

Study of Neutrino Mass Matrix with One Vanishing Subtrace using Trimaximal Mixing

Sangeeta Dey^{1,a}, Priyanka Kumar^{2,b} and Mahadev Patgiri^{3,a}

^aDepartment of Physics, Cotton University, Guwahati, India

^bDepartment of Physics, Dakshin Kamrup College, Guwahati, India

E-mail: phy1891006_sangeeta@cottonuniversity.ac.in

Abstract. In this work we study the phenomenological implications of neutrino mass matrix with one vanishing subtrace using trimaximal mixing matrix, TM_1 . Here, we have analysed only two patterns of one vanishing subtrace i.e., $S_{11} = 0$ and $S_{12} = 0$. We find that the two considered patterns accommodate the present neutrino oscillation data for both Normal Hierarchy and Inverted Hierarchy. For both the patterns, we present the correlation plots of the neutrino oscillation parameters and calculate the effective Majorana mass, hence study the neutrinoless double beta decay rate.

1 Introduction

Presence of neutrino oscillation demonstrated by numerous experiments conducted in the past few decades offered a strong proof of neutrino mass and pointed a way beyond the standard model. Therefore we need to go beyond the standard model to investigate the flavor structures of massive neutrinos as well as charged fermions. Even though neutrino physics has made great strides, there are still some unsolved issues such as the leptonic CP violation, neutrino mass hierarchy, origin of leptonic flavor structure, absolute mass-scale of neutrino, and the nature of neutrinos i.e, whether it is Majorana or Dirac particles. Nonetheless, the experiments such as JUNO [1], the DUNE [3] and the long baseline with Hyper-Kamiokande detector and J-PARK accelerator [2] can be used to study the mass hierarchy of the neutrinos. While some data regarding the upper limit of the neutrino mass-scale is derived from experiments conducted by KATRIN [4], GERDA [5], EXO [6], and KamLAND-ZEN [7] experiments. Primarily, solar, atmospheric, and reactor mixing angles, two neutrino mass-squared differences and the range of Dirac CP phase δ , have been measured with a considerable amount of precision to date [8–12].

However, on assuming the Majorana nature of the neutrinos, we can describe the neutrinos by a symmetric 3×3 mass matrix M_ν and a diagonalizing PMNS matrix V containing CP violating phases (α , β , and δ), in the basis, diagonal charged lepton mass matrix M_l . Now to comprehend the underlying flavor symmetry, multiple methods has been suggested by numerous researchers. These include the vanishing minors [13–20], zero trace [38–41], zero textures [21–32,64], vanishing



subtrace [43, 44], equality of elements/minors [33–37], zero determinant [42]. These methods have been proved to be very effective and useful since they minimize the total number of free parameters and helps in establishing simple and certain relationships among the neutrino mass ratios and the mixing angles. If these relationships prove to be feasible based on the light of current experimental data, in that case, they most likely have a very basic explanation and must stem from the fundamental flavor theory. Therefore, the phenomenological study of the textures of neutrino mass matrix makes a great deal of sense in the leptonic field of particle physics.

The neutrino oscillation experiment initially showed the reactor mixing angle to be vanishingly small and the atmospheric mixing angle to be maximal. This led to the proposal of several models for neutrino mixing, including hexagonal(HG), bimaximal mixing(BM) [45–50], tri-maximal mixing(TBM) [51–54], type-A, and type-B of the Golden ratio [55, 56]. Among the aforementioned forms, TBM is particularly intriguing due to its ability to anticipate the solar mixing angle $\sin^2 \theta_{12} = \frac{1}{3}$ and can be described on the basis of the smallest non-abelian discrete symmetry.

The Tri-maximal mixing matrix (TBM) can be written as [57]

$$U_{TBM} = \begin{pmatrix} \frac{\sqrt{2}}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}. \quad (1)$$

Nonetheless, a number of neutrino oscillation tests verified that θ_{13} is non zero, hence requiring modification to the TBM mixing pattern. This modification can be done by deviating two columns from TBM values, while keeping the other columns unchanged. If the first column of the TBM is unaltered, and the remaining two columns are changed, we can refer to the mixing matrix as TM_1 .

