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Lepton-flavour-violating tau decays from triality

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Motivated by flavour symmetry models, we construct theories based on a low-energy limit featuring lepton flavour triality that have the flavour-violating decays $\tau^\pm \to \mu^\pm\mu^\pm e^\mp$ and $\tau^\pm \to e^\pm e^\pm \mu^\mp$ as the main phenomenological signatures of physics beyond the standard model. These decay modes are expected to be probed in the near future with increased sensitivity by the Belle II experiment at the SuperKEKB collider. The simple standard model extensions featured have doubly-charged scalars as the mediators of the above decay processes. The phenomenology of these extensions is studied here in detail.

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I. INTRODUCTION

The search for charged-lepton flavour violation (cLFV) is an important component of the general program of seeking signals of physics beyond the standard model (BSM). The discovery of neutrino flavour oscillations established that the family lepton numbers L_e, L_μ and L_τ are not individually conserved. With the charged and neutral leptons coexisting in the same weak-isospin doublet, we thus expect that these quantities will also not be conserved in the charged-lepton sector. However, for such processes to be experimentally observable, lepton flavour violating (LFV) physics beyond that responsible for family-lepton number violating neutrino mass generation must exist.

There are stringent existing constraints on electron-muon cLFV processes such as $\mu \to e\gamma$ and $\mu \to eee$. Existing data on cLFV involving τ leptons are less constraining, but the sensitivity to such processes is expected to increase significantly as the Belle II experiment accumulates more data. The purpose of this paper is to construct and analyse some simple standard model (SM) extensions that have the decays

$$\tau^\pm \to \mu^\pm \mu^\pm e^\mp, \qquad \tau^\pm \to \mu^\mp e^\pm e^\pm \tag{1}$$

as the main phenomenological signatures of BSM physics. We summarise the current bounds and projected Belle II sensitivities in Table I.

To achieve our aim, a symmetry is needed that permits the above decays but also prevents other processes that are subject to strong constraints. A simple choice for such a symmetry is *lepton flavour triality*. This has, for example, been discussed in the context of A_4 flavour models [3–11] which are broken to a Z_3 subgroup in the charged lepton sector and Z_2 in the neutrino sector.

Motivated by an eventual embedding into a more complete flavour symmetry model, we introduce a Z_3 symmetry in the lepton sector which distinguishes the three families via their Z_3 *triality charges* (denoted T). The first (second) [third] generation of leptons has charge $T = 1$ (2) [3]. These charges correspond to the transformations

$$L \to \omega^T L, \quad e_R \to \omega^T e_R \tag{2}$$

where $\omega = e^{2\pi i/3}$ is the third root of unity, and the L and e_R are left-handed (LH) lepton doublets and right-handed (RH) charged lepton singlets, respectively. Thus all first-family leptons transform

Observable	Present constraint	Projected sensitivity
$\mathrm{BR}(\tau^- \to \mu^- \mu^- e^+)$	$< 1.7 \times 10^{-8}$ [1]	2.6×10^{-10} [2]
$\mathrm{BR}(\tau^- \to \mu^+ e^- e^-)$	$< 1.5 \times 10^{-8}$ [1]	2.3×10^{-10} [2]

TABLE I. We list the cLFV tau decays of interest, where the projected reach reflects the expected sensitivity with 50 ab^{-1} data from the Belle II collaboration [2] assuming a phase space distribution for the 3-body decay (see discussion in Sec. IV).

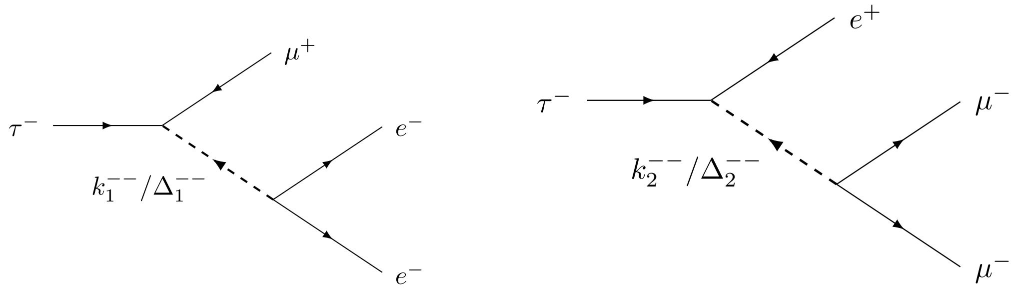

FIG. 1. Tree-level contributions to the process $\tau^- \to \mu^+ e^- e^-$ ($\tau^- \to \mu^- \mu^- e^+$) for the $T = 1$ ($T = 2$) electroweak singlet scalar models, utilising k_i, and triplet models, utilising Δ_i. Note that the singlet models couple to RH leptons, whereas the triplets couple to LH leptons.

via ω, all second-family leptons transform via ω^2, and all third-family leptons are triality singlets, transforming via $\omega^3 = 1$. A discussion of neutrino masses is postponed to Sec. V. The Higgs doublet, H, and all quark fields are also triality singlets.

Given these triality assignments, the leptonic Yukawa terms in the Lagrangian are

$$\mathcal{L} \supset y_{ei} \bar{L}_i e_{Ri} H + \text{h.c.}, \tag{3}$$

where $i, j = 1, 2, 3$ with repeated indices summed, and y_{ei} denote the charged lepton Yukawa couplings. The Z_3 symmetry forces the charged lepton mass matrix to be diagonal, and we thus identify the first (second) [third] generation of charged leptons with the electron (muon) [tau] lepton.

This simple model outlined above forms the basis for extensions that feature observable decays of the form of Eq. (1). The simplest models are based on scalar bileptons [12], in particular doubly-charged scalars[1] as illustrated in Fig. 1. In Sec. II we discuss the simplest realisation based on an electroweak singlet and in Sec. III we present a model based on an electroweak triplet. The phase space for the cLFV leptonic τ decays is discussed in Sec. IV and different possibilities to introduce neutrino masses are introduced in Sec. V and in Sec. VI we summarise and conclude. Technical details are reported in the appendices.

II. ELECTROWEAK SINGLET MODELS

The simplest model that has cLFV τ decays as the dominant BSM signature features a doubly-charged scalar weak-isospin singlet k_T with lepton triality charge T, hypercharge $Y(k_T) = 2$ and a mass m_{k_T}. Depending on the lepton triality assignment for the doubly-charged scalar k_T, there are different cLFV τ decay modes. Note that we only consider $T = 1$ and $T = 2$, because the $T = 0$ case does not result in cLFV τ decays.

[1] Doubly-charged scalars as mediators of cLFV tau decays have been studied, for example, in Refs. [13–18].

A. $T = 1$ singlet model

For triality $T = 1$ the Yukawa couplings of the doubly-charged scalar k_1 are

$$\mathcal{L}_{k_1} = \frac{1}{2}\left(2f_1\overline{(\tau_R)^c}\mu_R + f_2\overline{(e_R)^c}e_R\right)k_1 + \text{h.c.}. \tag{4}$$

which induce the decays $\tau^\pm \to \mu^\mp e^\pm e^\pm$ via the LH diagram in Fig. 1. Note that we may, without loss of generality, set the coupling constants $f_{1,2}$ to be real-valued and positive, which we do from now on. One of the phases of $f_{1,2}$ may be absorbed into k_1, and the other absorbed into a RH charged lepton. Ultimately, the leptonic CP violation can be taken to reside entirely in the PMNS matrix and the decay modes of the heavy neutral leptons.

For energies below the mass of k_1, we can employ standard model effective field theory (SMEFT) and match to low-energy effective field theory (LEFT), the concepts and notation of which are reviewed in Appendices A and B. The LEFT Wilson coefficient for the relevant cLFV decay mode

$$C^{\text{VRR}}_{ee,1312} = \frac{f_1 f_2}{4m^2_{k_1}} \tag{5}$$

contributes to the branching ratio[2]

$$\text{BR}(\tau^\pm \to \mu^\mp e^\pm e^\pm) = \frac{f_1^2 f_2^2}{64 G_F^2 m^4_{k_1}} \text{BR}(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau). \tag{6}$$

Applying the present constraint quoted in Table I and using Eq. (6), the following upper-bound results:

$$\sqrt{|f_1 f_2|} \lesssim 0.17 \, \frac{m_{k_1}}{\text{TeV}}. \tag{7}$$

This constraint is shown by the diagonal solid coloured lines in top panels of Fig. 2 for the benchmark masses $m_{k_1} = 1$ and 5 TeV. The coupling constant parameter space to the top-right of these lines is ruled out. The coloured dot-dashed lines show the expected reach of the Belle II experiment. The grey bands in Fig. 2 display the regions for which the coupling constants are non-perturbative, and hence irrelevant for our analysis.

The benchmark masses were chosen to be within the range that would produce an observable cLFV branching ratio at Belle II. By saturating the perturbativity conditions (i.e. $f_1^2 = f_2^2 = 4\pi$), the projected sensitivity quoted in Table I allows to probe k_1 masses:

$$m_{k_1} \lesssim 61 \;\; \text{TeV}. \tag{8}$$

Of course, for larger scalar masses the constraints considered become weaker, but so too does the observable effect in cLFV tau decays.

