

Evolution of Collisional Matter in Modified Teleparallel Theories

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Abstract. Here, we discuss the cosmic evolution in the presence of collisional matter (CM) with and without radiations within the framework of modified teleparallel theories. We opt $f(T, B)$ theory (where T stands for torsion scalar and B represents the boundary term associated to the divergence of torsion $2\nabla_\mu T^\mu = B$), which makes a good connection between $f(R)$ (R for Ricci Scalar) and $f(T)$ (T for torsion) theory under reasonable conditions. The power law and logarithmic $f(T, B)$ models are selected to discuss the behavior of deceleration parameter $q(z)$, Hubble parameter $H(z)$, Equation of state (EoS) for dark energy (DE), and effective EoS. We found the great oscillations of EoS for DE across the phantom divide line. Effective EoS also crossed the phantom divide line without any oscillations. The graphs for $H(z)$, $q(z)$, effective EoS are alike for NCM with radiations, CM without radiations and CM with radiations.

1. Introduction

Now a days, it's widely known fact that the cosmos is passing through the phase of accelerated expansion [2,3] which is confirmed by different observational schemes like, Supernovae SNeIa. Temperature anisotropies, baryon acoustic oscillation (BAO) [4] and cosmic microwave background (CMB) [5] given the strength to this cosmic acceleration. To explain this phenomena, we have two ways [6-14], first one is that, we consider the DE dominated universe which is accountable for this expansion due to negative pressure. In second approach by ignoring the DE, we amend the general relativity (GR) action as given by Einstein-Hilbert. Second approach leads us to the modified theories like $f(R)$ [15] (where R is Ricci scalar), $f(T)$ [16] (where T is torsion), $f(R, T)$ [17-25] (where R is Ricci scalar and T is trace of energy momentum tensor (EMT)), $f(T, B)$ [26,27], $f(R, T, R_{\mu\nu}T^{\mu\nu})$ [28-30] (where $T^{\mu\nu}$ is for EMT and $R_{\mu\nu}$ is Ricci tensor) and $f(R, G)$ [31-39]. DE can also be considered to be geometric component, specially in modified theories of gravity (MTG) because when we make the comparison of field equations in GR and MTG then LHS remains invariant. Recently, after the late time cosmic acceleration, we got confirmation about inflationary era [40,41] at early universe observed through B-mode power spectrum [42]. Different observational, experimental and indirect methods are introduced in the literature to show that how our universe develop according to cosmological time. The observations from the spectrum of direct DM scattering [43] was the indirect method to check inflationary era. In [44] authors observe the DM directly in the ground base laboratory. Therefore, to describe these two phenomena, we need a single theoretical framework. Observational data sustain the non zero cosmological constant ($\omega = -1$) [45] as well as flat universe ($p = 0$). Most recent Planck's data [46] also sustain the Λ CDM model.

Einstein introduced GR with source of gravity being scalar curvature but he also put in the concept of torsional formalism [47-50] which is noted as teleparallel equivalent of GR (TEGR). Lately, TEGR has been generalized as a function of torsion T known as $f(T)$ gravity [16]. Harko et al. [51] discussed the non-minimal interaction of torsion with matter in the Lagrangian density which leads to the more generalized $f(T)$ gravity. Authors [52,53] have also studied the efficacy of energy bounds for significant models and find the viable constraints on the associated free parameters and also explored the validity of thermodynamic laws. Kofinas and Saridakis [54-56] formulated a interesting and new theory noted as $f(T_G)$ gravity and then its extended form $f(T, T_G)$ theory. Zubair and Waheed also discussed the cosmological importance and energy conditions of this theory [57,58]. Further, Sebastian et al. [26] generalized the $f(T)$ theory by replacing the Lagrangian $f(T)$ with $f(T, B)$. One remarkable aspect of this theory is that, if we take particular form of Lagrangian like $f(-T + B)$ [26] then this theory has a good equivalence with $f(R)$ gravity. The big puzzle for cosmologists is that, how universe progress from decelerated phase [59] to accelerated phase? As we know that, the acceleration of universe is low that build two phases first is redshift transition and it's current value is $z_t = 0.46 \pm 0.13$ [9,10]. This transition from deceleration to acceleration is denoted by z_t . It is prime to be in touch with both the universe inflationary era and the late time acceleration.

MTG play a key role to discuss these both phenomena in a single theory by considering the CM. A very convenient question comes to mind that, why we have to study this CM and what are the effects of this CM? To explain this it is fascinating enough that we have strong evidence which shows the dark matter (DM) behaves like CM [60] at some extent. This is the reason that DM directly effects the dynamics of cosmos. Experiments like WMAP survey, PAMELA and ATIC showed that the production of electron-positron from supernova [60] is less than the actual production in the universe. This collisional nature of DM can also be found in direct searches of DM [44]. Most recent Planck's data [46] also confirmed that our universe made up of 26.8% DM which is very significant amount, DE contributes the 68.3% and visible matter is just accounts for 4.8% therefore, we must go for DM or CM in some extent. In this direction, Oikonomou and Karagiannakis [61,62] studied the CM in $f(R)$ gravity as well as they discussed the matter dominated era in the exponential MTG and discussed the DE oscillatory effects. They found, this study is purely model based. Zubair [63] also discussed the significant models of $f(T)$ theory by considering the CM and he showed the transition phase in his study for CM. Baffou et al. [64] also studied the CM in more generalized $f(R, T)$ gravity and they found the good consistency with Λ CDM model and recent observations. In recent work [65], Zubair et al. discussed the comic evolution of non-minimal coupled $f(R, T)$ models in collisional framework. It was found that cosmic evolution in the presence of CM was fairly similar to that of NCM case, but they found different transition points for CM and NCM. Furthermore, they [65] also found the variance in the acceleration rate for CM(with and without radiations) and NCM case.

