



Can extra dimensional effects allow wormholes without exotic matter?



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ABSTRACT

We explore the existence of Lorentzian wormholes in the context of an effective on-brane, scalar-tensor theory of gravity. In such theories, the timelike convergence condition, which is always violated for wormholes, has contributions, via the field equations, from on-brane matter as well as from an effective geometric stress energy generated by a bulk-induced radion field. It is shown that, for a class of wormholes, the required on-brane matter, as seen by an on-brane observer in the Jordan frame, is not exotic and does not violate the Weak Energy Condition. The presence of the effective geometric stress energy in addition to on-brane matter is largely responsible for creating this intriguing possibility. Thus, if such wormholes are ever found to exist in the Universe, they would clearly provide pointers towards the existence of a warped extra dimension as proposed in the two-brane model of Randall and Sundrum.

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1. Introduction

Ever since Einstein and Rosen [1] proposed the Einstein–Rosen bridge and Wheeler [2] coined the term *wormhole*, such geometries have been of great interest both in physics and in science fiction. One of the first non-singular wormhole solutions using a wrong-sign scalar field was found by Ellis in 1973 [3]. Subsequently, in the late 1980s, Morris, Thorne and Yurtsever [4] came up with a time-machine model using wormholes. This led to a host of articles on wormholery [5]. However, a major drawback of all work on static wormholes has been the proven fact that wormholes must violate the energy conditions [6]. Energy Conditions [7,8] are known to be sacred because they point towards physical requirements on matter. For example, we know that if the Weak Energy Condition is violated it implies a negative energy density in some frame of reference [6]. Within the framework of General Relativity, it is impossible to construct a static wormhole without violating the energy conditions. Thus, classically, such spacetimes cannot exist.

Many arguments may be given in support of the violation of the energy conditions. Some of them invoke quantum fields in curved spacetime – the renormalised stress energy tensor is known

to have such violations [9]. Attempts have been made towards avoiding violations or justifying them, in time-dependent spacetimes [10] and in alternative theories of gravity [11]. Further, there have been proposals regarding restricting the violation of the energy conditions over arbitrarily small regions [12].

In our work here we try to address the issue in a different way. We ask whether the existence of extra dimensions can in some way lead to the existence of Lorentzian wormholes which do not violate any energy condition. In other words, using the warped braneworld picture [13], can we say that an observer sitting on a 3-brane does not see a violation of the energy conditions for matter that threads a possible wormhole geometry? To make things more concrete, we use the effective, on-brane scalar-tensor theory constructed by Kanno and Soda [14] in the context of the two-brane Randall–Sundrum model, in a higher dimensional bulk spacetime [13]. In such a theory, the scalar radion field which encodes information about the extra dimensions and branes plays a crucial role. The radion is a measure of the proper distance between the branes. We find that for a class of wormhole geometries, the radion is everywhere finite and non-zero and the on-brane matter threading the wormhole is perfectly normal without any violation of the energy conditions. Therefore, if we ever see such a wormhole, we may be able to prop it up as a support for the existence of warped extra dimensions as proposed in the two-brane Randall–Sundrum model. We now elaborate in detail on this exciting possibility.

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2. Low energy, effective, on-brane gravity

Let us begin by writing down the four-dimensional, effective scalar-tensor theory due to Kanno and Soda [14]. The higher dimensional bulk spacetime is five dimensional with a warped extra dimension and two 3-branes located at $y = 0$ and $y = l$, where y denotes the extra dimension. The effective theory is a valid low energy theory as long as the on-brane matter energy density is much less than the brane tension. It is therefore clear that near singularities the theory will break down. However, since we are dealing with non-singular solutions (finite energy density and pressures) in this article the effective theory is valid everywhere. The field equations for this effective theory are given as:

$$G_{\mu\nu} = \frac{\bar{\kappa}^2}{l\Phi} T_{\mu\nu}^b + \frac{\bar{\kappa}^2(1+\Phi)}{l\Phi} T_{\mu\nu}^a + \frac{1}{\Phi} (\nabla_\mu \nabla_\nu \Phi - g_{\mu\nu} \nabla^\alpha \nabla_\alpha \Phi) - \frac{3}{2\Phi(1+\Phi)} \left(\nabla_\mu \Phi \nabla_\nu \Phi - \frac{1}{2} g_{\mu\nu} \nabla^\alpha \Phi \nabla_\alpha \Phi \right) \quad (1)$$

