solution of the reversibility versus irreversibility paradox proposed in the book is actually very simple and can be briefly summarized as follows.

(1) The fundamental laws of micro-physics are invariant under time reversal

(2) The lack of symmetry under time inversion that we routinely experience is a consequence of the very peculiar initial conditions from which the macroscopic systems we usually observe, evolve.

Indeed, the unavoidable mixing process between wine and water taking place in a glass ensues from the sharp separation of the two liquids at the initial time. Starting from this (largely out of equilibrium, hence statistically highly improbable) initial condition, the time evolution of the system molecules is then completely controlled by perfectly time reversible equations. From a thermodynamic point of view the system evolves towards its equilibrium state.[4] This means that repeating the experiments many times the final state of the system (i.e. the state reached after a very long time lapse) is within statistical fluctuations always the same.

A simple and direct check of the validity of this point of view is provided by the following simple but paradigmatic numerical example. Let us consider the "time" evolution of the stochastic variable represented by the mean value of the sum of $N = 100$ numbers, s_n, between $-1/2$ and $1/2$, randomly extracted from a uniform

[4] We are excluding here mathematical instances of systems with unrealistic initial conditions which would prevent the system to ever reach equilibrium, for instance a system of perfectly elastically interacting balls in a cubic box with perfectly reflecting walls, having initial velocities all equal and oriented perpendicularly to the box walls.

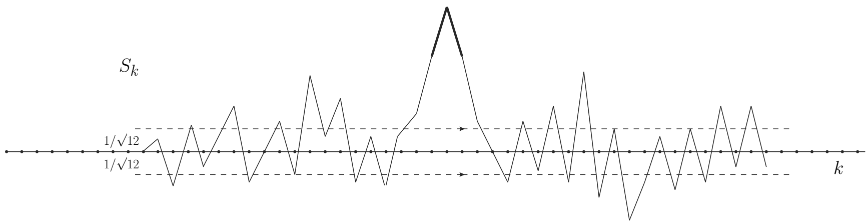

Fig. 4.4 The typical behaviour with "$k \sim$ time" of the stochastic variable defined in (4.13)

distribution, when successively a new term is added to replace the first one. In formulae we write

$$S_k = \sum_{n=k}^{99+k} s_n \,, \qquad k = 1, 2, \ldots, N_{config} \,. \tag{4.13}$$

The "time" history of this stochastic variable was followed for $N_{config} = \mathrm{O}(10^7)$ steps (the actual calculation was carried out with the historic UNIVAC machine located in the "Centro di Calcolo" of Sapienza University of Rome, at a time when codes were still written on punched cards). The "$k \sim$ time" evolution of S_k is very illuminating and it is sketched in Fig. 4.4.

For a perfectly flat distribution one expects the value of S_k to fluctuate around $\langle S \rangle = 0$ with $\langle S^2 \rangle = 1/12$. Looking at Fig. 4.4, one notices that there are many small fluctuations where S_k slightly differs from zero and a very few large fluctuations in which S_k is significantly different from zero. But the key aspect of the time behaviour of the system is that the latter is symmetric around any point where S_k is at a local maximum or minimum. As it follows from the second Law of Thermodynamics, the left and right slopes around those points can be shown to be equal giving rise there to cusps. This means that, looking at the behaviour of S_k, it is impossible to tell which way time is running. In other words the actual history of the system and the one obtained by time reversal are indistinguishable.

On the other hand, macro-physics looks irreversible because most often the system whose time behaviour we are observing is (almost) always largely out of equilibrium. Starting from a large fluctuation, the system, if left alone, (almost) inevitably moves away from this very low probability state, since the probability of moving towards an even larger fluctuation is totally negligible. As a result, a system finds itself (almost) always at a maximum of a fluctuation.

We conclude with an amusing observation. It appears that these considerations provide a practical way to check the degree of randomness of random numbers. In fact, since the fluctuation probability distribution for random number generation can be computed exactly, one can measure the probability $P[\langle S \rangle > \overline{S}\,]$ of finding $\langle S \rangle$ greater than a given threshold, say $\overline{S}$. In our 1969 simulation it turned out that the measured $P_{\rm exp}[\langle S \rangle > \overline{S}\,]$ didn't agree with theoretical expectation, meaning that the random numbers generated by our computer were not so random! Touschek was very upset by this situation to the point that he came up with a new algorithm aimed at "randomizing random numbers" [13].

