TN-64-61
July 1964

HEAT TRANSFER AND THERMAL STRESSES
IN TUBE-FOREST SLITS AND COLLIMATOR

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TABLE OF CONTENTS

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I Introduction

Further investigation to develop a slit and collimator design capable of absorbing stationary the full beam power of 2.18 MW at 20 GeV has led to a new concept. Recall that in SLAC TN-64-29, Ref. (1), two different collimator concepts were dealt with, namely a) the rotating drum-type collimator, capable of absorbing 2.18 MW beam power, and b) the stationary box-type collimator for start-up powers of 400 kW. It was shown that both devices were feasible from a heat transfer, thermal stress, and corrosion point of view, and that the lifetime of these devices was probably limited by radiation damage. The rotating drum-type collimator presented in addition some problems associated with the operation of bearings and seals in vacuum under high radiation doses. To get around these problems a third design is investigated, also capable of absorbing full beam power, in stationary operation. Recall that the power absorption of the 400 kW stationary device was limited by the wall thickness and the Z of the material. Careful investigation of materials with low Z has so far led to the acceptance of aluminum alloy. This is a good engineering material and is also quite corrosion resistant, Ref. (2). Thus, any new device capable of absorbing more power must minimize the wall thickness of the material to keep the temperature gradients down. This presents problems, since the pressure difference between the coolant and the vacuum is a necessity for functioning. Thus, the geometry remains the only variable. The thinnest possible wall thickness for a given pressure condition is obtained using the geometry of a hollow circular cylinder, i.e., a tube. Therefore, it should be possible to build a collimator using an array of tubes or what we might call

NOMENCLATURE

English Letter Symbols

- b - gap opening of collimator and slits
- C_1 ... C_4 - constants
- D - diameter
- k - thermal conductivity
- ℓ - length
- P - power
- p - pressure
- $\dot{q}'$ - heat transfer rate per unit length
- $\dot{q}''$ - heat transfer rate per unit area
- r - radius
- s_a, s_c - heat sources
- T - temperature

Greek Letter Symbols

- β - coefficient of linear thermal expansion
- ϵ - strain
- φ - angle
- σ - stress
- σ_b - standard deviation, effective beam radius

Subscripts

- a - aluminum
- c - copper
- ca - copper-aluminum interface
- hc - hot core
- i - inside
- o - outside
- th - thermal
- tot - total
- wb - water bulk conditions
- YT - yield tensile

a "tube-forest". This idea is pursued in this paper. Since the physical size of such a device should be kept as small as possible, in particular the length, variation of the tube wall thickness according to the development of the electromagnetic cascade shower is a necessity. Furthermore, as already mentioned in Ref. (1), low Z materials do not cast a dark shadow and a diffuse beam boundary results. To correct this it was suggested to add a block of copper at the end of slits and collimators, where power deposition allows introduction of a higher Z material. Therefore, one possibility to build a collimator would be to use an array of aluminum tubes, tailored in thickness according to shower development and terminated with a solid copper block. A variation of this concept was adopted in this paper. It consists again of an array of aluminum tubes, all tubes having the same original wall thickness. The tubes would have copper electrodeposited on the outside (vacuum interface) in varying thicknesses, according to shower development. Due to the high thermal conductivity the temperature rise across the copper would be small, but at the same time a relatively large number of equivalent radiation lengths of material could be introduced over a short distance. This method will minimize physical length and at the same time reduce the danger of overheating the copper block due to misaligmment of the collimator.