In teleparallel $f(T, B)$ gravity, Sebastian et al. [1] reconstructed the power law, logarithmic, and exponential models and they discussed the thermodynamics in this theory. They found equilibrium description of thermodynamics and they found that GSLT is valid for phantom era of universe. In this paper, our main focus is to discuss the constructed [1] power law model and logarithmic model for self interacting CM and radiations. We will make a comparison for evolution of deceleration parameter $q(z)$, Hubble parameter $H(z)$, EoS for DE ω_{DE} , and effective EoS ω_{eff} in the presence of NCM with radiation, CM without radiation and CM with radiations. We will study the phantom crossing behavior for these models in the presence of CM. This article has the following pattern: In section **II**, we will study the basic formalism of $f(T, B)$ gravity and its modifications. In section **III**, we will discuss the CM in $f(T, B)$ gravity and in section **IV**, we shall discuss the cosmic evolution of $H(z)$, $q(z)$, ω_{DE} , and ω_{eff} for NCM, and CM with and without radiation. Section **V** summarizes our results.

2. TEGR and its modifications

Now, we are going to discuss the basis of TEGR. The vital variable in this theory is given by the tetrad e_m^μ and E_m^μ is the inverse of the tetrad. This theory depends on the manifold which has a non-zero torsion but zero curvature. For the justification of this type of geometry, we have to make assumption that universe is globally flat, commonly known as Weitzenböck connection $W_\mu{}^a{}_\nu$. This another concept of gravity has an important fact that, the field equation of TEGR has equivalence with the field equation of GR. The following equation represents the relation between metric and the dynamical variable which is tetrad field,

$$g_{\mu\nu} = e_\mu^a e_\nu^b \eta_{ab}, \quad (1)$$

where the Minkowski metric with this signature $(-, +, +, +)$ denoted by η_{ab} . Keep in mind that, at every single point of the manifold the tetrad fields are orthonormal vectors, therefore they follow the preceding relations

$$E_m^\mu e_\mu^n = \delta_m^n, \quad (2)$$

$$E_m^\nu e_\mu^m = \delta_\mu^\nu. \quad (3)$$

The anti-symmetric part of the Weitzenböck connection gives us the torsion tensor as,

$$T_{\mu\nu}^a = W_\mu{}^a{}_\nu - W_\nu{}^a{}_\mu = \partial_\mu e_\nu^a - \partial_\nu e_\mu^a. \quad (4)$$

The torsion scalar gives us the teleparallel action as

$$S^{abc} = \frac{1}{4} \left(T^{abc} - T^{cab} - T^{bac} \right) + \frac{1}{2} \left(\eta^{ac} T^b - \eta^{ab} T^c \right), \quad (5)$$

with the torsion tensor $T = S_a{}^{bc} T_{bc}^a$. The contraction of torsion tensor $T_\mu = T^\nu{}_{\nu\mu}$ gives us the torsion vector.

Explicitly, the action reads

$$S_{\text{TEGR}} = \frac{1}{\kappa^2} \int e T d^4x + S_m, \quad (6)$$

where $\kappa^2 = 8\pi G$, and e stands for the determinant of the tetrad which is equal to $\sqrt{-g}$ and S_m represents the action for matter content. After discussing the above equation we are able to manifest that the torsion scalar is connected with Ricci scalar by

$$R = -T + \frac{2}{e} \partial_\mu (e T^\mu) = -T + B, \quad (7)$$

The only difference between Einstein-Hilbert action and TEGR action is boundary term “B”. Therefore, we got equivalence between the metric variations of the Einstein-Hilbert action and tetrad variations of the action (6). So, field equations shall be alike for both the action (6) and the Einstein-Hilbert action.

A significant generalization of the action (6) is made by replacing the T term with the function of T . This generalization is given below

$$S_{f(T)} = \frac{1}{\kappa^2} \int e f(T) d^4x + S_m, \quad (8)$$

the replacement of T with $f(T)$ makes it much stronger than before, that is actually second order theory. Now, this theory is alike $f(R)$ gravity. Nevertheless, these theories are not equal. For

the sake of the unification of both $f(T)$ and $f(R)$ gravity, authors [26] suggested the following action

$$S_{f(T,B)} = \frac{1}{\kappa^2} \int d^4x e f(t, B) + S_m, \quad (9)$$

this is the most generalized teleparallel gravity, and $f(T, B)$ has also dependence on boundary term B . If, we aim to reach the well noted theories of gravity like $f(R)$ we just simply need to replace the arbitrary function as $f = f(B - T) = f(R)$ and for $f(T)$ gravity just replace it with $f = f(T)$ as mentioned in [26]. The field equations of this generalized theory are found to be as by varying the action with respect to tetrad