4.4 Some Personal Recollections

The revision of the draft of the Italian version of the manuscript usually took place in Touschek's apartment situated in Via Pola either in the morning from 10 to lunch time or in the afternoon from 3 pm onwards. The reason was that too often we could not access the Physics Department because it was "occupato dal movimento studentesco". We are in 1968, in the hottest moment of the turmoil.

When I arrived, on Touschek's desk submerged by his typewritten notes with ashtrays full of butts, there was always a bottle of Chianti, open and ready, together with two glasses. According to Touschek "Chianti was the ideal magic potion making the brain work smoothly and brilliantly". Actually not mine! After a couple of hours my brain wasn't so much focused and "brilliant" any more. But it was very difficult to refuse to drink the wine repeatedly poured into my glass! Until, after a week or so, I discovered that the only working strategy I could put in place was to never empty the glass after the first shot.

These meetings were for me extraordinary occasions in which we didn't only talk about the book we were preparing and Physics in general, but also about every day life, philosophy, politics (remember this was in 1968). An argument which came up frequently was the issue of the famous "pompe di m...", a commercial activity in Rome (in Piazza Indipendenza) initially run by his aunt. With polemical irony those were indicated by Touschek as the real source of the income of his family, because he considered the meagre salary he was getting as "Professore Aggregato" like a kind of charity graciously handed by the Ministry of Education. One day, particularly angry because the procedure initiated to get him a tenured position as "Professore Ordinario" was being delayed, he put up on the door of his office his pay slip with the provocative purpose of denouncing the total failure of his fight against the dull Italian bureaucracy. Actually Touschek wanted also to complain about the low funding of research and the enormous delay in the construction of ADONE. The latter was blamed by Touschek mainly on the inability of the LNF (Laboratori Nazionali di Frascati) management to deal with the repeated strikes and "occupations" going on in those years.

In everyday relations Touschek was an exquisite person with an extraordinary sense of humor and a sort of disenchanted cynicism making people around him fascinated and disoriented. But at the same time he was always keen to talk and ready to patiently explain things. The time I spent working on the book not only served as a guide for my career as a Physicist, but was also a school of formation as a person.

4.5 Conclusions

Touschek was a great scientist, a brilliant teacher and an amazing person, and for me a source of invaluable inspiration. Interacting with him was a fantastic human and scientific adventure.

I want to conclude these considerations on the birth and the content of the book "Meccanica Statistica" with a few remarks that I believe are the key to explain the forthcoming developments of theoretical research in Italy.

The point I want to make is that, despite the fact that Touschek had been lecturing on the subject of Statistical Mechanics for only 4 or 5 years at Sapienza University of Rome (at a certain point he moved to the course of "Metodi Matematici della Fisica"), his cultural legacy had an enormous impact on the development of Theoretical Physics in Rome. One cannot consider it to be just a mere coincidence the fact that in the following years Statistical Mechanics has grown to be one of the major areas of investigation in Rome and in Italy, culminating in the award of this year's Nobel Prize for Physics to Giorgio Parisi. Indeed, the root of the many important contributions that Italian physicists have given to a number of research fields related to Statistical Mechanics (among which, besides the theory of Spin Glasses and Complex Systems [14, 15], I want to mention Turbulence [16], Lattice QCD [17, 18] and the emerging field of Biophysics [19]) can be traced back to the crucial influence that Touschek scientific and teaching activity had on a whole generation of physicists.

Finally, just allow me to repeat that a great regret for Touschek (as well as for myself) was that he didn't manage to publish an English version of the book. Is it too late today?

Acknowledgements I wish to thank the Organizers of this Memorial. It was not only a moving and intense occasion but also a stimulating overview of the recent developments in the theory of fundamental interactions and the prospects for the construction of future particle accelerators brought about by Touschek's far-reaching vision.

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