photonics

IMPACT FACTOR 2.4

CITESCORE 2.3

Communication

Experimental Direct Measurement of the Relative Entropy of Coherence

Xufeng Huang, Yuan Yuan, Yueping Niu and Shangqing Gong

Special Issue
Quantum Optics: Science and Applications

Edited by
Dr. Hua-Lei Yin, Dr. Peng Xu and Dr. Jie Chen

MDPI

https://doi.org/10.3390/photonics10091004 *photonics*

Communication

Experimental Direct Measurement of the Relative Entropy of Coherence

Xufeng Huang [1], Yuan Yuan [1,*], Yueping Niu [1,2,3] and Shangqing Gong [1,2,3]

[1] School of Physics, East China University of Science and Technology, Shanghai 200237, China; y30211287@mail.ecust.edu.cn (X.H.); niuyp@ecust.edu.cn (Y.N.); sqgong@ecust.edu.cn (S.G.)
[2] Shanghai Engineering Research Center of Hierarchical Nanomaterials, Shanghai 200237, China
[3] Shanghai Frontiers Science Center of Optogenetic Techniques for Cell Metabolism, Shanghai 200237, China
* Correspondence: yuanyuan2019@ecust.edu.cn

Abstract: Quantum coherence is the most distinguished feature of quantum mechanics, which characterizes the superposition properties of quantum states. It plays a critical role in various fields, ranging from quantum information technology to quantum biology. Although various coherence quantifiers have been proposed since the resource theory of coherence was established, there are a lack of experimental methods to estimate them efficiently, which restricts the applications of coherence. Relative entropy of coherence is one of the main quantifiers of coherence, and is frequently used in quantum information science. Such nonlinear properties of quantum states are usually calculated from full descriptions of the quantum state, although they are not related to all parameters that specify the state. Here, we experimentally measure the relative entropy of coherence for the arbitrary qubit states directly in the photonic system without using standard state tomography. In the experiment, we directly measure the von Neumann entropy of the quantum states through interference, thus obtaining the relative entropy of coherence, and finding that the experimental results are in good agreement with the theory. Our work provides a nice alternative experimental scheme for measuring the relative entropy of coherence.

Keywords: quantum optics; quantum coherence; relative entropy; quantum measurement

check for updates

Citation: Huang, X.; Yuan, Y.; Niu, Y.; Gong, S. Experimental Direct Measurement of the Relative Entropy of Coherence. *Photonics* **2023**, *10*, 1004. https://doi.org/10.3390/photonics10091004

Received: 16 July 2023
Revised: 28 August 2023
Accepted: 1 September 2023
Published: 2 September 2023

1. Introduction

Quantum coherence is an essential property that distinguishes quantum theory from the classical realm, characterizing the superposition properties of quantum systems. Being a fundamental property of quantum systems, it plays an important role in quantum thermodynamics [1–5], nanoscale physics [6], transport theory [7], and biological systems [8–10]. Since coherence was proposed in the framework of resource theory, the applications of quantum coherence in quantum information technology have been widely studied [11–16], such as quantum computation [17], quantum communication [18–20], quantum metrology [21], quantum algorithms [22,23], quantum uncertainty [24], and quantum channel discrimination [25,26]. Additionally, as a fundamental resource, it is also closely related to other quantum resources, including asymmetry [27], entanglement [28–31] and other quantum correlations [32]; the coherence distillation [33–36], coherence dilution, and conversion between coherence and quantum correlations have also been investigated [37–43].

Quantum coherence is a valuable feature of quantum systems; one of the primary contents of the resource-theoretical framework is its rigorous quantification. There are various methods for quantifying coherence, but they should be restricted by the conditions of the resource theory [11,15]. The most compelling method is based on state distance, and this method leads to the relative entropy of coherence and the ℓ_1 norm of coherence. Relative entropy plays an essential role in quantum information theory [44], so the relative entropy of coherence, as one of the main quantifiers of coherence, is frequently used in quantum information science, such as quantifying key rates in quantum key distribution [18], characterizing the wave nature of quantons [45,46], relating to the success probability of the Grover algorithm [23], etc. Especially, the relative entropy of coherence has significance in coherence distillation and conversion between coherence and other quantum correlations [39,42,43], and is often chosen as the measure of this property in these processes.

While many theoretical and experimental works were devoted to the relative entropy of coherence [37,47–51], an important issue is how to efficiently estimate it in experiments. Clearly, one can perform state tomography and then calculate the amount of coherence with the derived quantum density matrix according to the expressions of the coherence measures. However, this method contains redundant information, because not all elements of the density matrix are related to the coherence measures that we desire. Therefore, particular methods are proposed to estimate the quantum coherence without state tomography [49,52–59], and these methods apply to the measurement of the coherence for different quantifiers, respectively. However, for the relative entropy of coherence, most theoretical strategies only estimate its upper and lower bounds for unknown quantum states by mathematical calculations and numerical optimizations [49,53,59]. Thus, the lack of efficient and experiment-friendly methods for measuring the relative entropy of coherence severely limits their applications. In this work, we experimentally measure the relative entropy of coherence for the arbitrary qubit states directly in the photonic system without the complicated measurements and calculations. Specifically, for an arbitrary quantum state, we experimentally consider all possible measurement bases in state space and find the minimum Shannon entropy of the probability distribution of the measurement outcomes, thereby obtaining the von Neumann entropy of the state. Thus, the relative entropy of coherence can be easily obtained according to the expression. This is verified both in pure states and mixed states of the qubit in the experiment.