TM_1 mixing can be given by [57]

$$U_{TM_1} = \begin{pmatrix} \frac{\sqrt{2}}{\sqrt{3}} & \frac{1}{\sqrt{3}} \cos \theta & \frac{1}{\sqrt{3}} \sin \theta \\ -\frac{1}{\sqrt{6}} & \frac{\cos \theta}{\sqrt{3}} - \frac{e^{i\phi} \sin \theta}{\sqrt{2}} & \frac{\sin \theta}{\sqrt{3}} + \frac{e^{i\phi} \cos \theta}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{\cos \theta}{\sqrt{3}} + \frac{e^{i\phi} \sin \theta}{\sqrt{2}} & \frac{\sin \theta}{\sqrt{3}} - \frac{e^{i\phi} \cos \theta}{\sqrt{2}} \end{pmatrix}. \quad (2)$$

Similarly modifying the TBM matrix, we can give rise to mixing matrix TM_2 and TM_3 which is giving by

$$U_{TM_2} = \begin{pmatrix} \frac{\sqrt{2}}{\sqrt{3}} \cos \theta & \frac{1}{\sqrt{3}} & \frac{\sqrt{2}}{\sqrt{3}} \sin \theta \\ -\frac{\cos \theta}{\sqrt{6}} + \frac{e^{-i\phi} \sin \theta}{\sqrt{2}} & \frac{1}{\sqrt{3}} & -\frac{\sin \theta}{\sqrt{6}} - \frac{e^{-i\phi} \cos \theta}{\sqrt{2}} \\ -\frac{\cos \theta}{\sqrt{6}} - \frac{e^{-i\phi} \sin \theta}{\sqrt{2}} & \frac{1}{\sqrt{3}} & -\frac{\sin \theta}{\sqrt{6}} + \frac{e^{-i\phi} \cos \theta}{\sqrt{2}} \end{pmatrix}, \quad (3)$$

$$U_{TM_3} = \begin{pmatrix} \cos \theta & \sin \theta & 0 \\ -\frac{e^{-i\phi} \sin \theta}{\sqrt{2}} & \frac{e^{-i\phi} \cos \theta}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{e^{-i\phi} \sin \theta}{\sqrt{2}} & \frac{e^{-i\phi} \cos \theta}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \quad (4)$$

However, because TM_3 predicts $\theta_{13} = 0$, it is considered to be phenomenologically unviable. When $\theta = 0$ and $\phi = 0$, the mixing scheme U_{TM_1} and U_{TM_2} reduces to TBM scheme. In earlier works, phenomenological textures of neutrino mass matrix were studied with texture one zero, two zeroes etc., in the context of trimaximal mixing matrix [57–60]. On considering Majorana nature of the neutrinos and on the basis of diagonal neutrino mass matrix, M_l , a total of six textures of one vanishing subtrace which is denoted by $S_{ij} = 0$, ($i, j = 1, 2, 3$), can be considered. In this work, we study the phenomenology of only two cases ($S_{11} = 0, S_{12} = 0$), of one vanishing subtrace using Trimaximal mixing TM_1 . Here we explore the correlation plots for θ_{23} , δ and J_{cp}

with ϕ and further calculate $|m_{ee}|$, to study neutrinoless double beta decay rate for both the cases.

Here in this article, we have studied the texture of one vanishing subtrace with TM_1 mixing in Sec.2. In Sec.3 and 4 we have analysed both the considered cases of one vanishing subtrace. We present our discussion and conclusions in Sec.5.

2 TM_1 mixing and one vanishing subtrace

Corresponding to U_{TM_1} mixing, the neutrino mass matrix, M_ν can be given by

$$M_\nu = U_{TM_1} P M_{diag} P^T U_{TM_1}^T, \quad (5)$$

where

$$M_{diag} = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}, P_\nu = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i\beta} \end{pmatrix}. \quad (6)$$

Now we can express θ_{12} , θ_{13} and θ_{23} in terms of the free parameters θ and ϕ as

$$s_{12} = \sqrt{1 - \frac{2}{3 - \sin^2 \theta}}, s_{13} = \frac{\sin \theta}{\sqrt{3}}, s_{23} = \frac{1}{2} \left(1 + \frac{\sqrt{6} \cos \phi \sin 2\theta}{3 - \sin^2 \theta} \right). \quad (7)$$