Here $g_{\mu\nu}$ is the on-brane metric and the covariant differentiation is defined with respect to $g_{\mu\nu}$. $\bar{\kappa}^2$ is the 5D gravitational coupling constant. $T_{\mu\nu}^a$, $T_{\mu\nu}^b$ are the stress-energy on the Planck brane and the visible brane respectively. The appearance of $T_{\mu\nu}^a$ (matter energy momentum on the 'a' brane) in the field equations on the 'b' brane, inspired the usage of the term 'quasi-scalar-tensor theory' in this context [14]. Assuming $T_{\mu\nu}^a = 0$ we have a usual scalar-tensor theory. We will assume no matter on the 'a' brane in our future discussion.

Note the scalar Φ which appears in the equations. Φ is known as the radion field and it is a measure of the distance between the two branes. Φ depends on the brane coordinates which are collectively referred to as x . We have

$$\Phi(x) = e^{2\frac{d(x)}{l}} - 1 \quad (2)$$

where $d(x)$ is the proper distance between the two branes given as

$$d(x) = \int_0^l e^{\phi(x)} dy \quad (3)$$

with ϕ appearing in the five dimensional line element

$$ds_5^2 = e^{2\phi(x)} dy^2 + \tilde{g}_{\mu\nu}(y, x^\mu) dx^\mu dx^\nu \quad (4)$$

The scalar radion satisfies the field equation

$$\nabla^\alpha \nabla_\alpha \Phi = \frac{\bar{\kappa}^2}{l} \frac{T^a + T^b}{2\omega + 3} - \frac{1}{2\omega + 3} \frac{d\omega}{d\Phi} (\nabla^\alpha \Phi) (\nabla_\alpha \Phi) \quad (5)$$

with T^a , T^b being the traces of energy momentum tensors on Planck ('a') and visible ('b') branes, respectively. The coupling function $\omega(\Phi)$ is expressed in terms of Φ as

$$\omega(\Phi) = -\frac{3\Phi}{2(1+\Phi)} \quad (6)$$

It is crucial to have the following physical conditions on the $\Phi(x)$ (or the $d(x)$).

- Φ is never zero.
- Φ does not diverge to infinity at any finite value of the brane coordinates.

The above conditions imply that the branes do not collide and nowhere does the brane separation become infinitely large. One may say that if the above conditions are obeyed by Φ we have a stable radion.

3. Energy conditions

It is easy to see that the field equations (Eqn. (1)) for the line element can be formally written as:

$$G_{\mu\nu} = \frac{\bar{\kappa}^2}{l\Phi} T_{\mu\nu}^b + \frac{1}{\Phi} T_{\mu\nu}^\Phi \quad (7)$$

where $T_{\mu\nu}^\Phi$ constitutes the third and fourth terms (without the $\frac{1}{\Phi}$ factor) in the R. H. S. of Eqn. (1). Recall that the Raychaudhuri equation for the expansion Θ of timelike geodesic congruences is given as:

$$\frac{d\Theta}{d\lambda} + \frac{1}{3}\Theta^2 + \Sigma^2 - \Omega^2 = -R_{\mu\nu}u^\mu u^\nu \quad (8)$$