2. Theoretical Method

There are various ways to quantify quantum coherence within the framework of resource theory. Relative entropy of coherence as a main quantifier of coherence is expressed as [11]

$$C_r(\rho) = S(\rho_d) - S(\rho). \tag{1}$$

Here, $S(\rho) = -\mathrm{Tr}[\rho \log_2 \rho]$ is the von Neumann entropy, and ρ_d denotes the state obtained from ρ when we delete all off-diagonal elements. If we write the density matrix ρ in the diagonal form,

$$\rho = \sum_{j=1}^{n} b_j |b_j\rangle\langle b_j|, \tag{2}$$

the von Neumann entropy can be rewritten as $S(\rho) = -\sum_{j=1}^{n} b_j \log_2 b_j$. To measure the relative entropy of coherence for an unknown state directly, we need to achieve a direct measurement of the von Neumann entropy of the state. Taking on all possible measurement basis in the Hilbert space, search for the minimum Shannon entropy of the measurement results, which is the von Neumann entropy of the state [56]. Let us focus on the coherence measure for a qubit, which is in the state space composed of basis $\{|0\rangle, |1\rangle\}$. In principle, we should make measurements in all bases $\{|D_1\rangle, |D_2\rangle\}$, which are

$$\begin{aligned} |D_1\rangle &= \cos(\frac{\alpha}{2})|0\rangle + e^{i\phi}\sin(\frac{\alpha}{2})|1\rangle, \\ |D_2\rangle &= \sin(\frac{\alpha}{2})|0\rangle - e^{i\phi}\cos(\frac{\alpha}{2})|1\rangle. \end{aligned} \tag{3}$$

The experimental strategy is to test all measurement bases in Equation (3) and find the minimum Shannon entropy of the measurement results. The Shannon entropy is defined as $S = -\sum_{i=1}^{n} p_i \log_2 p_i$, and p_i is probability obtained from each measurement. In practice, testing all measurement bases in the Hilbert space seems a bit complicated. However, we can use the properties of von Neumann entropy to simplify this search process. If a unitary transformation operation U is performed on a quantum state, its von Neumann entropy remains unchanged, i.e., $S(\rho) = S(U\rho U^\dagger)$; thus, utilizing this property can greatly reduce the number of measurement bases that need to be tested in experiments. For an arbitrary qubit state in the Bloch sphere representation, one can perform a unitary operation to rotate it to the equatorial plane of the Bloch sphere. Since the probabilities of measuring the state on the equatorial plane measured with bases $|0\rangle$ and $|1\rangle$ are $1/2$, this provides a method to rotate an unknown state onto the equatorial plane in the experiment. After rotating the state to the equatorial plane, one only needs to look for the measurement basis located on the equatorial line of the Bloch sphere to minimize the Shannon entropy of the measurement results, i.e., $\alpha = \pi/2$, and vary the parameter ϕ of the bases. Thus, the measurement is simplified as $\Pi_1 = |D_1\rangle\langle D_1|$ and $\Pi_2 = |D_2\rangle\langle D_2|$, with measurement bases $|D_1\rangle$ and $|D_2\rangle$ are expressed as

$$\begin{aligned} |D_1\rangle &= \frac{1}{\sqrt{2}}(|0\rangle + e^{i\phi}|1\rangle), \\ |D_2\rangle &= \frac{1}{\sqrt{2}}(|0\rangle - e^{i\phi}|1\rangle). \end{aligned} \tag{4}$$

By varying the measurement bases, at a certain point, they will necessarily be consistent with the spectral decomposition states $|b_j\rangle$ of the density matrix in Equation (2). At this point, the most unbalanced distribution of measurement results can be observed in the experiment, which corresponds to the minimum Shannon entropy of the measurement outcome. Now, the quantity of $S(\rho)$ can be obtained.

In the next step, we need to measure the von Neumann entropy of the dephased state $S(\rho_d)$, which is a basis-dependent quantity. We consider the coherence of ρ with respect to the computational basis $\{|i\rangle\}$, thus $\rho_d = \sum_i |i\rangle\langle i|\rho|i\rangle\langle i|$. Obviously, the measurement of $S(\rho_d)$ is realized by giving a measurement in the basis $\{|i\rangle\}$. Therefore, having measured both $S(\rho)$ and $S(\rho_d)$, the relative entropy of coherence can be calculated by Equation (1). It is worthwhile mentioning that the 'direct' we mean is without the complicated measurements and calculations, and reconstruction of the density matrix is unnecessary.