$$\begin{aligned} 2e\delta_\nu^\lambda \square f_B + eB f_B \delta_\nu^\lambda - 2e\nabla^\lambda \nabla_\nu f_B + 4eS_\nu^{\mu\nu} [(\partial_\mu f_B) + (\partial_\mu f_T)] \\ - 4ef_T T_{\mu\nu}^\sigma S_\sigma^{\lambda\mu} + 4e_\nu^\mu \partial_\mu (eS_a^{\mu\nu}) f_T - ef\delta_\nu^\lambda = 16\pi e \mathcal{T}_\nu^\lambda, \end{aligned} \quad (10)$$

here $\mathcal{T}_\nu^\lambda = e_\nu^\mu \mathcal{T}_\mu^\lambda$ is the standard EMT, $f_T = \frac{\partial f}{\partial T}$, $f_B = \frac{\partial f}{\partial B}$ and $\square = \nabla^\mu \nabla_\mu$. Purely, this theory is tetrad formalism, and fourth-order.

3. Collisional Matter Model within $f(T,B)$ gravity

The investigated outcomes showed that the production of electron-positron from supernova [66-68] is less than the actual production in then universe. This conflict helps us to find the total destruction of Weakly interacting massive particles which are the candidate for DM and this is the collisional process as well [60]. The total matter content energy in our cosmos is less than the present energy in the universe and this problem can be solved by considering the collisional dark matter. Now, our discussion topic is the late time dynamics of the universe by considering CM. In [69] they discover the cosmic dynamics exclusively by considering CM in the Einstein gravity and realized that deceleration parameter q is not dependent of z and remains constant. Eventually, we are not able to discuss the transition phase from decelerated phase to accelerated one. The late time dynamics by taking into account CM is discussed by Oikonomou, V.K. et al. [61,62] and it was found that the CM is not the only one that can not describe the late-time acceleration but it can also amend the cosmic acceleration in $f(R)$ gravity. This discovery motivated us to the future evolution of cosmic parameters in $f(R, T)$ gravity [65]. Simply, DE EoS parameter ω_{DE} is able to cross the phantom divide line even in the presence of CM. We can see [69,70] that firstly CM was introduced and studied in GR and $f(R)$ gravity . Mostly, self interacting CM model is discussed with perfect fluid in which total mass-energy density is basic assumption, represented by ϵ_m which have the dependence on two terms that are given below

$$\epsilon_m = \rho_m + \rho_m \Pi, \quad (11)$$

where

$$\Pi = \Pi_0 + \omega \ln \left(\frac{\rho_m}{\rho_{m_0}} \right) \quad (12)$$

with ρ_{m_0} and Π_0 are constants. Although fluid is not dust like, but it has positive pressure and satisfies the EoS:

$$P_m = \omega \rho_m, \quad (13)$$

where ω represents the collisional nature of matter and range of ω is between $0 < \omega < 1$.

By putting the Equ.(12) into Equ.(11) we will get the relation for the total energy density of the cosmos as follows:

$$\epsilon_m = \rho_m \left(1 + \Pi_0 + \omega \ln \left(\frac{\rho_m}{\rho_{m_0}} \right) \right) \quad (14)$$

In the continuous medium the continuity equation for the motion of volume element is stated as:

$$\nabla^\nu T_{\mu\nu} = 0 \quad (15)$$

And the energy momentum tensor takes the following form

$$T_{\mu\nu} = (\epsilon_m + P_m) u_\mu u_\nu - P_m g_{\mu\nu}, \quad (16)$$

where $u_\mu = \frac{dx_\mu}{ds}$ is the four velocity and satisfies the equation $u_\mu u_\nu = 1$. It is noted that $P_m = p_m$ because of the negligence of the pressure of the ordinary matter. Using flat FLRW line element, Conservation equation given in (16) becomes

$$\dot{\epsilon}_m + 3\frac{\dot{a}}{a}(\epsilon_m + P_m) = 0. \quad (17)$$

Combining (13) and(14), gives the following result

$$\rho_m = \rho_{m0} \left(\frac{a_0}{a}\right)^3 \quad (18)$$

with a_0 the current scale factor.

Finally, collisional matter can be describe completely by Equ.(14) and Equ.(18). where Π_0 has the following relation

$$\Pi_0 = \left(\frac{1}{\Omega_M} - 1\right), \quad (19)$$

and its current numerical value is $\Pi_0 = 2.14169$.

There is an important case need to be discussed that our universe is filled with both CM and relativistic matter which is commonly known as radiations. To discuss this significant case ρ_{matt} is defined as following

$$\rho_{matt} = \epsilon_m + \rho_{r0} a^{-4}, \quad (20)$$

where ρ_{r0} represent the present energy density for radiation. The pressure for this case is given by

$$P_{matt} = p_m + p_r, \quad (21)$$

where p_m is the pressure for CM and p_r represents the pressure by radiations. By using the Equ.(14) and Equ.(18) we could get Equ.(20) as

$$\rho_{matt} = \rho_{m0} a^{-3} (1 + \Pi_0 - 3\omega \ln(a)) + \rho_{r0} a^{-4}. \quad (22)$$

We can also rewrite above equation (22) as

$$\rho_{matt} = \rho_{m0} (g(a) + \chi a^{-4}), \quad (23)$$

where χ has a relation of the form

$$\chi = \frac{\rho_{r0}}{\rho_{m0}}, \quad (24)$$

its numerical value is $\chi = 3.1 \cdot 10^{-4}$ and $g(a)$ represent the nature of CM and it is defined as following

$$g(a) = a^{-3} (1 + \Pi_0 - 3\omega \ln(a)). \quad (25)$$

Keep in mind that, if we will take $\omega = 0$, $\Pi_0 = 0$ in this formalism then we will get $g(a) = a^{-3}$ and its represent the NCM which is considered to be dust. As χ represents the radiations component so, if we take $\chi = 0$ then radiations will ignore and vice versa.