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[2] Neglecting the electron mass, the effect of the muon mass is the same for $\text{BR}(\tau^\pm \to \mu^\mp e^\pm e^\pm)$ and $\text{BR}(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau)$.

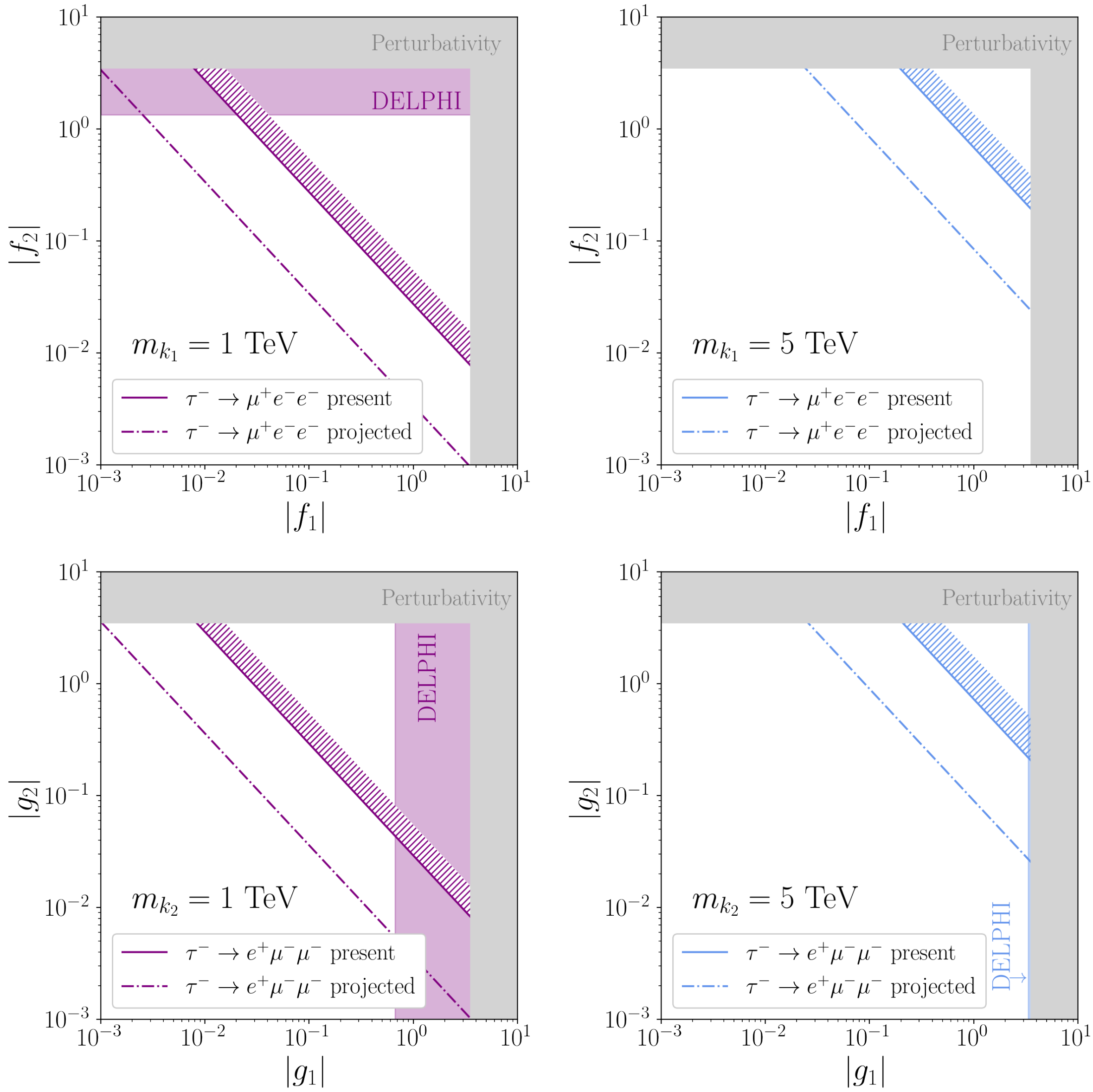

FIG. 2. An overview of the electroweak singlet scalar parameter space. The present constraint on cLFV tau decays rules out the parameter space bounded by solid lines, in the direction of the hashing. The reach of Belle II is indicated by dot-dashed lines, i.e. observation of the decay $\tau^- \to \mu^+ e^- e^-$ ($\tau^- \to e^+ \mu^- \mu^-$) would be compatible with parameter space in the upper-right in the $|f_1|$-$|f_2|$ ($|g_1|$-$|g_2|$) plane above this line. The grey-shaded region is ruled-out by perturbativity of the BSM couplings. The shaded region labelled 'DELPHI' is ruled-out by the constraints in Eq. (18), and where this is absent the constraint falls outside the perturbative regime.

B. $T = 2$ singlet model

For lepton triality $T = 2$ the relevant Yukawa couplings in the Lagrangian of the doubly-charged scalar k_2 are obtained by exchanging the lepton flavours $1 \leftrightarrow 2$ in the Lagrangian of Eq. (4), and are thus given by

$$\mathcal{L}_{k_2} = \frac{1}{2} \left(2 g_1 \overline{(\tau_R)^c} e_R + g_2 \overline{(\mu_R)^c} \mu_R \right) k_2 + \text{h.c.} \tag{9}$$

where we have set $g_{1,2}$ to be real and positive, as per the discussion in Sec. II B. The RH diagram in Fig. 1 leads to the decay modes $\tau^\pm \to \mu^\pm\mu^\pm e^\mp$. These may be parameterised by the LEFT Wilson coefficient

$$C^{\rm VRR}_{ee,2321} = \frac{g_1 g_2}{4m_{k_2}^2} \tag{10}$$

leading to the branching ratio

$$\mathrm{BR}(\tau^\pm \to \mu^\pm\mu^\pm e^\mp) = \frac{g_1^2 g_2^2}{64 G_F^2 m_{k_2}^4} \tilde{I}\left(\frac{m_\mu^2}{m_\tau^2}\right) \mathrm{BR}(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau). \tag{11}$$

The effect of the muon mass appears in the factor $\tilde{I}(r) = I_2(r)/I(r)$,

$$I(r) = 1 - 8r + 8r^3 - r^4 - 12r^2 \ln(r) \tag{12}$$

$$I_2(r) = \sqrt{1-4r}\left(1 - 2r(7 + r + 6r^2)\right) + 24r^2(1-r^2)\ln\left(\frac{1+\sqrt{1-4r}}{1-\sqrt{1-4r}}\right). \tag{13}$$

$I(r)$ is the usual muon mass effect in $\mathrm{BR}(\tau^- \to \mu^-\bar{\nu}_\mu\nu_\tau)$ and $I_2(r)$ the corresponding factor for $\mathrm{BR}(\tau^\pm \to e^\mp\mu^\pm\mu^\pm)$, neglecting the electron mass.

Similarly to the k_1 case, we may derive a bound on k_2 parameters using the present constraint quoted in Table I,

$$\sqrt{|g_1 g_2|} \lesssim 0.17 \, \frac{m_{k_2}}{\mathrm{TeV}}. \tag{14}$$

This is plotted in the lower two panels of Fig. 2 for the benchmark values of m_{k_2}. As with the k_1 case, the expected reach of Belle II is indicated by the dot-dashed lines. Fig. 2 clearly demonstrates the strong constraints of cLFV leptonic τ decays on the electroweak singlet scalar and the improved sensitivity of the Belle II experiment.

The same benchmark masses are employed for the k_2 parameter study. Saturating the perturbativity conditions, the projected sensitivity in Table I is able to probe models with an observable branching ratio at Belle II with

$$m_{k_2} \lesssim 59 \text{ TeV}. \tag{15}$$

C. Direct searches

The only available decay channels for the doubly-charged scalars are to pairs of same-sign leptons. ATLAS [19] searched for pair production of doubly-charged scalar singlets which subsequently decay to the $e^\pm e^\pm$, $e^\pm\mu^\pm$, or $\mu^\pm\mu^\pm$ same-sign dilepton final states. Their results provide the most stringent direct search constraints. Assuming similarly sized Yukawa couplings, the doubly-charged scalar k_1 (k_2) would have 50% branching ratio to electrons (muons) and 50% to $\tau\mu$ (τe) which maximises the product of the Yukawa couplings. So assuming that all events with a τ lepton in the final state are missed by the detector, there are lower bounds on the masses of the doubly-charged scalars given by

$$m_{k_1} \geq 0.62 \text{ TeV}, \qquad m_{k_2} \geq 0.57 \text{ TeV}, \tag{16}$$

as per published data of Fig. 14 in Ref. [19], rounded to two significant figures. For a 100% branching ratio to electrons (muons) the lower bounds become

$$m_{k_1} \geq 0.66 \text{ TeV}, \qquad m_{k_2} \geq 0.72 \text{ TeV}. \tag{17}$$

D. Lepton scattering constraints

The doubly-charged scalar also mediates $2 \to 2$ scattering of leptons via t-channel exchange. In particular, k_1 contributes to $e^+e^- \to e^+e^-$ and k_2 contributes to $e^+e^- \to \tau^+\tau^-$, both of which have been constrained by the DELPHI experiment [20]. Translating the results in Ref. [21], we find the following lower limits on the k_1 and k_2 masses as a function of the Yukawa couplings with electrons,

$$\frac{m_{k_1}}{|f_2|} \geq 0.74\,\text{TeV}, \qquad \frac{m_{k_2}}{|g_1|} \geq 1.5\,\text{TeV}. \tag{18}$$

These constraints are indicated by coloured bands in Fig. 2. For larger $m_{k_{1,2}}$ masses, the DELPHI constraint lies outside of the perturbative regime for the coupling constants.

