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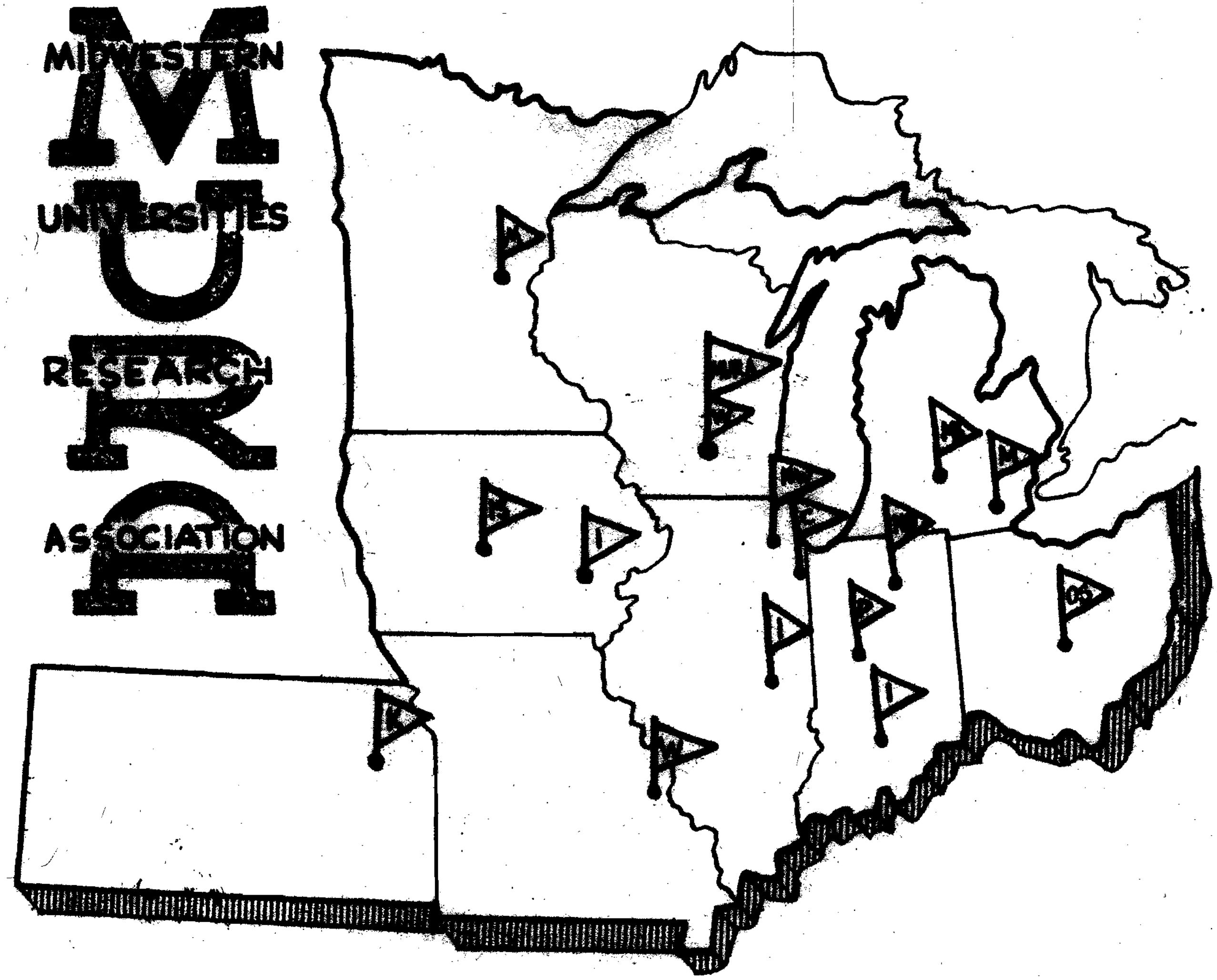

BEAM EXTRACTION FROM AN FFAG SYNCHROTRON

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ABSTRACT

A combination pulsed and DC magnetic field system is described which will extract the beam from an FFAG spiral sector synchrotron with essentially no loss in betatron phase density, permitting the use of this type of synchrotron as a high intensity injector into a pair of colliding beam storage rings.

