Contributions for the kaon pair from $\rho(770)$, $\omega(782)$ and their excited states in the $B \to K\bar{K}h$ decays

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We study the resonance contributions for the kaon pair originating from the intermediate states $\rho(770, 1450, 1700)$ and $\omega(782, 1420, 1650)$ for the three-body hadronic decays $B \to K\bar{K}h$ in the perturbative QCD approach, where $h = (\pi, K)$. The branching fractions of the virtual contributions for $K\bar{K}$ from the Breit-Wigner formula tails of $\rho(770)$ and $\omega(782)$, which have been ignored in experimental and theoretical studies for these decays, are found larger than the corresponding contributions from the resonances $\rho(1450, 1700)$ and $\omega(1420, 1650)$. The differential branching fractions for $B \to \rho(770)h \to K\bar{K}h$ and $B \to \omega(782)h \to K\bar{K}h$ are found nearly unaffected by the quite different values of the full widths for $\rho(770)$ and $\omega(782)$ in this paper. The predictions in this work for the branching fractions of the quasi-two-body decays $B^+ \to \pi^+\rho(1450)^0 \to \pi^+K^+K^-$ and $B^+ \to \pi^+\rho(1450)^0 \to \pi^+\pi^+\pi^-$ meet the requirement of $SU(3)$ symmetry relation.

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I. INTRODUCTION

Charmless three-body hadronic B meson decays provide us a field to investigate different aspects of weak and strong interactions. The underlying weak decay of the b quark is simple and can be described well by the effective Hamiltonian [1], but the strong dynamics in these three-body processes is very complicated, owing to the hadron-hadron interactions, the three-body effects [2,3] and the rescattering processes [4–7] in the final states, and also on account of the resonant contributions, which are related to the scalar, vector, and tensor resonances, and are commonly described by the relativistic Breit-Wigner (BW) formula [8] as well as the nonresonant contributions which are the rest at the amplitude level for the relevant decay processes. The experimental efforts for the three-body B decays by employing Dalitz plot technique [9] within the isobar formalism [10–12] have revealed valuable information on involved strong and weak dynamics. But the *a priori* model with all reliable and correct strong dynamical components is needed for the Dalitz plot analyses [13]. The expressions of the decay amplitudes for those three-body decays without or have wrong factors for certain intermediate states will have negative impacts on the observables such as the branching fractions and CP violations for the relevant decay processes.

Recently, in the amplitude analysis of the three-body decays $B^\pm \to \pi^\pm K^+K^-$, LHCb collaboration reported an unexpected large fit fraction $(30.7 \pm 1.2 \pm 0.9)\%$ in Ref. [14] for the resonance $\rho(1450)^0$ decaying into charged kaon pair. This fit fraction implies a branching fraction $(1.60 \pm 0.14) \times 10^{-6}$ for the quasi-two-body decay $B^+ \to \pi^+\rho(1450)^0 \to \pi^+K^+K^-$ [15]; this is in view of the branching fractions $(5.38 \pm 0.40 \pm 0.35) \times 10^{-6}$ from Belle [16] and $(5.0 \pm 0.5 \pm 0.5) \times 10^{-6}$ presented by *BABAR* [17] for the $B^+ \to K^+K^-\pi^+$ decays. While in the ρ dominant decay modes $B^\pm \to \pi^\pm\pi^+\pi^-$, the contribution for $\pi^+\pi^-$ pair from the intermediate state $\rho(1450)^0$ was found to be small but consistent with the theoretical expectation in Ref. [18] by LHCb in their recent works [19,20].

In Ref. [21], within flavor $SU(3)$ symmetry, we predicted the branching fraction for $B^+ \to \pi^+\rho(1450)^0 \to \pi^+K^+K^-$ to be about one tenth of that for the decay $B^+ \to \pi^+\rho(1450)^0 \to \pi^+\pi^+\pi^-$ and much smaller than the corresponding result in [14,15], and our prediction got the supports from the theoretical analyses in Ref. [22]. In addition, the virtual contribution [23–27] for K^+K^- from the BW formula [8] tail of the resonance $\rho(770)^0$, which has been ignored by the experimental analysis was found to be the same order but larger than the contribution of $\rho(1450)^0 \to K^+K^-$ [21]. In this work, we shall

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systematically study the contributions for the kaon pair from the resonances $\rho(770, 1450, 1700)$ and $\omega(782, 1420, 1650)$ in the $B \to K\bar{K}h$ decays within the perturbative QCD (PQCD) approach [28–31], where h is the bachelor state pion or kaon. As for the other $J^{PC} = 1^{--}$ isovector resonances, like $\rho(1570)$, $\rho(1900)$, and $\rho(2150)$, we will leave their possible contributions for kaon pair to the future studies in view of their ambiguous nature [15].

The contributions for $K\bar{K}$ from the tails of $\rho(770)$ and $\omega(782)$ in the charmless three-body hadronic B meson decays have been ignored in both the theoretical studies and the experimental works. But in the processes of $\pi^- p \to K^- K^+ n$ and $\pi^+ n \to K^- K^+ p$ [32,33], $\bar{p}p \to K^+ K^- \pi^0$ [34,35], $e^+e^- \to K^+K^-$ [36–44], and $e^+e^- \to K_S^0 K_L^0$ [45–50], the resonances $\rho(770)$ and $\omega(782)$ along with their excited states are indispensable for the formation of the kaon pair. In addition, the resonances $\rho(770, 1450)^\pm$ are the important intermediate states for the $K^\pm K_S^0$ pair in the final state of hadronic τ decays [51–54]. The subprocesses $\rho(1450, 1700) \to K\bar{K}$ be concerned for the decay $J/\psi \to K^+K^-\pi^0$ in Refs. [55–58] could be mainly attributed to the observation of a resonant broad structure around 1.5 GeV in the K^+K^- mass spectrum in [59]. While for the decays $B \to KKK$ [60–65] and $B \to KK\pi$ [17,66], the unsettled $f_X(1500)$ which decaying into K^+K^- channel could probably be related to the resonance $\rho(1450)^0$ [67].

For the three-body decays $B \to K\bar{K}h$, the subprocesses $\rho \to K\bar{K}$ and $\omega \to K\bar{K}$ can not be calculated in the PQCD approach and will be introduced into the distribution amplitudes of the $K\bar{K}$ system via the kaon vector timelike form factors. The intermediate $\rho(770)$, $\omega(782)$ resonances and their excited states are generated in the hadronization of the light quark-antiquark pair $q\bar{q}^{(\prime)}$ with $q^{(\prime)} = (u, d)$ as demonstrated in Fig. 1 where the factorizable and non-factorizable Feynman diagrams have been merged for the sake of simplicity. In the first approximation one can neglect the interaction of the $K\bar{K}$ pair originating from the intermediate states with the bachelor h, and study the decay processes $B \to \rho(770, 1450, 1700)h \to K\bar{K}h$ and $B \to \omega(782, 1420, 1650)h \to K\bar{K}h$ in the quasi-two-body framework [68–70]. The $\pi\pi \leftrightarrow KK$ rescattering effects were found have important contributions for $B^\pm \to \pi^\pm K^+K^-$ [14], which would be investigated in a subsequent work. The final state interaction effect for the $\rho(1450, 1700) \to K\bar{K}$ were found to be suppressed in [55] and will be neglected in the numerical calculation of this work. The quasi-two-body framework based on PQCD approach has been discussed in detail in [68], which has been followed in Refs. [18,21,67,71–78] for the quasi-two-body B meson decays in recent years. Parallel analyses for the related three-body B meson processes within QCD factorization can be found in Refs. [22,79–91], and for relevant work within the symmetries one is referred to Refs. [92–100].