II Heat Transfer

For wall thicknesses small compared to the beam size we can assume uniform heat source distribution. Then the temperature distribution in a composite copper-aluminum tube is given for steady state condition by:

$$\begin{cases} \dfrac{d^2T}{dr^2} + \dfrac{1}{r}\dfrac{dT}{dr} + \dfrac{s_c}{k_c} = 0 & r_{ca} \leq r \leq r_o \quad (1) \\[2ex] \dfrac{d^2T}{dr^2} + \dfrac{1}{r}\dfrac{dT}{dr} + \dfrac{s_a}{k_a} = 0 & r_i \leq r \leq r_{ca} \quad (2) \end{cases}$$

Boundary conditions

$$@\ r = r_i \qquad T = T_i \qquad (3)$$

$$@\ r = r_{ca} \qquad T = T_a = T_c = T_{ca} \qquad (4)$$

$$@\ r = r_{ca} \qquad q' = k_c 2\pi r_{ca}\left(\frac{dT}{dr}\right)_{r_{ca}} \qquad (5)$$

$$= \pi\,(r_o^2 - r_{ca}^2)s_c$$

$$@\ r = r_i \qquad q' = k_a\ 2\pi r_i\left(\frac{dT}{dr}\right)_{r_i}$$

$$= \pi\left(r_{ca}^2 - r_i^2\right)s_a + \pi(r_o^2 - r_{ca}^2)s_c \qquad (6)$$

The general solution to Eq. (1) is

$$T = C_1 \ln r - \frac{s_c}{4k_c} r^2 + C_2 \qquad (7a)$$

and Eq. (2) yields

$$T = C_3 \ln r - \frac{s_a}{4k_a} r^2 + C_4 \qquad (8a)$$

Using B.C. Eq. (3) and Eq. (8a), one obtains

$$C_4 = T_i - C_3 \ln r_i + \frac{s_a}{4k_a} r_i^2$$

and have

$$T = T_i + C_3 \ln\left(\frac{r}{r_i}\right) - \frac{s_a}{4k_a}(r^2 - r_i^2) \tag{8b}$$

In the same way one obtains, using B.C. Eq. (4) and Eq. (7a)

$$T = T_{ca} + C_1 \ln\left(\frac{r}{r_{ca}}\right) - \frac{s_c}{4k_c}(r^2 - r_{ca}^2) \tag{7b}$$

Differentiating Eq. (7b) and substituting B.C. Eq. (5) yields

$$C_1 = \frac{s_c}{2k_c} r_o^2$$

and Eq. (7b) writes as

$$T = T_{ca} + \frac{s_c}{2k_c} r_o^2 \ln\left(\frac{r}{r_{ca}}\right) - \frac{s_c}{4k_c}(r^2 - r_{ca}^2) \tag{7c}$$

and the maximum temperature drop across the copper is

$$\Delta T_c = \frac{s_c}{2k_c} r_o^2 \ln\left(\frac{r_o}{r_{ca}}\right) - \frac{s_c}{4k_c}(r_o^2 - r_{ca}^2) \tag{7d}$$

Next use B.C. Eq. (6) and Eq. (8b) and C_3 is obtained to be

$$C_3 = \frac{1}{2k_a}[s_a r_{ca}^2 + s_c(r_o^2 - r_{ca}^2)]$$

which results in

$$T = T_i + \frac{1}{2k_a}[s_a r_{ca}^2 + s_c(r_o^2 - r_{ca}^2)] \ln\left(\frac{r}{r_i}\right) - \frac{s_a}{4k_a}(r^2 - r_i^2) \tag{8c}$$

and the maximum temperature drop across the aluminum is

$$\Delta T_a = \frac{1}{2k_a}[s_a r_{ca}^2 + s_c(r_o^2 - r_{ca}^2)] \ln\left(\frac{r_{ca}}{r_i}\right) - \frac{s_a}{4k_a} r_{ca}^2 - r_i^2 \tag{8d}$$

and the total temperature drop across the tube is

$$\Delta T_{tot} = \frac{1}{2k_a}\{s_a r_{ca}^2 + s_c(r_o^2 - r_{ca}^2)\}\ell n\left(\frac{r_{ca}}{r_i}\right) - \frac{s_a}{4k_a}(r_o^2 - r_i^2)$$

$$+ \frac{s_c}{2k_c} r_o^2 \,\ell n\left(\frac{r_o}{r_{ca}}\right) - \frac{s_c}{4k_c}(r_o^2 - r_{ca}^2) \tag{9}$$

