Equation of State of Hot Neutron Star Matter

S. Sahoo[1], S. P. Pattnaik[1,2], P. Bano[1], Z. Naik[1], S. Padhiary[1], K. Madhuri[2] and *T. R. Routray[1,*]*

[1]School of Physics, Sambalpur University, Jyotivihar-768 019, INDIA

[2]Deptartment of Physics, Govt. Women's College, Sambalpur-768004, INDIA

** Email: trr1@rediffmail.com*

Introduction

The quantities given as input in the model calculations of the events of high-energy astrophysical systems, like supernovae explosion [1], protoneutron stars (PNSs) [2] and binary neutron stars mergers (BNSM) [3] are the pressure (P), energy density (H), baryonic density (ρ) and temperature (T). On the contrary, the composition of the compact stars is not well understood. So, it is the common trend to start with the n+p+e+μ composition for the neutron stars (NSs). There are works taking into account the hyperons as well as quark matter [4]. In this work, we shall work out the equation of state (EOS) of neutron star matter (NSM) under the n+p+e+μ composition for the typical NSs which are in isothermal neutrino-free condition.

Formalism

Under the parabolic approximation (PA) of the EOS of the nucleonic part, the beta equilibrium condition in hot NSM of n+p+e+μ composition is given by [5],

$$4(1 - 2Y_p)\, F_{sym}(\rho, T) = \mu_e(\rho_e, T) = \mu_\mu(\rho_\mu, T) \quad (1)$$

where, F_{sym} is the free symmetry energy, $Y_p = (\rho_p/\rho)$ is the proton fraction, μ_i, ρ_i, i= e, μ, are the chemical potentials and densities of electron and muon systems. The free symmetry energy F_{sym} is computed as,

$$F_{sym} = (F_{PNM} - F_{SNM})/\rho = ((H_{PNM} - TS_{PNM}) - (H_{SNM} - TS_{SNM}))/\rho \quad (2)$$

where, F_{SNM}, F_{PNM} are the free energy densities in symmetric nuclear matter (SNM) and pure neutron matter (PNM) and (H_{SNM}, S_{SNM}), (H_{PNM}, S_{PNM}) are the corresponding energy densities and entropy densities. Thus, the study of hot NSM under PA identically reduces to independent evaluation of the EOSs of SNM and PNM at finite T. The leptonic EOS has been calculated under the relativistic Fermi gas model. The nucleonic and leptonic distribution functions at finite T are described by Fermi-Dirac (FD) distribution $f_T(k)$ given by

$$f_T(k) = \left[1 + \exp\left\{\frac{\left(\varepsilon^i(k,\rho_i,T) - \mu_i(\rho_i,T)\right)}{T}\right\}\right]^{-1} \quad (3)$$

subject to the phase-space normalization,

$$\rho_i = \frac{\xi}{(2\pi)^3}\int f_T(k)\, d^3k, \quad (4)$$

where, ε^i, i= n, p, e, μ are the respective single particle energies and ξ is the spin-isospin degeneracy factor.

The charge neutrality of the NSM is given by

$$Y_p = Y_e + Y_\mu\,, \quad (5)$$

where $Y_e = (\rho_e/\rho)$ and $Y_\mu = (\rho_\mu/\rho)$ are the electron and muon fractions, respectively.

At a given T, the particle fractions Y_p, Y_e and Y_μ are obtained from the simultaneous solution of Eqs. (1) and (5) as a function of density ρ, and the EOS of hot NSM is computed from the expressions,

$$H_{NSM} = H_N(\rho,Y_p,T)+H_e(\rho_e,Y_e,T)+H_\mu(\rho_\mu,Y_\mu,T) \quad (6)$$

$$F_{NSM} = F_N(\rho,Y_p,T)+F_e(\rho_e,Y_e,T)+F_\mu(\rho_\mu,Y_\mu,T) \quad (7)$$

$$P_{NSM} = P_N(\rho,Y_p,T)+P_e(\rho_e,Y_e,T)+P_\mu(\rho_\mu,Y_\mu,T) \quad (8)$$

where, H, F and P with the index in the subscript, are the respective energy density, free energy density and pressure. The subscript N represents the nucleonic contributions which have been computed from the PA of the quantities,

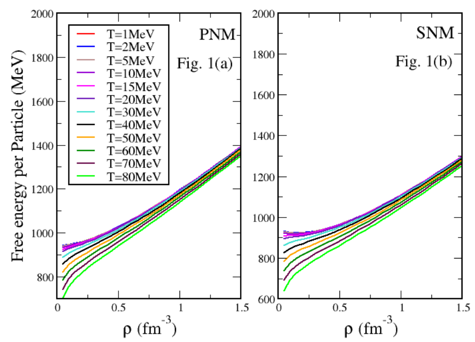
$$H_N(\rho,Y_p,T) = H_{SNM} + (1- 2Y_p)^2\,[H_{PNM} - H_{SNM}] \quad (9)$$