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A. Introduction

The use of a conventional AG synchrotron as an injector into a pair of colliding beam storage rings has been proposed as an alternative to the two-way Ohkawa FFAG accelerator.[1] A spiral sector FFAG synchrotron could also be used as such an injector, with the advantage of a considerably shorter buildup time of the stacked circulating beam in the storage rings, because of the higher repetition rate (about a factor of 50). Both storage rings systems would have larger (a factor of 7 or so) circulating current densities than the two-way machine if space charge is taken as a limiting factor. There is, however, the extra beam transfer step in the storage ring process, so if the system is to be competitive with the two-way machine, the phase density loss during transfer must be kept less than the factor 7. O'Neill[1] has suggested the use of a pulsed field one turn ejection (and subsequent injection) system for the transfer process for his AG synchrotron. Such a system will be used here for the FFAG synchrotron extraction. Extraction from a non-linear FFAG machine, however, involves the additional possibility of phase area distortion--and phase density loss--due to the non-linearity. Section B discusses a pulsed extraction system for a spiral sector FFAG synchrotron in which the non-linearities introduce negligible loss in the extraction process.

In addition to the one-turn beam extraction with high phase density, it is very desirable that there be means of adiabatic extraction from FFAG synchrotrons in order to give a high duty cycle for experiments with the external beam. The problems and possibilities of this adiabatic extraction will be discussed briefly in Section C.

1. O'Neill, G. K., Concentric Storage Rings, GKO'N-11 (Jan. 1958) Princeton-Penn. Accelerator Project.

B. A Pulsed One-Turn Beam Extraction System for a Spiral Sector FFAG Synchrotron

If large radial oscillations are employed in the extraction process from a spiral sector machine, the non-linearities will cause the radial and vertical phase areas to distort, the amount, of course, depending on a large number of factors. If the distortion is excessive, the average beam density over a useful phase area will decrease considerably from the original peak density. If the radial and vertical oscillations are both kept small enough, however, both phase areas of the beam can be kept reasonably shaped and the average density near the peak density. Any process involving large displacements with a high rate of distortion must be confined to a short region. The following system is in line with the above requirements.

A vertical magnetic field which is uniform over a small azimuthal region is pulsed on in a period short compared to one revolution. This perturbation produces a small amplitude of oscillation, but large enough at its maximum to have displaced the entire beam into a DC high field region. This high field bump bends the beam enough to send it down the spiral and out of the machine in less than one-half sector and with negligible phase area distortion.

The extraction system was designed for a particular spiral sector machine. The machine parameters are indicated in Table I.

TABLE I

Spiral Sector Machine Parameters

Median Plane Field : $H = (1 + x)^k \left[1 + \cos\left(N\theta - \frac{1}{w}\ \ln\ (1 + x)\right)\right]$

$N = 30$	Frequencies
$k = 53$	$\sigma_x = .56\pi$
$\frac{1}{w} = 301$	$\sigma_y = .48\pi$
$\tan \gamma = N_w = .1$	Total Stable Phase Areas (coupled motion)
	$\Delta x \Delta p_x \cong 2 \times 10^{-5}$
	$\Delta y \Delta p_y \cong 4 \times 10^{-6}$

where:
x in units of R
$p_x = \frac{d(Rx)}{ds} = \frac{dx}{d\theta}$

Phase plots for the coupled motion obtained from runs on the MURA IBM 704 computer are shown in Fig. 1. The plots are made at the azimuthal positions $N\theta = \pi, 3\pi, \ldots$, ie., midway between the magnets. The curves rotate, of course, in going through the sectors. The phase areas listed in Table I correspond to the curves (A). It is evident that there is considerable scattering in the plots indicating a considerable loss in phase density. The motion becomes unstable if either the x- or y-amplitude is much larger than the curves (A). The curves (B) resemble invariant curves more closely. These areas are $\Delta x \, \Delta p_x = 6 \times 10^{-6}$ and $\Delta y \, \Delta p_y = 10^{-6}$. Within the curves (B) the oscillation frequencies do not change appreciably with amplitude.