The heat transfer rate is readily obtained from B.C. Eq. (5)

$$\dot{q}_c'' = \frac{\dot{q}_c'}{2\pi r_{ca}} = \frac{s_c}{2r_{ca}}(r_o^2 - r_{ca}^2) \tag{10}$$

and at the aluminum-water interface from B.C. Eq. (6)

$$\dot{q}_{tot}'' = \frac{\dot{q}_c' + \dot{q}_a'}{2\pi r_i} = \frac{s_a}{2r_i}(r_{ca}^2 - r_i^2) + \frac{s_c}{2r_i}(r_o^2 - r_{ca}^2) \tag{11}$$

III Description of Apparatus and Characteristic Dimensions

As already indicated in Section I, the basic structural component is the tube. The tube diameter of $D = 1.92$ cm (≈ 0.75 inch) was set with two questions in mind:

a) What tube diameter compared to the beam diameter is necessary to reduce the temperature differences in circumferential direction, i.e., to maintain

$$\frac{T(r,\varphi)}{T(r,180+\varphi)}$$ close to unity?

b) How can the total number of tubes be minimized in order to reduce the cost for production of the unit?

The wall thickness of the aluminum tube was selected to be the standard 0.035 inch. The tubes will have copper electrodeposited on the outside (vacuum side), tailored in thickness according to the heat transfer requirements imposed by the electromagnetic cascade shower development.

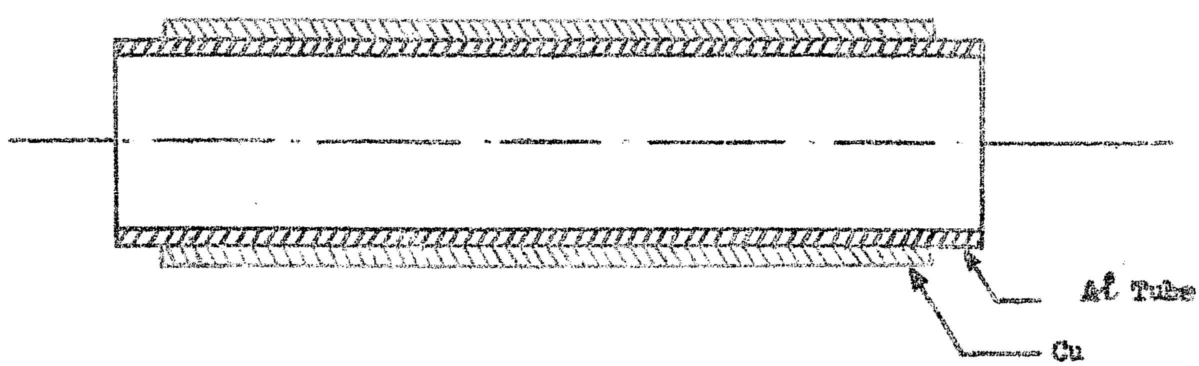

Fig. 1. Aluminum Tube with Copper Deposition.

Since it is desirable to defocus the beam as much and as fast as possible and since low Z materials spread the beam more than high Z materials, if expressed in actual length (not in rℓ), one introduces first a number of tube columns with no copper deposited, up to past the shower maximum. (Note, rows are referred to as being parallel to the beam's axis and columns as being normal).

A total of 95 columns of plain aluminum tubing defocus the beam enough and bring the beam past the shower maximum. From column 96 to 190 copper is added in varying thicknesses, increasing from 96 to 190. Column 96 has a 0.001 inch thick copper deposition, column 190 one of 0.400 inch, see Table 1. The minimum possible radiation length in the system is 31.05, obtained by adding up the partial rℓ of Cu, Aℓ and H_2O for a line through the tube centers of a row, see Fig. 2. As can be seen from Fig. 2, any other line, particularly the one through the wall on the periphery of the tubes and parallel to the row axes will give more rℓ per unit of actual length. Figure 2 gives a distribution of rℓ introduced by 1 tube unit, for various wall thicknesses and copper deposition thicknesses.