where u^μ is the tangent vector to the central geodesic in the congruence, $\Sigma^2 = \Sigma_{ij}\Sigma^{ij}$ (Σ_{ij} is the shear), $\Omega^2 = \Omega_{ij}\Omega^{ij}$ (Ω_{ij} is the rotation) and λ is the affine parameter. We know [7,8] that geodesics focus within a finite value of the affine parameter provided $R_{\mu\nu}u^\mu u^\nu \geq 0$ (the timelike convergence condition). In General Relativity, using the Einstein field equations, the timelike convergence condition becomes the Strong Energy Condition ($T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T)u^\mu u^\nu \geq 0$). Other versions of the energy conditions include the Weak Energy Condition $T_{\mu\nu}u^\mu u^\nu \geq 0$ or the Null Energy Condition, $T_{\mu\nu}k^\mu k^\nu \geq 0$ (k^μ being the tangent to null geodesics). Such energy conditions are deemed important since they lead to physical requirements on matter. For example, the Weak Energy Condition for a diagonal energy momentum tensor reduces to the set of inequalities $\rho \geq 0, \rho + \tau \geq 0, \rho + p \geq 0$ where ρ, τ and p correspond to the energy density and the radial and tangential pressures, respectively. It can be shown that this set of WEC inequalities imply that the energy density is never negative in any frame of reference [6]. Thus, independent of the timelike convergence condition, we can assume these conditions as requirements that all known energy momentum tensors of matter must obey.

In a theory of gravity which is not General Relativity, the relation between the timelike convergence condition and the energy condition (say WEC) is not direct [15]. Let us now look at this aspect from the standpoint of the effective scalar-tensor theory we are considering. We work in the Jordan frame. We also assume that the line elements we will be considering are those for which the Ricci scalar $R = 0$. The convergence condition then becomes

$$R_{\mu\nu}u^\mu u^\nu = \frac{\bar{\kappa}^2}{l\Phi} T_{\mu\nu}^b u^\mu u^\nu + \frac{1}{\Phi} T_{\mu\nu}^\Phi u^\mu u^\nu \geq 0 \quad (9)$$

It therefore becomes possible to satisfy the convergence condition but, at the same time, have a violation of the WEC for $T_{\mu\nu}^b$. Similarly one can violate the convergence condition but still satisfy the WEC. Such freedom arises entirely due to the presence of the extra term, i.e. the effective geometric stress energy, $T_{\mu\nu}^\Phi$, due to the radion scalar.

The important question here is, what does an observer on the brane see? Obviously, such an observer in the Jordan frame, will only see $T_{\mu\nu}^b$ [15]. But the focusing, defocusing of geodesic congruences will be decided by the nature of $R_{\mu\nu}u^\mu u^\nu$. Why isn't the radion effective stress energy visible and measurable to the brane observer? The answer is similar to the motivation behind introducing a scalar field, in the original Brans–Dicke theory, where it was responsible for generating the gravitational constant G [16]. Here too, the presence of the radion signals the existence of extra dimensions and has nothing to do with the ordinary matter which is seen by the Jordan frame observer.

We will exploit the above arguments while constructing our on-brane Lorentzian wormhole spacetime.

4. The Lorentzian wormhole

It is known that, in General Relativity, static wormholes cannot satisfy the energy conditions on matter. The wormhole throat acts as a defocusing lens which leads to the violation of the time-like/null convergence condition. We shall consider here a known wormhole solution with $R = 0$ [17]. In Schwarzschild coordinates, such a wormhole is given by the line element

$$ds^2 = - \left(\kappa + \lambda \sqrt{1 - \frac{2m}{r'}} \right)^2 dt^2 + \frac{dr'^2}{1 - \frac{2m}{r'}} + r'^2 (d\theta^2 + \sin^2 \theta d\phi^2) \quad (10)$$

Note there is no horizon or singularity in this line element (i.e. g_{00} never equal to zero) as long as κ, λ are both either positive (or negative) with $|\kappa| > |\lambda|$. In our work here, we choose $\kappa > \lambda > 0$. The spatial section of the geometry is identical to that of Schwarzschild spacetime. $r' = 2m$ is the location of the wormhole throat. Using the isotropic coordinate r where $r' = r \left(1 + \frac{m}{2r}\right)^2$, the line element becomes

$$ds^2 = - \left(\kappa + \lambda \frac{1 - \frac{m}{2r}}{1 + \frac{m}{2r}} \right)^2 dt^2 + \left(1 + \frac{m}{2r}\right)^4 (dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2) \quad (11)$$

