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The Terrell-Penrose effect when photographing a sphere at rest with a moving camera

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Abstract. The paper deals with the Terrell-Penrose effect, the component parts of which are the turn of the moving sphere and independence of the form of the sphere, as recorded by a camera, from the speed of motion of the sphere. The present work shows that the independence of the form of the moving sphere from the speed as well as its turn in the photograph are due to the discrepancy between the purpose of the photograph and the choice of the angle of photographing. In examining the Terrell-Penrose effect in this paper preference is given to a reference frame, where the sphere is at rest, whereas the camera (camera obscura) is in motion. With this choice of the reference frame the mentioned optical effects may be accounted for by aberration of light (reflected from the quiescent sphere), which has formed the image on the photoreceptor of the moving camera. In the case of semitransparency of the sphere and the presence of markers at the points of the sphere, which lie on the line of its motion, the Lorentz contraction of the moving sphere would also be revealed by way of photography.

1. Introduction

The Terrell-Penrose effect in question [1-2] relates to relativistic effects, which, paradoxical as they may seem outwardly, attract attention of those dealing with special relativity. The component parts of this effect are the turn of the fast-moving sphere and independence of its form from the speed of motion being recorded by a photoreceptor. This behavior of the fast-moving sphere does not contradict to the Lorentz contraction, having in mind that it is theoretically described by Terrell and Penrose having taken the Lorentz contraction of the moving sphere into account. There is information about the experimental confirmation of the preservation of the shape of a moving sphere registered by photographic means [3].

The imperfection of the description of the effect given by Terrell and Penrose lies in the fact that it was carried out only in a reference frame with the quiescent camera and the moving sphere. This choice of a reference frame results from a tradition that examination of relativistic effects is to be carried out in reference frames wherein observers or measuring instruments are at rest and the objects of observation are in motion. Aberration of light is perhaps the only phenomenon, the explanation of which allows a moving observer or a moving instrument. At the same time, as regards description of the physical side of relativistic phenomena and effects it is often expedient to use a reference frame wherein the observed object is at rest and the observation facility is in motion as Feynman and Einstein did [4-5]. In the present work preference is given just to the reference frame wherein the sphere is at rest and the camera (camera

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obscura) is in motion. Such choice of a reference frame makes the Terrell-Penrose effect simpler and more visual.

2. Photography of stationary rod and sphere with a stationary camera obscura

Assume that between points A and B of X-axis on the plane XZ within a Cartesian coordinate frame K the rod *d* long lies. The ends of the rod are located symmetrically to the origin O of coordinates of the frame K (Fig.1).

Let us assume now that besides the rod AB in the coordinate frame K there is a semi-transparent sphere with the center at the origin O of coordinates, through which the rod is visible. The radius of the sphere is ½*d*.

If the photography is performed from a remote point F by a pinhole camera at rest then the image of the latter has the form of a circle, points A and B of which are connected by a rod viewed through it (fig. 1a). But if the photographing is performed from the point G then the contour of the sphere is created by the beams, synchronously emitted some time before by the points of the sphere, which lie on the large circle formed by the surface of the sphere and by the diametrical section located perpendicular to the line OG. Points C and H of the diameter d_{CH} shown in fig. 1 also belong to these points.