4. Late time cosmological evolution in $f(T, B)$ gravity

4.1. Deceleration parameter

In the following section, we will discuss the flat FLRW metric with this signature $(-, +, +, +, +)$ given as

$$ds^2 = -dt^2 + a(t)^2(dx^2 + dy^2 + dz^2), \quad (26)$$

where $a(t)$ is for scale factor. In these coordinates, the tetrad field can be studied as

$$e_\mu^a = \text{diag}(1, a(t), a(t), a(t)). \quad (27)$$

if we take a look on above equation and consider that FLRW universe is filled with perfect fluid then the field equation (10) for $f(T, B)$ gravity will be

$$-3H^2(3f_B + 2f_T) + 3H\dot{f}_B - 3\dot{H}f_B + \frac{1}{2}f(T, B) = \kappa^2\rho_m, \quad (28)$$

$$-3H^2(3f_B + 2f_T) - \dot{H}(3f_B + 2f_T) - 2H\dot{f}_T + \ddot{f}_B + \frac{1}{2}f(T, B) = -\kappa^2p_m. \quad (29)$$

Here, Hubble parameter represented by $H = \dot{a}/a$ and dots refers the time derivative. Furthermore, p_m and ρ_m are for pressure and energy density of the matter content respectively. One can simply found the values of T and B by using FLRW metric as, $T = 6H^2$ and $B = 6(\dot{H} + 3H^2)$. In the same way, we can justify $R = -T + B = 6(2H^2 + \dot{H})$. Now, Eqs. (28) and (29) can also be represented in a fluid form,

$$3H^2 = \kappa_{\text{eff}}^2\rho_{\text{eff}}, \quad (30)$$

$$-3H^2 - 2\dot{H} = \kappa_{\text{eff}}^2P_{\text{eff}}. \quad (31)$$

The equations (30) and (31) are identical to standard FLRW equations as in GR. The ρ_{eff} and p_{eff} in $f(T, B)$ gravity are found as:

$$\rho_{\text{eff}} = \epsilon_m + \frac{1}{\kappa^2} \left[-3H\dot{f}_B + (3\dot{H} + 9H^2)f_B - \frac{1}{2}f(T, B) \right], \quad (32)$$

$$P_{\text{eff}} = P_m + \frac{1}{\kappa^2} \left[\frac{1}{2}f(T, B) + \dot{H}(2f_T - 3f_B) - 2H\dot{f}_T - 9H^2f_B + \ddot{f}_B \right], \quad (33)$$

With the energy density $\rho_{DE} = \rho_{\text{eff}} - \epsilon_m$ and the pressure $P_{DE} = P_{\text{eff}} - P_m$. Conservation law with the effective energy density will be in the form

$$\frac{d(k_{\text{eff}}^2\rho_{\text{eff}})}{dt} + 3Hk_{\text{eff}}^2(\rho_{\text{eff}} + P_{\text{eff}}) = 0 \quad (34)$$

where, we have $k_{\text{eff}}^2 = -\frac{\kappa^2}{2f_T}$.

We get the following equation by solving the equation (34) and using equation (17)

$$f - 2\epsilon_m - 6f_B\dot{H} + 6H^2(-3f_B - 2f_T + 12(3f_{BB} + f_{TB})\dot{H}) + 36Hf_{BB}\ddot{H} = 0 \quad (35)$$

using redshift relation $a = \frac{1}{1+z}$, we will get equation (35) as

$$\begin{aligned} \frac{d^2H}{dz^2} &= \frac{5}{1+z} \frac{dH}{dz} - \frac{1}{H} \left(\frac{dH}{dz} \right)^2 \\ &- \frac{f - 2\epsilon_m + 6(1+z)H \frac{dH}{dz}(f_B - 12H^2f_{TB}) - 6H^2(3f_B + 2f_T)}{36(1+z)^2H^3f_{BB}} \end{aligned} \quad (36)$$

Now, we have all the required components to study the different models in the presence of CM and radiations in $f(T, B)$ gravity.