The fact that the Ricci scalar is identically zero inspires us to see if this is a viable line element in the Kanno–Soda effective theory of gravity. The general form of a spherically symmetric static line element in isotropic coordinates is assumed as,

$$ds^2 = - \frac{f^2(r)}{U^2(r)} dt^2 + U^2(r) [dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2] \quad (12)$$

where $U(r)$ and $f(r)$ are the unknown functions to be determined by solving the field equations. Using the above line element ansatz and the assumption that Φ is a function of r alone, we obtain the following field equations,

$$-2 \frac{U''}{U} + \left(\frac{U'}{U} \right)^2 - 4 \frac{U'}{Ur} = - \frac{\Phi'^2}{4\Phi(1+\Phi)} + \left(\frac{U'}{U} - \frac{f'}{f} \right) \frac{\Phi'}{\Phi} + \frac{\bar{\kappa}^2}{l\Phi} \rho \quad (13)$$

$$- \left(\frac{U'}{U} \right)^2 + 2 \frac{f'}{f} \left(\frac{U'}{U} + \frac{1}{r} \right) = - \frac{3\Phi'^2}{4\Phi(1+\Phi)} - \frac{U'}{U} \frac{\Phi'}{\Phi} - \frac{2\Phi'}{\Phi r} - \frac{f'}{f} \frac{\Phi'}{\Phi} + \frac{\bar{\kappa}^2}{l\Phi} \tau \quad (14)$$

$$\left(\frac{U'}{U} \right)^2 + \frac{f''}{f} - 2 \frac{f'}{f} \frac{U'}{U} + \frac{f'}{f} \frac{1}{r} = \frac{\Phi'^2}{4\Phi(1+\Phi)} + \frac{U'}{U} \frac{\Phi'}{\Phi} + \frac{\Phi'}{\Phi r} + \frac{\bar{\kappa}^2}{l\Phi} p \quad (15)$$

where ρ, τ and p correspond to on-brane matter and, using the tracelessness condition, we have $-\rho + \tau + 2p = 0$. We have absorbed a factor of U^2 in the definitions of ρ, τ and p .

On the other hand, the scalar (Φ) field equation becomes

$$\Phi'' + \frac{f'}{f} \Phi' + 2 \frac{\Phi'}{r} = \frac{\Phi'^2}{2(1+\Phi)} \quad (16)$$

The above equation can be integrated once to get

$$\frac{\Phi'}{\sqrt{1+\Phi}} = \frac{2C_1}{r^2 f} \quad (17)$$

where C_1 is a positive, non-zero constant. Notice that radion field equation has no contribution from on-brane matter, essentially because $T_{\mu\nu}^b$ is assumed traceless.

Further, the tracelessness requirement leads to a single equation for the metric functions, given as

$$\frac{U''}{U} + \frac{f''}{f} - \frac{f'}{f} \frac{U'}{U} + 2 \frac{f'}{fr} + 2 \frac{U'}{Ur} = 0 \quad (18)$$

For the $R = 0$ line element mentioned earlier (see Eqns. (10), (11)), and using isotropic coordinates, we have

$$f(r) = \left(1 + \frac{m}{2r}\right) \left[\kappa \left(1 + \frac{m}{2r}\right) + \lambda \left(1 - \frac{m}{2r}\right) \right] \quad (19)$$

$$U(r) = \left(1 + \frac{m}{2r}\right)^2 \quad (20)$$

We can check that the above f and U satisfy the tracelessness condition ($R = 0$) given in Eqn. (18). Earlier work on similar $R = 0$ solutions (with $f = 1$) in the context of KS effective theory can be found in [18].