3. Experimental Implementation

We experimentally measure the relative entropy of coherence for the arbitrary qubit states directly in the photonic system. Figure 1 presents the experimental device, which is composed of a single-photon source module, a state preparation module, and a measurement module. In the single-photon source module, a 404 nm wavelength laser with 50 mW pumps a type-II beamlike phase-matching beta-barium-borate (BBO, $6.0 \times 6.0 \times 2.0$ mm^3, $\theta = 40.98°$) crystal to produce a pair of 808-nm photons. One of the photons is detected by a single photon detector (SPD) as a trigger signal.

In the state preparation module, we prepare a series of pure states $|\psi_{\theta,\varphi}\rangle = \cos\frac{\theta}{2}|0\rangle + \sin\frac{\theta}{2}e^{i\varphi}|1\rangle$ and mixed states $\rho = a|+\rangle\langle+| + (1-a)|1\rangle\langle 1|$, where $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$. The qubit is encoded with the polarization degree of freedom of the photon. We define $|0\rangle \equiv |H\rangle$ and $|1\rangle \equiv |V\rangle$, where $|H\rangle(|V\rangle)$ denotes the horizontal (vertical) polarized state of the single photon. An unbalanced Mach–Zehnder (M-Z) interferometer composed of a polarization beam splitter (PBS) and a beam splitter (BS), together with two half-wave plates (H_1 and H_2) is employed to prepare the mixed state ρ, where H_1 adjusts parameter a and H_2 is 22.5°. When we set H_1 and H_2 to 0°, after passing a half-wave plate (H_3) and quarter-wave plate (Q_1) with deviation angles, the photon is prepared as desired pure state $|\psi_{\theta,\varphi}\rangle$.

In the measurement module, a half-wave plate H_4 is used to rotate an unknown qubit state to the equatorial plane of the Bloch sphere. Beam displacers (BDs) cause the vertical polarized photons to be transmitted directly, and the horizontal polarized photons undergo a 4-mm lateral displacement; hence, it can split and combine the photons depending on their polarizations. An M-Z interferometer composed of BDs and the half-wave plates (H_5 and H_6), together with a PBS and a half-wave plate (H_7), is used to realize measurement bases

D_1 and D_2, where H_5 and H_6 are 45°, H_7 is 22.5°. The parameter ϕ in the measurement bases is achieved by adjusting the phase of the interferometer.

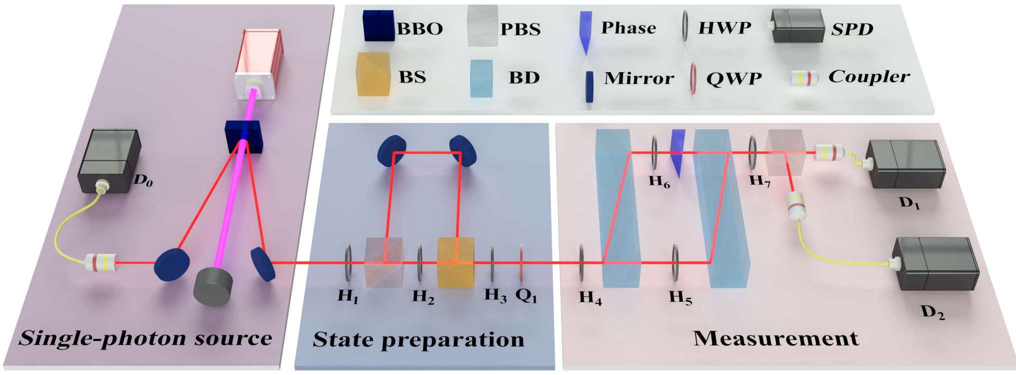

Figure 1. Experimental setup for measuring the relative entropy of coherence for a qubit state. The experimental device consists of a single photon source section, a state preparation section, and a measurement section. In the single photon source section, a 404 nm laser pumps a BBO crystal to generate photon pairs with a wavelength of 808 nm through a spontaneous parameter down conversion, and produced photon pairs are coupled to two single-mode fibers, respectively. One of the photons is detected by a single photon detector (SPD) D_0 as a trigger. In the state preparation section, we use the polarization degree of freedom of the photons to encode the qubit states. An unbalanced Mach–Zehnder (M-Z) interferometer with two half-wave plates (H_1 and H_2) is employed to prepare the mixed state ρ. H_3 and a quarter-wave plate (Q_1) prepare the pure state $|\psi_{\theta,\varphi}\rangle$. In the measurement section, a combination of beam displacers (BDs) and half-wave plates (H_5 and H_6) with certain angular settings composed of an M-Z interferometer is used to realize measurement bases D_1 and D_2, where H_5 and H_6 are 45°, H_7 is 22.5°. H_4 rotates the polarization state of the photon towards the equatorial plane of the Bloch sphere.