If the beam is injected at 50 Mev into the phase areas of curve (B) by 15 Bev the betatron phase areas will each have damped a factor $\frac{p}{p_o} = \frac{\beta\gamma}{\beta_o\gamma_o}$ $= 50$ giving $\Delta x \, \Delta p_x = 1.2 \times 10^{-7}$ and $\Delta y \, \Delta p_y = 2 \times 10^{-8}$ (assuming no loss in betatron phase density during acceleration).

Fig. 2 shows the effect on the damped x-ellipse of an impulsive x-momentum change. In the Fig. 2 Δp_x is 3.3×10^{-3}. One sector later (again midway between the magnets) the beam center has moved out $\Delta x = 4 \times 10^{-4}$. (The new x-phase area shape can be found by noting that radial lines transform into radial lines and the area remains constant.) Since the beam is inside the curves (B), the y-phase area will be essentially unchanged by the x-perturbation. The x-space between the edges of the perturbed and unperturbed beam is about 3×10^{-4}. For a machine radius of 70 meters this corresponds to a radial separation of 2 cm, enough room for a current sheet to be there to generate the DC field for extraction.

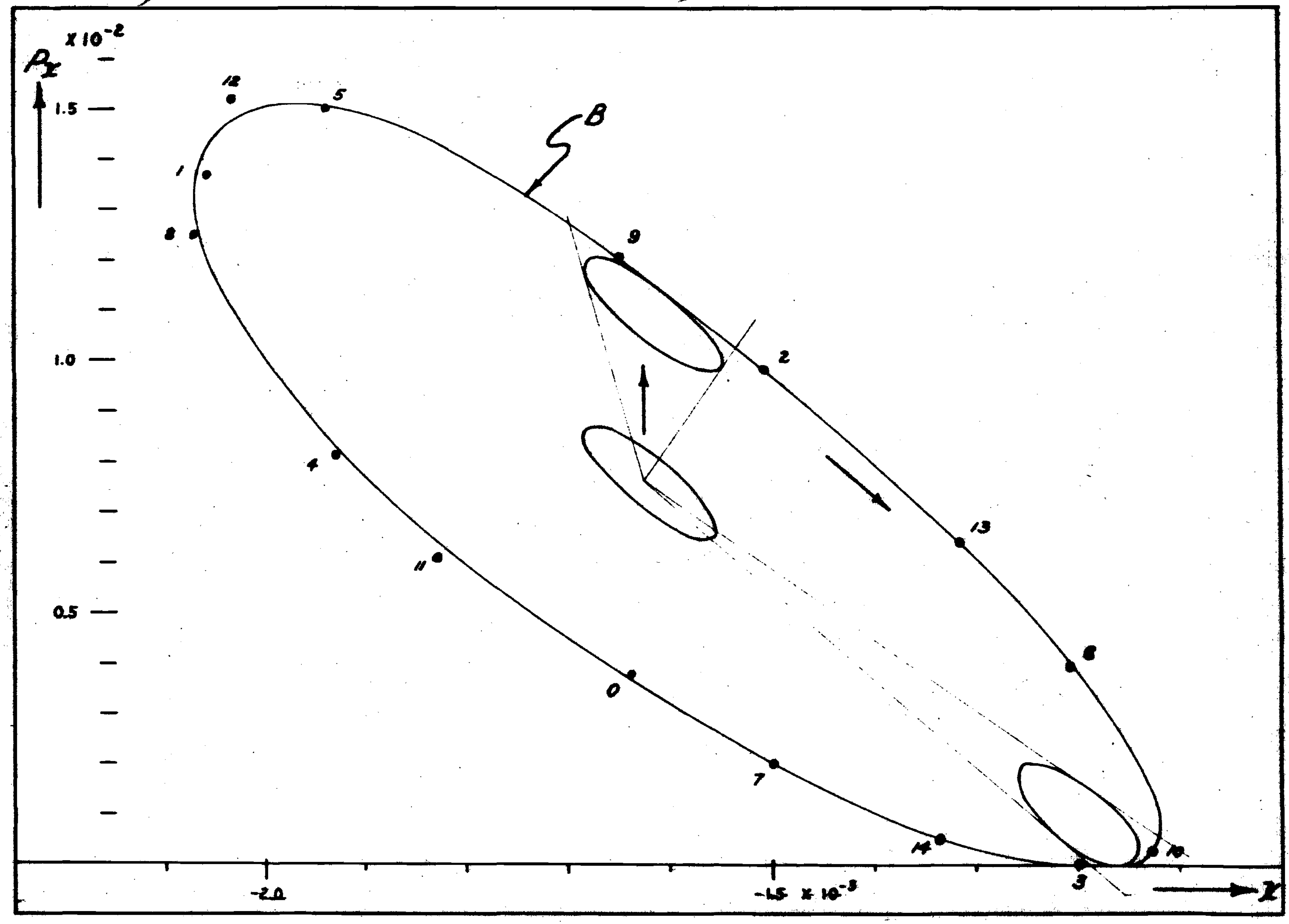


Fig. 2. Radial displacement produced by a field bump between two of the magnets. The large ellipse (B) is from Fig. 1. The small ellipse is curve B damped in going from 50 Mev to 15 Bev.