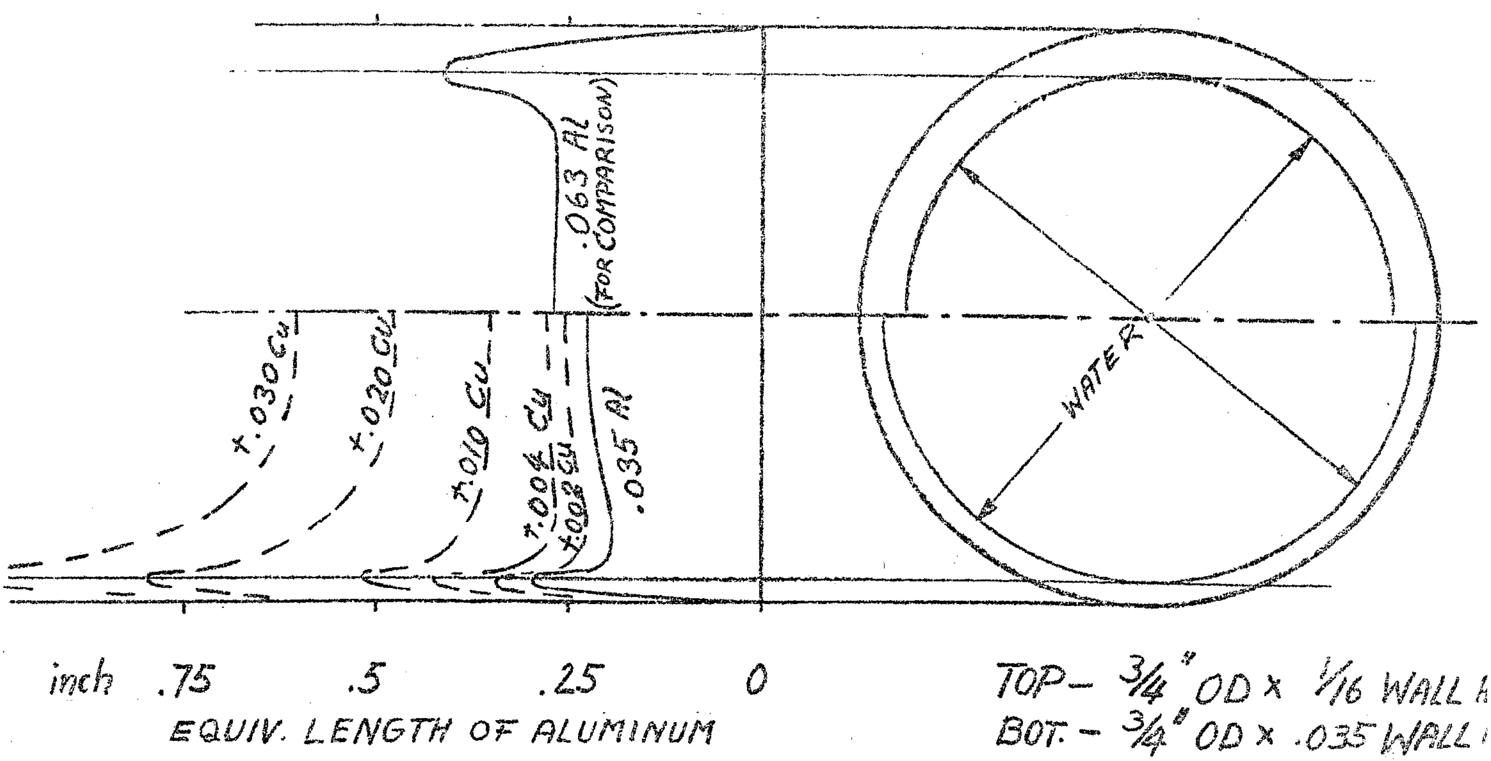

Fig. 2. Tube Configurations and Length Equivalents.