Firstly, we measure the relative entropy of coherence for the qubit pure states. For an arbitrary qubit pure state, after rotating it to the equatorial plane and performing measurements Π_1 and Π_2, the probability distribution of the outcomes can be expressed as follows:

$$\begin{aligned} P_1 &= \frac{1}{2} + \frac{1}{2}\cos(\varphi - \phi), \\ P_2 &= \frac{1}{2} - \frac{1}{2}\cos(\varphi - \phi). \end{aligned} \tag{5}$$

Thus, the most unbalanced measurement outcome distribution can be observed by adjusting the phase of the interferometer in the experiment, i.e., the interference visibility is maximum, which corresponds to the minimum Shannon entropy of the measurement outcomes, and the quantity of von Neumann entropy $S(\rho)$ can be obtained. The quantity of $S(\rho_d)$ is measured in the basis $\{|0\rangle, |1\rangle\}$. The relative entropy of coherence $C_r(\rho)$ can be obtained from the difference between these two quantities. Experimental results for measuring $S(\rho_d)$, $S(\rho)$, and $C_r(\rho)$ are shown in Figure 2. Figure 2a–c shows the results of measuring the states $|\psi_{\theta,\varphi=0}\rangle$ with θ ranging from 0 to π. Figure 2d–f shows the results of measuring the states $|\psi_{\pi/2,\varphi}\rangle$ with φ ranging from 0 to π. As we see from the figure, the relative entropy of coherence for the qubit pure states depends on $S(\rho_d)$, because of its von Neumann entropy is 0.

Furthermore, we also measure the relative entropy of coherence for the qubit mixed states. For the state ρ that we choose, after rotating it to the equatorial plane and performing measurements Π_1 and Π_2, the probability distribution of the outcomes is

$$\begin{aligned} P_1' &= \frac{1}{2}(1 - \sqrt{2a^2 - 2a + 1}\cos\phi), \\ P_2' &= \frac{1}{2}(1 + \sqrt{2a^2 - 2a + 1}\cos\phi). \end{aligned} \tag{6}$$

Similarly, we adjust the phase of the interferometer find the most unbalanced measurement outcome, and obtain the quantity of von Neumann entropy $S(\rho)$. Experimental results for measuring $S(\rho_d)$, $S(\rho)$, and $C_r(\rho)$ for qubit mixed states are shown in Figure 3a–c, respectively. The experimental results are in good agreement with the theoretical prediction.

This experimental method accurately achieves direct measurement of relative entropy coherence in both qubit pure states and mixed states. For a pure state of the qubit, its relative entropy coherence is only related to $S(\rho_d)$. If it is known in advance that the state to be measured is a qubit pure state in the experiment, to save resources, it is unnecessary to measure the von Neumann entropy of the state. According to the experimental results in Figure 2d–f, we can see that $S(\rho_d)$, $S(\rho)$, and $C_r(\rho)$ are independent to the parameter φ. The results of the mixed state are different from those of the pure state, and the main point is the von Neumann entropy of the mixed state $S(\rho)$ is no longer equal to zero. As the parameter a increases, it first increases and then decreases, reaching its maximum value at $a = 0.5$. This experimental method has good robustness for both pure states and mixed states, providing a nice alternative experimental scheme for measuring the relative entropy of coherence.

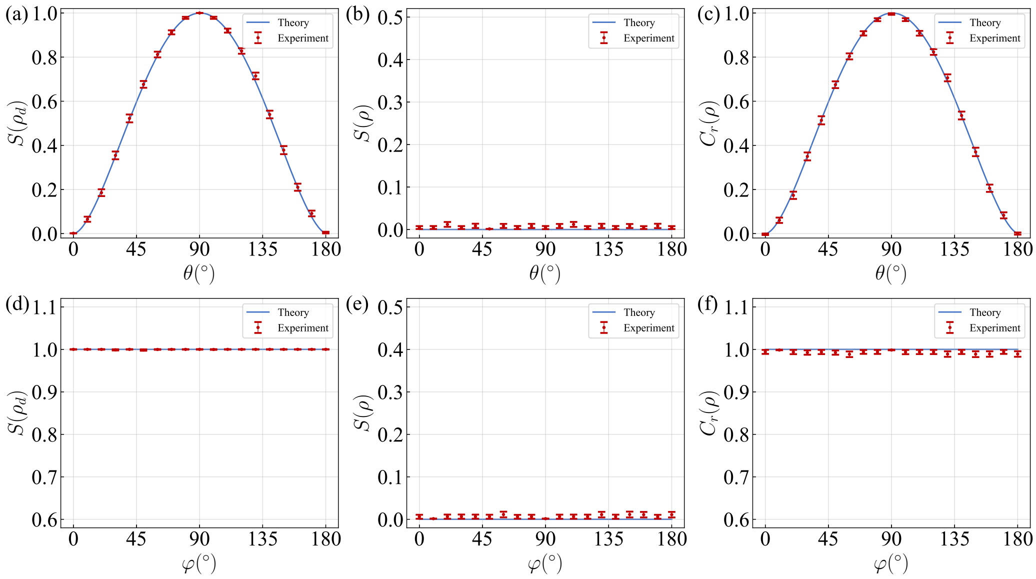

Figure 2. Experimental results for measuring the relative entropy of coherence for the qubit pure states. (**a**–**c**) show the results of $S(\rho_d)$, $S(\rho)$, and $C_r(\rho)$ by measuring the states $|\psi_{\theta,\varphi=0}\rangle$ with θ ranging from 0 to π. (**d**–**f**) show the results by measuring the states $|\psi_{\pi/2,\varphi}\rangle$ with φ ranging from 0 to π. The error bar denotes the standard deviation.