Now,

$$J_{CP} = \frac{1}{8} \sin 2\theta_{23} \sin 2\theta_{12} \cos \theta_{13} \sin 2\theta_{13} \sin \delta. \quad (8)$$

which represents the Jarlskog Invariant, J_{CP} [61,63]. Again, the Jarlskog invariant, in the context of TM_1 mixing can be written as

$$J_{CP} = \frac{1}{6\sqrt{6}} \sin 2\theta \sin \phi. \quad (9)$$

From Eq(8) and Eq(9) we have

$$\sin^2 \phi = \left(1 - \frac{6 \sin^2 2\theta \cos \phi}{(3 - \sin^2 \theta)^2} \right) \sin^2 \delta. \quad (10)$$

$|m_{ee}|$ i.e., the effective neutrino mass can be written as [57]

$$|m_{ee}| = \left| \frac{(2m_1 + m_2 \cos^2 \theta e^{2i\alpha} + m_3 \sin^2 \theta e^{2i\beta})}{3} \right|. \quad (11)$$

Now, we write the elements of the Majorana neutrino mass matrix as

$$(m_\nu)_{ab} = \sum_{i=1}^3 U_{ai}^* U_{bi} \lambda_i. \quad (12)$$

where $\lambda_1 = m_1$, $\lambda_2 = m_2 e^{2i\alpha}$, $\lambda_3 = m_3 e^{2i\beta}$. The constraint condition for vanishing subtrace is given as

$$(m_\nu)_{ab} + (m_\nu)_{cd} = 0. \quad (13)$$

On simplifying Eq(13) we obtain

$$Am_1 + Bm_2 e^{2i\alpha} + Cm_3 e^{2i\beta} = 0. \quad (14)$$

here, $A=U_{a_1}U_{b_1} + U_{c_1}U_{b_1}$, $B=U_{a_2}U_{b_2} + U_{c_2}U_{d_2}$ and $C=U_{a_3}U_{b_3} + U_{c_3}U_{d_3}$. We define the ratio of the neutrino masses as

$$\rho = \frac{m_1}{m_2}, \sigma = \frac{m_1}{m_3}. \quad (15)$$

Now we calculate m_1 , m_2 and m_3 from the following equations

$$m_1 = \sqrt{\Delta m_{21}^2} \sqrt{\frac{\rho^2}{1-\rho^2}}, m_2 = \sqrt{\Delta m_{21}^2} \sqrt{\frac{1}{1-\rho^2}}, m_3 = \sqrt{\Delta m_{21}^2} \sqrt{\frac{\rho^2}{\sigma^2(1-\rho^2)}}. \quad (16)$$

The ratio between the solar and the atmospheric mass squared splitting is given as

$$r = \left| \frac{\Delta m_{21}^2}{\Delta m_{32}^2} \right|. \quad (17)$$

The six textures of one vanishing subtrace, defined by S_{ij} are given below:

$$\begin{aligned} S_{11} = 0 &\Rightarrow m_{22} + m_{33} = 0; & S_{12} = 0 &\Rightarrow m_{21} + m_{33} = 0. \\ S_{13} = 0 &\Rightarrow m_{21} + m_{32} = 0; & S_{22} = 0 &\Rightarrow m_{11} + m_{33} = 0. \\ S_{23} = 0 &\Rightarrow m_{11} + m_{32} = 0; & S_{33} = 0 &\Rightarrow m_{11} + m_{22} = 0. \end{aligned}$$

Now for TM_1 mixing, we find the range of the free parameter θ lies between $(9.9739^\circ, 10.946^\circ)$ at 3σ level. Now we check the range of ϕ by plotting s_{23} Vs ϕ . In Fig.1, we show the correlation plots of s_{23} , δ and J_{cp} with ϕ .

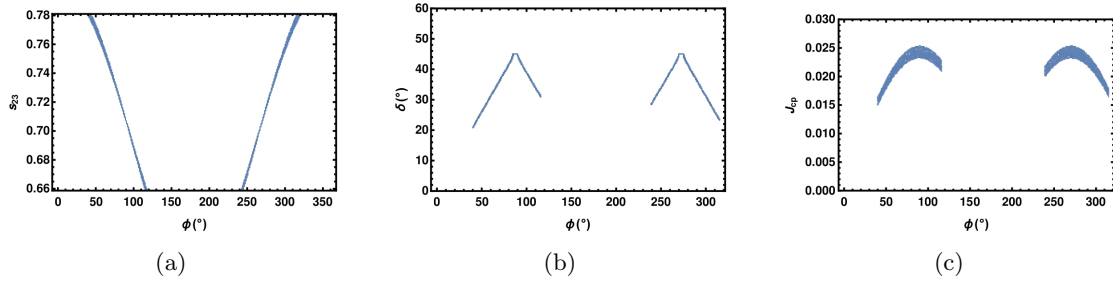


FIG. 1. (a) Correlation plot of s_{23} with ϕ , (b) Correlation plot of δ and ϕ and (c) Correlation plot of J_{cp} and ϕ for TM_1 .