This paper is organized as follows. In Sec. II, we review the kaon vector timelike form factors, which are the crucial inputs for the quasi-two-body framework within PQCD and decisive for the numerical results of this work. In Sec. III, we give a brief introduction of the theoretical framework for the quasi-two-body B meson decays within PQCD approach. In Sec. IV, we present our numerical results of the branching fractions and direct CP asymmetries for the quasi-two-body decays $B \to \rho(770, 1450, 1700)h \to K\bar{K}h$ and $B \to \omega(782, 1420, 1650)h \to K\bar{K}h$, along with some necessary discussions. A summary of this work is given in Sec. V. The wave functions and factorization formulas for the related decay amplitudes are collected in the Appendixes.

II. KAON TIMELIKE FORM FACTORS

The electromagnetic form factors for the charged and neutral kaon are important for the precise determination of the hadronic loop contributions to the anomalous magnetic moment of the muon and the running of the QED coupling to the Z boson mass [43,101,102] and are also valuable for the measurements of the resonance parameters [38,40,41,43,46,49,50]. The kaon electromagnetic form factors have been extensively studied in Refs. [54, 103–106] on the theoretical side. Up to now the experimental information on these form factors comes from the measurements of the reactions $e^+e^- \to K^+K^-$ [38,39,44] and $e^+e^- \to K^+K^-(\gamma)$ [41]. Since $K\bar{K}$ is not an eigenstate of isospin, both isospin 0 and 1 resonances need to be considered in components of the form factors of kaon [41]. The combined analysis of the $e^+e^- \to K^+K^-$ and $e^+e^- \to K_SK_L$ cross sections and the spectral function in the $\tau^- \to K^-K^0\nu_\tau$ decay allows one to extract the isovector and isoscalar electromagnetic form factors for kaons [107].

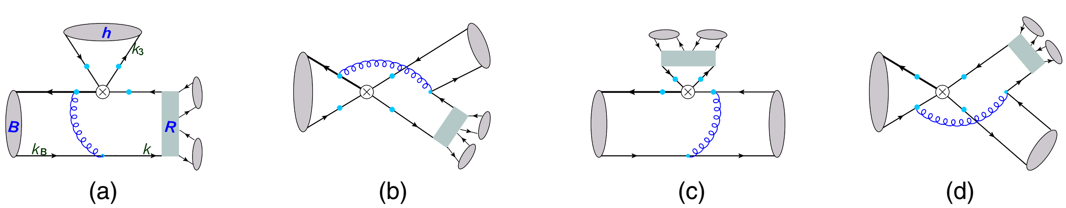

FIG. 1. Typical Feynman diagrams for the processes $B \to Rh \to K\bar{K}h$, with R representing the resonances ρ, ω and their excited states. The dots on the quarks connecting the weak vertex $\otimes$ are the switchable vertices for the hard gluons.

TABLE I. The fitted results of c_R^Ks in Refs. [104,106,107]. The column fit 1 (fit 2) contains the values of the constrained (unconstrained) fits.

c_R^K	Fit 1 [104]	Fit 2 [104]	Fit 1 [106]	Fit 2 [106]	Model I [107]	Model II [107]
$c^K_{\rho(770)}$	1.195 ± 0.009	1.139 ± 0.010	1.138 ± 0.011	1.120 ± 0.007	1.162 ± 0.005	1.067 ± 0.041
$c^K_{\omega(782)}$	1.195 ± 0.009	1.467 ± 0.035	1.138 ± 0.011	1.37 ± 0.03	1.26 ± 0.06	1.28 ± 0.14
$c^K_{\rho(1450)}$	-0.112 ∓ 0.010	-0.124 ∓ 0.012	-0.043 ± 0.014	-0.107 ± 0.010	-0.063 ± 0.014	-0.025 ± 0.008
$c^K_{\omega(1420)}$	-0.112 ∓ 0.010	-0.018 ∓ 0.024	-0.043 ± 0.014	-0.173 ± 0.003	-0.13 ± 0.03	-0.13 ± 0.02
$c^K_{\rho(1700)}$	-0.083 ∓ 0.019	-0.015 ∓ 0.022	-0.144 ± 0.015	-0.028 ± 0.012	-0.160 ± 0.014	-0.234 ± 0.013
$c^K_{\omega(1650)}$	-0.083 ∓ 0.019	-0.449 ∓ 0.059	-0.144 ± 0.015	-0.621 ± 0.020	-0.37 ± 0.05	-0.234 ± 0.013

The vector timelike form factors for charged and neutral kaons are defined by the matrix elements [85,108]

$$\langle K^+(p_1)K^-(p_2)|\bar{q}\gamma_\mu(1-\gamma_5)q|0\rangle = (p_1-p_2)_\mu F^q_{K^+K^-}(s), \tag{1}$$

$$\langle K^0(p_1)\bar{K}^0(p_2)|\bar{q}\gamma_\mu(1-\gamma_5)q|0\rangle = (p_1-p_2)_\mu F^q_{K^0\bar{K}^0}(s), \tag{2}$$

with the invariant mass square $s = p^2$ and the $K\bar{K}$ system momentum $p = p_1 + p_2$. These two form factors $F^q_{K^+K^-}$ and $F^q_{K^0\bar{K}^0}$ can be related to kaon electromagnetic form factors F_{K^+} and F_{K^0}, which are defined by [104]

$$\langle K^+(p_1)K^-(p_2)|j^{em}_\mu|0\rangle = (p_1-p_2)_\mu F_{K^+}(s), \tag{3}$$

$$\langle K^0(p_1)\bar{K}^0(p_2)|j^{em}_\mu|0\rangle = (p_1-p_2)_\mu F_{K^0}(s), \tag{4}$$

and have the forms [104]

$$F_{K^+}(s) = +\frac{1}{2}\sum_{\iota=\rho,\rho',\dots} c^K_\iota \mathrm{BW}_\iota(s) + \frac{1}{6}\sum_{\varsigma=\omega,\omega',\dots} c^K_\varsigma \mathrm{BW}_\varsigma(s) + \frac{1}{3}\sum_{\kappa=\phi,\phi',\dots} c^K_\kappa \mathrm{BW}_\kappa(s), \tag{5}$$

$$F_{K^0}(s) = -\frac{1}{2}\sum_{\iota=\rho,\rho',\dots} c^K_\iota \mathrm{BW}_\iota(s) + \frac{1}{6}\sum_{\varsigma=\omega,\omega',\dots} c^K_\varsigma \mathrm{BW}_\varsigma(s) + \frac{1}{3}\sum_{\kappa=\phi,\phi',\dots} c^K_\kappa \mathrm{BW}_\kappa(s), \tag{6}$$

with the electromagnetic current $j^{em}_\mu = \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d - \frac{1}{3}\bar{s}\gamma_\mu s$ carried by the light quarks u, d, and s [109]. The BW formula in $F_{K^+}(s)$ and $F_{K^0}(s)$ has the form [20,110]

$$\mathrm{BW}_R = \frac{m_R^2}{m_R^2 - s - i m_R \Gamma_R(s)}, \tag{7}$$

where the s-dependent width is given by

$$\Gamma_R(s) = \Gamma_R \frac{m_R}{\sqrt{s}} \frac{|\vec{q}|^3}{|\vec{q_0}|^3} X^2(|\vec{q}| r^R_{\mathrm{BW}}). \tag{8}$$