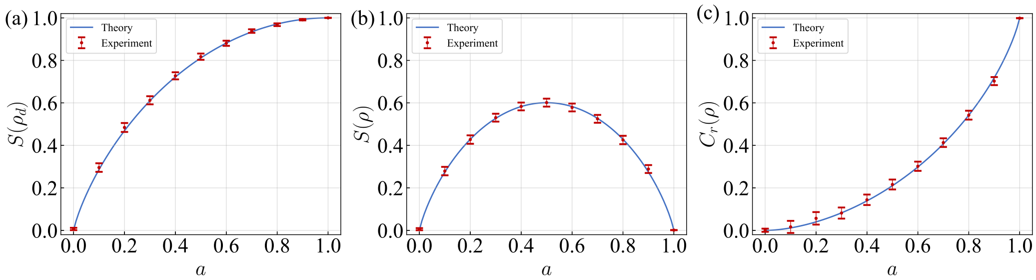

Figure 3. Experimental results for measuring the relative entropy of coherence for the qubit mixed states. (**a**–**c**) show the results of $S(\rho_d)$, $S(\rho)$, and $C_r(\rho)$ by measuring the mixed states with parameter a ranging from 0 to 1. The error bar denotes the standard deviation.

4. Discussions

In this section, we compare our method with standard tomographic reconstruction. It is fair to compare the estimation accuracy for relative entropy of coherence induced by the two methods when the same number of copies of the states are consumed, and given a fixed total number of copies of the state $N = 1200$. Our method is mainly divided into three steps, and each step measurement consuming the number of copies of the state are N_1, N_2, and N_3, respectively, $N_1 + N_2 + N_3 = N$. Step 1: perform the measurement in σ_z bases, and calculate $S(\rho_d)$ using the obtained probability distribution. Step 2: rotate the state to the equatorial plane and measure the phase of the interferometer. Step 3: adjust the interference visibility is maximum and measure the most unbalanced measurement outcome distribution, which corresponds to the minimum Shannon entropy of the measurement outcomes, and the quantity of von Neumann entropy $S(\rho)$ can be obtained. As a standard tomographic reconstruction, $N/3$ copies of the state are used for the measurements in bases of three Pauli operators, respectively. Reconstruct the density matrix based on the measurement results, and substitute it into the mathematical expression to calculate the value of relative entropy of coherence. In the process of the numerical simulations, we take the state $\cos\theta|0\rangle + \sin\theta|1\rangle$ as an example and choose the mean squared error $\Delta^2 C_r^{est} := E[(C_r^{est} - C_r)^2]$ as an estimation precision quantifier, where C_r is the actual coherence value and C_r^{est} is the estimated value. We set different combinations $N = [N_1, N_2, N_3] = 1200$ to simulate the estimation accuracy, and the simulation results are shown in Figure 4, which shows that the mean squared error induced by our method is smaller than that of the standard tomographic reconstruction.

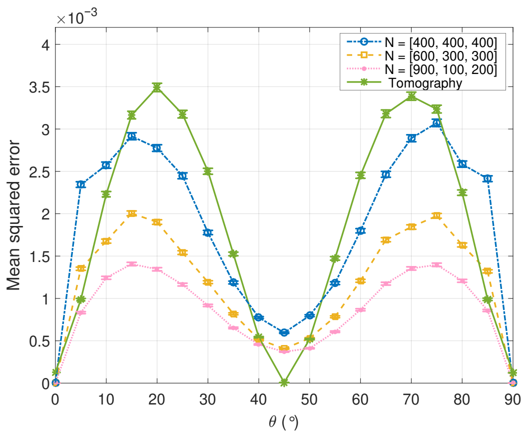

Figure 4. The simulation results for comparing our method with standard state tomography. Mean squared error is chosen as an estimation precision quantifier. The figure shows the mean squared error induced by our method and standard state tomography. We set different combinations $N = [N_1, N_2, N_3] = 1200$ to simulate the estimation accuracy separately.