E. Other observables

Other observables include leptonic Higgs and Z boson decays. We find that the most striking signal would be the cLFV Z decay to four leptons.[3] The dominant contribution to these comes from decays to $\tau^+\tau^-$ followed by a cLFV τ decay to three leptons. Higgs decays are not as sensitive as they are suppressed by the τ Yukawa coupling, and thus we focus on Z boson decays.

1. Flavour-violating Z decays

There are two contributions to cLFV Z boson decays: the decay via two off-shell scalar electroweak singlets, or Z decays to $\tau^+\tau^-$ followed by a leptonic cLFV τ decay. The former is highly suppressed due to the constraint on the electroweak singlet scalar mass. For example, for the scalar k_1, this can be seen from the branching ratio

$$\text{BR}(Z \to k_1^{++}k_1^{--} \to e^+e^+\mu^-\tau^-) = \frac{f_1^2 f_2^2 \alpha_{\text{em}}}{2^{15}\cdot 3\cdot 5^2\cdot 7\pi^4}\frac{(T_3 - Q s_w^2)^2}{s_w^2 c_w^2}\frac{m_Z^9}{m_{k_1}^8 \Gamma_Z}, \tag{19}$$

[3] Decays of the Higgs and Z bosons to two charged leptons receive corrections at 1-loop level, which is left for future work.

where Q denotes the electric charge of the scalar, and T_3 its third component of weak-isospin. Therefore it is justified to neglect this contribution, and to approximate cLFV Z boson decays by

$$\begin{aligned} \mathrm{BR}(Z \to \tau^+\tau^- \to e^+e^+\mu^-\tau^-) &= \mathrm{BR}(Z \to \tau^+\tau^-) \cdot \mathrm{BR}(\tau^+ \to e^+e^+\mu^-), \\ &= 0.036\,\mathrm{BR}(\tau^+ \to e^+e^+\mu^-). \end{aligned} \tag{20}$$

A similar argument can be made for k_2. Searching for the cLFV Z boson decay $Z \to e^+e^+\mu^-\tau^-$ thus provides an interesting probe, directly related to the cLFV in τ decays at the focus of this work. Conversely, constraining the cLFV τ decays also provides indirect constraints for cLFV Z boson decays. As the upper limits for these τ decays are generally stronger than those for the corresponding Z boson decays, we do not expect competitive constraints from the latter.

2. Anomalous magnetic moments

For completeness, we note that the doubly-charged scalar also contributes to lepton anomalous magnetic moments [21, 22] through

$$0 \le \Delta a_\ell = \frac{m_\ell^2}{24\pi^2 m_{k_T}^2} \begin{cases} f_1^2 & \text{for } T = 1, \\ g_2^2 & \text{for } T = 2. \end{cases} \tag{21}$$

This effect is too small to be of practical phenomenological interest.

III. ELECTROWEAK TRIPLET MODELS

The construction above can be mirrored to produce alternative models where the doubly-charged scalar is embedded in a $Y = 1$ weak-isospin triplet, Δ_T, where (as before) T denotes lepton triality. As for the singlet case, only $T = 1, 2$ are relevant for models of cLFV τ decays. The triplet models have richer phenomenology than the singlet models, due mainly to including effects of the weak-isospin partners of the doubly-charged scalars.

As well as the doubly-charged scalar Δ_T^{++}, such a triplet also contains a singly-charged scalar Δ_T^+ and a neutral complex scalar Δ_T^0. It is convenient to represent this complex triplet using a traceless 2×2 matrix of the form

$$\boldsymbol{\Delta}_T = \begin{pmatrix} \frac{\Delta_T^+}{\sqrt{2}} & \Delta_T^{++} \\ \Delta_T^0 & -\frac{\Delta_T^+}{\sqrt{2}} \end{pmatrix} = \Delta_T^i \frac{\sigma_i}{\sqrt{2}}, \tag{22}$$

where σ_i denotes the Pauli matrices. A weak-isospin transformation is represented through

$$\boldsymbol{\Delta}_T \to U \boldsymbol{\Delta}_T U^\dagger \tag{23}$$

where U is in the fundamental representation of SU(2). The normalisations have been chosen so that the quadratic invariant

$$\mathrm{tr}(\boldsymbol{\Delta}_T^\dagger \boldsymbol{\Delta}_T) = \Delta_T^{--}\Delta_T^{++} + \Delta_T^-\Delta_T^+ + \Delta_T^{0*}\Delta_T^0 \tag{24}$$

produces standard normalisations for the component fields.

A. $T = 1$ triplet model

For lepton triality $T = 1$ the relevant Yukawa coupling Lagrangian is

$$\mathcal{L}_{\Delta_1} = \frac{1}{2}\left(2f_1\,\overline{L_3^c}\;i\sigma_2\boldsymbol{\Delta}_1\,L_2 + f_2\,\overline{L_1^c}\;i\sigma_2\boldsymbol{\Delta}_1\,L_1\right) + \text{h.c.} \tag{25}$$

where

$$\overline{L_3^c}\;i\sigma_2\boldsymbol{\Delta}_1\,L_2 = -\overline{(\tau_L)^c}\,\mu_L\,\Delta_1^{++} - \frac{1}{\sqrt{2}}\left[\overline{(\tau_L)^c}\,\nu_{\mu L} + \overline{(\nu_{\tau L})^c}\,\mu_L\right]\Delta_1^{+} + \overline{(\nu_{\tau L})^c}\,\nu_{\mu L}\,\Delta_1^0, \tag{26}$$

$$\overline{L_1^c}\;i\sigma_2\boldsymbol{\Delta}_1\,L_1 = -\overline{(e_L)^c}\,e_L\,\Delta_1^{++} - \sqrt{2}\,\overline{(e_L)^c}\,\nu_{eL}\,\Delta_1^{+} + \overline{(\nu_{eL})^c}\,\nu_{eL}\,\Delta_1^0. \tag{27}$$

Similarly to the singlet models, the phases of these Yukawa coupling constants can be absorbed by field redefinitions, so we set them to be real-valued and positive from now on without loss of generality. Note that this is the same triplet that appears in the type-II seesaw mechanism [23–26], whose contribution to neutrino masses is briefly discussed in Sec. V.

For energies below the mass of Δ_1, the Wilson coefficient[4] relevant for the $\tau^\pm \to \mu^\mp e^\pm e^\pm$ decays is

$$C^{\text{VLL}}_{ee,1213} = \frac{f_1 f_2}{4m_{\Delta_1}^2} \tag{28}$$

with branching ratios given by

$$\text{BR}(\tau^\pm \to \mu^\mp e^\pm e^\pm) = \frac{f_1^2 f_2^2}{64 G_F^2 m_{\Delta_1}^4}\text{BR}(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau). \tag{29}$$

The current constraint from the non-observation of these decays is identical to the k_1 singlet cases, i.e. Eq. (7) holds with m_{k_1} replaced by m_{Δ_1}

$$\sqrt{|f_1 f_2|} \lesssim 0.17\,\frac{m_{\Delta_1}}{\text{TeV}}. \tag{30}$$

Similarly, the projected Belle II mass reach is roughly 61 TeV. These constraints are depicted by the diagonal solid coloured lines in the top row of Fig. 3 for three benchmark masses, with the region to the top-right being ruled out. The projected reach of Belle II is indicated by the dot-dashed lines. The same benchmark masses are studied here as were studied for the electroweak singlet scalar models.

B. $T = 2$ triplet model

Taking the electroweak triplet scalar with lepton triality $T = 2$, the Yukawa coupling Lagrangian for Δ_2 is

$$\mathcal{L}_{\Delta_2} = \frac{1}{2}\left(2g_1\overline{L_3^c}\,i\sigma_2\boldsymbol{\Delta}_2\,L_1 + g_2\overline{L_2^c}\,i\sigma_2\boldsymbol{\Delta}_2\,L_2\right) + \text{h.c.} \tag{31}$$

[4] The matching of the electroweak triplet scalar model has been derived in Refs. [27, 28] up to 1-loop order.