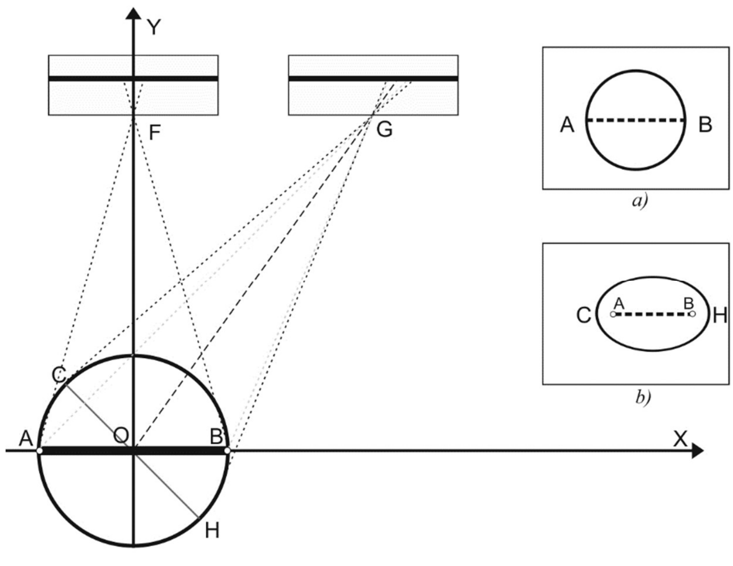

Fig. 1. Photography of the rod AB from points F and G with a camera obscura at rest. The photograph *a* shows a geometrically undistorted image of the rod AB and of the sphere in the center of the frame. The photograph *b* shows the undistorted image of the rod AB and the stretched image of the sphere that are shifted from the center of the frame.

Because of the inclination of the line CH to the plane of the photoreceptor the longitudinal diameter of the circle in the photograph turns out to be longer than the transverse diameter 1/cos φ times, and the circle takes the form of the ellipsis elongated in the direction of the X- axis. However, the image of the rod AB will not be distorted because of the parallelism of the rod AB and the photoreceptor and the similarity of the triangles AGB and A′GB′. The image as a whole is shifted from the center.

3. Photography of stationary rod and sphere with a moving camera obscura

Let us pass on to examination of the photography of the rod and the sphere at the above-mentioned points F and G with a camera at a speed *v* moving parallel to the *X*-axis and staying at the distance *l* from the X-axis (fig. 2). The camera moves together with the reference frame K′ constantly being on the axis Y′ at the point with ordinate $l' = l$.

IOP Conf. Series: Journal of Physics: Conf. Series **1251** (2019) 012035 doi:10.1088/1742-6596/1251/1/012035

Assume now that at the moment of time $t = t' = 0$, when the Y and Y′ axes coincide, the camera obscura (showed by the dotted line) hitting the surveying point F momentarily triggers the shutter. Let us note that the information, which is at the given point passing through the momentarily opened aperture of the camera obscura and coming to the photoreceptor, does not depend on whether the camera is in motion or not. The light information practically instantly passing through the opening of the quiescent camera shown in fig. 1 as well as the light information practically instantly passing through the opening of the moving camera shown in fig. 2 are identical. With all that, the images of the quiescent object photographed with the cameras that are at rest and in motion, are different. The photography results are primarily affected by the Lorentz contraction of the photoreceptor and its displacement during the motion of the light beams from the opening of the camera obscura to the photoreceptor.

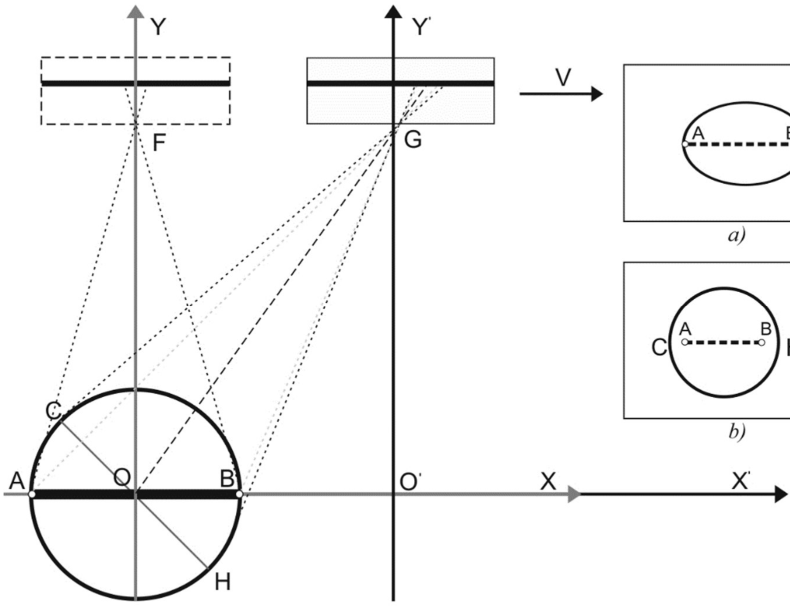

Fig. 2. Photography of the rod AB and the sphere from points F and G with the camera obscura moving at a speed *v*. The photograph *a* shows the stretched image of the rod and the stretched image of the sphere that are shifted from the center of the frame. The photograph *b* shows the shortened image of the rod and the undistorted sphere in the center of the frame.