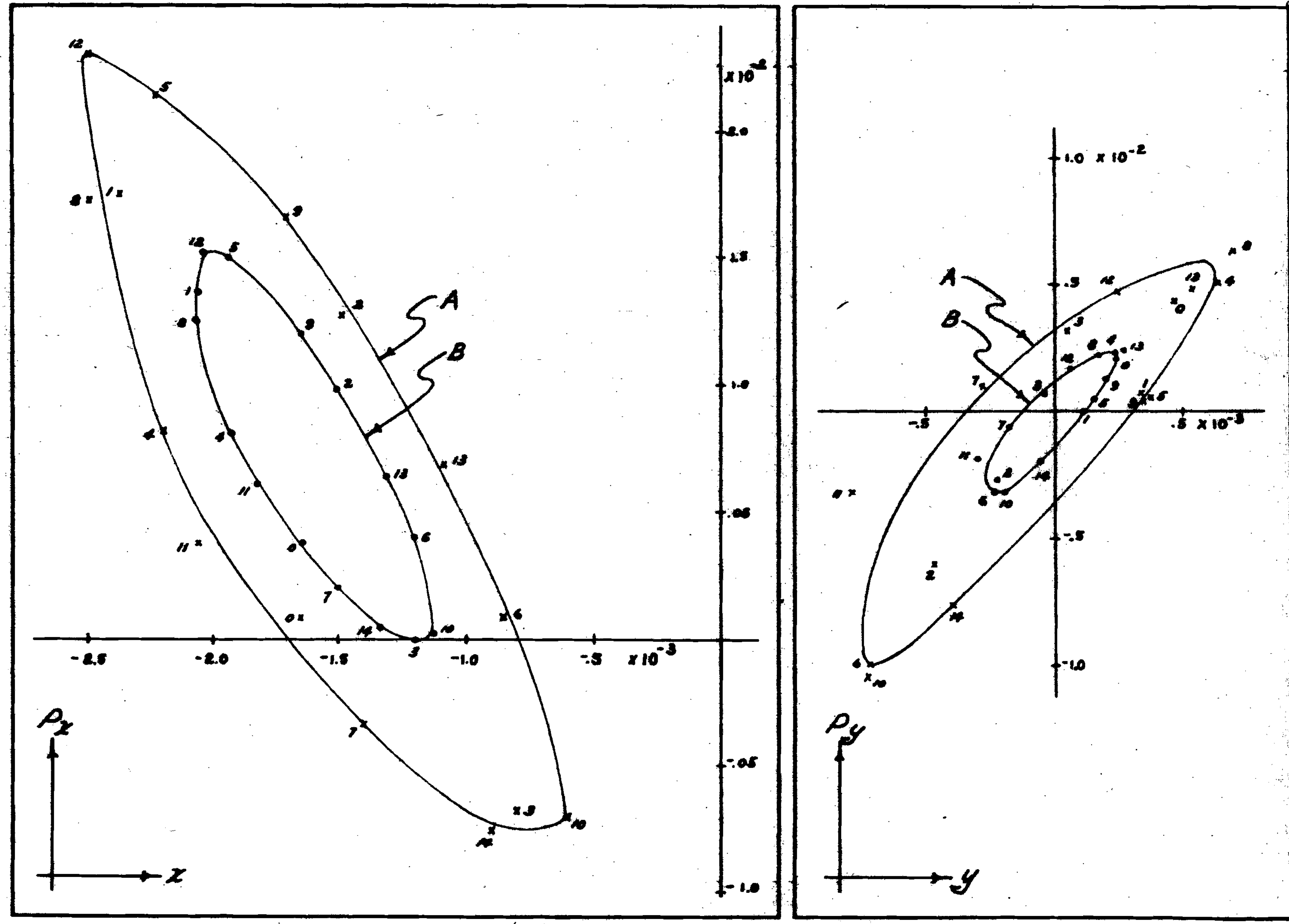


Fig. 1. ~~Phase plots~~ for coupled motion in the accelerator of Table I, at $N\theta = \pi, 3\pi$ ... Curves lettered the same are the same computer run.

An arbitrary figure of $x = 6 \times 10^{-3}$ was taken as the machine edge--slightly more than one quarter of a radial wave of the magnetic field. The iron will probably have to extend this far to make the equilibrium orbit fields correct. A series of computer runs then indicated that an impulsive momentum change to a $p_x \geq 3 \times 10^{-2}$ at $N\theta = \pi$ (the spiral angle is 10×10^{-2}) would bring the beam out to $x = 6 \times 10^{-3}$. With $p_x = 2 \times 10^{-2}$ the beam oscillated back before reaching 6×10^{-3} and the y-amplitude started to grow. With $p_x = 5 \times 10^{-2}$ the beam reached $x = 5.8 \times 10^{-3}$ in $\Delta N\theta = 13/16\,\pi$ radians. Here p_x final $= 11 \times 10^{-2}$, a decent exit angle. The beam has moved out over a quarter wave of magnetic field in less than half sector in azimuth so the beam has not quite entered between the poles of the next magnet before it exits.

Designs of a pulsed field system to give the above impulsive momentum change of $\Delta p_x = 3.3 \times 10^{-3}$ and a DC extraction field to change p_x to 5×10^{-2} are given in the Appendix.

To examine the behavior of the phase areas during the extraction process after Fig. 2, the DC extraction field was treated as an impulsive perturbation, occurring at $N\theta = \pi$, moving the x-phase area from its final position in Fig. 2 to a position around the point $p_x = 5 \times 10^{-2}$, with the same x-values. A computer run then mapped a sample x- and y-phase area, starting about the point $p_x = 5 \times 10^{-2}$, at $N\theta = \pi$, to the position $N\theta = (1 + 13/16)\pi$. The x- and y-phase areas at the starting point ($N\theta = \pi$) are shown in Fig. 3a. The phase areas just before the beam leaves the machine, at $N\theta = (1 + 13/16)\pi$, are shown in Fig. 3b. The phase areas after exit from the machine depend on the field fall off in the radial direction. If the edge is taken as "hard" (instantaneous