To proceed we now need to know the $\Phi(r)$. With the above $f(r)$ we can obtain $\xi = \sqrt{1+\Phi}$ quite easily. This leads to

$$\xi' = \frac{C_1}{r^2(\kappa - \lambda)} \frac{1}{\left(1 + \frac{m}{2r}\right)\left(q + \frac{m}{2r}\right)} \quad (21)$$

and

$$\xi = \frac{C_1}{m\lambda} \ln \frac{2rq + m}{2r + m} + C_4 \quad (22)$$

where $q = \frac{\kappa + \lambda}{\kappa - \lambda}$. To get a well-behaved radion, we need a Φ which is never zero or infinite. The above requirement implies that we choose $q > 1$ with m, C_4 and C_1 positive and non-zero. It is useful to note that Φ can be greater than zero even if $C_4 = 0$ as long as $q > 1$ and the other constants are suitably adjusted. We will use this fact later, in this article.

If we assume $\kappa = 0$ and $\lambda = 1$, then we have Schwarzschild geometry ($q = -1$). In this case, the $\xi(r)$ turns out to be

$$\xi = \frac{C_1}{m} \ln \frac{2r - m}{2r + m} + C_4 \quad (23)$$

It is easy to see that there is always a zero (infact two zeros) of $\Phi(r)$ for Schwarzschild. The only possibility then is to take a constant Φ , which is trivial [19].

5. Checking the weak energy condition

We now need to verify the nature of the on-brane matter that threads the wormhole geometry. From the field equations given earlier (Eqns. (13), (14), (15)), it is easy to obtain the ρ, τ and p and verify the Weak Energy Condition inequalities. We shall now explicitly write down the L. H. S. of these inequalities.

$$\frac{\bar{\kappa}^2}{l} \rho = \frac{16\beta x^4}{m^2(1+x)^2(q+x)^2} [\beta + (q-1)\xi] \quad (24)$$

$$\begin{aligned} \frac{\bar{\kappa}^2}{l} (\rho + \tau) &= \frac{8x^3}{m^2(1+x)^2(q+x)^2} \\ &\times \left[4\beta q \xi + q(1+q) - q(1+q)\xi^2 \right. \\ &+ x \left(8\beta^2 + (1+q)^2 - (1+q)^2 \xi^2 \right) \\ &\left. + x^2 \left((1+q) - (1+q)\xi^2 - 4\beta \xi \right) \right] \quad (25) \end{aligned}$$

$$\frac{\bar{\kappa}^2}{l} (\rho + p) = \frac{4x^3}{m^2(1+x)^2(q+x)^2} \left[-4q\xi\beta + q(q+1)(\xi^2 - 1) + x(8(q-1)\xi\beta + (1+q)^2(\xi^2 - 1)) + x^2(4\xi\beta + (1+q)(\xi^2 - 1)) \right] \tag{26}$$

In the above expression, we have used the following re-definitions.

$$x = \frac{m}{2r} ; \quad C_1 = \alpha m ; \quad \beta = \frac{\alpha}{\kappa - \lambda} \tag{27}$$

The wormhole throat is at $r = \frac{m}{2}$ (or $r' = 2m$). Hence, the domain of x is from $x = 0$ to $x = 1$. One can check that the above stress energy is traceless.

The radion field as a function of x is given in terms of ξ where ξ is written as:

$$\xi(x) = \sqrt{1 + \Phi} = \frac{\alpha}{\lambda} \ln \frac{q+x}{1+x} + C_4 \tag{28}$$

We have $\beta = \frac{\alpha}{\kappa - \lambda}$. Defining $\frac{\kappa}{\lambda} = \nu$, we get $q = \frac{\nu+1}{\nu-1}$. Also $\beta = \frac{\alpha}{\lambda} \frac{1}{\nu-1}$. Since $\nu = \frac{q+1}{q-1}$, we get $\frac{\alpha}{\lambda} = \frac{2\beta}{q-1}$. Hence all the inequalities as well as the radion field are now defined in terms of the parameters q, β and C_4 .