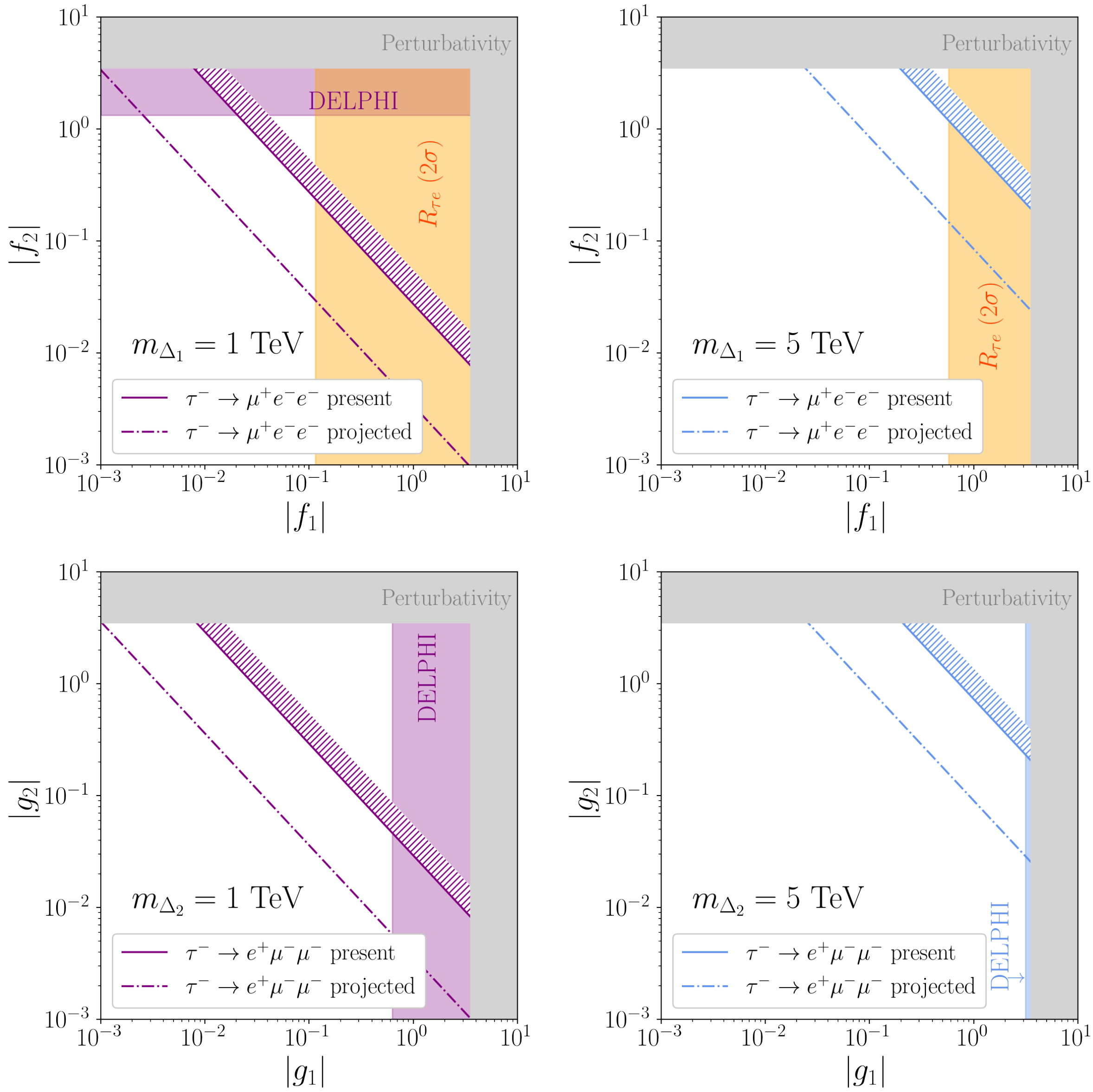

FIG. 3. An overview of the electroweak triplet scalar parameter space. The present constraint on cLFV tau decays rules out the parameter space bounded by solid lines, in the direction of the hashing. The reach of Belle II is indicated by dot-dashed lines, i.e. observation of the decay $\tau^- \to \mu^+ e^- e^-$ ($\tau^- \to e^+ \mu^- \mu^-$) would be compatible with parameter space in the upper-right in the $|f_1|$-$|f_2|$ ($|g_1|$-$|g_2|$) plane above this line. The grey-shaded region is ruled-out by perturbativity of the BSM couplings. The shaded region labelled 'DELPHI' is ruled-out by the constraints in Eq. (39), and where this is absent the constraint falls outside the perturbative regime. For the lepton universality ratios double ratios $R_{\ell_i \ell_j}$ of leptonic τ decays, the shaded region is the constraint derived when theory is required to match experiment at the 2σ level, as labelled. For Δ_2 in the lower row, these constraints are not shown because they are weaker than those from DELPHI.

with real and positive Yukawa couplings $g_{1,2}$. These expand out to give Eq. (26) with the substitution $\mu \to e$ for the g_1 term, and Eq. (27) with $e \to \mu$ for the g_2 term:

$$\overline{L_3^c}\, i\sigma_2 \boldsymbol{\Delta}_2\, L_1 = -\overline{(\tau_L)^c}\, e_L\, \Delta_2^{++} - \frac{1}{\sqrt{2}} \left[\overline{(\tau_L)^c}\, \nu_{eL} + \overline{(\nu_{\tau L})^c}\, e_L \right] \Delta_2^+ + \overline{(\nu_{\tau L})^c}\, \nu_{eL}\, \Delta_2^0, \tag{32}$$

$$\overline{L_2^c}\, i\sigma_2 \boldsymbol{\Delta}_2\, L_2 = -\overline{(\mu_L)^c}\, \mu_L\, \Delta_2^{++} - \sqrt{2}\, \overline{(\mu_L)^c}\, \nu_{\mu L}\, \Delta_2^+ + \overline{(\nu_{\mu L})^c}\, \nu_{\mu L}\, \Delta_2^0. \tag{33}$$

The Wilson coefficient

$$C^{VLL}_{ee,2321} = \frac{g_1 g_2}{4m^2_{\Delta_2}} \tag{34}$$

parametrises the strength of the decays $\tau^\pm \to \mu^\pm\mu^\pm e^\mp$, with the branching ratio being

$$\mathrm{BR}(\tau^\pm \to \mu^\pm\mu^\pm e^\mp) = \frac{g_1^2 g_2^2}{64 G_F^2 m^4_{\Delta_2}} \tilde{I}\left(\frac{m_\mu^2}{m_\tau^2}\right) \mathrm{BR}(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau). \tag{35}$$

As for the Δ_1 model, the current constraint from the non-observation of these decays is the same as for the singlet k_2 case, i.e. Eq. (14) holds with the m_{k_2} replaced by m_{Δ_2},

$$\sqrt{|g_1 g_2|} \lesssim 0.17 \, \frac{m_{\Delta_2}}{\mathrm{TeV}} \tag{36}$$

and the Belle II mass reach is roughly 59 TeV. This bound and the Belle II reach are illustrated in the bottom row of Fig. 3.

C. Direct searches

As for the singlet cases, there are constraints from direct searches for doubly-charged scalars decaying to a pair of same-sign leptons. A difference between the singlet and triplet models is that the former involve RH leptons, while the latter feature LH leptons. From the published data of Fig. 13 in Ref. [19] we infer lower limits on the masses of the doubly-charged scalars $\Delta^{++}_{1,2}$ given by

$$m_{\Delta_1} \geq 0.69 \text{ TeV} \quad \text{and} \quad m_{\Delta_2} \geq 0.73 \text{ TeV} \tag{37}$$

for the case of 50% BR to electrons and muons, respectively, with the final states containing a τ which is assumed to be unobserved. For a 100% branching ratio to electrons (muons) the lower bounds become

$$m_{\Delta_1} \geq 0.77 \text{ TeV} \quad \text{and} \quad m_{\Delta_2} \geq 0.85 \text{ TeV} \tag{38}$$

according to the published data of Fig. 10 in Ref. [19].

D. Lepton scattering constraints

As for the singlet cases, the DELPHI experiment [20] places constraints on the doubly-charged scalar mass m_{Δ_1} and coupling constant f_2 from $e^+e^- \to e^+e^-$, and m_{Δ_2} and g_1 from $e^+e^- \to \tau^+\tau^-$. Translating the results in Ref. [21], we obtain the bounds

$$\frac{m_{\Delta_1}}{|f_2|} \geq 0.75\,\mathrm{TeV}, \qquad \frac{m_{\Delta_2}}{|g_1|} \geq 1.6\,\mathrm{TeV}. \tag{39}$$

Note that these constraints are very slightly different from the analogous singlet bounds because the opposite chiral structure changes the details of the interference between the SM and triplet-exchange diagrams.

E. Leptonic processes involving neutrinos

The electroweak triplet scalars introduce new contributions to leptonic processes with neutrinos. We follow the discussion in Refs. [21, 22, 29] and focus on the most stringent electroweak physics constraints [21, 22], namely lepton flavour universality, neutrino trident and shifts of the Fermi constant and its impact on the weak mixing angle, the W boson mass and CKM unitarity. The latter three do not receive direct contributions at tree level, but are modified indirectly via the shift of the Fermi constant extracted in muon decay, $G_{F,\mu}$.