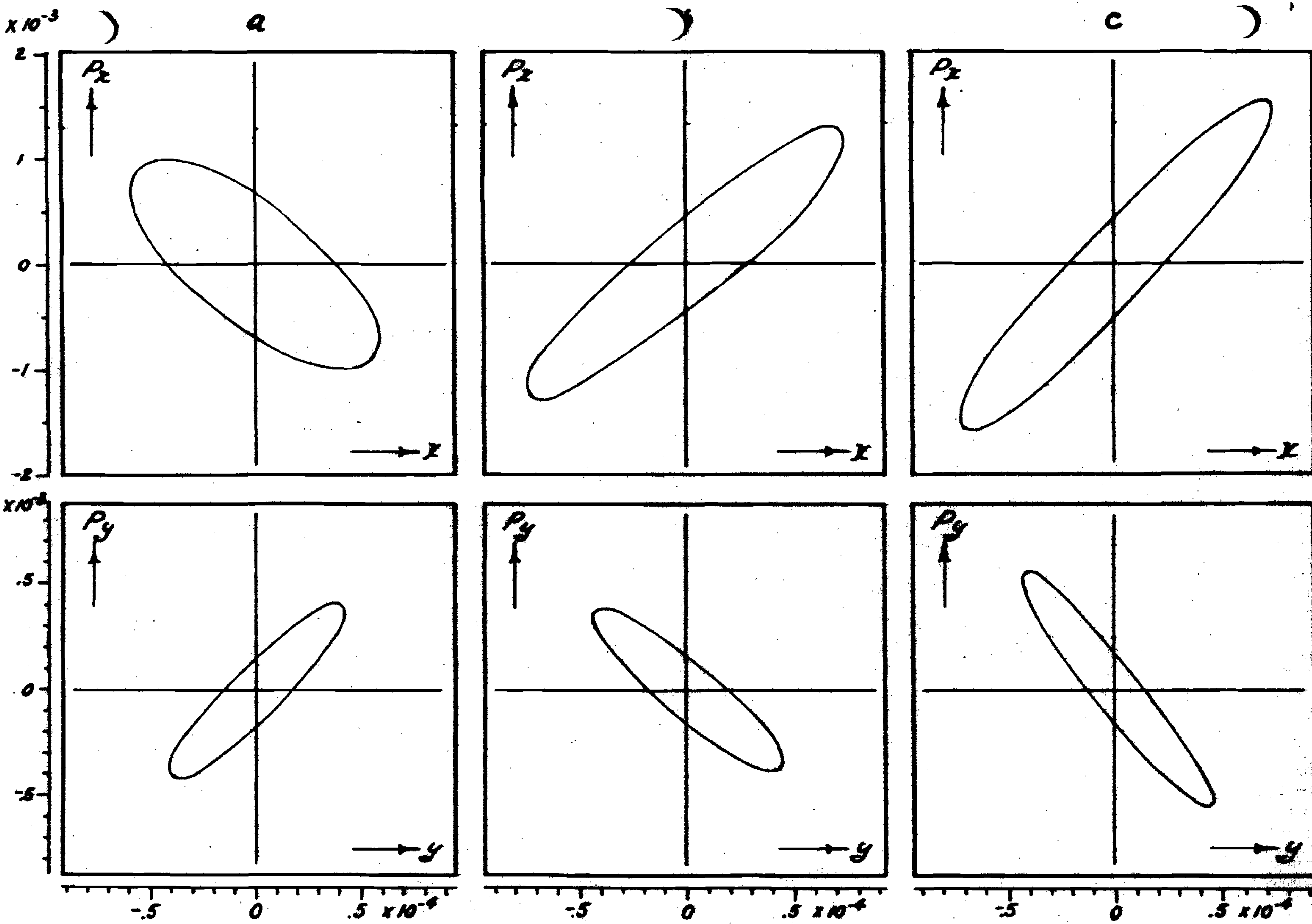


Fig. 3. Transformation of phase areas on extraction.

Fig. 3a shows the damped phase areas at $N\theta = \pi$, at the final position of Fig. 2.

Fig. 3b shows the same areas just before leaving the machine at $N\theta = (1 + {}^{13}\!/_{8})\pi$. The central point is $x = 5.8 \times 10^{-3}$, $p_x = 1.1 \times 10^{-1}$.

Fig. 3c indicates the change of the areas after going through the discontinuous field edge at $x = 6 \times 10^{-3}$.

fall off), the resultant phase areas are as shown in Fig. 3c. This "hard edge" is a fairly good approximation, with the exit angle of $p_x = .11$ radians and the small vertical aperture ($y = 10^{-3}$) of these machines.

These phase areas are reasonable and there has been little loss in phase density, so it appears that such an extraction scheme with subsequent similar reinjection into an AG storage ring appears feasible, at least as regards phase density. Reinjection into the storage ring adds another problem, however, if the beam is to be stacked there in the storage ring. The pulsed inflector field must be designed not to disturb the previously stacked beam. (Pulsed ejection was a one-turn process with no other beam close by to worry about.)

It is possible also to stack in the spiral machine up to the point where the differences in radial position and momentum can be handled by the extraction system. The system presented above, with the same fields and oscillation amplitudes, could handle an extra radial increment due to the energy spread of $\Delta x = 10^{-4}$ (.7 cm) getting it past a 1 cm current sheet into the extraction channel. The difference in momentum corresponding to this radial spread is small $\frac{\Delta P}{P} = k\,\Delta x = 53 \times 10^{-4} = 1/2\,\%$. Due to the momentum range, however, if the above uniform bending fields were used, there would be an extra spread in p_x of $\frac{\Delta P}{P}$ times the bending angle $\cong 5 \times 10^{-3} \times .1 = 5 \times 10^{-4}$ radians, causing around a 40 % loss in radial phase density. This loss could be eliminated by making the deflection field match the momentum of the particles by the addition of a gradient.