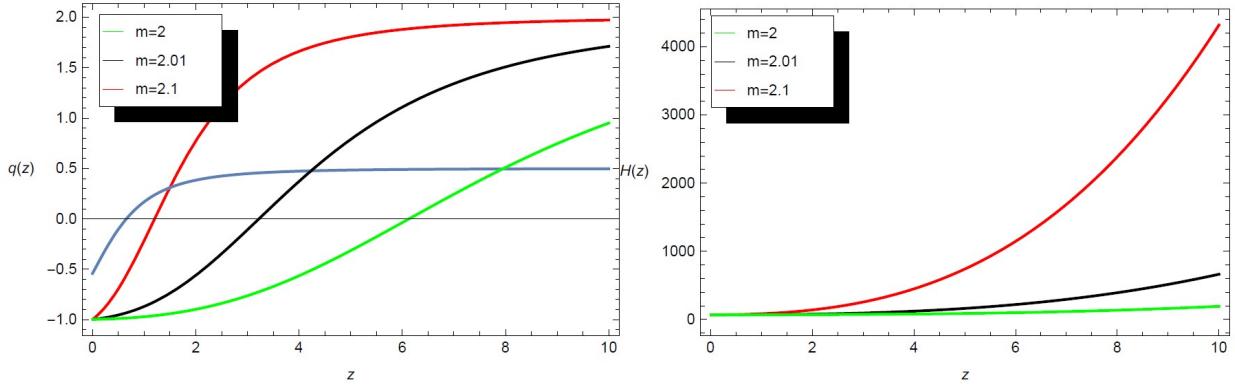


Figure 1. The LHS graph represent the evolution of deceleration parameter whereas graph on RHS represent the evolution of $H[z]$. Herein, we choose $H_0 = 68.3$, $\Omega = 0.3183$, $\alpha = 5$, $\beta = 5$, $n = -0.5$, $\omega = 0$ for NCM includes radiations, $\omega = 0.6$ for CM except radiation and CM+radiation.

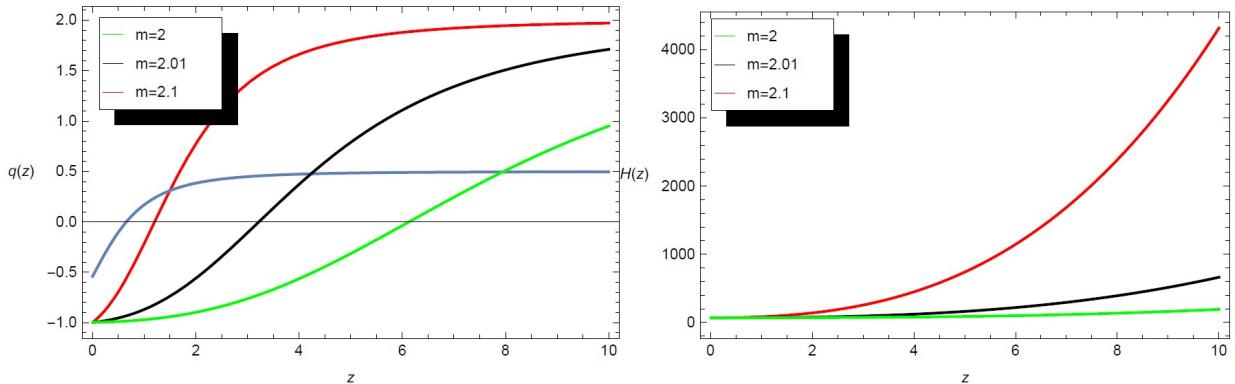


Figure 2. The LHS graph represent the evolution of effective EoS whereas graph on RHS represent the evolution of EoS for dark energy. Herein, we choose $H_0 = 68.3$, $\Omega = 0.3183$, $\alpha = 5$, $\beta = 5$, $n = -0.5$, $\omega = 0$ for NCM includes radiations, $\omega = 0.6$ for CM except radiation and CM+radiation.

4.2. $f(T, B) = \alpha T + \beta B^m T^n$

We choose $H_0 = 68.3$, $\Omega = 0.3183$, $\alpha = 5$, $\beta = 5$, $n = -0.5$, $\omega = 0$ for NCM with radiations, $\omega = 0.6$ for CM except radiations, and $\omega = 0.6$ CM includes radiations. We will numerically discuss this $f(T, B)$ model by using equation of motion (36). In Figure 1, LHS represent the evolution of deceleration parameter for all considered cases in term of redshift. Blue curve represent the standard Λ CDM model, green curve is for NCM includes radiations, black curve is for CM except radiation, red curve is for combine CM and radiations. We can clearly see from LHS of Figure 1 that each curve shows the different behavior for different cases, Green curve shows the transition phase for larger value almost at $z_t = 6.1$, black curve shows the transition phase earlier as compare to green curve which is equal to $z_t = 3.1$ and red curve shows the most closer value to observed value for redshift transition phase which is equal to $z_t = 1.1$. We can also conclude that as we increase the value of m (model parameter) then the value for redshift transition phase is more closer to observed value. In the LHS of the Figure 2, we represent the evolution of effective EoS for NCM with radiation, CM without radiation, and for radiations