In particular, the partial width for the decay $\ell_\alpha \to \ell_\beta \nu_i \bar{\nu}_j(\gamma)$ in terms of relevant Wilson coefficients is given by [30–33]

$$\Gamma(\ell_\alpha \to \ell_\beta \nu_i \bar{\nu}_j(\gamma)) = \frac{G_F^2 m_\alpha^5}{192\pi^3} I\left(\frac{m_\beta^2}{m_\alpha^2}\right) R_W R_\gamma \sum_{i,j} \frac{|C^{VLL}_{\nu e,ij\beta\alpha}|^2 + |C^{VLR}_{\nu e,ij\beta\alpha}|^2}{8G_F^2}, \tag{40}$$

where the functions R_W and I parameterise corrections from finite lepton masses, while R_γ parametrises the emission of soft photons. The function I is defined in Eq. (12) and the other functions are given by

$$R_W = 1 + \frac{3}{5}\frac{m_\alpha^2}{m_W^2} + \frac{9}{5}\frac{m_\beta^2}{m_W^2}, \qquad R_\gamma = 1 + \frac{\alpha(m_\alpha)}{2\pi}\left(\frac{25}{4} - \pi^2\right)$$
$$\text{with} \quad \alpha(m_\alpha)^{-1} = \alpha^{-1} - \frac{2}{3\pi}\ln\left(\frac{m_\alpha}{m_e}\right) + \frac{1}{6\pi}. \tag{41}$$

The SM prediction for the LEFT Wilson coefficients is

$$C^{\rm VLL,SM}_{\nu e,ij\beta\alpha} = -2\sqrt{2}G_F\left(-\frac{1}{2} + s_w^2\right)\delta_{ij}\delta_{\alpha\beta} - 2\sqrt{2}G_F\delta_{i\alpha}\delta_{j\beta}, \quad C^{\rm VLR,SM}_{\nu e,ij\beta\alpha} = -2\sqrt{2}G_F s_w^2 \delta_{ij}\delta_{\alpha\beta}. \tag{42}$$

The electroweak triplet contributes via the 4-lepton SMEFT operators O^{ll} to $C^{VLL}_{\nu e}$ as described by the LEFT matching conditions in App. B and thus modify the leptonic muon and tau decays.

1. *Lepton flavour universality*

The most direct probe is provided by the lepton flavour universality double ratios,

$$R_{\mu e} = \frac{\Gamma(\tau \to \mu + \text{inv})}{\Gamma(\tau \to e + \text{inv})}\frac{\Gamma_{\rm SM}(\tau \to e + \text{inv})}{\Gamma_{\rm SM}(\tau \to \mu + \text{inv})}, \tag{43}$$
$$R_{\tau\mu} = \frac{\Gamma(\tau \to e + \text{inv})}{\Gamma(\mu \to e + \text{inv})}\frac{\Gamma_{\rm SM}(\mu \to e + \text{inv})}{\Gamma_{\rm SM}(\tau \to e + \text{inv})}, \tag{44}$$
$$R_{\tau e} = \frac{\Gamma(\tau \to \mu + \text{inv})}{\Gamma(\mu \to e + \text{inv})}\frac{\Gamma_{\rm SM}(\mu \to e + \text{inv})}{\Gamma_{\rm SM}(\tau \to \mu + \text{inv})}, \tag{45}$$

of which only two are independent. For the triplet Δ_1 we find

$$R_{\tau e} = R_{\mu e} = \frac{\left(1 - \frac{f_1^2}{8\sqrt{2}G_F m_{\Delta_1}^2}\right)^2}{1 + \frac{(f_1 f_2)^2}{2^7 G_F^2 m_{\Delta_1}^4}} \approx 1 - \frac{f_1^2}{4\sqrt{2}G_F m_{\Delta_1}^2}, \qquad R_{\tau\mu} = 1 \tag{46}$$

and for Δ_2

$$R_{\tau\mu} = R_{\mu e}^{-1} = \frac{\left(1 - \frac{g_1^2}{8\sqrt{2}G_F m_{\Delta_2}^2}\right)^2}{1 + \frac{(g_1 g_2)^2}{2^7 G_F^2 m_{\Delta_2}^4}} \approx 1 - \frac{g_1^2}{4\sqrt{2}G_F m_{\Delta_2}^2}, \qquad R_{\tau e} = 1, \tag{47}$$

where we neglected terms quartic in the Yukawa couplings in the approximation.

Using the experimental values in Ref. [34], we find for the double ratios [21]

$$R_{\mu e} = 1.0034(32), \qquad R_{\tau\mu} = 1.0022(29), \qquad R_{\tau e} = 1.0056(29). \tag{48}$$

Each of these is to be compared to the SM prediction of unity. We note that there is a tension with the SM prediction for $R_{\tau e}$ which cannot be alleviated at the central value by the electroweak triplet scalar. Requiring that the model agrees with the values in Eq. (48) to within 2σ, the strongest constraints from these ratios come from $R_{\tau e}$ for Δ_1 and $R_{\tau\mu}$ for Δ_2. The excluded regions are indicated by the vertical coloured bands in the top row of Fig. 3. The region excluded by $R_{\tau\mu}$ is not shown in the bottom row because $|g_1|$ is more strongly constrained by the DELPHI measurement.

2. Trident process

The singly-charged scalar Δ_2^+ contributes at tree-level via the g_2 Yukawa interaction to the trident process $\nu_\mu N \to \nu_\mu \mu^+ \mu^- N$, where N represents a nucleon. This is parameterised by the Wilson coefficient

$$C_{\nu e,2222}^{\text{VLL,NP}} = \frac{g_2^2}{4m_{\Delta_2}^2}, \tag{49}$$

which gives the ratio of the modified cross section σ to the SM cross section σ_{SM} to be

$$\frac{\sigma}{\sigma_{\text{SM}}} = \frac{(1 + 4\sin^2\theta_w - C_{\nu e,2222}^{\text{VLL,NP}}/(\sqrt{2}G_F))^2 + (1 - C_{\nu e,2222}^{\text{VLL,NP}}/(\sqrt{2}G_F))^2}{(1 + 4\sin^2\theta_w)^2 + 1}. \tag{50}$$

As the Wilson coefficient $C_{e\nu,2222}^{\text{VLL,NP}}$ is strictly positive, the electroweak triplet contribution reduces the ratio to be less than one.

The CHARM-II [35], CCFR [36] and NuTeV [37] experiments have measured this ratio obtaining 1.58 ± 0.64, 0.82 ± 0.28 and $0.72^{+1.73}_{-0.72}$, respectively. In order to estimate how neutrino trident production $\nu_\mu N \to \nu_\mu \mu^+ \mu^- N$ constrains the electroweak triplet scalar Δ_2, we combine the experimental measurements[5] to obtain 0.94 ± 0.25, which constrains

$$-0.15 \le \frac{C_{\nu e,2222}^{\text{VLL,NP}}}{\sqrt{2}G_F} \le 0.28 \qquad \Longrightarrow \qquad |g_2| \lesssim 4.3\,\frac{m_{\Delta_2}}{\text{TeV}}. \tag{51}$$

[5] For the combination we assume Gaussian distributions and utilise the larger upper error for the NuTeV result.

Input	Value
m_Z	91.1876(21) GeV
$G_{F,\mu}$	$1.1663787(6) \times 10^{-5}$ GeV^{-2}
α^{-1}	137.035999180(10)

TABLE II. Input parameters for electroweak observables, taken from Ref. [34].

This may be compared with the bound in Eq. (36). Numerically it is a weak constraint, with the upper bound on g_2 in the non-perturbative regime and thus moot. However, it is worth noting that the constraint is purely on g_2 rather than the product $g_1 g_2$, and so in principle the trident process provides a complementary constraint.

As this constraint is rather weak in this model (and also orthogonal to the main discussion of cLFV leptonic τ decays) we do not discuss the sensitivity of neutrino trident processes at DUNE. DUNE is able to measure other trident processes, including lepton-flavour-violating neutrino trident processes. See Refs. [38, 39] for a detailed discussion of neutrino tridents at DUNE.

3. Fermi constant

These models generate new contributions to muon decay, which is used to determine the Fermi constant G_F. We denote the Fermi constant determined through muon decay $G_{F,\mu}$. Using the input parameters for the SM electroweak observables listed in Table II, these contributions introduce a shift in $G_{F,\mu}$,

$$G_{F,\mu}^2 = \frac{1}{8}\sum_{i,j}\left(|C^{VLL}_{\nu e,ij12}|^2 + |C^{VLR}_{\nu e,ij12}|^2\right), \tag{52}$$

in contrast to the SM value $G_{F,0} = (\sqrt{2}v^2)^{-1/2}$. Here v is the vacuum expectation value (VEV) of the Higgs field $\langle H\rangle = v/\sqrt{2}$. We define $G_{F,0} = G_{F,\mu}(1+\delta G_F)$, and for Δ_1 and Δ_2 we find that

$$\delta G_F = \begin{cases} -\frac{f_1^2 f_2^2}{1024 G_{F,\mu}^2 m_{\Delta_1}^4} & \text{for } \Delta_1, \\ -\frac{g_1^2 g_2^2}{1024 G_{F,\mu}^2 m_{\Delta_2}^4} & \text{for } \Delta_2. \end{cases} \tag{53}$$

As the correction to the Fermi constant only occurs at quartic order in the Yukawa couplings, we do not expect strong constraints from δG_F.