If the photography is performed from the point F then the light from the ends of the rod and from the large circle of the sphere lying on the plane XZ hits the point F and enters the opening of the moving camera obscura. Instantly the light falls on the photoreceptor, forming the image. In the coordinate frame K the light beams synchronously reflected from the ends of the rod AB synchronously fall on the moving photoreceptor. Because of the motion of the photoreceptor the image on it over the time of the light passage of the distance from the opening of the camera obscura to the photoreceptor shifts by the distance vh/c, where h is the distance between the hole and the photoreceptor and c is the speed of light.

Because of the Lorentz contraction of the photoreceptor moving in the coordinate frame K the image of the rod and the sphere obtained after visualization will prove to be stretched lengthwise (in the direction of motion) by $1\big/\sqrt{1-v^2/c^2}$ times (fig. 2*a*, the photograph left). The length d'_k of the image of the rod captured on the photoreceptor (the proper length d'_k in the coordinate frame K′) will prove to be increased and related to the value d_k by the ratio

IOP Conf. Series: Journal of Physics: Conf. Series **1251** (2019) 012035 doi:10.1088/1742-6596/1251/1/012035

$$d'_k = d_k \big/ \sqrt{1 - v^2/c^2} \qquad (1)$$

If the photography of the sphere and the rod is performed with the camera moving in the coordinate frame K not from the point F, but from the point G, then the beams that have arrived from the ends of the rod and that at the moment of triggering the shutter have passed through the opening of the camera obscura asynchronously (in the reference frame K) fall on this plane because of the inclination to the plane of the photoreceptor. If the photoreceptor were not in motion, then the length of its image would be equal d_k because of the mutual parallelism of the photoreceptor and the rod. However, the camera obscura and the photoreceptor move at a speed v. The motion of the photoreceptor has a dual effect on the image. On the one hand, because of the Lorentz contraction the image on the photoreceptor becomes by $1\big/\sqrt{1-v^2/c^2}$ times stretched, in the result of which the length of the image of the rod on the photoreceptor must become equal to $d_k\big/\sqrt{1-v^2/c^2}$, as in the case given in fig. 1*b*. On the other hand, over the time Δt_k between the incidence of the beam coming from point A, and the incidence of the beam coming from point B, the displacement of the photoreceptor equals $v\Delta t_k$. It is easy to show that the difference in the time of propagation of light from the opening of the camera to the front and to the rear edges of the image is equal to $\frac{d_k}{c}\sin\varphi$, if the angle of incidence of light equals φ. Given this equality the displacement $v\Delta t_k$ of the photoreceptor can be represented by the formula

$$v\Delta t_k = \frac{d_k v}{c}\sin\varphi\,. \qquad (2)$$

The displacement $v\Delta t_k$ shortens the length $d_k\big/\sqrt{1-v^2/c^2}$ of the image of the rod on the photoreceptor by the value $\frac{d_k v}{c\sqrt{1-v^2/c^2}}\sin\varphi$, and the length d'_k of the image of the rod AB falling on the moving photoreceptor, captured in the coordinate frame K (the proper length d'_k in the coordinate frame K′) becomes equal to

$$d'_k = \frac{d_k}{\sqrt{1-v^2/c^2}}\left(1 - \frac{v}{c}\sin\varphi\right). \qquad (3)$$

If the angle $\sin\varphi$ equals v/c , then (1) is transformed to

$$d'_k = d_k\sqrt{1-v^2/c^2}\,. \qquad (4)$$

The diameter d_{AB} of the sphere coincides with the rod AB, therefore its length $d'_{k,AB}$ in the photograph is expressed by the formula

$$d'_{k,AB} = d_{k,AB}\sqrt{1-v^2/c^2}\,. \qquad (5)$$

As regards the photograph of the sphere, at the moment of triggering the shutter of the camera obscura the beams come to the opening of the camera. These beams are synchronously emitted by the points of the sphere lying on a large circle formed by the surface of the sphere and the central section located perpendicular to the line OG.