The Blatt-Weisskopf barrier factor [111] with barrier radius $r^R_{\mathrm{BW}} = 4.0\ \mathrm{GeV}^{-1}$ [20] is given by

$$X(z) = \sqrt{\frac{1+z_0^2}{1+z^2}}. \tag{9}$$

The magnitude of the momentum

$$|\vec{q}| = \frac{1}{2\sqrt{s}}\sqrt{[s-(m_K+m_{\bar{K}})^2][s-(m_K-m_{\bar{K}})^2]}, \tag{10}$$

and the $|\vec{q_0}|$ is $|\vec{q}|$ at $s = m_R^2$. One should note that $\bar{c}\gamma_\mu c$ can also contribute to F_{K^+} and F_{K^0} in the high-mass region [41,112,113] and the BW formula for the ρ family could be replaced with the Gounaris-Sakurai (GS) model [114] as in Refs. [104,106,115]. The F_{K^+} and F_{K^0} can be separated into the isospin $I=0$ and $I=1$ components as $F_{K^{+(0)}} = F^{I=1}_{K^{+(0)}} + F^{I=0}_{K^{+(0)}}$, with the $F^{I=0}_{K^+} = F^{I=0}_{K^0}$ and $F^{I=1}_{K^+} = -F^{I=1}_{K^0}$, and one has $\langle K^+(p_1)\bar{K}^0(p_2)|\bar{u}\gamma_\mu d|0\rangle = (p_1-p_2)_\mu 2F^{I=1}_{K^+}(s)$ [70,104].

When concern only the contributions for K^+K^- and $K^0\bar{K}^0$ from the resonant states $\iota = \rho(770, 1450, 1700)$ and $\varsigma = \omega(782, 1420, 1650)$, we have [85]

$$F^u_{K^+K^-}(s) = F^d_{K^0\bar{K}^0}(s) = +\frac{1}{2}\sum_\iota c^K_\iota \mathrm{BW}_\iota(s) + \frac{1}{2}\sum_\varsigma c^K_\varsigma \mathrm{BW}_\varsigma(s), \tag{11}$$

$$F^d_{K^+K^-}(s) = F^u_{K^0\bar{K}^0}(s) = -\frac{1}{2}\sum_\iota c^K_\iota \mathrm{BW}_\iota(s) + \frac{1}{2}\sum_\varsigma c^K_\varsigma \mathrm{BW}_\varsigma(s). \tag{12}$$

For the $K^+\bar{K}^0$ and K^0K^- pairs, which have no contribution from the neutral resonances $\omega(782, 1420, 1650)$, we have [54,103,104]

$$F_{K^+\bar{K}^0}(s) = F_{K^0K^-}(s) = F_{K^+}(s) - F_{K^0}(s) = \sum_\iota c_\iota^K \mathrm{BW}_\iota(s). \tag{13}$$

One should note that the different constants in Eqs. (11)–(12) and Eqs. (5)–(6) reveal the different definitions of the vector timelike and electromagnetic form factors for kaons in this work.

The c_R^K (with $R = \iota,\ \varsigma,\ \kappa$) is proportional to the coupling constant $g_{RK\bar{K}}$, and the coefficients have the constraints [107]

$$\sum_{\iota=\rho,\rho',\ldots} c_\iota^K = 1, \qquad \frac{1}{3}\sum_{\varsigma=\omega,\omega',\ldots} c_\varsigma^K + \frac{2}{3}\sum_{\kappa=\phi,\phi',\ldots} c_\kappa^K = 1 \tag{14}$$

to provide the proper normalizations $F_{K^+}(0) = 1$ and $F_{K^0}(0) = 0$, but the possibility of $SU(3)$ violations are allowed, which will become manifest in differences between the fitted normalization coefficients [104]. In Refs. [104,106,107], the coefficients c_R^Ks for the resonances $\rho(770)$, $\omega(782)$, and $\phi(1020)$ and their excited states have been fitted to the data, the results for $\rho(770, 1450, 1700)$ and $\omega(782, 1420, 1650)$ are summarized in Table I, from which one can find that the fitted values for the $c^K_{\rho(1450)}$, $c^K_{\rho(1700)}$, $c^K_{\omega(1420)}$, or $c^K_{\omega(1650)}$ are quite different in Refs. [104,106,107].

With the relations [104]

$$c^K_{\omega(782)} \approx \sqrt{2}\cdot\frac{f_{\omega(782)}g_{\omega(782)K^+K^-}}{m_{\omega(782)}},$$
$$g_{\omega(782)K^+K^-} = \frac{1}{\sqrt{2}}g_{\phi(1020)K^+K^-}, \tag{15}$$

and $\Gamma_{\omega(782)\to ee} = 0.60 \pm 0.02$ keV, $\Gamma_{\phi(1020)} = 4.249 \pm 0.013$ MeV, the branching fraction $(49.2 \pm 0.5)\%$ for the decay $\phi(1020) \to K^+K^-$ and the masses for $K^\pm$, $\omega(782)$, and $\phi(1020)$ in [15], it is easy to obtain the result 1.113 ± 0.019 for the coefficient $c^K_{\omega(782)}$, where the error comes from the uncertainties of $\Gamma_{\omega(782)\to ee}$ and $\Gamma_{\phi(1020)}$, while the errors come from the uncertainties of the relevant masses are very small and have been neglected. Similarly, we have $c^K_{\rho(770)} = 1.247 \pm 0.019$ with $g_{\rho(770)K^+K^-} = g_{\omega(782)K^+K^-}$ [104] and the decay constant $f_{\rho(770)} = 216 \pm 3$ MeV [116], where the error comes from the uncertainties of $f_{\rho(770)}$ and $\Gamma_{\phi(1020)}$. Our estimations for $c^K_{\omega(782)}$ and $c^K_{\rho(770)}$ are consistent with the results in [104,106,107]. But unlike the results of fit 2 in Refs. [104,106] and the values in [107], we have $c^K_{\omega(782)}$ as slightly less than $c^K_{\rho(770)}$, because the decay constant (mass) for $\omega(782)$ is slightly smaller (larger) than that for $\rho(770)$. Supposing $f_{\rho(770)} = f_{\omega(782)}$ and $m_{\rho(770)} = m_{\omega(782)}$, one will have $c^K_{\omega(782)} = c^K_{\rho(770)}$ with Eq. (15) and then back to the point of the constrained fit in [104,106]. To be sure, the violation of the relation $g_{\rho(770)K^+K^-} = g_{\omega(782)K^+K^-} = \frac{1}{\sqrt{2}}g_{\phi(1020)K^+K^-}$ will modify our estimations for $c^K_{\omega(782)}$ and $c^K_{\rho(770)}$, but the violation was found quite small [43].