TABLE I. Latest parameters of the neutrino oscillation from global fits [62]. Here $\Delta m_{3l}^2 = \Delta m_{31}^2 > 0$ and $\Delta m_{3l}^2 = \Delta m_{32}^2 < 0$ are for normal and inverted hierarchy respectively.

Parameter	Normal Hierarchy		Inverted Hierarchy	
	best fit $\pm 1\sigma$	3σ range	best fit $\pm 1\sigma$	3σ range
θ_{12}°	$33.45_{-0.75}^{+0.77}$	31.27 – 35.86	$33.45_{-0.75}^{+0.78}$	31.27 – 35.87
θ_{23}°	$42.1_{-0.9}^{+1.1}$	39.7 – 50.9	$49.0_{-1.3}^{+0.9}$	39.8 – 51.6
θ_{13}°	$8.62_{-0.12}^{+0.12}$	8.25 – 8.98	$8.61_{-0.12}^{+0.14}$	8.24 – 9.02
δ_{cp}°	230_{-25}^{+36}	144 – 350	278_{-30}^{+22}	194 – 345
$\Delta m_{21}^2/10^{-5}eV^2$	$7.42_{-0.20}^{+0.21}$	6.82 – 8.04	$7.42_{-0.20}^{+0.21}$	6.82 – 8.04
$ \Delta m_{3l}^2 /10^{-3}eV^2$	$2.510_{-0.027}^{+0.027}$	2.430 – 2.593	$2.490_{-0.028}^{+0.026}$	-2.574 – -2.410

3 Case $S_{11} = 0$

Here we consider the vanishing subtrace of element m_{11} , therefore we have

$$m_{22} + m_{33} = 0. \tag{18}$$

Now we have the expressions of A, B, C as

$$A = \frac{1}{3}, \tag{19}$$

$$B = \frac{1}{3}(2 \cos^2 \theta + 3e^{2i\phi} \sin^2 \theta)e^{2i\alpha}, \tag{20}$$

$$C = \frac{1}{3}(2 \sin^2 \theta + 3e^{2i\phi} \cos^2 \theta)e^{2i\beta}. \tag{21}$$

From Eqs.(18,19,20,21) we have the expressions of ρ and σ as

$$\rho = \frac{m_1}{m_2} = \frac{(2 \cos^2 \theta \cos 2\alpha + 3 \sin^2 \theta \cos 2(\phi + \alpha))(2 \sin^2 \theta \sin 2\beta + 3 \cos^2 \theta \sin 2(\beta + \phi)) - (2 \sin^2 \theta \cos 2\beta + 3 \cos^2 \theta \cos 2(\phi + \beta))(2 \cos^2 \theta \sin 2\alpha + 3 \sin^2 \theta \sin 2(\alpha + \phi))}{-(2 \sin^2 \theta \sin 2\beta + 3 \cos^2 \theta \sin (2(\beta + \phi)))}, \tag{22}$$

$$\sigma = \frac{m_1}{m_3} = \frac{(2 \cos^2 \theta \cos 2\alpha + 3 \sin^2 \theta \cos 2(\phi + \alpha))(2 \sin^2 \theta \sin 2\beta + 3 \cos^2 \theta \sin(2(\beta + \phi))) - (2 \sin^2 \theta \cos 2\beta + 3 \cos^2 \theta \cos(2(\phi + \beta)))(2 \cos^2 \theta \sin 2\alpha + 3 \sin^2 \theta \sin 2(\alpha + \phi))}{(2 \cos^2 \theta \sin 2\alpha + 3 \sin^2 \theta \sin(2(\alpha + \phi)))}. \tag{23}$$

Now we draw the correlation plots of m_1 , m_2 , and m_3 with ϕ and also study the neutrinoless double beta decay by plotting $|m_{ee}|$ with respect to ϕ .