Several measurements are sensitive to the Fermi constant and thus provide constraints on δG_F. Here we consider the weak mixing angle, the W boson mass and CKM unitarity as a subset of these measurements. For each of these, we use the tree-level SM expression to determine how the shift in G_F affects the observable, and then derive a constraint on the shift δG_F the electroweak fit from Ref. [40] (which includes loop-level SM corrections). This fit provides SM predictions for the different observables (without including the measurements) which are then compared to the experimental measurements to obtain constraints on δG_F.

The effective leptonic weak mixing angle does not receive direct corrections at tree-level[6] because the Z boson couplings to leptons are not modified. Therefore, we find

$$\bar{s}_\ell^2 = s_{w,0}^2 \left(1 - \frac{1 - s_{w,0}^2}{1 - 2s_{w,0}^2}\delta G_F\right), \tag{54}$$

which has been obtained from the tree-level SM prediction of the weak mixing angle

$$s_{w,0}^2 = \frac{1}{2}\left(1 - \sqrt{1 - \frac{4\pi\alpha}{\sqrt{2}G_F m_Z^2}}\right). \tag{55}$$

Contrasting this with the experimental result $\bar{s}_{\ell,\mathrm{exp}}^2(\mathrm{LEP}) = 0.23153(4)$ [41] with the loop-corrected SM prediction for the weak mixing angle based on the fit in [40], $\bar{s}_{\ell,\mathrm{SM}}^2 = 0.231534(41)$, results in

$$\delta G_F = 12(170) \times 10^{-6}. \tag{56}$$

A shift in the Fermi constant also translates to a shift in the W boson mass

$$m_W^2 = m_{W,0}^2 \left(1 + \frac{s_{w,0}^2}{1 - 2s_{w,0}^2}\delta G_F\right) \tag{57}$$

which has been derived from the tree-level SM prediction

$$m_{W,0} = m_Z\sqrt{1 - s_{w,0}^2}. \tag{58}$$

A comparison of the experimental global fit to the W boson mass excluding the new CDF measurement $m_{W,\mathrm{exp}} = 80.377(12)$ [34] [and a combination of all Tevatron measurements by themselves $m_{W,\mathrm{Tevatron}}$ =80.4274(89) [42]] with the SM prediction $m_W = 80.3545(42)$ GeV [40] results in

$$\delta G_F = 0.00130(73)\ [0.00421(57)]. \tag{59}$$

A shift in the Fermi constant also leads to an apparent violation of CKM unitarity. We find for the unitarity relation of the first row CKM matrix elements that

$$\sum_\beta |V_{u\beta}|^2 = 1 + 2\delta G_F. \tag{60}$$

The global fit in Ref. [34] requires $\sum_\beta |V_{u\beta}|^2 = 0.9985 \pm 0.0007$. If we conservatively demand consistency at 3σ (to include the SM prediction), we find

$$\delta G_F = -0.00075(35). \tag{61}$$

Note that the different observables are in tension with each other: the leptonic weak mixing angle prefers no correction to δG_F, the W boson mass indicates a positive δG_F, and CKM unitarity

[6] We neglect loop-level corrections to $\Pi_{\gamma Z}(m_Z^2)$ in the analysis.

negative δG_F. As δG_F in the electroweak triplet model is proportional to the fourth power of Yukawa couplings, none of the observables constraining δG_F provide a competitive constraint. Even the sensitivity of the leptonic weak mixing angle (which probes δG_F at the level of 10^{-4}) is only sensitive to scales $m_{\Delta_T} \lesssim 0.50$ TeV for Yukawa couplings of order unity, which is already excluded by direct searches at the LHC (see Sec. III C).

F. Other observables

As discussed for the electroweak singlet scalar, there are new contributions to leptonic Higgs and Z boson decays and the anomalous magnetic moment [21, 22]. These are too small to have any measurable phenomenological implications.

IV. PHASE SPACE

In the caption of Table I, we alluded to upper limits placed by experiment on cLFV leptonic τ decays depending on the assumed distribution of signal events. The limits quoted in this table are extracted by the experimental collaborations assuming that the leptons from the τ-decay follow a phase space distribution, i.e. no kinematic dependence in the matrix element. The model-discriminating power of three-body phase space in tau decays has been studied, for example, in Refs. [43–46].

In the both the electroweak singlet and triplet models, the differential decay rate for $\tau^- \to \ell_i^- \ell_i^- \ell_j^+$ is given by

$$\frac{d^2\Gamma(\tau^- \to \ell_i^- \ell_i^- \ell_j^+)}{dm_{--}^2\, dm_{+-}^2} = \frac{1}{256\pi^3 m_\tau^3}\overline{|\mathcal{M}|}^2 = \frac{|C_{i3ij}|^2}{64\pi^3 m_\tau^3}(m_{--}^2 - 2m_{\ell_i}^2)(m_\tau^2 + m_{\ell_j}^2 - m_{--}^2), \tag{62}$$

where $m_{--}^2 = (p_{\ell_i} + p_{\ell_i'})^2$ is the invariant mass of the system of two same-sign leptons in the final state, $m_{+-}^2 = (p_{\ell_i} + p_{\ell_j})^2$ is that of two oppositely charged final state leptons, and $\overline{|\mathcal{M}|}^2$ is the spin-averaged matrix element for the process. The squared Wilson coefficient is given by $|C_{i3ij}|^2 = |C_{ee,i3ij}^{\mathrm{VRR}}|^2$ in the electroweak singlet models and by $|C_{i3ij}|^2 = |C_{ee,i3ij}^{VLL}|^2$ in the electroweak triplet models. Thus all models feature the same distributions for the differential decay rates. A so-called *phase space distribution* for the differential decay rate discussed in this section corresponds to setting $\overline{|\mathcal{M}|}^2 = 1$.

From Eq. (62) there is a flat distribution in m_{+-}^2 and a peak in m_{--}^2 at $m_{--}^2 = \frac{1}{2}(2m_{\ell_i}^2 + m_{\ell_j} + m_\tau^2) \approx \frac{1}{2}m_\tau^2$, where the latter approximation corresponds to neglecting the mass of leptons in the final state. This is illustrated by the Dalitz plots in the top row of Figure 4. The structure of these Dalitz distributions is characteristic of models in which the dominant BSM contribution to these processes is via a RH or LH vector operator, respectively. Ultimately if these decays are detected