5. Conclusions

We experimentally measure the relative entropy of coherence for an unknown qubit state directly in the photonic system without carrying out standard state tomography, including pure states and mixed states. In the experiment, we directly measure the von Neumann entropy of the quantum states through interference, and find the maximum interference visibility, i.e., at this point, the most unbalanced distribution of measurement outcome is observed, which corresponds to the minimum Shannon entropy of the measurement outcome, and then the von Neumann entropy is obtained. Finally, the relative entropy of coherence is acquired through simple calculation, and we find a good agreement between theory and experiment. This experimental scheme has several advantages with respect to other techniques. In particular, this method is explicit and does not need any optimization procedures or complicated calculations. Interference, as the physical effect of the coherence, is also reflected during the measurement process. Our work provides a valuable alternative experimental scheme to the measurement of relative entropy of coherence. Moreover, in addition to being a widely used quantifier of quantum coherence, relative entropy is also used to quantify entanglement, so measuring this quantity experimentally is important to quantum information science. Our work may have potential applications in the estimation of entanglement entropy.

Author Contributions: Conceptualization, Y.Y. and Y.N.; methodology, X.H. and S.G.; data curation, X.H. and Y.Y.; writing—original draft preparation, X.H.; writing—review and editing, Y.Y., Y.N. and S.G.; supervision, Y.Y. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by the National Natural Science Foundation of China (Grant No. 12004113, 12034007), and the Natural Science Foundation of Shanghai (Grant No. 22ZR1418100, 21XD1400700).

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: The data presented in this study are available on request from the corresponding author.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Narasimhachar, V.; Gour, G. Low-temperature thermodynamics with quantum coherence. *Nat. Commun.* **2015**, *6*, 7689. [CrossRef] [PubMed]
2. Lostaglio, M.; Jennings, D.; Rudolph, T. Description of quantum coherence in thermodynamic processes requires constraints beyond free energy. *Nat. Commun.* **2015**, *6*, 6383. [CrossRef] [PubMed]
3. Korzekwa, K.; Lostaglio, M.; Oppenheim, J.; Jennings, D. The extraction of work from quantum coherence. *New J. Phys.* **2016**, *18*, 023045. [CrossRef]
4. Gour, G. Role of quantum coherence in thermodynamics. *PRX Quantum* **2022**, *3*, 040323. [CrossRef]
5. Santos, J.P.; Céleri, L.C.; Landi, G.T.; Paternostro, M. The role of quantum coherence in non-equilibrium entropy production. *npj Quantum Inf.* **2019**, *5*, 23. [CrossRef]
6. Karlström, O.; Linke, H.; Karlström, G.; Wacker, A. Increasing thermoelectric performance using coherent transport. *Phys. Rev. B* **2011**, *84*, 113415. [CrossRef]
7. Herranen, M.; Kainulainen, K.; Rahkila, P.M. Kinetic transport theory with quantum coherence. *Nucl. Phys. A* **2009**, *820*, 203c–206c. [CrossRef]
8. Lambert, N.; Chen, Y.N.; Cheng, Y.C.; Li, C.M.; Chen, G.Y.; Nori, F. Quantum biology. *Nat. Phys.* **2013**, *9*, 10–18. [CrossRef]
9. Romero, E.; Augulis, R.; Novoderezhkin, V.I.; Ferretti, M.; Thieme, J.; Zigmantas, D.; Van Grondelle, R. Quantum coherence in photosynthesis for efficient solar-energy conversion. *Nat. Phys.* **2014**, *10*, 676–682. [CrossRef]
10. Huelga, S.F.; Plenio, M.B. A vibrant environment. *Nat. Phys.* **2014**, *10*, 621–622. [CrossRef]
11. Baumgratz, T.; Cramer, M.; Plenio, M.B. Quantifying coherence. *Phys. Rev. Lett.* **2014**, *113*, 140401. [CrossRef] [PubMed]
12. Levi, F.; Mintert, F. A quantitative theory of coherent delocalization. *New J. Phys.* **2014**, *16*, 033007. [CrossRef]
13. Winter, A.; Yang, D. Operational resource theory of coherence. *Phys. Rev. Lett.* **2016**, *116*, 120404. [CrossRef] [PubMed]
14. Yadin, B.; Ma, J.; Girolami, D.; Gu, M.; Vedral, V. Quantum processes which do not use coherence. *Phys. Rev. X* **2016**, *6*, 041028. [CrossRef]
15. Streltsov, A.; Adesso, G.; Plenio, M.B. Colloquium: Quantum coherence as a resource. *Rev. Mod. Phys.* **2017**, *89*, 041003. [CrossRef]
16. Hu, M.L.; Hu, X.; Wang, J.; Peng, Y.; Zhang, Y.R.; Fan, H. Quantum coherence and geometric quantum discord. *Phys. Rep.* **2018**, *762*, 1–100. [CrossRef]
17. Matera, J.M.; Egloff, D.; Killoran, N.; Plenio, M.B. Coherent control of quantum systems as a resource theory. *Quantum Sci. Technol.* **2016**, *1*, 01LT01. [CrossRef]
18. Ma, J.; Zhou, Y.; Yuan, X.; Ma, X. Operational interpretation of coherence in quantum key distribution. *Phys. Rev. A* **2019**, *99*, 062325. [CrossRef]
19. Yin, H.L.; Fu, Y.; Li, C.L.; Weng, C.X.; Li, B.H.; Gu, J.; Lu, Y.S.; Huang, S.; Chen, Z.B. Experimental quantum secure network with digital signatures and encryption. *Natl. Sci. Rev.* **2023**, *10*, nwac228. [CrossRef]
20. Gu, J.; Cao, X.Y.; Fu, Y.; He, Z.W.; Yin, Z.J.; Yin, H.L.; Chen, Z.B. Experimental measurement-device-independent type quantum key distribution with flawed and correlated sources. *Sci. Bull.* **2022**, *67*, 2167–2175. [CrossRef]
21. Li, L.; Wang, Q.W.; Shen, S.Q.; Li, M. Quantum coherence measures based on Fisher information with applications. *Phys. Rev. A* **2021**, *103*, 012401. [CrossRef]
22. Hillery, M. Coherence as a resource in decision problems: The Deutsch-Jozsa algorithm and a variation. *Phys. Rev. A* **2016**, *93*, 012111. [CrossRef]