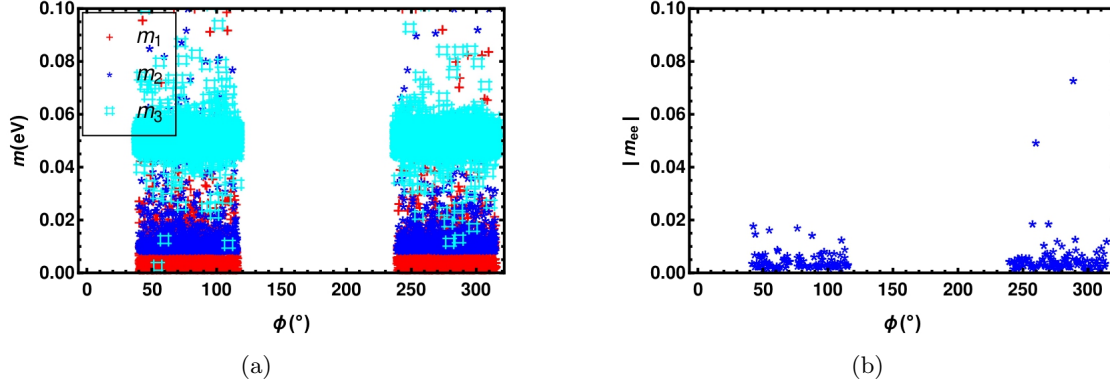


FIG. 2. (a) Correlation plot of m_1 , m_2 , m_3 with ϕ , and (b) $|m_{ee}|$ and ϕ plot for vanishing subtrace of m_{11} .

In Fig.2(a) we observe case $S_{11} = 0$ of one vanishing subtrace allows Normal as well as Inverted Hierarchy. Fig.2(b) displays the plot for $|m_{ee}| - \phi$ and we observe $|m_{ee}|$ bounded within the viable range. Therefore we can say that this texture is allowed under the study of $0\nu\beta\beta$ (neutrinoless double beta decay).

4 Case $S_{12} = 0$

Here we consider the vanishing subtrace of element m_{12} , therefore we have

$$m_{21} + m_{33} = 0. \quad (24)$$

Now we obtain the expressions of A, B and C as

$$A = \frac{-1}{6}, \quad (25)$$

$$B = \left[\left(\frac{\cos \theta}{\sqrt{3}} - \frac{e^{i\phi} \cos \theta}{\sqrt{2}} \right) \frac{1}{\sqrt{3}} \cos \theta + \left(\frac{\cos \theta}{\sqrt{3}} + \frac{e^{i\phi} \sin \theta}{\sqrt{2}} \right)^2 \right] e^{2i\alpha}, \quad (26)$$

$$C = \left[\left(\frac{\sin \theta}{\sqrt{3}} + \frac{e^{i\phi} \sin \theta}{\sqrt{2}} \right) \frac{1}{\sqrt{3}} \sin \theta + \left(\frac{\sin \theta}{\sqrt{3}} - \frac{e^{i\phi} \cos \theta}{\sqrt{2}} \right)^2 \right] e^{2i\beta}. \quad (27)$$

From Eqs.24,25,26,27 we have the expressions of ρ and σ as

$$\rho = \frac{m_1}{m_2} = \frac{(4\sqrt{6} \cos^2 \theta \cos 2\alpha + 3 \sin 2\theta \cos (2\alpha + \phi) + 3\sqrt{6} \sin^2 \theta \cos (2(\alpha + \phi))) (3 \sin 2\theta \sin (2\beta + \phi) - 3\sqrt{6} \cos^2 \theta \sin 2(\beta + \phi) - 4\sqrt{6} \sin^2 \theta \sin 2\beta) + (4\sqrt{6} \sin^2 \theta \sin 2\beta - 3 \sin 2\theta \sin (2\beta + \phi) + 3\sqrt{6} \cos^2 \theta \sin (2(\beta + \phi))) (4\sqrt{6} \cos^2 \theta \sin 2\alpha + 3 \sin 2\theta \sin 2(\alpha + \phi) + 3\sqrt{6} \sin^2 \theta \sin 2(\alpha + \phi))}{3 \sin (2\beta + \phi) \sin 2\theta - 3\sqrt{6} \cos^2 \theta \sin (2(\beta + \phi)) - 4\sqrt{6} \sin^2 \theta \sin 2\beta}, \quad (28)$$