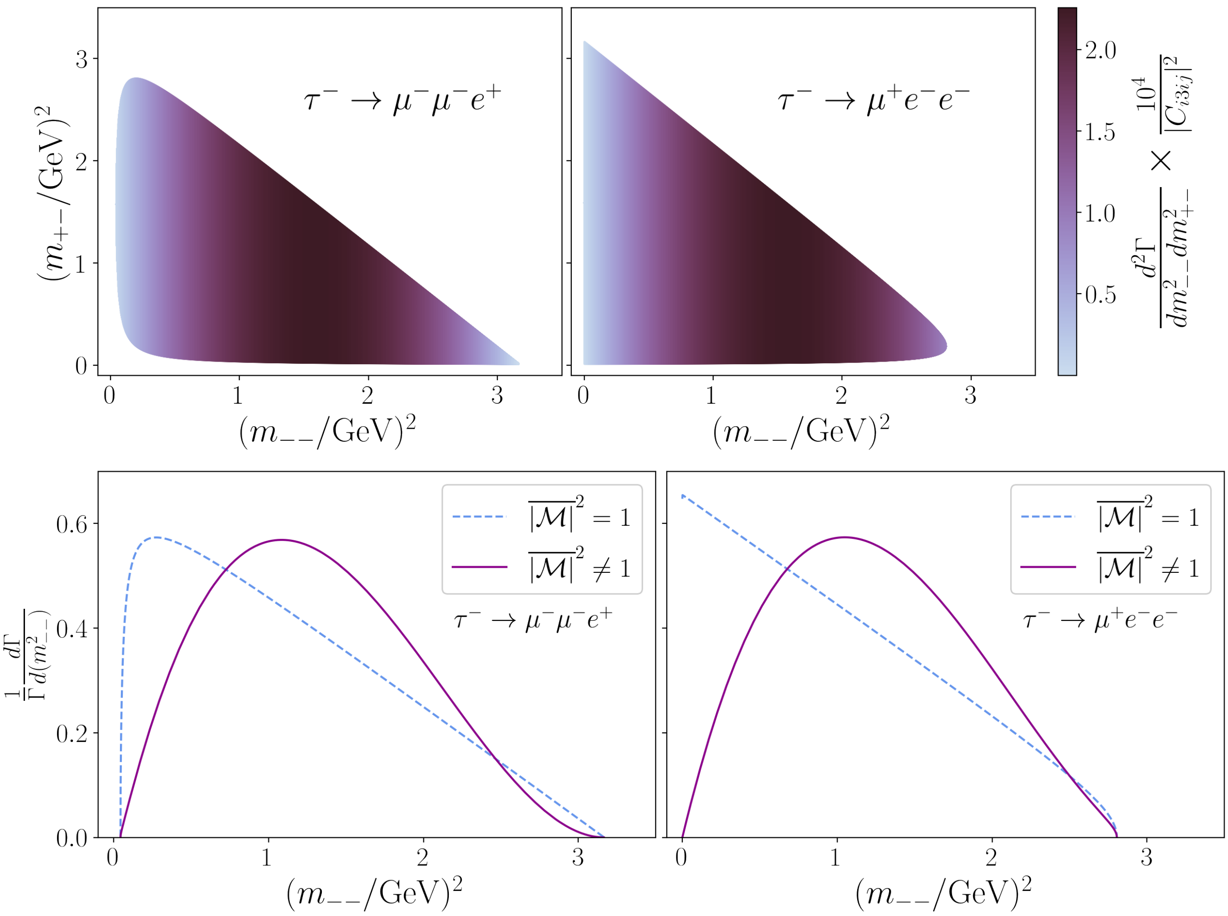

FIG. 4. A study of key cLFV tau decay phase space. The top row shows Dalitz plots for the two key decay processes $\tau^- \to \mu^-\mu^-e^+$ (left) and $\tau^- \to \mu^+e^-e^-$ (right) assuming the dominant contribution is via the left or right-handed vector operator C_{i3ij}, as appears in Eq. (62). The bottom row shows the integrated differential distributions with respect to the observable m^2_{--}, illustrating the difference between the kinematic distribution of this model ($\overline{|\mathcal{M}|}^2 \neq 1$ as per Eq. (62), solid purple line) and the phase space distribution ($\overline{|\mathcal{M}|}^2 = 1$, blue dashed line).

with significant multiplicity, an analysis of the three-body phase space could discriminate between different BSM explanations [43].

In the bottom row of Figure 4 we illustrate the corresponding $\frac{d\Gamma(\tau^-\to\ell_i^-\ell_i^-\ell_j^+)}{dm^2_{--}}$ distribution (solid purple), comparing it to a phase space distribution (dashed blue). Integration over m^2_{+-} results in a factor

$$m^2_{+-,\text{max}} - m^2_{+-,\text{min}} = \frac{\lambda^{1/2}(m^2_\tau, m^2_{--}, m^2_{\ell_j})\lambda^{1/2}(m^2_{--}, m^2_{\ell_i}, m^2_{\ell_i})}{m_{--}}, \tag{63}$$

where $\lambda^{1/2}$ is the square-root of the Källén function[7], and the invariant mass m^2_{--} takes values in the range $4m^2_{\ell_i} < m^2_{--} < (m_\tau - m_{\ell_j})^2$. Integration of the differential decay width results in the expressions shown in Eqs. (6) and (11).

Even with non-observation in these channels, the expected number of events in different kinematic regions is shown in Figure 4 to vary model-dependently (this was also emphasised in Refs. [43–

[7] $\lambda(x, y, z) \equiv x^2 + y^2 + z^2 - 2xy - 2yx - 2zx$.

46], although they did not study these two decay channels). As such, an exclusion should be weighted accordingly to provide the most accurate measure of the present and projected constraint – although it is not feasible for an experimental collaboration to present constraints on each individual model or even at the EFT level, where interference effects would also need to be considered. Without access to internal Belle/ Belle II information, one cannot easily recast the branching-ratio limits to a specific model, and so we still adopt the values derived using the assumption of a phase space distribution (Table I). In the absence of events, recasting the limits based on the phase space distribution requires the detection efficiency as a function of the invariant masses m^2_{--} and m^2_{+-}.

V. NEUTRINO MASSES

So far, we have mainly focused on the charged-lepton sector. In this section we turn to neutrino masses and discuss a few possible scenarios how non-zero neutrino masses may be incorporated into these models.

The most straightforward way to generate non-zero neutrino masses in both the electroweak singlet and triplet models is by introducing three RH sterile neutrinos with similar Z_3 triality charges. The first (second) [third] generation of sterile neutrinos has charge $T = 1$ (2) [3]. These charges correspond to the transformations

$$\nu_R \to \omega^T \nu_R \tag{64}$$

where ν_R are the RH neutrinos. Given these assignments, the neutrino Yukawa and mass terms in the Lagrangian are

$$-\mathcal{L} \supset y_{\nu i} \bar{L}_i \nu_{Ri} \tilde{H} + \frac{1}{2} M_{ij} \overline{(\nu_{Ri})^c}\, \nu_{Rj} + \text{h.c.}, \tag{65}$$

where $i, j = 1, 2, 3$ with repeated indices summed, $\tilde{H} = i\tau_2 H^*$, $y_{\nu i}$ denotes the neutrino Yukawa couplings, and M is the RH neutrino Majorana mass matrix. The neutrino Dirac mass matrix is diagonal. With exact triality, the RH neutrino Majorana mass matrix is constrained to the form

$$M = \begin{pmatrix} 0 & M_{12} & 0 \\ M_{12} & 0 & 0 \\ 0 & 0 & M_{33} \end{pmatrix} . \tag{66}$$

This is incompatible with the neutrino oscillation data, given that we also have a diagonal neutrino Dirac mass matrix. Therefore lepton triality *must* be broken. This breaking could be achieved by via explicit soft-breaking operators, which then generate the remaining entries of the RH neutrino mass matrix. Alternatively, it can be achieved by introducing a SM singlet complex scalar S with $T = 1$ (so that $S \to \omega S$ and $S^* \to \omega^2 S^*$) which leads to the additional triality-preserving Yukawa coupling terms

$$\mathcal{L} \supset \frac{1}{2}\left[x_{11}\overline{(\nu_{R1})^c}\,\nu_{R1} + x_{23}\left(\overline{(\nu_{R2})^c}\,\nu_{R3} + \overline{(\nu_{R3})^c}\,\nu_{R2}\right)\right] S + \frac{1}{2}\left[x_{22}\overline{(\nu_{R2})^c}\,\nu_{R2} + x_{13}\left(\overline{(\nu_{R1})^c}\,\nu_{R3} + \overline{(\nu_{R3})^c}\,\nu_{R1}\right)\right] S^* + \text{h.c.}\ . \tag{67}$$

Triality is then spontaneously broken by a nonzero VEV for S, and the zero entries in Eq. (66) are now all generated. A diagonal neutrino Dirac mass matrix together with a general RH neutrino Majorana mass matrix is able to accommodate the neutrino oscillation data.

Adopting the type-I seesaw mechanism requires the VEV-generated triality-breaking terms to be of a high scale, not dissimilar to M_{12} and M_{33}.[8] Note that the S and S^* Yukawa terms combine to explicitly break lepton-number conservation. Lepton number is also explicitly broken by a cubic (aS^3 + h.c.) triality-preserving term in the scalar potential, which means that the phase of S is not a Goldstone boson. Neutrino masses can equally well generated using the type-III seesaw mechanism [47] with electroweak triplet fermions instead of electroweak singlet fermions.

As was mentioned in the introduction, lepton triality is motivated by discrete flavour symmetries, which break the flavour group to a Z_3 subgroup in the charged lepton sector and to a Z_2 subgroup in the neutrino sector. The misalignment between the two sectors explains the leptonic mixing matrix with the prime example for this construction being the A_4 flavour group. Assuming that the additional BSM physics related to the flavour symmetry is sufficiently decoupled, the A_4 flavour symmetry models for neutrino masses mentioned in the introduction (Refs. [3–11]) also yield the phenomenology of charged leptons discussed in this paper.

Aside from triality, the electroweak triplet scalar is that of the type-II seesaw mechanism. A nonzero VEV for Δ_T^0 would therefore contribute some direct Majorana mass terms for the light neutrinos. The VEV is naturally suppressed because the cubic Higgs coupling $H^T i\sigma_2 \boldsymbol{\Delta}_T^\dagger H$ softly breaks lepton triality. Similarly, the electroweak singlet scalar features in the Zee-Babu model [48, 49]. See [14, 50, 51] for recent phenomenological studies.

As it becomes evident from the discussion of the seesaw mechanisms, lepton triality has to be broken to achieve the observed leptonic mixing pattern [52, 53]. This can also be seen more generally by considering the Weinberg operator [54],

$$\mathcal{L}_5 = -\frac{\kappa_{ij}}{4}(L_i H)^T C (L_j H) + \text{h.c.}\ , \tag{68}$$

where the SU(2) indices are contracted within each pair of parentheses. Lepton triality constrains the Wilson coefficient to take the form[9]

$$\kappa = \begin{pmatrix} 0 & \kappa_{12} & 0 \\ \kappa_{12} & 0 & 0 \\ 0 & 0 & \kappa_{33} \end{pmatrix} . \tag{69}$$

[8] Given the high spontaneous breaking scale for the discrete Z_3 symmetry, the resulting cosmological domain wall problem could be solved by inflating-away the domain walls.