This radial increment of $\Delta x = 10^{-4}$ corresponds to a fair number of injected pulses. The energy spread corresponding to this Δx at 15 Bev is 75 Mev. Assuming no rf phase density loss, a 50 kilovolt spread injected at 50 Mev would become .15 Mev at 15 Bev, or there would be $75/.15 = 500$ pulses in the radial increment. With a factor of four rf density loss (as in the MURA proposal), there would be 125 pulses. These pulses can be made to spatially superimpose in the storage ring, giving a net relative interaction beam density, (including the factor of 7 increase in spatial density due to the smaller radius and larger ν_y) of 7 x 125 = 875, practically equal to that of 1000 injected pulses of the MURA proposal. If larger oscillations are allowed in the extraction process, which is certainly feasible up to a point, the interaction density can also be made larger.

C. Comments on Adiabatic Extraction from FFAG Synchrotrons

The type of system discussed in the previous section can be employed in any FFAG synchrotron, either spiral or radial sector, to extract the beam quickly. Bringing the beam out slowly, however, to give a long duty cycle for experiments with an external beam, appears to be a more difficult problem.

The Piccioni scheme, being considered for the AGS, is not easy to apply here. In FFAG accelerators with positive k a beam traversal of the energy loss foil causes the equilibrium orbit to be displaced to a smaller radius, where, however, no extraction channel can be placed because beam must be accelerated through that region. No current sheet peeler can exist there. It is conceivable, however, that one might add such a current sheet or small localized magnetic bump, just before extraction, giving the beam deflected there by the foil enough of a kick to send it to an extractor at a larger radius.

Another possibility suggested by Symon is to use the energy loss of the foil as a means of driving the beam beyond the stability limit, and some fraction at least, into an extraction channel. Adiabatically increasing a perturbing field (or adiabatically decreasing the stability limit) might also be used for the same purpose.

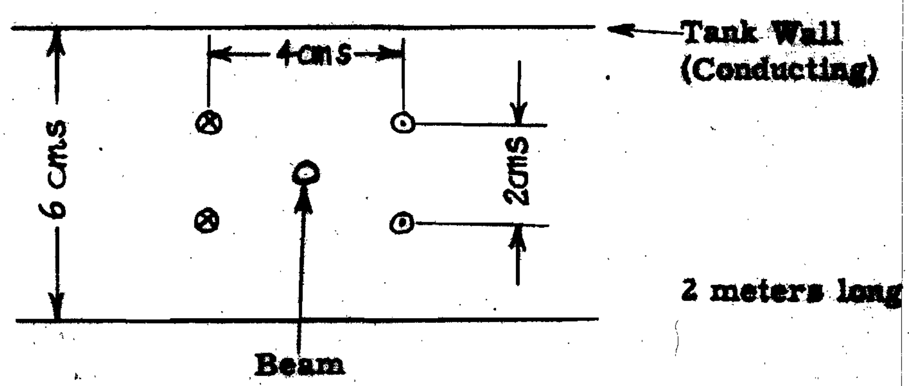
DC regenerative extraction, of the type employed in cyclotrons, is difficult to use here because of the small pitch from the energy gain per turn (of order 1 mill). To increase a pitch this small to a useful amount, by say, making the x-tune, and the amplitude of a driven x-oscillation, depend on energy, would require sitting practically on top of a resonance for a large number of revolutions, and does not appear feasible.