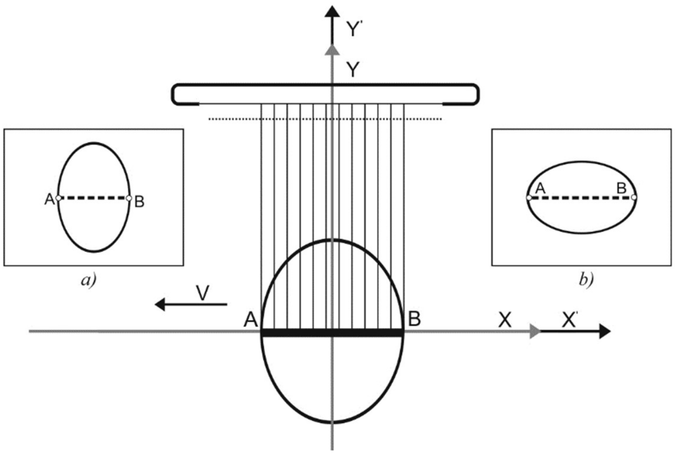

Fig. 3. A sphere with rods or laser beams replacing rods moves with velocity *v* in the coordinate frame K′ to the left. In the photograph *a* on the left is a shortened image of a moving sphere, obtained by simultaneously in the coordinate frame K′ triggering the shutter cells. In the photograph *b* on the right there is an image of a sphere at rest taken by a moving camera (the cells of the camera are triggered simultaneously in time of the coordinate frame K). With a small distance between the shutter and the photoreceptor the image shift is small.

In contrast to the rod AB, the image of the segment CH, incident on the moving photoreceptor, stretches in $1/\cos\varphi$ time. This leads to the fact that the segment CH on the image turns out to be by $1/\cos\varphi$ times longer than the diameter d_{CH}, i.e. the image of the sphere is undistorted. The absence of distortion refers not to the longitudinal size d_{AB} of the sphere, but to its size expressed by the diameter d_{CH}. The longitudinal size of the sphere is reproduced by the size between the labels of the sphere coinciding with the ends of the rod, i.e. the sphere is reduced in the direction of its movement by a factor of $1/\sqrt{1-v^2/c^2}$.

4. Photography of a moving sphere in parallel beams

In [2], Terrell stated: "The camera is assumed to be at rest… Of course, it would make no difference if the camera were, instead, considered to move at high speed past stationary object…" Indeed, the shape and content of the images of the sphere taken by the camera during their relative movement at points F and G do not depend on what we consider to be at rest and what is moving. However, declaring that there is no difference between shooting a moving and a quiescent sphere, Terrell and Penrose do not pay attention to the fact that the shooting a stationary sphere with a moving camera needs to be performed at point F, but the shooting a moving sphere with a stationary camera at point G. It is precisely because of the shooting of a sphere by a camera from point G that the sphere in the photo retains its shape and "turns". A strange element of the argument when choosing a point for shooting a moving sphere is the statement that a moving sphere, having arrived at some time at point O′ on the Y′ axis can be seen by an observer only after a time Δt equal to l/c.

The strangeness is that if not a moving sphere appears at the point O′ of closest approach but a moving camera appears at the point F of closest approach, then the sphere can be seen by an observer with a camera at the same moment of time, albeit visually shifted in the direction of motion of the camera due to aberration of light. Why can't the light-beam information about the hemisphere facing point F permanently present at this point enter the camera at the moment this point F coincides with the hole of the camera obscura? Why is the image of the sphere, obtained at point F, stretched regardless of what we consider to be moving?