In principle, the c_R^K for the couplings can be calculated with the formula [106,117]

$$c^K_{R_n} = \frac{(-1)^n\Gamma(\beta_R^K - 1/2)}{\alpha'\sqrt{\pi}m^2_{R_n}\Gamma(n+1)\Gamma(\beta_R^K - 1 - n)}, \tag{16}$$

with $\alpha' = 1/(2m^2_{R_0})$, and $n = 0$ for the ground states $\rho(770)$, $\omega(782)$, and $\phi(1020)$, $n \geq 1$ for their radial excitations. The parameters β_R^K could be deduced from Eq. (16) with the fitted $c^K_{R_0}$ [106]. With Eq. (16) one will deduce the results $c^K_{\rho(1450)} = -0.156 \pm 0.015$ and $c^K_{\omega(1420)} = -0.066 \pm 0.014$. The $c^K_{\rho(1450)}$ here is consistent with the result of fit 2 in [104] but some larger than the latter for the magnitude. If we take into account the relation $g_{\omega(1420)K^+K^-} \approx g_{\rho(1450)K^+K^-}$, the big difference between $c^K_{\omega(1420)}$ and $c^K_{\rho(1450)}$ seems not reasonable. In view of the consistency for the coefficient $c_{\rho(1450)}$ of the pion electromagnetic form factor F_π in Refs. [115,118–121] by different collaborations, we here propose a constraint for $c^K_{\rho(1450)}$ from the coefficient $c^\pi_{\rho(1450)}$ of F_π. With the relation $g_{\rho(1450)K^+K^-} \approx \frac{1}{2}g_{\rho(1450)\pi^+\pi^-}$ within flavor $SU(3)$ symmetry [104], one has

$$|c^K_{\rho(1450)}| \approx \sqrt{2}\cdot\frac{|f_{\rho(1450)}g_{\rho(1450)K^+K^-}|}{m_{\rho(1450)}}$$
$$\approx \frac{|f_{\rho(1450)}g_{\rho(1450)\pi^+\pi^-}|}{\sqrt{2}m_{\rho(1450)}} \approx |c^\pi_{\rho(1450)}|, \tag{17}$$

where the different definitions for the coefficient $c^\pi_{\rho(1450)}$ in [115,118–121] and the differences for the BW and GS models should be taken into account. In view of the results for $c^K_{\rho(1450)}$ in [104] and $c^\pi_{\rho(1450)}$ in Refs. [115,118–121], we adopt the $c^K_{\rho(1450)} = -0.156 \pm 0.015$ deduced from Eq. (16) in our numerical calculation. In Ref. [122], with the analyses of the e^+e^- annihilation data, $\Gamma_{\omega'\to ee}$ was estimated to be 0.15 keV, implies the decay constant $f_{\omega(1420)} = 131$ MeV. With the $f_{\rho(1450)} = 182 \pm 5$ MeV in [123] and the masses for $\omega(1420)$ and $\rho(1450)$ in [15], one can estimate the ratio between $c^K_{\omega(1420)}$ and $c^K_{\rho(1450)}$ as 0.748 ± 0.040, then one has $c^K_{\omega(1420)} = -0.117 \pm 0.013$, which agree with the constrained result in [104] and the corresponding values in [107] as shown in Table I.

The results for $c^K_{\rho(1700)}$ vary dramatically in Table I, from -0.015 ∓ 0.022 [104] to -0.234 ± 0.013 [107]. A reliable reference value should come from the measurements of F_π rather than the result deduced from Eq. (16) since $\rho(1700)$

is believed to be a 1^3D_1 state in ρ family [15,122,124]. With Eq. (17) and the replacement $\rho(1450) \to \rho(1700)$ one has $|c^K_{\rho(1700)}| \approx 0.081$ with the result $|c_{\rho''}| = 0.068$ for F_π in [115]. The difference between the $|c^K_{\rho(1700)}|$ and $|c_{\rho''}|$ is induced by the differences of the BW and GS models and the different definitions for them. Then we adopt the fitted result -0.083 ∓ 0.019 for $c^K_{\rho(1700)}$ [104] in the numerical calculation in this work. As for the coefficient $c^K_{\omega(1650)}$, we employ the value -0.083 ∓ 0.019 of the constrained fits in [104] because of insufficiency of the knowledge for the properties of $\omega(1650)$.

III. KINEMATICS AND DIFFERENTIAL BRANCHING FRACTION

In the light cone coordinates, the momentum p_B for the initial state B^+, B^0, or B^0_s with the mass m_B is written as $p_B = \frac{m_B}{\sqrt{2}}(1, 1, 0_\mathrm{T})$ in the rest frame of B meson. In the same coordinates, the bachelor state pion or kaon in the concerned processes has the momentum $p_3 = \frac{m_B}{\sqrt{2}}(1-\zeta, 0, 0_\mathrm{T})$, and its spectator quark has the momentum $k_3 = (\frac{m_B}{\sqrt{2}}(1-\zeta)x_3, 0, k_{3\mathrm{T}})$. For the resonances ρ, ω and their excited states, and the $K\bar{K}$ system generated from them by the strong interaction, we have the momentum $p = \frac{m_B}{\sqrt{2}}(\zeta, 1, 0_\mathrm{T})$ and the longitudinal polarization vector $\epsilon_L = \frac{1}{\sqrt{2}}(-\sqrt{\zeta}, 1/\sqrt{\zeta}, 0_\mathrm{T})$. It is easy to check the variable $\zeta = s/m_B^2$ with the invariant mass square $s = m^2_{K\bar{K}} \equiv p^2$. The spectator quark comes out from B meson and goes into the intermediate states in hadronization shown in Fig. 1(a) has the momenta $k_B = (\frac{m_B}{\sqrt{2}}x_B, 0, k_{B\mathrm{T}})$ and $k = (0, \frac{m_B}{\sqrt{2}}x, k_\mathrm{T})$ before and after it passes through the hard gluon vertex. The x_B, x, and x_3, which run from zero to one in the numerical calculation, are the momentum fractions for the B meson, the resonances and the bachelor final state, respectively.

For the P-wave $K\bar{K}$ system along with the subprocesses $\rho \to K\bar{K}$ and $\omega \to K\bar{K}$, the distribution amplitudes are organized into [21,68,71]

$$\phi^{P\text{-wave}}_{K\bar{K}}(x,s) = \frac{-1}{\sqrt{2N_c}}[\sqrt{s}\not{\epsilon}_L\phi^0(x,s) + \not{\epsilon}_L\not{p}\phi^t(x,s) + \sqrt{s}\phi^s(x,s)], \tag{18}$$

with

$$\phi^0(x,s) = \frac{3C_X F_K(s)}{\sqrt{2N_c}}x(1-x)[1 + a^0_R C^{3/2}_2(1-2x)], \tag{19}$$

$$\phi^t(x,s) = \frac{3C_X F^t_K(s)}{2\sqrt{2N_c}}(1-2x)^2[1 + a^t_R C^{3/2}_2(1-2x)], \tag{20}$$

$$\phi^s(x,s) = \frac{3C_X F^s_K(s)}{2\sqrt{2N_c}}(1-2x)[1 + a^s_R(1-10x+10x^2)], \tag{21}$$

where F_K is employed as the abbreviation of the vector timelike form factors in Eqs. (11)–(13) and gain different component for different resonance contribution from to the expressions of the Eqs. (11)–(13) in the concerned decay processes. Moreover, we have factored out the normalization constant C_X to make sure the proper normalizations for the timelike form factors for kaon, and C_X are given by