The present design calls for a tube length of 1 ft. The copper is deposited uniformly over the whole tube surface, except for a narrow band on either tube end, 0.5" wide. The actually usable length, i.e., the length over which the beam can impinge on the tube is 7 inch (≈ 18 cm).

It is suggested that the collimator should be indexed within this range on a short-time interval basis and according to ergs/gr deposited in a particular position. This way radiation damage to the material would be more or less uniformly spread over the whole usable range, and when the material finally fails at one location, it is about to fail over the whole 7-inch length.

A 4.5 inch high flow header adds to the height on either side of the tube sheet. The depth of the tube forest, i.e., the number of rows, is presently set to be 8 for both collimator and slit. This figure is conservative wirh respect to the collimator requirements, but in the case of the slits one may want to add another row of tubes, depending on the beam profile.

The spacing of the tubes is chosen to be 1/32" minimum distance to allow for good pump-out of the vacuum system. This will result in a depth of about 6.5 inch (≈16.5 cm) and a length of ≈ 13.6 ft (≈418 cm).

For fabrication purposes, it is suggested that the unit is assembled from modular sections, which are made individually and bolted to a strong-back, such as an I-beam or a box. The box could contain water, which would act as moderator for existing neutrons.

Since aluminum suffers from pitting corrosion in stagnating water, provision for complete drainage of the unit in the case of a long shut-down has to be made. To accomplish this, the 4 jaws of the collimator will operate under 45° to the horizontal and vertical.

The minimum allowable maximum gap opening is according to established criteria for the collimator b = 10 cm, and for the slits b = 15 cm.

The jaws open symmetrically away from the beam centerline with a drive that holds them within .005" of symmetry. Adjustments on the base move the vacuum envelope and jaws in the vertical and horizontal directions. These motions permit centering the jaws on the beam and also the exposure of different areas of the tubes to radiation damage.

IV COMPUTATIONAL RESULTS

The data used to evaluate actual temperatures and heat transfer rates were extracted from Guiragossian, Ref. (3). The equations developed in Section II were programmed and a digital computer was used for the computation.

The treatment is conservative. Only a hot core of the beam is analyzed, which is sufficient to show that the device, i.e., the tube, will operate under full beam power. The size of this hot core is arbitrarily defined to extend from r=0 to that radius where the power deposition is 80% of the peak value occurring at r=0, and evaluated at the shower maximum. In the case of a chosen beam size of σ_b=0.3 cm, this corresponds to r_{hc}=0.26 cm and for a σ_b=0.1 cm beam, one obtains r_{hc}=0.1 cm. Guiragossian, Ref. (3), gives the radial shower development only at 3 locations. Since one wants to tailor the tube size according to shower development, additional values were obtained by interpolation and extrapolation. Figure 3 is the tool to obtain the desired values. It gives (Percent of Power Deposition/Unit Length) vs Radiation Length for σ_b=0.3 cm and aluminum (for r_{hc}=0.26). For convenience' sake, all materials are expressed in terms of equivalent $r\ell$ of aluminum. It should be pointed out that this gives conservative values in the front section up to past the shower maximum due to the presence of water with a lower Z than aluminum. With the introduction of copper the opposite would be true if it were not for the fact that with increasing depth the slope of a similar copper curve approaches that of the given aluminum curve. With respect to beam size and radial energy distribution, one should point out that at the beginning of the shower development a significant dependence on σ_b exists. For increasing depth, these factors become less important and scattering and shower spread dominate. For an equal size of the above-mentioned hot core the two curves approach each other.