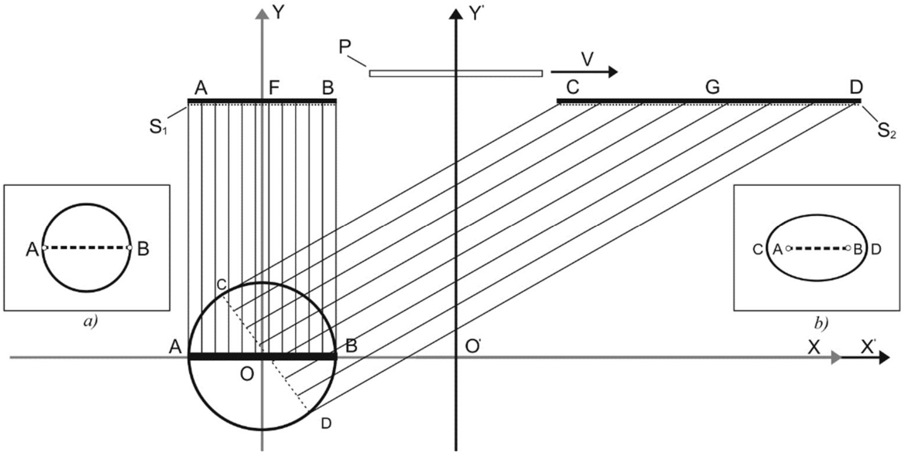

Fig. 4. The upper part of the hemisphere is covered with lasers with vertically directed beams. The hemisphere facing the screen S_2 is covered with lasers, the rays of which, being parallel, are directed to the screen S_2. The photograph *a* shows the image on the screen S_1, the photograph *b* shows the image on the screen S_2. Dotted lines in the sphere show a rod AB, if it were visible or projected with laser beams through the sphere. Photoreceptor P moves passing over the screens.

To answer these questions, let's digress from shooting the sphere with a pinhole camera and imagine that the upper hemisphere of the sphere in question is equipped with a set of thin rods parallel to the Y axis. If at the moment of time $t' = 0$, observers of the coordinate frame K′ fix the position of thin rods, they will detect the contraction of the sphere (the result of observation).

Now imagine that the thin rods are replaced by green beams of lasers located on the upper hemisphere. Note that if all the photons of a beam move in a certain coordinate frame along a straight line, then they are on this straight line in any other inertial coordinate frame. In particular, if photons move along a vertical line in the reference frame K, forming a beam lying on this line, then in the K′ frame each of these photons will remain on this line at any given time, changing only the direction of motion according to the wave vector transformation.

If at the moment of time t′ = 0 the observers of the coordinate frame K′ fix the location of each ray, then again they will find a contraction of the sphere (the result of the observation). To do this, they can, for example, simultaneously registering the positions of the rays with devices or projecting the rays onto a photoreceptor through a flat shutter, all of which cells simultaneously (in the K′ frame) open for a brief instant. In the latter case, the observers will get a sphere image compressed in the direction of motion, but the reduced image on the color positive photoreceptor will not be red shifted, but blue shifted.

Consider now a structure consisting of a sphere whose center is at the origin O of the coordinate frame K, two screens S_1 and S_2, and two sets of lasers whose rays illuminate the screens S_1 and S_2 (fig. 4). One of the sets of lasers, the rays of which are directed upwards and fall perpendicularly to the screen S_1, is located in the upper part of the sphere, the other set of lasers, the rays of which are obliquely falling on the screen S_2, are located on the hemisphere facing the screen S_2. The points C and D are located on the large circle of the hemisphere facing the screen S_2. The “photograph” *a* of fig. 4 shows the undistorted image of the hemisphere displayed on the screen S_1 with vertically directed laser beams. The photograph *b* of fig. 4 shows the stretched image of a hemisphere projected onto the screen S_2.

The rod AB in both images, if it is visible or projected with laser beams, is not distorted due to the parallelism of the rod and the plane of the photoreceptor.