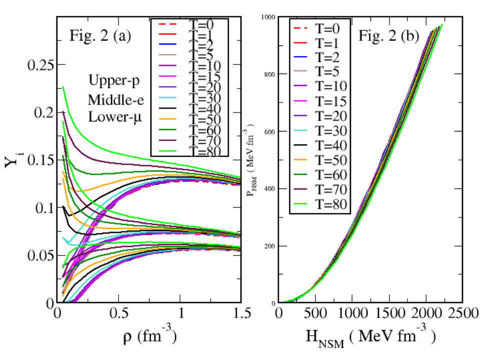
and similarly for F_N and P_N for the equilibrium values of Y_p. The calculation has been done using the finite range SEI [6].

Results and discussion

The EOS of SEI used for the calculation corresponds to γ =1/2. We have computed the free energy per particle F/ρ in PNM (F_{PNM}/ρ) and SNM (F_{SNM}/ρ) which is shown in the two panels of Fig. 1(a) and 1(b) respectively, as a function of ρ for T in the range 1-80 MeV. In both the systems, PNM and SNM, F/ρ is a decreasing function of T, and the magnitude of decrease is larger in the lower density. The difference of F/ρ in PNM and SNM gives the free symmetry energy F_{sym} (ρ, T) which is used in Eq. (1). Evaluation of this Eq. (1) together with the charge neutrality in

Eq. (5) gives the equilibrium particle fractions, Y_i, i= p, e and μ which is shown as a function of ρ for T = 1 to 80 MeV in Fig. 2 (a). For the composition thus obtained, the EOS of the NSM is computed from the Eqs. (6, 7, 8). In panel (b) of Fig. 2 the pressure P_{NSM} is shown as a function of energy density H_{NSM}, which shows that unlike particle fractions, the EOS does not show much variation with the T from its zero temperature values. Due to this, the bulk properties like mass and radius of maximum mass NS at T = 0 and at finite T will remain almost the same.





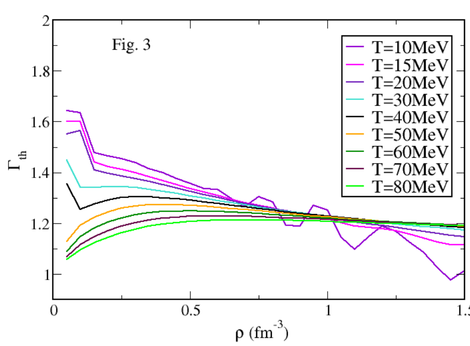


Fig. 1: F/ρ as a function of ρ (PNM-1(a), SNM-1(b)), Fig. 2: Y_i in NSM as a function of ρ in (a), P_{NSM} as a function of H_{NSM} in (b), Fig. 3: Γ_{th} as a function of ρ.

The thermal index Γ_{th} which is the measure of the thermal evolution of the EOS over its T = 0 values is defined as,

$$\Gamma_{th} = 1 + \frac{[P_{NSM}(\rho,T) - P_{NSM}(\rho,T=0)]}{[H_{NSM}(\rho,T) - H_{NSM}(\rho,T=0)]} \qquad (10)$$

This is a crucial parameter given as input in the simulation studies of BNSM and PNSs. Usually, a constant value for Γ_{th} is given [7]. We have shown Γ_{th} as a function of ρ for T = 10-80 MeV in Fig. 3. In the lower density range at below two times the normal NM density Γ_{th} is found to increase and after attaining a maximum decreases. This decreasing trend continues in the high-density range also particularly, for T < 10 MeV. But for T > 10 MeV, all the Γ_{th} curves approach a constant value, a result similar to the one found by Moustakidis in Ref. [5].

Conclusion

The isothermal neutrino free hot n+p+e+μ is studied using SEI under the parabolic approximation of the EOS of ANM, where the free symmetry energy is taken as the difference of free energy per particle in PNM and SNM. The composition of the hot n+p+e+μ NSM is found to vary widely upon the variation of T but has marginal influence on the EOS of hot NSM implying that the maximum mass and radius of the NS at finite T will change little from the T = 0 results. We have calculated the thermal index Γ_{th} which predicts an almost constant behavior for T > 10 MeV, a result similar to the one found in MDI [5]. We, now, plan to extend the present study to the neutrino-trapped isothermal and isoentropic hot NSM which have crucial relevance in the studies of BNSM and PNSs.

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