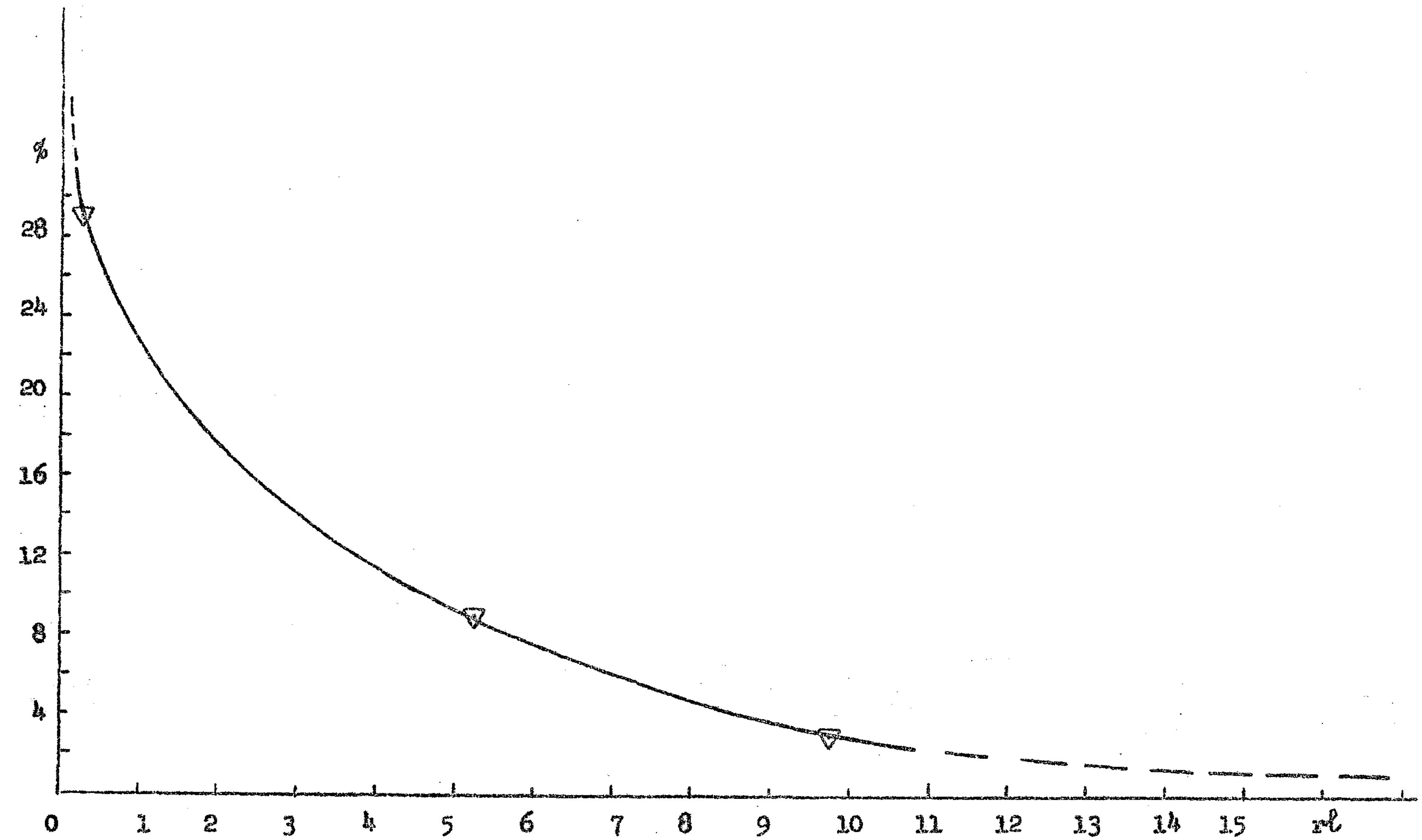

Fig. 3. Percent of Total Power/Unit Length Deposited within r_{hc} = 0.26 cm.

For P=2.1MW and σ_b=0.3 cm, one obtains in the front section at the shower maximum and for no copper deposition a maximum possible temperature rise across the tube wall of $\Delta T_{metal}=43^{\circ}C$. This is for the case where the beam's axis is either tangential to the tube or intersects it. The heat transfer rate at the aluminum-water interface is for this particular case $\dot{q}''=1.37\ kW/cm^2$.