We must now explicitly check the Weak Energy Condition inequalities $\rho \geq 0, \rho + \tau \geq 0$ and $\rho + p \geq 0$. To this end, we plot the graphs of these quantities for some sample values of the various parameters. A general proof for all parameters is not easy. We first note that ρ is always greater than zero, irrespective of our choice of β, q or C_4 as long as $\beta > 0, q > 1$ and $C_4 > 0$. Fig. 1 shows the plot for ρ as a function of x . For the other two inequalities, let us look at the values of the term inside the square brackets in (25) and (26), for $x = 0$. Note that the values are exactly opposite to each other. The relevant term is

$$-4q\beta\xi(x=0) + q(q+1)(\xi^2(x=0) - 1) \tag{29}$$

which, at $x = 0$, is positive in the $\rho + p$ expression and negative in the $\rho + \tau$ expression. Therefore, unless this term is identically zero at $x = 0$, there will be a violation of either inequality in the vicinity of $x = 0$. Note, at $x = 0$, by virtue of the overall factor x^3 , in the full expression, the value will be zero. But the approach to zero will be from the positive side for one inequality and from the negative side for the other.

What happens if we choose β such that this term is identically zero at $x = 0$? From the above equation, we find this sets up a relation between β^2 and q , given as,

$$\beta^2 = \frac{(q^2 - 1)(q - 1)}{4[(q + 1)(\ln q)^2 - 2(q - 1) \ln q]} \tag{30}$$

For $q = 3$ we have

$$\beta^2 = \frac{1}{(\ln 3)^2 - (\ln 3)} \tag{31}$$

With this choice for β we will have a zero value for the term $-4q\beta\xi + q(q+1)(\xi^2 - 1)$ at $x = 0$. Hence, we fix the following values for the parameters:

$$m = 1 ; \quad q = 3 ; \quad \beta^2 = \frac{1}{(\ln 3)^2 - (\ln 3)} ; \quad C_4 = 0 \tag{32}$$

We have taken the positive square root of β^2 . With the above choices, we now plot the L. H. S. of the inequalities as functions of x . Since we are plotting the L. H. S. as functions of x we must remember that infinity is at $x = 0$ and the wormhole throat (minimum r) is at $x = 1$. The domain $0 \leq x \leq 1$ covers the entire domain $\frac{m}{2} \leq r \leq \infty$.

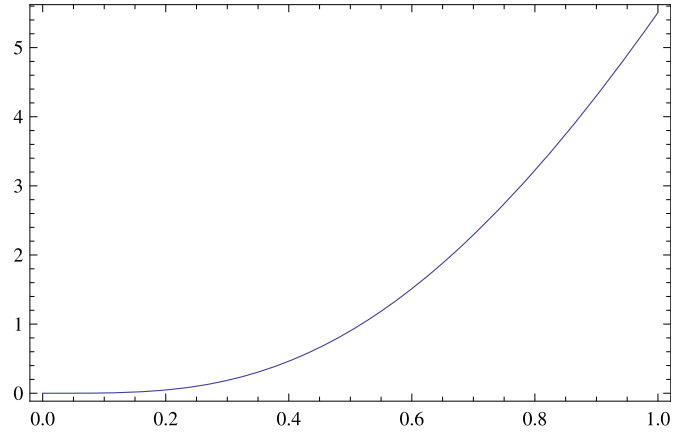


Fig. 1. ρ vs. x .