23. Shi, H.L.; Liu, S.Y.; Wang, X.H.; Yang, W.L.; Yang, Z.Y.; Fan, H. Coherence depletion in the Grover quantum search algorithm. *Phys. Rev. A* **2017**, *95*, 032307. [CrossRef]
24. Sun, Y.; Luo, S. Coherence as uncertainty. *Phys. Rev. A* **2021**, *103*, 042423. [CrossRef]
25. Piani, M.; Cianciaruso, M.; Bromley, T.R.; Napoli, C.; Johnston, N.; Adesso, G. Robustness of asymmetry and coherence of quantum states. *Phys. Rev. A* **2016**, *93*, 042107. [CrossRef]
26. Napoli, C.; Bromley, T.R.; Cianciaruso, M.; Piani, M.; Johnston, N.; Adesso, G. Robustness of coherence: An operational and observable measure of quantum coherence. *Phys. Rev. Lett.* **2016**, *116*, 150502. [CrossRef]
27. Marvian, I.; Spekkens, R.W. How to quantify coherence: Distinguishing speakable and unspeakable notions. *Phys. Rev. A* **2016**, *94*, 052324. [CrossRef]
28. Streltsov, A.; Singh, U.; Dhar, H.S.; Bera, M.N.; Adesso, G. Measuring quantum coherence with entanglement. *Phys. Rev. Lett.* **2015**, *115*, 020403. [CrossRef]
29. Chitambar, E.; Hsieh, M.H. Relating the resource theories of entanglement and quantum coherence. *Phys. Rev. Lett.* **2016**, *117*, 020402. [CrossRef]
30. Theurer, T.; Satyajit, S.; Plenio, M.B. Quantifying dynamical coherence with dynamical entanglement. *Phys. Rev. Lett.* **2020**, *125*, 130401. [CrossRef]
31. He, R.D.; Wu, K.D.; Xiang, G.Y.; Li, C.F.; Guo, G.C. Experimental quantification of dynamical coherence via entangling two qubits. *Opt. Express* **2022**, *30*, 10346–10353. [CrossRef] [PubMed]
32. Tan, K.C.; Kwon, H.; Park, C.Y.; Jeong, H. Unified view of quantum correlations and quantum coherence. *Phys. Rev. A* **2016**, *94*, 022329. [CrossRef]
33. Regula, B.; Fang, K.; Wang, X.; Adesso, G. One-shot coherence distillation. *Phys. Rev. Lett.* **2018**, *121*, 010401. [CrossRef] [PubMed]
34. Regula, B.; Lami, L.; Streltsov, A. Nonasymptotic assisted distillation of quantum coherence. *Phys. Rev. A* **2018**, *98*, 052329. [CrossRef]
35. Zhao, Q.; Liu, Y.; Yuan, X.; Chitambar, E.; Winter, A. One-shot coherence distillation: Towards completing the picture. *IEEE Trans. Inf. Theory* **2019**, *65*, 6441–6453. [CrossRef]
36. Xiong, S.J.; Sun, Z.; Su, Q.P.; Xi, Z.J.; Yu, L.; Jin, J.S.; Liu, J.M.; Nori, F.; Yang, C.P. Experimental demonstration of one-shot coherence distillation: Realizing N-dimensional strictly incoherent operations. *Optica* **2021**, *8*, 1003–1008. [CrossRef]
37. Zhao, Q.; Liu, Y.; Yuan, X.; Chitambar, E.; Ma, X. One-shot coherence dilution. *Phys. Rev. Lett.* **2018**, *120*, 070403. [CrossRef]
38. Ma, J.; Yadin, B.; Girolami, D.; Vedral, V.; Gu, M. Converting coherence to quantum correlations. *Phys. Rev. Lett.* **2016**, *116*, 160407. [CrossRef]
39. Chitambar, E.; Streltsov, A.; Rana, S.; Bera, M.; Adesso, G.; Lewenstein, M. Assisted distillation of quantum coherence. *Phys. Rev. Lett.* **2016**, *116*, 070402. [CrossRef]
40. Streltsov, A.; Rana, S.; Bera, M.N.; Lewenstein, M. Towards resource theory of coherence in distributed scenarios. *Phys. Rev. X* **2017**, *7*, 011024. [CrossRef]
41. Streltsov, A.; Chitambar, E.; Rana, S.; Bera, M.N.; Winter, A.; Lewenstein, M. Entanglement and coherence in quantum state merging. *Phys. Rev. Lett.* **2016**, *116*, 240405. [CrossRef] [PubMed]
