Application of R/S method in EPOS4 simulated pp collisions at $\sqrt{s} = 2.76$ TeV

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Introduction

Over the last few years, complex network analysis has gained significant attention in studying heavy-ion collisions. Albert and Barabasi pioneered the construction of scale-free networks [1]. Later, a network-based approach was applied to analyze time series , leading to the development of techniques for examining long-range relations and fractal structures. To analyze time series with self-similarity, we can establish a relation between the fractal dimension D and the Hurst exponent H using the equation $D = 2 - H$. In a one-dimensional case, D typically falls between 1 and 2. The Hurst exponent, which ranges from 0 to 1, quantifies the degree of long-term memory in a time series, acting as a measure of the smoothness of fractal structures. A value of $H > 0.5$ suggests persistence in the time series, while $H < 0.5$ indicates anti-persistence. Thus, higher values of H imply less volatility and greater smoothness. The Hurst exponent is a valuable tool for evaluating the fractal characteristics of a time series, as it also provides insight into persistence and correlation. Furthermore, H can be determined using rescaled range (R/S) analysis, which assesses the statistical variability of a time series by examining how variability changes over different time spans. For the study purpose, we have used the EPOS4 model generated two sets of data, one with default setup (i.e. without any extra consideration) and the other with default + hydro (i.e. with considering full hydrodynamic evolution) of pp interactions at $\sqrt{s} = 2.76$ TeV. For a detailed view of the physics included in the model and the selection of parameters during data generation one can follow Ref.[2]

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Methodology

Hurst [3] introduced the R/S analysis method, later popularized by Wallis and Mandelbrot, which is central to complex network analysis. R/S statistics examine deviations of sequences from the mean, rescaled by the standard deviation. The key steps are:

- Define a data set $X_N = (x_i)$ with N nodes. A sub-series $Y_M = (y_j)$ with $M = sN$, $s \in (0, 1)$, is derived.
- Calculate the mean: $\bar{y_s} = \frac{1}{M}\sum_{k=1}^{M} y_k$.
- Create a cumulative series: $z_i = \sum_{k=1}^{i} y_k \sim \bar{y_s}$, $i = 1, 2, \ldots, M$.
- Obtain the range: $R_s = \max(z_i) - \min(z_i)$.
- Rescale the range by standard deviation: $(R/S)_s = \frac{R_s}{\sigma_s}$, where $\sigma_s = \left[\frac{1}{M}\sum_{k=1}^{M}(y_k - \bar{y_s})^2\right]^{\frac{1}{2}}$.

Discussion

For a randomly selected subsample of events, the q-norm is used to calculate the rescaled range (R/S) for varying values of the scale s. The results are then averaged over the subsample. This procedure is repeated for each event, and the averages are then computed across all events. Finally, the logarithm of the rescaled range, $\ln(R/S)$, is plotted against the logarithm of the scale, $\ln(s)$.

Fig. 1 shows the variation of $\ln(R/S)$ and $\ln(s)$ for different values of q, i.e., $q = 1, 2, 3, 4, 5$ & 6 for EPOS4 generated pp interactions dataset at $\sqrt{s} = 2.76$ TeV. The figure clearly shows the absence of a crossover region, suggesting that the multiparticle production dynamics are scale-independent. This also implies the presence of correlations among the produced particles. However, if the process exhibited scale dependence, the graph would display a crossover region, characterized by the intersection of two lines with different slopes, preventing it from being uniquely characterised by a power law.

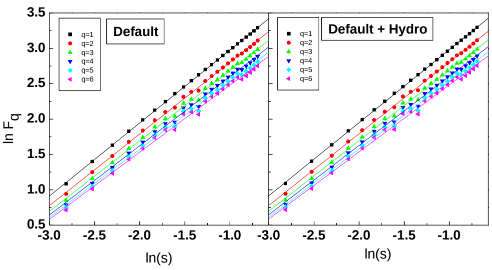

FIG. 1: Variations of ln(R/S) with ln(s) for pp interactions at $\sqrt{s} = 2.76$ TeV.

In Fig. 2(a), we illustrate the variation of H_q versus q for EPOS4 generated pp interactions dataset at $\sqrt{s} = 2.76$ TeV. From the figure it has been observed that the values of H_q decrease as q increases for both the data sets i.e. default and default + hydro, respectively. The values of $H(q)$ for both the conditions are below unity, suggesting that the multiparticle production process is the self-affine process and is anisotropic in nature. In Fig. 2(b), we show how the fractal dimension $D(q)$ varies with respect to q. We have compared the results in both the conditions. However, no significant differences have been noticed between default and default + hydro setups. A similar study with AMPT, UrQMD and experimental emulsion data has published recently[5].

We have also calculated the values of the degrees of multifractality (ω) by using the relation $\omega = D_{max} - D_{min}$, through which one can gain information about the multifractal nature of the system. The nonzero values of (ω) indicates the presence of multifractal behavior [4]. The values of ω, $H(2)$ and $D(2)$ are given in Table 1.

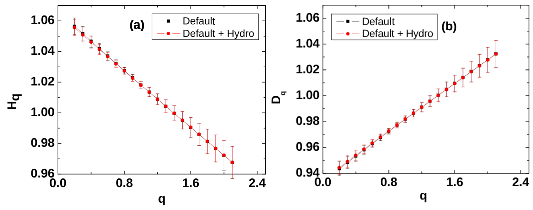

FIG. 2: (a) Variation of H_q vs. q and (b) Variation of D_q vs. q for pp interactions at $\sqrt{s} = 2.76$ TeV.

TABLE I: Value of the Hurst exponent (H) and Hausdorff Dimension (D) and degrees of multifractality (ω) for pp collision at EOPS4 Model

Parameters	$\sqrt{s} = 2.76$ TeV	
	Default	Default + Hydro
$H(2)$	0.972 ± 0.009	0.972 ± 0.009
$D(2)$	1.028 ± 0.009	1.028 ± 0.009
ω	0.477 ± 0.077	0.479 ± 0.077

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