A comparison of the tube size to the radial power distribution shows that conditions supporting boiling heat transfer mechanism prevail over the whole circumference of the tube at the axial location of beam impingement.

The effective temperature for the maximum stress development is then based on p=10 psig and $T_{wb}=40^{\circ}C$

$$\Delta T_{tot} = \Delta T_{metal} + \Delta T_{film}$$

$$= 43 + 74 = 117^{\circ}C.$$

The maximum metal temperature on the vacuum interface of the tube is then $157^{\circ}C \approx 315^{\circ}F$.

The yield strength for aluminum alloy 5052-H32* at $315^{\circ}F$ is

$$\sigma_{YT} \approx 25{,}000\ psi$$

The maximum possible compressive stress in this constrained tube is

$$\sigma_{th} = E\beta\Delta T_{tot} = 10.2\times10^{6}\times24\times10^{-6}\times117$$

$$= 28{,}500\ psi.$$

Note: This is a principal stress and the actually effective stress is a few percent lower.

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*Recent investigations have shown that 5052 is superior to 6061 with respect to vacuum tight welds. A note summarizing this investigation is under preparation.

Longitudinal conduction effects and the fact that the tube under consideration is not really fully constrained will reduce this stress substantially, possibly as much as 40%. There is no problem from a stress point of view. Even if the stress should be higher than σ_{YT} for the case of $P > 2.1MW$ and/or $a_p < 0.3$ cm, small local yielding will cause no immediate danger for the structure. After shut-off of the power, the net result would be a residual tensile stress. Another temperature rise of the same amount would in the most result in a compressive stress of the magnitude of the yield strength. The temperature fluctuations due to the pulsing nature of the beam cause only small deviations from the equilibrium temperature and from the steady state stress level in the case of aluminum and can therefore be neglected for thermal fatigue considerations. The cycling due to turn-on and shut-off of the machine has to be considered. The number of cycles, however, is small from a fatigue point of view. It will probably not exceed a few thousand per year and they are more or less uniformly distributed over the whole 7" length. The problem therefore is not really governed by fatigue characteristics but by usual yield and tensile properties of the material.

A location of considerable interest in addition to the shower maximum is at the very beginning of the shower development, i.e, the first few tubes. At that location the beam has hardly spread yet and $T(r, \phi)/T(r, 180 + \phi)$ is far from being unity.

If the beam's axis, for example, is tangent on the tube, one obtains at that location $\Delta T_{metal} \approx 30^\circ C$ and a stress effective temperature difference might be $\Delta T_{tot} \approx \Delta T_{metal} + \Delta T_{film} \approx 90^\circ C$. The opposite side of the tube is essentially at bulk water temperature. In the case of a 1 ft long tube, fixed on either end in a tube sheet, there might be a tendency for buckling (one of the Eulerian buckling cases). It is of considerable interest to know how much buckling one has to expect for $\Delta T \approx 90^\circ C$, since the tube could bend into the beam and increase the momentum spread $\Delta p/p$. The strain for an unrestrained tube and ΔT=90° would be on the hot side

$$\epsilon = \beta\Delta T = 24\times10^{-6}\times90$$
$$= 2.16\times10^{-3} \text{cm/cm.}$$

The resulting increase in length of a longitudinal fiber on the side of the tube might be

$$\Delta\ell = 1.3\times10^{-3}\text{cm} (\approx 5\times10^{-4} \text{inch}).$$

$\Delta\ell$ for the opposite tube side is essentially zero. The resulting buckling is negligible for all practical purposes. It is much less than fabrication tolerances for the collimating face of a jaw.