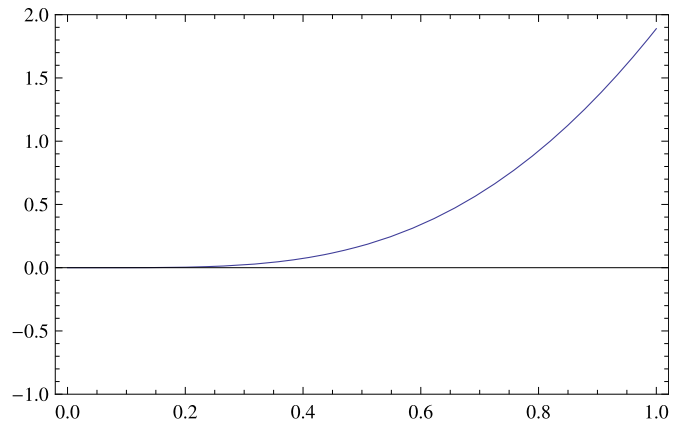


Fig. 2. $\rho + \tau$ vs. x .

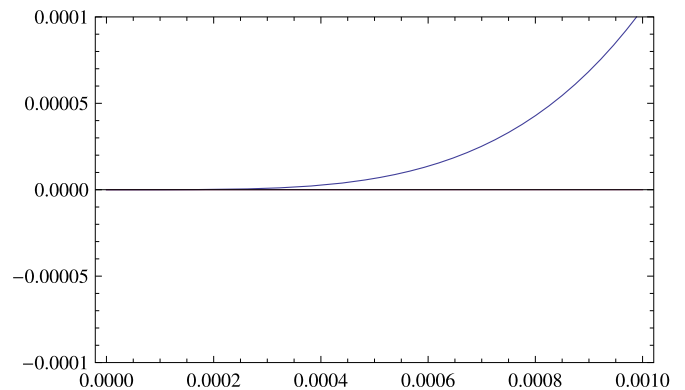


Fig. 3. $10^6(\rho + p)$ vs. x .

Fig. 2 shows a plot of $\rho + \tau$ versus x . In Figs. 3 and 4, we have plotted $\rho + p$ in various ranges and with overall constant scale factors, so that we do not miss any negativity. The radion field as a function of x is shown in Fig. 5. The radion is never zero and it stabilises to an almost constant value for large r . The relation between β and q is shown in Fig. 6.

The on-brane matter stress energy is seen to satisfy all the energy condition inequalities. This is important because wormholes are known to violate the energy conditions. It is the effective radion contribution to the total stress energy which enables the satisfaction of the Weak Energy Condition for the on-brane mat-

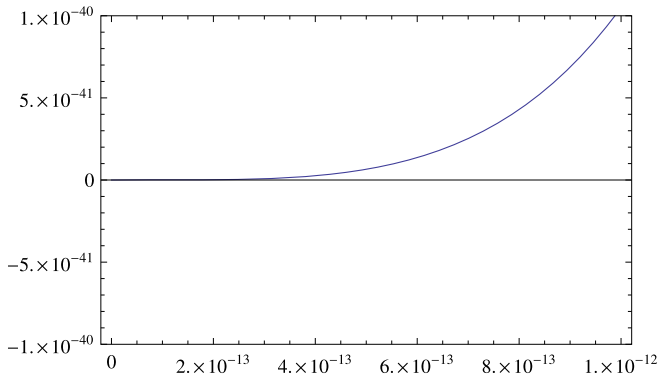


Fig. 4. $10^6(\rho + p)$ vs. x , near $x = 0$.

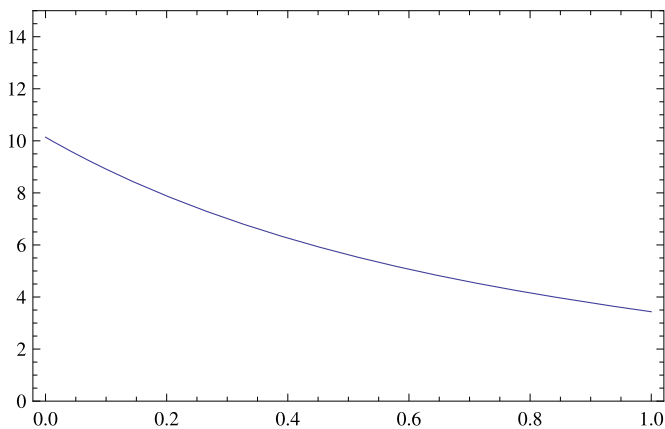


Fig. 5. Radion $\Phi(x)$ vs. x .