APPENDIX

1. A Pulsed Magnetic Deflector

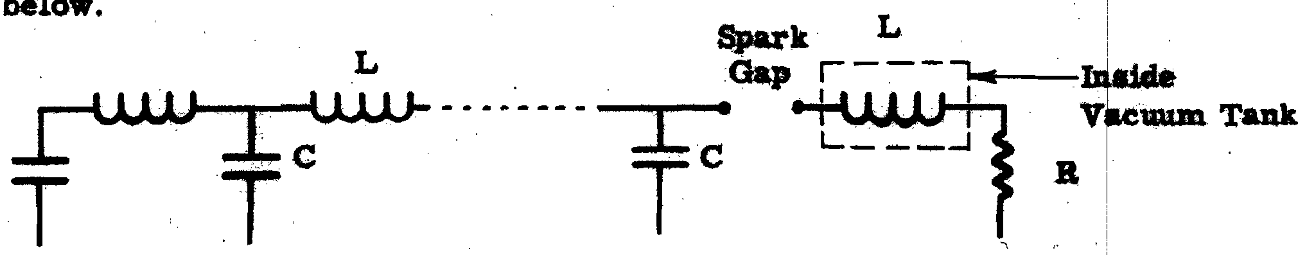
A 15 Bev beam can be deflected an angle of 3.3×10^{-3} radians with a bending impulse of $Hs = 3300\, pc\Delta\varphi = 3300 \times 16 \times 10^3 \times 3.3 \times 10^{-3} = 1.75 \times 10^5$ gauss cm. The system described here to give a crude estimate of the parameters involved has a 1 kg field over a 2 meter azimuthal region.

The pulsed field can be produced by a current system inside the vacuum tank, as indicated below.



The total current required is $I = \frac{Hl}{.4\pi} \cong \frac{10^3 \times 12}{.4\pi} = 10{,}000$ amps. The stored energy in the field is $\frac{H^2}{8\pi} \times \text{Vol} \cong \frac{(10^3)^2 \times 200 \times 4 \times 6}{8\pi} = 20$ Joules, and the self inductance of the system, taken as a one-turn loop, is $L = \frac{2 \times \text{Energy}}{I^2} = .4\,\mu$h.

The pulse must be on for one revolution, 1.5 μsec for R = 70 meters, and be turned on in a small fraction of this time. This can be achieved reasonably well with a delay line system, with the inductance L as an element of the line, as indicated below.

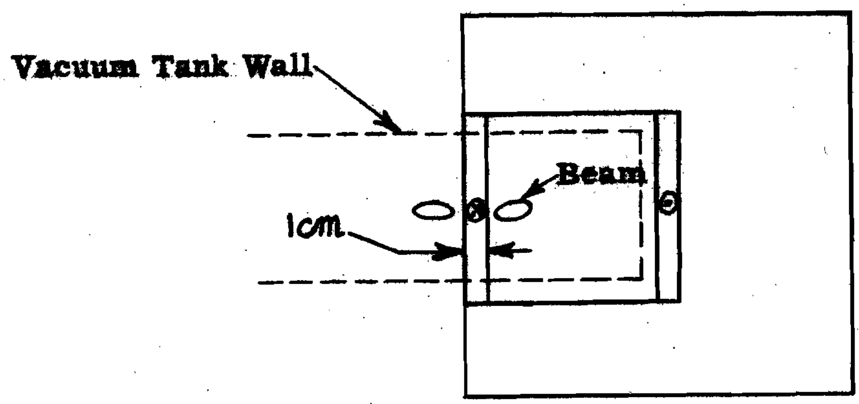


With a 15 kv pulse, with $I = 10,000$ amps, $R = 1.5$ ohms, so C, given by $R = \sqrt{\frac{L}{C}}$, is $\sim .2\,\mu f$. The rise time, approximately the time delay per section $= \sqrt{LC} \cong .3\,\mu$sec. There are five sections for a $1.5\,\mu$sec pulse. For a faster rise time, C can be decreased and the system run at a higher voltage with more sections or the inductive load can be split into parts driven by separate delay lines.

2. DC Extraction Field

To bend the 15 Bev beam 5×10^{-2} radians requires 2.64×10^{6} gauss cm or 10 kg for 2.6 meters. (The azimuthal distance between the spiral magnets is 7 meters.) To change a field from zero to 10 kg in one cm, the copper width chosen earlier, requires a current sheet of density 8,000 amps/cm^2, an extremely high density, but one that can be handled over a limited region with a high flow rate water cooling system.

The vacuum tank can be between the poles of the extraction magnet, with the current sheet through it, as shown below.



Or, the tank can be made large enough at this position to enclose the magnet, which can be made quite small.