$$\sigma = \frac{m_1}{m_3} = \frac{(4\sqrt{6} \cos^2 \theta \cos 2\alpha + 3 \sin 2\theta \cos (2\alpha + \phi) + 3\sqrt{6} \sin^2 \theta \cos 2(\alpha + \phi)) (3 \sin (2\beta + \phi) \sin 2\theta - 3\sqrt{6} \cos^2 \theta \sin 2(\beta + \phi) - 4\sqrt{6} \sin^2 \theta \sin 2\beta) + (4\sqrt{6} \sin^2 \theta \sin 2\beta - 3 \sin 2\theta \sin (2\beta + \phi) + 3\sqrt{6} \cos^2 \theta \sin 2(\beta + \phi)) (4\sqrt{6} \cos^2 \theta \sin 2\alpha + 3 \sin 2\theta \sin (2\alpha + \phi) + 3\sqrt{6} \sin^2 \theta \sin 2(\alpha + \phi))}{4\sqrt{6} \cos^2 \theta \sin 2\alpha + 3 \sin 2\theta \sin (2\alpha + \phi)}. \quad (29)$$

Now we draw the correlation plots of m_1 , m_2 , and m_3 with ϕ and also study the neutrinoless double beta decay by plotting $|m_{ee}|$ with respect to ϕ .

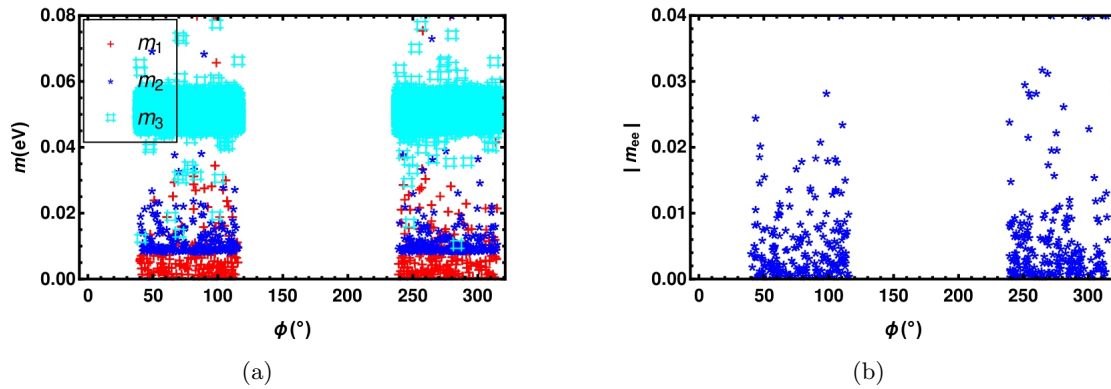


FIG. 3. (a) Correlation plot of m_1 , m_2 , m_3 with ϕ , and (b) $|m_{ee}|$ and ϕ plot for vanishing subtrace of m_{12} .

In Fig.3(a), we see that case $S_{12} = 0$ of one vanishing subtrace allows for Normal Mass Hierarchy as well as Inverted Mass Hierarchy. Fig.3(b) displays the plot for $|m_{ee}| - \phi$ and we observe that $|m_{ee}|$ remains within the viable range. Therefore, we can say that this texture is viable under the study of $0\nu\beta\beta$.

5 Discussion and Conclusions

Here we have carried out the phenomenology of one vanishing subtrace in neutrino mass matrix using Trimaximal mixing TM_1 . Out of six cases of one vanishing subtrace, the study of only two cases i.e., $S_{11} = 0$ and $S_{12} = 0$ is being carried out in this paper. Firstly, we have explored the correlation between $s_{23} - \phi$ and found that ϕ is constrained in the range $(40^\circ, 116^\circ) \oplus (239^\circ, 315^\circ)$. We also observed the correlation between $\delta - \phi$ and $J_{cp} - \phi$ for TM_1 mixing. On plotting the Majorana masses m_1 , m_2 , m_3 for both the cases $S_{11} = 0$ and $S_{12} = 0$, we observe that both the considered cases allows Normal Mass Hierarchy as well as Inverted Mass hierarchy. We also found that both the cases are viable for $0\nu\beta\beta$.

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