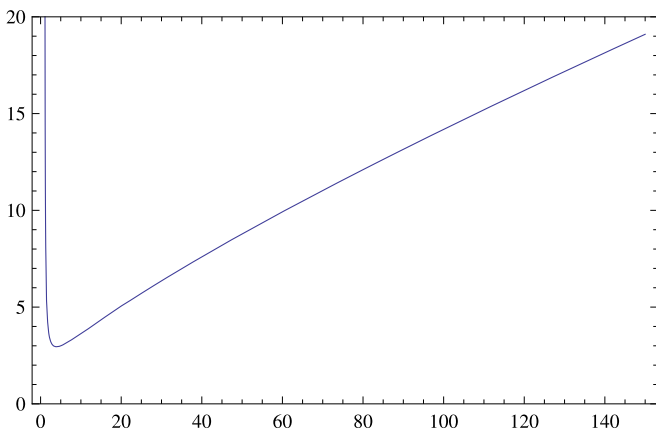


Fig. 6. β vs. q .

ter. In addition, it is also the requirement of a well-behaved radion which makes this possible.

One must note that the choice of β is very crucial. Any value of β which does not respect the relation between β and q given above will lead to a violation of either the $\rho + \tau$ inequality or the $\rho + p$ inequality. Further, one must choose $q > 1$ and also $C_4 = 0$. Fortunately, with $C_4 = 0$ we still have a well-behaved radion – the Φ never equals zero. We mention here that for $C_4 > 0$, it is possible to have a well-behaved radion but, we have checked, using plots over a wide range of values of the various parameters, that the energy conditions are indeed violated. At this point, it seems unlikely that the choice of $C_4 = 0$ carries any physical meaning,

though we will not be surprised if it does, in a way of which we are presently not aware.

6. Remarks

In summary, we emphasise that we have been able to construct an example of a wormhole which can exist on the visible brane, with matter, as seen by a Jordan frame, on-brane observer, not violating the Weak Energy Condition. It must be noted that the timelike convergence condition is indeed violated – thus, there is no conflict with the geometric conclusions that emerge from the Raychaudhuri equation for the expansion. As stated earlier, the existence of such a wormhole is made possible by the presence of the radion field which is a measure of the distance between the branes, in the two-brane Randall–Sundrum model. The radion, being an extra dimensional entity, provides an effective, geometric stress energy which enables the satisfaction of the WEC for on-brane matter. In a sense, we have succeeded in ‘transferring’ all the WEC violation into the radion stress energy, thereby ensuring that the WEC holds for the observed matter on the brane.

Several years ago, in [20], the non-local term in the single brane effective theory of Shiromizu–Maeda–Sasaki [21] was used to propose $R = 0$ wormholes. However, the work presented here is based on the Kanno–Soda two-brane effective theory which does not have any non-local term and where, the presence of the extra dimension is manifest only through the radion scalar.

In our investigations here, we have largely focused on a single family of wormhole spacetimes. Obviously, this family is not unique. One can try out various extensions of this work by looking at other possibilities with the Ricci scalar $R = 0$ and also by removing the $R = 0$ restriction. Further, given the wormhole geometry on the brane it might be useful to study the behaviour of timelike or null trajectories and derive interesting physical consequences which can provide some idea about observable signatures for such spacetimes.

Finally, in a broader perspective, the possible existence of a wormhole with non-exotic matter could be thought of as similar to *missing energies* in collider phenomenology which are expected to provide signals of the existence of extra dimensions [22]. As noted at the beginning of this article, the existence of such an on-brane wormhole without exotic matter can also be a signature of the existence of a warped extra dimension.

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