Next, let us consider the case where σ_b=0.1 cm. The tube sizes shall be the same as above. The highest temperature rise across the tube wall increases from ΔT_{metal}=43°C to $\Delta T_{metal}\approx$140°C. Addition of $T_{film}\approx$120°C results in T=260°C V. ($\approx$500°F). The yield strength of 5052-H32 is down to $\sigma_{YT, 500^\circ F}\approx$7,500 psi and aluminum is not a suitable engineering material at such temperatures. A few pulses will not cause any damage but for continuous operation one must reduce the beam power, i.e., the rep-rate.

A situation comparable to σ_b=0.3 cm and P=2.18MW would be for σ_b=0.1 P≈1MW, i.e., a rep-rate of 150 to 180 pulses/sec at 20 GeV and 50% beam loading.

The present tube size distribution, heat transfer rates and temperature rises across the tubes based on an incident beam of σ_b=0.3 cm and P=2.18MW are summarized in Table 1. Note that the radiation lengths are based on a line through the centers of the first row of tubes.

TABLE 1 - SUMMARY OF RESULTS

Tube No.	Cu Deposition	Eq. Rad. Length	Total Length	Totl Heat Transf Rate	ΔT_{metal}
--	Inch	cm/Al	cm	KW/cm²	°C
15	0.0	0.984	30.8	0.424	13.2
30	0.0	1.969	61.7	0.916	28.5
45	0.0	2.954	92.6	1.213	37.8
55	0.0	3.611	113.1	1.360	42.4
65	0.0	4.268	133.7	1.366	42.6
75	0.0	4.924	154.3	1.283	40.0
85	0.0	5.581	174.9	1.148	35.8
95	0.0	6.237	195.5	0.942	29.3
96	0.001	6.307	197.5	1.144	41.6
105	0.001	6.936	216.1	0.870	31.9
106	0.002	7.009	218.2	0.978	38.5
115	0.002	7.662	236.8	0.776	30.7
116	0.004	7.742	238.9	0.997	44.4
125	0.004	8.459	257.6	0.687	30.9
126	0.006	8.545	259.6	0.844	40.9
135	0.006	9.325	278.4	0.564	27.5
136	0.008	9.418	280.5	0.673	34.6
145	0.008	10.261	299.4	0.461	23.7
146	0.012	10.368	301.5	0.612	33.7
155	0.012	11.336	320.6	0.375	20.7
156	0.020	11.473	322.8	0.573	31.7
160	0.020	12.021	331.4	0.366	22.0
161	0.030	12.193	333.6	0.521	33.6
165	0.030	12.88	342.5	0.400	25.8
166	0.050	13.123	344.8	0.651	46.6
170	0.050	14.095	354.0	0.318	22.7
171	0.070	14.409	356.4	0.447	34.8
173	0.070	15.038	361.3	0.335	26.1
174	0.100	15.458	363.8	0.487	42.2
179	0.100	17.560	376.6	0.206	17.7
180	0.200	18.336	379.7	0.449	50.6
183	0.200	20.661	388.9	0.068	7.6
184	0.400	22.146	393.0	0.163	25.8
190	0.400	31.054	417.6	~ 0.0	~ 0.0

V Conclusion

It was shown that a stationary operating collimator can be built which will absorb 2.2MW continuously, provided that $\sigma_b \geq 0.3$ cm. In the case of $\sigma_b = 0.1$ cm the power has to be decreased to about 1MW in order to obtain conditions comparable to $\sigma_b = 0.3$ cm. The basic structural component is a thin-walled aluminum tube, water-cooled on the inside. A large number of tubes are assembled in what is called a "tube-forest". Aluminum alloy 5052-H32 is the suggested material. To keep the length short and also to cast a dense, dark shadow the aluminum tubes in the rear section will have copper electrodeposited on the outside. The thickness of the copper is determined by the electromagnetic cascade shower development and heat transfer limitations. The expected lifetime of a unit is probably several years, unless radiation damage to the aluminum turns out to be more severe than presently assumed.

References

1.) "Heat Transfer and Thermal Stress Analysis of Slits and Collimators", D.R. Walz, SLAC TN-64-29, March, 1964.

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