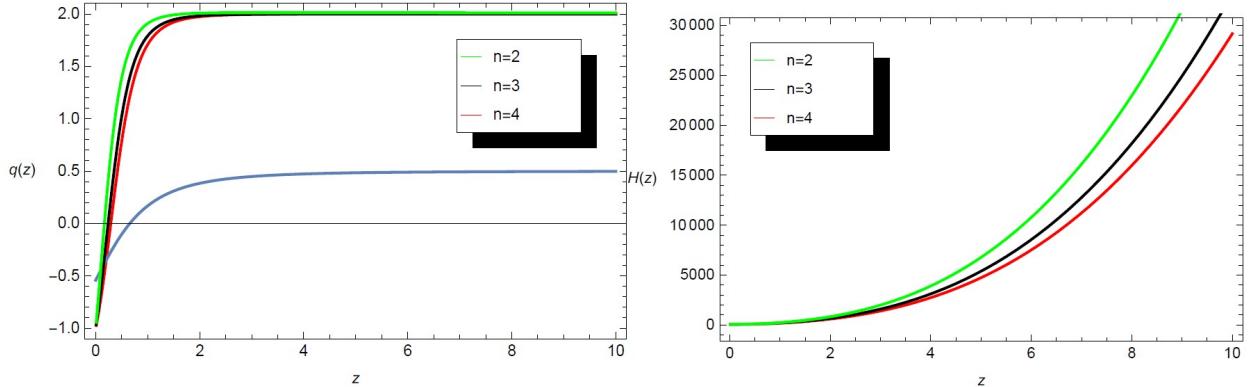


Figure 3. The LHS graph represent the evolution of deceleration parameter whereas graph on RHS represent the evolution of $H[z]$. Herein, we choose $H_0 = 68.3$, $\Omega = 0.3183$, $m = 2$, $\omega = 0$ for NCM includes radiations, $\omega = 0.6$ for CM except radiation and CM+radiation.

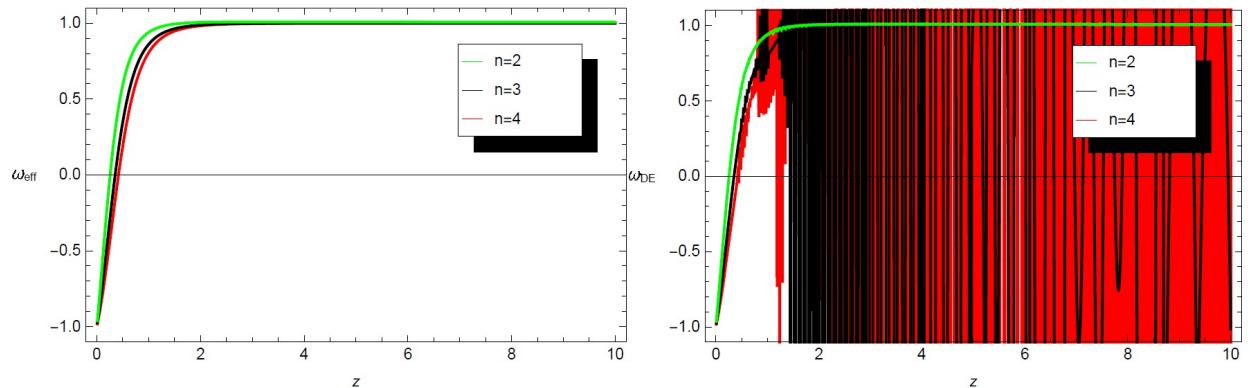


Figure 4. The LHS graph represent the evolution of effective EoS whereas graph on RHS represent the evolution of dark energy. Herein, we choose $H_0 = 68.3$, $\Omega = 0.3183$, $m = 2$, $\omega = 0$ for NCM includes radiations, $\omega = 0.6$ for CM except radiation and CM+radiation.

plus CM case. Keep in mind that the color scheme is same as for deceleration parameter. The graphs show that in all cases ω_{eff} approaches to -1 and all curves are crossing the phantom line [71-73] without showing any oscillation. The RHS of Figure 1 represents the evolution of $H(z)$ which is numerically calculated for this model and we found the numerical value of $H(z)$ which have the good correspondence with latest planck's data. For this model the present value ($z = 0$) is found to be 68.4 for all cases of NCM, CM. Green curve for $H(z)$ shows the almost constant behavior and black curve has variance for higher values of redshift as compare to the red curve. The RHS of Figure 2 represents the evolution of EoS for DE, it approaches to -1 for NCM, CM and CM plus radiation. One can clearly see the oscillation for higher values of redshift in the CM plus radiation case.

4.3. $f(T, B) = T^m + B^n \log[T]$

In Figure 3, the LHS represent the evolution of deceleration parameter for all considered cases in term of redshift. The Color scheme is same as for previous model. In LHS of Figure 3, we can clearly see that the deceleration parameter shows the same behavior for all considered cases.

There is a slight difference for transition phase and this model shows the opposite behavior as compare to power law model. The red curve has a largest value as compare to other curves but for all cases value for the redshift transition is less than the observed value. We can also conclude that if we increase the value of n (model parameter) then the value for the redshift transition phase is more closer to observed value. In the LHS of the Figure 4, we represent the evolution of effective EoS for all considered NCM and CM cases. The graphs show that in all cases ω_{eff} approaches to -1 and all curves are crossing the phantom line [71-73] without showing any oscillation. The RHS of Figure 3 represents the evolution of $H(z)$ which is numerically calculated for this model and we found the current value of $H(z)$ for this model at $z = 0$ is equal to 68.4 for all cases NCM, CM, and CM plus radiation. All curves for $H(z)$ shows a different variance and none of any graph is constant here as previous model. The RHS of Figure 4 represents the evolution of EoS for DE, it is approaches to -1 for NCM, CM and CM plus radiation. One can clearly see the oscillation in DE graphs and if we increase the value of n then number of oscillation also increases across the phantom divide line. For $n = 4$, we can see the huge number of oscillation as compare to $n = 3$.