$$C_{\rho^0} = C_\omega = \sqrt{2}, \qquad C_{\rho^\pm} = 1. \tag{22}$$

The Gegenbauer polynomial $C^{3/2}_2(\chi) = 3(5\chi^2 - 1)/2$ for the distribution amplitudes ϕ^0 and ϕ^t, and the Gegenbauer moments have been catered to the data in Ref. [68] for the quasi-two-body decays $B \to K\rho \to K\pi\pi$. Within flavor $SU(2)$ symmetry, we adopt the same Gegenbauer moments for the P-wave $K\bar{K}$ system originating from the intermediate states ω and ρ in this work. The vector timelike form factors F^t_K and F^s_K for the twist-three distribution amplitudes are deduced from the relations $F^{t,s}_K(s) \approx (f^T_\rho/f_\rho)F_K(s)$ and $F^{t,s}_K(s) \approx (f^T_\omega/f_\omega)F_K(s)$ [68] with the result $f^T_\rho/f_\rho = 0.687$ at the scale $\mu = 2$ GeV [125]. The relation $f^T_\rho/f_\rho \approx f^T_\omega/f_\omega$ [116] is employed because of the lack of a lattice QCD determination for f^T_ω.

In PQCD approach, the factorization formula for the decay amplitude $\mathcal{A}$ of the quasi-two-body decays $B \to \rho h \to K\bar{K}h$ and $B \to \omega h \to K\bar{K}h$ is written as [126,127]

$$\mathcal{A} = \phi_B \otimes \mathcal{H} \otimes \phi^{P\text{-wave}}_{K\bar{K}} \otimes \phi_h \tag{23}$$

and, according to Fig. 1, is at leading order in the strong coupling α_s. The hard kernel $\mathcal{H}$ here contains only one hard gluon exchange, and the symbol $\otimes$ means convolutions in parton momenta. For the B meson and bachelor final state h in this work, their distribution amplitudes ϕ_B and ϕ_h are the same as those widely adopted in the PQCD approach, we attach their expressions and parameters in Appendix A.

For the CP averaged differential branching fraction ($\mathcal{B}$), one has the formula [15,21,84]

$$\frac{d\mathcal{B}}{d\zeta} = \tau_B \frac{|\vec{q}|^3|\vec{q}_h|^3}{12\pi^3 m_B^5}\overline{|\mathcal{A}|^2}, \tag{24}$$

where τ_B is the mean lifetime for B meson. The magnitude of the momentum $|\vec{q}_h|$ for the state h in the rest frame of the intermediate states is written as

$$|\vec{q}_h| = \frac{1}{2\sqrt{s}}\sqrt{[m_B^2 - (s+m_h)^2][m_B^2 - (s-m_h)^2]}, \tag{25}$$

TABLE II. Masses for the relevant particles, the full widths for $\rho(770, 1450, 1700)$ and $\omega(782, 1420, 1650)$ (in units of GeV) and the Wolfenstein parameters [15].

$m_{B^0} = 5.280$	$m_{B^\pm} = 5.279$	$m_{B_s^0} = 5.367$	$m_{K^0} = 0.498$	$m_{K^\pm} = 0.494$
$m_{\pi^0} = 0.135$	$m_{\pi^\pm} = 0.140$	$m_{\rho(770)} = 0.775$	$\Gamma_{\rho(770)} = 0.149$	$m_{\omega(782)} = 0.783$
$\Gamma_{\omega(782)} = 0.00849$	$m_{\omega(1420)} = 1.410 \pm 0.060$	$\Gamma_{\omega(1420)} = 0.290 \pm 0.190$		
$m_{\rho(1450)} = 1.465 \pm 0.025$	$\Gamma_{\rho(1450)} = 0.400 \pm 0.060$	$m_{\omega(1650)} = 1.670 \pm 0.030$		
$\Gamma_{\omega(1650)} = 0.315 \pm 0.035$	$m_{\rho(1700)} = 1.720 \pm 0.020$	$\Gamma_{\rho(1700)} = 0.250 \pm 0.100$		
$\lambda = 0.22650 \pm 0.00048$	$A = 0.790^{+0.017}_{-0.012}$	$\bar{\rho} = 0.141^{+0.016}_{-0.017}$	$\bar{\eta} = 0.357 \pm 0.01$	

with m_h as the mass for the bachelor meson pion or kaon. When $m_K = m_{\bar{K}}$, the Eq. (10) has a simpler form

$$|\vec{q}| = \frac{1}{2}\sqrt{s - 4m_K^2}. \tag{26}$$

Note that the cubic $|\vec{q}|$ and $|\vec{q}_h|$ in Eq. (24) are caused by the introduction of the Zemach tensor $-2\vec{q}\cdot\vec{q}_h$, which is employed to describe the angular distribution for the decay of spin 1 resonances [128]. The direct CP asymmetry $\mathcal{A}_{CP}$ is defined as

$$\mathcal{A}_{CP} = \frac{\mathcal{B}(\bar{B}\to\bar{f}) - \mathcal{B}(B\to f)}{\mathcal{B}(\bar{B}\to\bar{f}) + \mathcal{B}(B\to f)}. \tag{27}$$

The Lorentz invariant decay amplitudes according to Fig. 1 for the decays $B\to\rho h\to K\bar{K}h$ and $B\to\omega h\to K\bar{K}h$ are given in Appendix B.

IV. NUMERICAL RESULTS AND DISCUSSIONS

In the numerical calculation, we employ the decay constants $f_B = 0.189$ GeV and $f_{B_s} = 0.231$ GeV for the $B^{0,\pm}$ and B_s^0 mesons [129], respectively, and the mean lifetimes $\tau_{B^0} = (1.519\pm0.004)\times10^{-12}$ s, $\tau_{B^\pm} = (1.638\pm 0.004)\times10^{-12}$ s and $\tau_{B_s^0} = (1.515\pm0.004)\times10^{-12}$ s [15]. The masses for the relevant particles in the numerical calculation of this work, the full widths for the resonances $\rho(770, 1450, 1700)$ and $\omega(782, 1420, 1650)$, and the Wolfenstein parameters of the Cabbibo-Kobayashi-Maskawa (CKM) matrix are presented in Table II.