Let a flat photoreceptor P (fig. 4) fly by in the vicinity of the screens S_1 and S_2 above them. Suppose that the screens S_1 and S_2 can function as liquid crystal shutters, the cells of which are capable of simultaneously and instantaneously opening and transmitting the light incident on them from the lasers. Imagine that at the moment, when the photoreceptor is located above one of the screens, the cells of the liquid crystal shutter open simultaneously for an instant, and the laser beams form an image on the photoreceptor. If at the moment, when the center of the photoreceptor is located at the point F over the shutter S_1, the cells simultaneously open according to the clock of the coordinate frame K′ (the sphere moves in the frame K′ and the photoreceptor is at rest), then the image of the sphere and the rod will be shortened, like the rod in fig. 2*a*. If the cells of the shutter S_1 open simultaneously according to the clock of the coordinate frame K (the sphere is at rest, and the photoreceptor is moving in frame K), then the image of the sphere and the rod AB will be stretched, like the image of the sphere and the rod AB in fig. 2*b*.

If the photo is taken at the moment, when the center of the photoreceptor is located at the point G above the shutter S_2, then, if in the K′ frame the cells of the shutter S_2 are triggered simultaneously (the sphere moves in the K′ frame, the photoreceptor is at rest) then the image in the photo is similar to the image in Fig. 2*b*. The color of the image is red shifted.

What is the reason that, when taking pictures from the point F of the frame K, you can only get a snapshot of a stretched sphere and of a rod, similar to the picture of the stationary sphere, obtained by parallel projecting with the moving camera (when the shutter cells are synchronous in the coordinate frame K)? Why is it impossible to obtain from the point F an image of a moving sphere, similar to the image of the moving sphere, obtained by the method of parallel projecting with the camera at rest (when the shutter cells are synchronous in the coordinate frame K′)?

Answers to these questions can be obtained by using the concepts of concretized instantly existing objects considered in [6].

Let us imagine that in the center of a sphere R being at rest in the coordinate frame K is a pulsed light source. Let at some moment in time the source send a short light pulse, and after a time $\frac{1}{2}d/c$ the light for a moment simultaneously illuminates the surface of the sphere. We introduce the concept of the luminous sphere R_0(flash), meaning by this the totality of the sphere elements and events representing the simultaneous illumination of these elements at some point in time. Speaking about the illumination of the sphere R_0(flash) at a given time, we will have in mind not the visual perception, but the instrumental recording (observation in Terrell's terminology) of the glow at different points of the sphere at a given time. Such the sphere R_0(flash) being instantly existing object, appears momentarily and immediately disappears. In terms of the luminous sphere R_0(blink) we will not include a sphere with a partially illuminated surface at the moment If, for example, a pulsed light source is located not at the center, but at a point located between points O and A on the X axis, then the flash of the source does not create the stationary sphere R_0(flash) since there is no the time at which in the coordinate frame K, where the sphere R and the rod AB are at rest, a simultaneous luminescence of the surface of the sphere can be registered.

But it is easy to show that there is a reference frame in which such an instantly partially illuminated at rest sphere is instantly fully illuminated moving at a speed *v*. If we consider such a fully illuminated sphere as a separate object and designate it as Rv(flash) then we can speak of the existence of this object only in a state of motion with velocity *v*. Thus, the object R_0(flash) is at rest, and the object Rv(flash) in the state of motion with speed v.

The state of rest and movement of objects R_0(flash) and Rv(flash) does not depend on the choice of reference system. On the contrary, the choice of the reference system depends on which of these objects must be registered. Objects R_0(flash) and Rv(flash) are concrete forms of an abstract illuminated sphere without specifying the degree of illumination of its surface at some moment of time.

Using similar concepts it is not difficult to show that the photography from the point F allows to capture only the sphere and the rod AB at rest, but the photography from the point G allows to capture the moving rod AB and moving sphere although not from the angle that is required to discover Lorentz contraction of the sphere. The hole of the camera obscura at the point F plays the role of a shutter, all

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the cells of which are activated simultaneously in the reference system K. The hole of the camera obscura at the point G plays the role of a shutter, all the cells of which are activated simultaneously in the reference system K′.

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