5. Conclusion

In this paper, we took three significant models of $f(T, B)$ theory of gravity. First model is known as power law model and second is logarithmic model. We mainly focused on cosmic evolution of Hubble parameter $H(z)$, deceleration parameter $q(z)$, effective EoS, EoS for DE in the presence of self interacting CM and radiations. Special thing in this paper is that, we discussed the new form of matter with positive pressure which satisfy the EoS for the relation $0 < \omega < 1$ other than the usual or ordinary matter (pressure less matter $p = 0$) and DE. If we increase the value of ω from 0 to 1 then the collisional nature of matter is also increases. Commonly know that ordinary matter and DE has interaction with each other but actually they are selfinteracting as well. Therefore, we focused on this self-interacting property of the matter besides interaction with matter and DE. We expressed the dynamical equation in terms of Hubble parameter and convert all equations in the function of redshift for the sake of detailed numerical discussion. We found the interesting results while discussing the EoS for DE ω_{DE} , we found the oscillatory behavior of DE in the presence of CM rather then NCM. Although, we found large number of oscillations across phantom divide line for DE in the presence of CM, DE approaches to -1 for all cases which represents the observationally consistent Λ CDM model. The crossing of phantom divide line is viable from observational data [74-78] and significant aspect to be studied in modified theories of gravity. We also numerically solved the Hubble parameter and its numerical value approaches to 68.4, which is according to latest Planck's data [46]. We found transition phase in deceleration parameter $q(z)$ for all considered cases NCM, CM without radiation and CM with radiation and successfully compare the deceleration parameter with Λ CDM model for all considered models (Power law and logarithmic). Effective EoS crossed the phantom divide line in all cases NCM, CM, CM with radiation. We also found that our study is totally model dependent. When we changed our model, then results are totally changed just like make a comparison of power law and logarithmic model. The power law model shows a considerable variance when we change the parameters of model but logarithmic model does not shows a such variance in its graphs. There is minor difference in graphs for NCM, CM and CM plus radiation when we choose logarithmal model as compare to power law. Although, we have good results but model dependent so its not easy to say that the effects of CM on cosmic evolution makes a better results or worse. We need a more discussion in this direction to reach a final conclusion but to discuss the CM and radiations in the MTG is really worthy and able to make a novel contribution in further research.

References

- [1] Bahamonde S, Zubair M and Abbas G 2018 *Physics of Dark Universe* **19** pp 78-90
- [2] Perlmutter S et al 1999 *Astrophys. J.* **517** 565
- [3] Riess A G et al 1998 *Astron. J.* **116** 1009
- [4] Spergel D N et al 2007 *Astrophys. J. Suppl.* **170** 377
- [5] Eisenstein D J et al 2005 *Astrophys. J.* **633** 560
- [6] Bennett C L et al 2003 *Astrophys. J. Suppl. Ser.* **148** 1
- [7] Perlmutter S et al 1997 *Astrophys. J.* **483** 565
- [8] Perlmutter S et al 1998 *Nature* **391** 51
- [9] Riess A G et al 2004 *Astrophys. J.* **607** 665
- [10] Riess A G et al 2007 *Astrophys. J.* **659** 98
- [11] Hawkins E et al 2003 *Mon. Not. R. Astron. Soc.* **346** 78
- [12] Tegmark M et al 2004 *Phys. Rev. D* **69** 103501
- [13] Cole S et al 2005 *Mon. Not. R. Astron. Soc.* **362** 505
- [14] Jain B and Taylor A 2003 *Phys. Rev. Lett.* **91** 141302
- [15] Buchdahl H A 1970 *Mon. Not. R. Astron. Soc.* **150** 1
- [16] Ferraro R and Fiorini F 2007 *Phys. Rev. D* **75** 084031
- [17] Harko T et al 2011 *Phys. Rev. D* **84** 024020
- [18] Moraes P H R S and Sahoo P K 2017 *Eur. Phys. J. C* **77** 480
- [19] Moraes P H R S, Correa R A C and Lobato R V 2017 *JCAP* **07** 029
- [20] Moraes P H R S and Sahoo P K 2017 *Phys. Rev. D* **96** 044038
- [21] Sharif M and Zubair M 2012 *JCAP* **03** 028
- [22] Baffou E H, Salako I G and Houndjo M J S 2017 *Int. J. Geom. Meth. Mod. Phys.* **14** 1750051
- [23] Barrientos E, Lobo F S N, Mendoza S, Olmo G J and Rubiera-Garcia D 2018 *Phys. Rev. D* **97** 104041
- [24] Pradhan A and Jaiswal R 2018 *Int. J. Geom. Meth. Mod. Phys.* **15** 1850076
- [25] Shabani H and Ziae A H 2018 *Eur. Phys. J. C* **78** 397
- [26] Bahamonde S, Bohmer C G and Wright M 2015 *Phys. Rev. D* **92** 104042
- [27] Bahamonde S and Capozziello S 2017 *Eur. Phys. J. C* **77** 107
- [28] Odintsov S D and Saez-Gomez D 2013 *Phys. Lett. B* **725** 437
- [29] Haghani Z, Harko T, Lobo F S N, Sepangi H R and Shahidi S 2013 *Phys. Rev. D* **88** 044023
- [30] Zubair M and Zeeshan M 2018 *Astrophys. Space. Sci.* **363** 248
- [31] Nojiri S, Odintsov S D and Sasaki M 2005 *Phys. Rev. D* **71** 123509
- [32] Koivisto T and Mota D F 2007 *Phys. Lett. B* **644** 104
- [33] Koivisto T and Mota D F 2007 *Phys. Rev. D* **75** 023518
- [34] Kawai S, Sakagami M A and Soda J 1998 *Phys. Lett. B* **437** 284
- [35] Nojiri S and Odintsov S D 2005 *Phys. Lett. B* **631** 1
- [36] De Felice A and Tsujikawa S 2009 *Phys. Lett. B* **675** 1
- [37] Uddin K, Lidsey J E and Tavakol R 2009 *Gen. Relativ. Gravit.* **41** 2725
- [38] Cognola G, Elizalde E, Nojiri S, Odintsov S D and Zerbini S 2006 *Phys. Rev. D* **73** 084007
- [39] De Felice A, Mota D F and Tsujikawa S 2010 *Phys. Rev. D* **81** 023532
- [40] Mukhanov V 2005 *Physical Foundations of Cosmology* (Cambridge University Press, Cambridge)
- [41] Gorbunov D S and Rubakov V A 2011 *Introduction to the Theory of the Early Universe: Cosmological Perturbations and Inflationary Theory* (World Scientific, Hackensack)
- [42] Ade P A R et al 2014 *Phys. Rev. Lett.* **112** 241101
- [43] Cheung Y K E and Vergados J D 2016 *Preprint* 1410.5710
- [44] Oikonomou V K, Vergados J D and Moistakidis Ch C 2007 *Nucl. Phys. B* **773** 19
- [45] Weinberg S 1989 *Rev. Mod. Phys.* **61** 1
- [46] Ade P A R et al 2014 *Astronomy and Astrophysics* **571** A16
- [47] Einstein A 1930 *Sitzungsber. Preuss. Akad. Wiss. Phys. Math. K1* 24 p 401
- [48] Hayashi K and Shirafuji T 1979 *Phys. Rev. D* **19** 3524
- [49] Arcos H I and Pereira J G 2004 *Int. J. Mod. Phys. D* **13** 2193
- [50] Maluf J W 2013 *Ann. Phys.* **525** 339
- [51] Harko T, Lobo F S N, Otalora G and Saridakis E N 2014 *Phys. Rev. D* **89** 124036
- [52] Zubair M and Waheed S 2014 *Astrophys. Space Sci.* **355** 361
- [53] Zubair M and Waheed S 2014 *Astrophys. Space Sci.* **360** 68
- [54] Kofinas G and Saridakis E N 2014 *Phys. Rev. D* **90** 084044
- [55] Kofinas G and Saridakis E N 2014 *Phys. Rev. D* **90** 084045
- [56] Kofinas G, Leon G and Saridakis E N 2014 *Class. Quantum Grav.* **31** 175011
- [57] Zubair M and Waheed S 2014 *Astrophys. Space Sci.* **359** 47