Utilizing the differential branching fractions the Eq. (24) and the decay amplitudes collected in Appendix B, we obtain the CP averaged branching fractions and the direct CP asymmetries in Tables III, IV, and V for the concerned quasi-two-body decay processes $B\to\rho(770, 1450, 1700)h\to K\bar{K}h$ and $B\to\omega(782, 1420, 1650)h\to K\bar{K}h$. For these PQCD predictions, the uncertainties of the Gegenbauer moments $a_R^0 = 0.25\pm0.10$, $a_R^t = -0.50\pm0.20$ and $a_R^s = 0.75\pm0.25$ along with the decay widths of the intermediate states contribute the first error. The second error for each result in Tables III, IV, and V comes from the shape parameter $\omega_B = 0.40\pm0.04$ or $\omega_{B_s} = 0.50\pm0.05$ in Eq. (A2) for the $B^{+,0}$ or B_s^0 meson. The third one is induced by the chiral scale parameters $m_0^h = \frac{m_h^2}{m_q+m_{q'}}$ with $m_0^\pi = 1.4\pm0.1$ GeV and $m_0^K = 1.9\pm 0.1$ GeV [130] and the Gegenbauer moment $a_2^h = 0.25\pm 0.15$ for the bachelor final state pion or kaon. The fourth one comes from the Wolfenstein parameters A and $\bar{\rho}$ listed in Table II. The uncertainties of $c^K_{\rho(770)} = 1.247\pm0.019$, $c^K_{\omega(782)} = 1.113\pm0.019$, $c^K_{\rho(1450)} = -0.156\pm0.015$, $c^K_{\omega(1420)} = -0.117\pm0.013$ and $c^K_{\omega(1650),\rho(1700)} = -0.083\pm0.019$ result in the fifth error for the predicted branching fractions in this work, while these coefficients c_R^K, which exist only in the kaon timelike form factors, will not change the direct CP asymmetries for the relevant decay processes. There are other errors for the results in Tables III, IV, and V, which come from the masses and the decay constants of the initial and final states, from the parameters in the distribution amplitudes for bachelor pion or kaon, from the uncertainties of the Wolfenstein parameters λ and $\bar{\eta}$, etc., are small and have been neglected.

The PQCD predictions are omitted in Tables III, IV, and V for those quasi-two-body decays with the subprocesses $\rho(770, 1450, 1700)^0\to K^0\bar{K}^0$ and $\omega(782, 1420, 1650)\to K^0\bar{K}^0$. The variations caused by the small mass difference between $K^\pm$ and K^0 for the branching fraction and direct CP asymmetry of a decay mode with one of these intermediate states decaying into $K^0\bar{K}^0$ or K^+K^- are tiny. As the examples, we calculate the branching fractions for the decays $B^+\to\pi^+\rho(770)^0$, $B^+\to K^+\rho(770)^0$, $B^+\to\pi^+\omega(782)$, and $B^+\to K^+\omega(782)$ with the resonances $\rho(770)^0$ and $\omega(782)$ decay into the final state $K^0\bar{K}^0$. Their four branching fractions with the same sources for the errors as these results in Table III are predicted to be

$$\mathcal{B}(B^+\to\pi^+\rho(770)^0\to\pi^+K^0\bar{K}^0) = 1.40^{+0.26+0.17+0.10+0.06+0.04}_{-0.24-0.17-0.10-0.06-0.04}\times10^{-7}, \tag{28}$$

$$\mathcal{B}(B^+\to K^+\rho(770)^0\to K^+K^0\bar{K}^0) = 5.08^{+0.92+1.00+0.70+0.25+0.15}_{-0.83-0.97-0.65-0.20-0.15}\times10^{-8}, \tag{29}$$

$$\mathcal{B}(B^+\to\pi^+\omega(782)\to\pi^+K^0\bar{K}^0) = 4.14^{+1.64+1.02+0.07+0.20+0.14}_{-1.32-0.94-0.08-0.16-0.14}\times10^{-8}, \tag{30}$$

TABLE III. PQCD predictions of the CP averaged branching fractions and the direct CP asymmetries for the quasi-two-body $B \to \rho(770)h \to K\bar{K}h$ and $B \to \omega(782)h \to K\bar{K}h$ decays. The decays with the subprocess $\rho(770)^0 \to K^0\bar{K}^0$ or $\omega(782) \to K^0\bar{K}^0$ have the same results as their corresponding decay modes with $\rho(770)^0 \to K^+K^-$ or $\omega(782) \to K^+K^-$.

Decay modes	$\mathcal{B}$	$\mathcal{A}_{CP}$
$B^+ \to \pi^0[\rho(770)^+ \to]K^+\bar{K}^0$	$2.01^{+0.38+0.29+0.24+0.10+0.06}_{-0.35-0.26-0.20-0.07-0.06} \times 10^{-8}$	$-0.16^{+0.18+0.20+0.10+0.00}_{-0.20-0.18-0.10-0.00}$
$B^+ \to \pi^+[\rho(770)^0 \to]K^+K^-$	$1.43^{+0.26+0.19+0.11+0.06+0.04}_{-0.25-0.17-0.10-0.05-0.04} \times 10^{-7}$	$-0.22^{+0.04+0.01+0.01+0.01}_{-0.04-0.01-0.01-0.01}$
$B^+ \to \pi^+[\omega(782) \to]K^+K^-$	$4.21^{+1.67+1.03+0.08+0.21+0.14}_{-1.34-0.96-0.08-0.17-0.14} \times 10^{-8}$	$0.02^{+0.01+0.01+0.02+0.00}_{-0.01-0.01-0.01-0.00}$
$B^+ \to K^0[\rho(770)^+ \to]K^+\bar{K}^0$	$2.21^{+0.51+0.51+0.34+0.10+0.07}_{-0.45-0.46-0.29-0.08-0.07} \times 10^{-7}$	$0.17^{+0.04+0.04+0.01+0.00}_{-0.05-0.03-0.02-0.00}$
$B^+ \to K^+[\rho(770)^0 \to]K^+K^-$	$5.15^{+0.91+0.99+0.69+0.25+0.16}_{-0.85-0.98-0.66-0.21-0.16} \times 10^{-8}$	$0.39^{+0.03+0.04+0.04+0.00}_{-0.04-0.04-0.05-0.01}$
$B^+ \to K^+[\omega(782) \to]K^+K^-$	$8.92^{+1.67+2.33+1.19+0.43+0.30}_{-1.47-2.18-1.07-0.34-0.30} \times 10^{-8}$	$0.22^{+0.04+0.05+0.04+0.00}_{-0.04-0.04-0.04-0.00}$
$B^0 \to \pi^-[\rho(770)^+ \to]K^+\bar{K}^0$	$1.02^{+0.21+0.28+0.14+0.06+0.03}_{-0.17-0.25-0.13-0.05-0.03} \times 10^{-7}$	$0.15^{+0.04+0.04+0.00+0.00}_{-0.03-0.03-0.00-0.00}$
$B^0 \to \pi^+[\rho(770)^- \to]K^-K^0$	$9.59^{+3.25+1.96+0.22+0.46+0.29}_{-2.90-1.88-0.19-0.33-0.29} \times 10^{-8}$	$-0.27^{+0.11+0.02+0.02+0.00}_{-0.08-0.01-0.02-0.00}$
$B^0 \to \pi^0[\rho(770)^0 \to]K^+K^-$	$1.47^{+0.96+0.53+0.19+0.13+0.04}_{-0.78-0.49-0.14-0.07-0.04} \times 10^{-9}$	$0.19^{+0.17+0.07+0.06+0.05}_{-0.15-0.06-0.04-0.05}$
$B^0 \to \pi^0[\omega(782) \to]K^+K^-$	$4.96^{+0.73+1.25+0.63+0.24+0.17}_{-0.87-1.36-0.65-0.22-0.17} \times 10^{-9}$	$0.58^{+0.19+0.11+0.14+0.04}_{-0.18-0.11-0.14-0.04}$
$B^0 \to K^+[\rho(770)^- \to]K^-K^0$	$1.77^{+0.30+0.41+0.27+0.08+0.05}_{-0.25-0.39-0.25-0.06-0.05} \times 10^{-7}$	$0.20^{+0.07+0.03+0.03+0.00}_{-0.08-0.02-0.03-0.00}$
$B^0 \to K^0[\rho(770)^0 \to]K^+K^-$	$5.44^{+0.88+1.26+0.82+0.24+0.17}_{-0.81-1.19-0.76-0.18-0.17} \times 10^{-8}$	$-0.01^{+0.01+0.01+0.00+0.00}_{-0.01-0.01-0.01-0.00}$
$B^0 \to K^0[\omega(782) \to]K^+K^-$	$5.99^{+1.15+1.60+0.88+0.22+0.20}_{-0.96-1.39-0.75-0.19-0.20} \times 10^{-8}$	$0.01^{+0.02+0.00+0.01+0.00}_{-0.02-0.00-0.01-0.00}$
$B_s^0 \to \pi^-[\rho(770)^+ \to]K^+\bar{K}^0$	$2.31^{+0.75+0.51+0.30+0.11+0.07}_{-0.62-0.39-0.26-0.08-0.07} \times 10^{-9}$	$-0.66^{+0.17+0.04+0.03+0.01}_{-0.16-0.06-0.03-0.01}$
$B_s^0 \to \pi^+[\rho(770)^- \to]K^-K^0$	$5.43^{+1.47+0.57+0.79+0.24+0.17}_{-1.45-0.48-0.77-0.20-0.17} \times 10^{-9}$	$0.04^{+0.03+0.01+0.01+0.00}_{-0.04-0.01-0.01-0.00}$
$B_s^0 \to \pi^0[\rho(770)^0 \to]K^+K^-$	$1.63^{+0.98+0.46+0.18+0.07+0.05}_{-0.81-0.41-0.16-0.06-0.05} \times 10^{-9}$	$-0.35^{+0.13+0.05+0.12+0.03}_{-0.14-0.06-0.14-0.03}$
$B_s^0 \to \pi^0[\omega(782) \to]K^+K^-$	$8.17^{+3.83+2.37+1.22+0.51+0.28}_{-3.28-2.14-1.21-0.45-0.28} \times 10^{-11}$	$0.11^{+0.03+0.00+0.02+0.00}_{-0.04-0.00-0.02-0.01}$
$B_s^0 \to K^-[\rho(770)^+ \to]K^+\bar{K}^0$	$2.04^{+0.03+0.43+0.22+0.11+0.06}_{-0.02-0.41-0.21-0.09-0.06} \times 10^{-7}$	$0.25^{+0.04+0.03+0.00+0.01}_{-0.04-0.03-0.00-0.01}$
$B_s^0 \to \bar{K}^0[\rho(770)^0 \to]K^+K^-$	$1.03^{+0.63+0.19+0.18+0.08+0.03}_{-0.45-0.17-0.16-0.05-0.03} \times 10^{-9}$	$0.60^{+0.24+0.03+0.16+0.02}_{-0.22-0.04-0.14-0.04}$
$B_s^0 \to \bar{K}^0[\omega(782) \to]K^+K^-$	$1.39^{+0.68+0.17+0.12+0.07+0.05}_{-0.57-0.14-0.14-0.07-0.05} \times 10^{-9}$	$-0.34^{+0.29+0.06+0.01+0.03}_{-0.21-0.06-0.03-0.03}$