42. Wu, K.D.; Hou, Z.; Zhong, H.S.; Yuan, Y.; Xiang, G.Y.; Li, C.F.; Guo, G.C. Experimentally obtaining maximal coherence via assisted distillation process. *Optica* **2017**, *4*, 454–459. [CrossRef]
43. Wu, K.D.; Hou, Z.; Zhao, Y.Y.; Xiang, G.Y.; Li, C.F.; Guo, G.C.; Ma, J.; He, Q.Y.; Thompson, J.; Gu, M. Experimental cyclic interconversion between coherence and quantum correlations. *Phys. Rev. Lett.* **2018**, *121*, 050401. [CrossRef] [PubMed]
44. Vedral, V. The role of relative entropy in quantum information theory. *Rev. Mod. Phys.* **2002**, *74*, 197. [CrossRef]
45. Bagan, E.; Bergou, J.A.; Cottrell, S.S.; Hillery, M. Relations between coherence and path information. *Phys. Rev. Lett.* **2016**, *116*, 160406. [CrossRef]
46. Yuan, Y.; Hou, Z.; Zhao, Y.Y.; Zhong, H.S.; Xiang, G.Y.; Li, C.F.; Guo, G.C. Experimental demonstration of wave-particle duality relation based on coherence measure. *Opt. Express* **2018**, *26*, 4470–4478. [CrossRef]
47. Zhu, H.; Hayashi, M.; Chen, L. Coherence and entanglement measures based on Rényi relative entropies. *J. Phys. A-math. Theor.* **2017**, *50*, 475303. [CrossRef]
48. Bu, K.; Singh, U.; Fei, S.M.; Pati, A.K.; Wu, J. Maximum relative entropy of coherence: An operational coherence measure. *Phys. Rev. Lett.* **2017**, *119*, 150405. [CrossRef]
49. Smith, G.; Smolin, J.A.; Yuan, X.; Zhao, Q.; Girolami, D.; Ma, X. Quantifying coherence and entanglement via simple measurements. *arXiv* **2017**, arXiv:1707.09928. [CrossRef].
50. Horová, N.; Stárek, R.; Mičuda, M.; Kolář, M.; Fiurášek, J.; Filip, R. Deterministic controlled enhancement of local quantum coherence. *Sci. Rep.* **2022**, *12*, 22455. [CrossRef]
51. Horová, N.; Stárek, R.; Mičuda, M.; Fiurášek, J.; Kolář, M.; Filip, R. Experimental mutual coherence from separable coherent qubits. *Phys. Rev. A* **2022**, *106*, 012440. [CrossRef]
52. Wang, Y.T.; Tang, J.S.; Wei, Z.Y.; Yu, S.; Ke, Z.J.; Xu, X.Y.; Li, C.F.; Guo, G.C. Directly measuring the degree of quantum coherence using interference fringes. *Phys. Rev. Lett.* **2017**, *118*, 020403. [CrossRef]
53. Zhang, D.J.; Liu, C.; Yu, X.D.; Tong, D. Estimating coherence measures from limited experimental data available. *Phys. Rev. Lett.* **2018**, *120*, 170501. [CrossRef] [PubMed]
54. Yu, X.D.; Gühne, O. Detecting coherence via spectrum estimation. *Phys. Rev. A* **2019**, *99*, 062310. [CrossRef]

55. Yuan, Y.; Hou, Z.; Tang, J.F.; Streltsov, A.; Xiang, G.Y.; Li, C.F.; Guo, G.C. Direct estimation of quantum coherence by collective measurements. *npj Quantum Inf.* **2020**, *6*, 46. [CrossRef]
56. de Lima Bernardo, B. Proposal for a direct measurement of the von Neumann entropy and the relative entropy of coherence. *Phys. Scr.* **2020**, *95*, 045104. [CrossRef]
57. Ma, Z.; Zhang, Z.; Dai, Y.; Dong, Y.; Chengjie, Z. Detecting and estimating coherence based on coherence witnesses. *Phys. Rev. A* **2021**, *103*, 012409. [CrossRef]
58. Dai, Y.; Dong, Y.; Xu, Z.; You, W.; Zhang, C.; Gühne, O. Experimentally accessible lower bounds for genuine multipartite entanglement and coherence measures. *Phys. Rev. Appl.* **2020**, *13*, 054022. [CrossRef]
59. Sun, L.L.; Yu, S. Universal method to estimate quantum coherence. *Phys. Rev. A* **2022**, *106*, 042428. [CrossRef]