[9] Note that different neutrino flavours can be singled out using a shift of all triality charges. The maximal 2-3 lepton mixing motivates to shift all triality charges by 2 which results in a Majorana neutrino ν_1 and a Dirac pair $\nu_{2,3}$.

Like for the case of the seesaw mechanism, lepton triality has to be broken to explain the observed lepton mixing matrix which may be achieved by introducing a SM singlet complex scalar S with $T = 1$ and an effective dimension-6 operators $S^{(*)}(L_iH)^T C(L_jH)$. A similar argument can be made for Dirac neutrinos, because the Yukawa interaction $\bar{L}_i \nu_{Rj} \tilde{H}$ and thus the neutrino Dirac mass term contain at most 3 non-zero entries for the lepton triality charges defined in Eq. (2) irrespective of the lepton triality charges of the RH neutrinos.

VI. CONCLUSIONS

Charged lepton flavour-violating decays and other related processes are an important probe of physics beyond the standard model. The discovery of neutrino flavour oscillations implies that such processes must occur at some level, but for them to be observable in practice physics in addition to that responsible for neutrino mass generation must exist. Importantly, significant advances in the search for flavour-violating τ decays will be made by the Belle II experiment over the next few years, thus opening a new discovery window. In this paper we presented very simple models based on the flavour symmetry structure of lepton triality that feature the decays $\tau^\pm \to \mu^\pm \mu^\pm e^\mp$ and $\tau^\pm \to e^\pm e^\pm \mu^\mp$ as the dominant signals of BSM physics. These decays are driven by the tree-level exchange of doubly-charged scalars shown Fig. 1. As illustrated in Figs. 2 and 3, the eventual Belle II sensitivity to these processes will see significantly more parameter space explored compared to the present situation, either discovering evidence of new physics or further constraining the possibilities.

These models are simple examples of minimal SM extensions that single out the τ sector for the dominant phenomenological signatures. The fact that it proves so easy to do this highlights the importance and relevance of the on-going experimental searches. We expect that our minimal models may be embedded in more complete theories of flavour symmetry and, beyond this, that quite different schemes could also be constructed to achieve a similar purpose.

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Appendix A: SMEFT

In addition to the renormalisable part of the Lagrangian we introduce dimension-6 SMEFT operators. We are particularly interested in operators which violate lepton flavour and are consistent with the Z_3 symmetry. The only relevant operators in the Warsaw basis [55] are the 4-lepton operators

$$\mathcal{L}_6 = C^{ll}(\bar{L}\gamma_\mu L)(\bar{L}\gamma^\mu L) + C^{ee}(\bar{e}_R\gamma_\mu e_R)(\bar{e}_R\gamma^\mu e_R) + C^{le}(\bar{L}\gamma_\mu L)(\bar{e}_R\gamma^\mu e_R), \tag{A1}$$

where we do not explicitly specify the flavour indices. In case flavour indices are important, they are written as subscripts in the order of the fermion flavours in the operators, e.g. C^{ll}_{abcd} is the Wilson coefficient of operator $(\bar{L}_a\gamma_\mu L_b)(\bar{L}_c\gamma^\mu L_d)$.

The electroweak singlet models lead to

$$C^{ee}_{3322} = \frac{|f_1|^2}{2m_{k_1}^2} \qquad C^{ee}_{1111} = \frac{|f_2|^2}{8m_{k_1}^2} \qquad C^{ee}_{1312} = C^{ee*}_{3121} = \frac{f_1 f_2^*}{4m_{k_1}^2} \tag{A2}$$

$$C^{ee}_{3311} = \frac{|g_1|^2}{2m_{k_2}^2} \qquad C^{ee}_{2222} = \frac{|g_2|^2}{8m_{k_2}^2} \qquad C^{ee}_{2321} = C^{ee*}_{3212} = \frac{g_1 g_2^*}{4m_{k_2}^2}, \tag{A3}$$

where we removed equivalent Wilson coefficients and only keep one of the equivalent flavour combinations. For the electroweak triplet models we find

$$C^{ll}_{3322} = C^{ll}_{3223} = \frac{|f_1|^2}{4m_{\Delta_1}^2} \qquad C^{ll}_{1111} = \frac{|f_2|^2}{8m_{\Delta_1}^2} \qquad C^{ll}_{1312} = C^{ll*}_{3121} = \frac{f_1 f_2^*}{4m_{\Delta_1}^2} \tag{A4}$$

$$C^{ll}_{3311} = C^{ll}_{3113} = \frac{|g_1|^2}{4m_{\Delta_2}^2} \qquad C^{ll}_{2222} = \frac{|g_2|^2}{8m_{\Delta_2}^2} \qquad C^{ll}_{2321} = C^{ll*}_{3212} = \frac{g_1 g_2^*}{4m_{\Delta_2}^2}. \tag{A5}$$

As lepton triality protects the flavour structure of the operators, no operator violating lepton triality is generated by renormalisation group corrections. Thus in this analysis we neglect renormalisation group corrections.

Appendix B: LEFT

As the Z_3 symmetry only allows lepton-flavour-violating 4-fermion operators at dimension-6 in SMEFT, we consider only leptonic 4-fermion interactions in LEFT [56]:

$$\begin{aligned}\mathcal{L} = {} & C^{VLL}_{ee}(\bar{e}\gamma^\mu e_L)(\bar{e}\gamma_\mu e_L) + C^{VRR}_{ee}(\bar{e}\gamma^\mu e_R)(\bar{e}\gamma_\mu e_R) + C^{VLR}_{ee}(\bar{e}\gamma^\mu e_L)(\bar{e}\gamma_\mu e_R) \\ & + C^{VLL}_{\nu e}(\bar{\nu}\gamma^\mu \nu_L)(\bar{e}\gamma_\mu e_L) + C^{VLR}_{\nu e}(\bar{\nu}\gamma^\mu \nu_L)(\bar{e}\gamma_\mu e_R).\end{aligned} \tag{B1}$$

Similarly to the SMEFT operators, we do not explicitly specify the flavour indices, unless needed. In case flavour indices are important, they are written as subscripts in the order of the fermion flavours in the operator, e.g. $C^{VLL}_{ee,abcd}$ is the Wilson coefficient of $(\bar{e}_a\gamma^\mu P_L e_b)(\bar{e}_c\gamma_\mu P_L e_d)$. We do not include the 4-neutrino operator, because it is not relevant for the discussion of the phenomenology.

Other lepton-flavour-violating Wilson coefficients are strongly suppressed by the unitarity of the PMNS mixing matrix and are neglected.

The matching to LEFT operators is given by [56]

$$C^{VLL}_{ee,ijkl} = C^{ll}_{ijkl} - \frac{g_Z^2}{4m_Z^2}[Z_{e_L}]_{ij}[Z_{e_L}]_{kl} - \frac{g_Z^2}{4m_Z^2}[Z_{e_L}]_{il}[Z_{e_L}]_{kj}, \tag{B2}$$

$$C^{VRR}_{ee,ijkl} = C^{ee}_{ijkl} - \frac{g_Z^2}{4m_Z^2}[Z_{e_R}]_{ij}[Z_{e_R}]_{kl} - \frac{g_Z^2}{4m_Z^2}[Z_{e_R}]_{il}[Z_{e_R}]_{kj}, \tag{B3}$$

$$C^{VLR}_{ee,ijkl} = C^{le}_{ijkl} - \frac{g_Z^2}{m_Z^2}[Z_{e_R}]_{ij}[Z_{e_R}]_{kl}, \tag{B4}$$

$$C^{VLL}_{\nu e,ijkl} = C^{ll}_{ijkl} + C^{ll}_{klij} - \frac{g^2}{2m_W^2}[W_l]_{il}[W_l]^*_{jk} - \frac{g_Z^2}{m_Z^2}[Z_\nu]_{ij}[Z_{e_L}]_{kl}, \tag{B5}$$

$$C^{VLR}_{\nu e,ijkl} = C^{le}_{ijkl} - \frac{g_Z^2}{m_Z^2}[Z_\nu]_{ij}[Z_{e_R}]_{kl}, \tag{B6}$$

with the SM contributions via W and Z boson exchange

$$[Z_\nu]_{pr} = \frac{1}{2}\delta_{pr}, \qquad [Z_{e_L}]_{pr} = \left(-\frac{1}{2} + s_w^2\right)\delta_{pr}, \qquad [Z_{e_R}]_{pr} = s_w^2\delta_{pr}, \qquad [W_l]_{pr} = \delta_{pr}. \tag{B7}$$

The Fermi constant in the SM is given by $2\sqrt{2}G_F = \frac{g^2}{2m_W^2} = \frac{g_Z^2}{2m_Z^2}$ and thus Eqs. (B5,B6) result in the SM contribution shown in Eq. (42). Renormalisation group corrections are dominated by QED running and thus generally small, so we neglect them throughout this analysis.

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