TABLE IV. PQCD predictions of the CP averaged branching ratios and the direct CP asymmetries for the quasi-two-body $B \to \rho(1450)h \to K\bar{K}h$ and $B \to \omega(1420)h \to K\bar{K}h$ decays. The decays with the subprocess $\rho(1450)^0 \to K^0\bar{K}^0$ or $\omega(1420) \to K^0\bar{K}^0$ have the same results as their corresponding decay modes with $\rho(1450)^0 \to K^+K^-$ or $\omega(1420) \to K^+K^-$.

Decay modes	$\mathcal{B}$	$\mathcal{A}_{CP}$
$B^+ \to \pi^0[\rho(1450)^+ \to]K^+\bar{K}^0$	$1.27^{+0.26+0.22+0.10+0.06+0.24}_{-0.22-0.18-0.12-0.04-0.24} \times 10^{-8}$	$-0.14^{+0.24+0.21+0.11+0.00}_{-0.22-0.17-0.09-0.00}$
$B^+ \to \pi^+[\rho(1450)^0 \to]K^+K^-$	$9.46^{+1.79+1.16+0.72+0.49+1.82}_{-1.65-1.14-0.69-0.38-1.82} \times 10^{-8}$	$-0.22^{+0.04+0.01+0.01+0.01}_{-0.04-0.01-0.01-0.01}$
$B^+ \to \pi^+[\omega(1420) \to]K^+K^-$	$1.62^{+0.61+0.45+0.03+0.08+0.36}_{-0.52-0.39-0.02-0.07-0.36} \times 10^{-8}$	$0.01^{+0.01+0.02+0.01+0.01}_{-0.02-0.02-0.02-0.01}$
$B^+ \to K^0[\rho(1450)^+ \to]K^+\bar{K}^0$	$1.20^{+0.29+0.24+0.17+0.05+0.23}_{-0.25-0.23-0.16-0.04-0.23} \times 10^{-7}$	$0.20^{+0.04+0.03+0.02+0.00}_{-0.05-0.02-0.02-0.00}$
$B^+ \to K^+[\rho(1450)^0 \to]K^+K^-$	$3.36^{+0.62+0.67+0.47+0.16+0.65}_{-0.56-0.64-0.43-0.13-0.65} \times 10^{-8}$	$0.42^{+0.03+0.04+0.05+0.01}_{-0.03-0.03-0.05-0.01}$
$B^+ \to K^+[\omega(1420) \to]K^+K^-$	$3.09^{+0.64+0.80+0.42+0.15+0.69}_{-0.57-0.73-0.37-0.12-0.69} \times 10^{-8}$	$0.32^{+0.05+0.05+0.03+0.01}_{-0.05-0.05-0.03-0.01}$
$B^0 \to \pi^-[\rho(1450)^+ \to]K^+\bar{K}^0$	$7.39^{+1.58+2.20+1.01+0.41+1.42}_{-1.31-1.86-0.96-0.33-1.42} \times 10^{-8}$	$0.16^{+0.02+0.05+0.01+0.01}_{-0.03-0.03-0.01-0.01}$
$B^0 \to \pi^+[\rho(1450)^- \to]K^-K^0$	$6.94^{+2.04+1.40+0.14+0.33+1.33}_{-1.94-1.38-0.14-0.25-1.33} \times 10^{-8}$	$-0.27^{+0.12+0.02+0.02+0.00}_{-0.08-0.01-0.02-0.00}$
$B^0 \to \pi^0[\rho(1450)^0 \to]K^+K^-$	$8.48^{+5.96+3.07+0.81+0.68+1.63}_{-5.14-3.01-0.78-0.49-1.63} \times 10^{-10}$	$0.20^{+0.21+0.10+0.09+0.06}_{-0.17-0.08-0.07-0.05}$
$B^0 \to \pi^0[\omega(1420) \to]K^+K^-$	$2.08^{+0.32+0.58+0.28+0.10+0.46}_{-0.37-0.66-0.32-0.08-0.46} \times 10^{-9}$	$0.58^{+0.17+0.10+0.11+0.02}_{-0.16-0.11-0.09-0.02}$
$B^0 \to K^+[\rho(1450)^- \to]K^-K^0$	$1.18^{+0.20+0.27+0.18+0.05+0.23}_{-0.17-0.25-0.17-0.04-0.23} \times 10^{-7}$	$0.22^{+0.08+0.03+0.04+0.00}_{-0.08-0.02-0.04-0.00}$
$B^0 \to K^0[\rho(1450)^0 \to]K^+K^-$	$3.69^{+0.67+0.84+0.55+0.16+0.71}_{-0.60-0.82-0.51-0.12-0.71} \times 10^{-8}$	$-0.01^{+0.01+0.01+0.00+0.00}_{-0.02-0.01-0.01-0.00}$
$B^0 \to K^0[\omega(1420) \to]K^+K^-$	$2.07^{+0.48+0.48+0.29+0.08+0.46}_{-0.46-0.45-0.26-0.06-0.46} \times 10^{-8}$	$-0.02^{+0.04+0.03+0.01+0.00}_{-0.02-0.03-0.01-0.00}$
$B_s^0 \to \pi^-[\rho(1450)^+ \to]K^+\bar{K}^0$	$1.55^{+0.39+0.30+0.16+0.07+0.30}_{-0.33-0.28-0.14-0.05-0.30} \times 10^{-9}$	$-0.66^{+0.15+0.04+0.05+0.02}_{-0.16-0.08-0.04-0.01}$
$B_s^0 \to \pi^+[\rho(1450)^- \to]K^-K^0$	$4.54^{+1.30+0.37+0.69+0.20+0.87}_{-1.27-0.40-0.67-0.16-0.87} \times 10^{-9}$	$0.04^{+0.03+0.01+0.02+0.00}_{-0.05-0.01-0.01-0.00}$
$B_s^0 \to \pi^0[\rho(1450)^0 \to]K^+K^-$	$1.15^{+0.72+0.35+0.09+0.05+0.22}_{-0.59-0.30-0.12-0.04-0.22} \times 10^{-9}$	$-0.36^{+0.12+0.05+0.10+0.02}_{-0.16-0.04-0.14-0.03}$
$B_s^0 \to \pi^0[\omega(1420) \to]K^+K^-$	$3.67^{+1.59+1.17+0.65+0.21+0.82}_{-1.38-0.97-0.58-0.19-0.82} \times 10^{-11}$	$0.14^{+0.03+0.00+0.01+0.00}_{-0.02-0.01-0.01-0.01}$
$B_s^0 \to K^-[\rho(1450)^+ \to]K^+\bar{K}^0$	$1.49^{+0.07+0.31+0.16+0.08+0.29}_{-0.06-0.30-0.15-0.06-0.29} \times 10^{-7}$	$0.25^{+0.04+0.03+0.00+0.01}_{-0.04-0.03-0.00-0.01}$
$B_s^0 \to \bar{K}^0[\rho(1450)^0 \to]K^+K^-$	$6.86^{+4.09+0.81+1.03+0.44+1.32}_{-3.56-0.75-0.94-0.39-1.32} \times 10^{-10}$	$0.64^{+0.29+0.02+0.08+0.05}_{-0.27-0.01-0.12-0.07}$
$B_s^0 \to \bar{K}^0[\omega(1420) \to]K^+K^-$	$5.79^{+3.28+0.53+0.63+0.34+1.29}_{-2.39-0.42-0.57-0.31-1.29} \times 10^{-10}$	$-0.54^{+0.29+0.13+0.05+0.01}_{-0.33-0.12-0.05-0.03}$