- [58] Zubair M and Waheed S 2015 *Astrophys. Space Sci.* **360** 11
- [59] Perivolaropoulos L 2007 in The Invisible Universe: Dark Matter and Dark Energy ed E Papantonopoulos *Lecture Notes in Physics* **720** (Springer-Verlag, New York) p 257
- [60] Spergel D N and Steinhardt P J 2000 *Phys. Rev. Lett.* **84** 3760
- [61] Oikonomou V K and Karagiannakis N 2015 *Class. Quantum. Grav.* **32** 085001
- [62] Oikonomou V K, Karagiannakis N and Park M 2015 *Phys. Rev. D* **91** 064029
- [63] Zubair M 2016 *Int. J. Mod. Phys. D* **25** 1650057
- [64] Baffou E H et al 2015 *Phys. Rev. D* **92** 084043
- [65] Zubair M, Zeeshan M, Hasan S S and Oikonomou V K 2018 *Symmetry* **10** 463
- [66] Chang J and et al ATIC Collaboration 2008 *Nature* **456** 362
- [67] Adriani O and et al PAMELA Collaboration 2009 *Nature* **458** 607
- [68] Hooper D et al 2007 *Phys. Rev. D* **76** 083012
- [69] Kleidis K and Spyrou N K 2011 *Astron. Astrophys.* **529** A26
- [70] Fock V 1959 *The Theory of Space, Time and Gravitation* (Pergamon Press, London) pp 81 and 91
- [71] Bamba K, Geng C-Q, Nojiri S and Odintsov S D 2009 *Phys. Rev. D* **79** 083014
- [72] Du Y, Zhang H and Li X-Z 2011 *Eur. phys. J. C* **71** 1660
- [73] Bamba K, Geng C-Q and Lee C-C 2011 *Int. J. Mod. Phys. D* **20** pp 1339-45
- [74] Alam U, Sahni V and Starobinsky A A 2004 *J. Cosmol. Astropart. Phys.* **0406** 008
- [75] Nesseris S and Perivolaropoulos L 2007 *J. Cosmol. Astropart. Phys.* **0701** 018
- [76] Wu P and Yu H 2006 *Phys. Lett. B* **643** 315
- [77] Alam U, Sahni V and Starobinsky A A 2007 *J. Cosmol. Astropart. Phys.* **0702** 011
- [78] Jassal H K, Bagla J S and Padmanabhan T 2010 *Mon. Not. R. Astron. Soc.* **405** 2639