TABLE V. PQCD predictions of the CP averaged branching ratios and the direct CP asymmetries for the quasi-two-body $B \to \rho(1700)h \to K\bar{K}h$ and $B \to \omega(1650)h \to K\bar{K}h$ decays. The decays with the subprocess $\rho(1700)^0 \to K^0\bar{K}^0$ or $\omega(1650) \to K^0\bar{K}^0$ have the same results as their corresponding decay modes with $\rho(1700)^0 \to K^+K^-$ or $\omega(1650) \to K^+K^-$.

Decay modes	$\mathcal{B}$	$\mathcal{A}_{CP}$
$B^+ \to \pi^0[\rho(1700)^+ \to]K^+\bar{K}^0$	$1.03^{+0.21+0.20+0.09+0.05+0.47}_{-0.18-0.17-0.10-0.04-0.47} \times 10^{-8}$	$-0.15^{+0.22+0.23+0.13+0.01}_{-0.23-0.21-0.12-0.00}$
$B^+ \to \pi^+[\rho(1700)^0 \to]K^+K^-$	$8.71^{+1.47+1.20+0.61+0.46+3.99}_{-1.34-1.17-0.59-0.36-3.99} \times 10^{-8}$	$-0.25^{+0.03+0.02+0.01+0.01}_{-0.03-0.01-0.01-0.01}$
$B^+ \to \pi^+[\omega(1650) \to]K^+K^-$	$1.48^{+0.42+0.32+0.02+0.01+0.68}_{-0.34-0.28-0.02-0.01-0.68} \times 10^{-9}$	$0.02^{+0.01+0.00+0.00+0.00}_{-0.01-0.00-0.00-0.00}$
$B^+ \to K^0[\rho(1700)^+ \to]K^+\bar{K}^0$	$1.08^{+0.27+0.21+0.18+0.05+0.49}_{-0.25-0.19-0.15-0.03-0.49} \times 10^{-7}$	$0.21^{+0.05+0.04+0.03+0.00}_{-0.06-0.03-0.02-0.00}$
$B^+ \to K^+[\rho(1700)^0 \to]K^+K^-$	$2.85^{+0.50+0.49+0.35+0.14+1.30}_{-0.49-0.48-0.32-0.11-1.30} \times 10^{-8}$	$0.47^{+0.02+0.04+0.05+0.01}_{-0.02-0.03-0.05-0.01}$
$B^+ \to K^+[\omega(1650) \to]K^+K^-$	$2.81^{+0.53+0.66+0.36+0.13+1.29}_{-0.47-0.59-0.32-0.10-1.29} \times 10^{-8}$	$0.36^{+0.03+0.05+0.05+0.01}_{-0.04-0.05-0.05-0.01}$
$B^0 \to \pi^-[\rho(1700)^+ \to]K^+\bar{K}^0$	$4.38^{+0.80+1.17+0.50+0.23+2.01}_{-0.73-1.06-0.48-0.19-2.01} \times 10^{-8}$	$0.18^{+0.03+0.03+0.01+0.01}_{-0.02-0.03-0.01-0.01}$
$B^0 \to \pi^+[\rho(1700)^- \to]K^-K^0$	$6.66^{+1.78+1.41+0.13+0.32+3.05}_{-1.69-1.40-0.12-0.24-3.05} \times 10^{-8}$	$-0.29^{+0.12+0.02+0.02+0.01}_{-0.08-0.02-0.02-0.01}$
$B^0 \to \pi^0[\rho(1700)^0 \to]K^+K^-$	$8.11^{+5.46+3.02+0.82+0.68+3.71}_{-4.98-2.97-0.80-0.54-3.71} \times 10^{-10}$	$0.18^{+0.20+0.08+0.07+0.04}_{-0.18-0.06-0.07-0.04}$
$B^0 \to \pi^0[\omega(1650) \to]K^+K^-$	$1.48^{+0.31+0.44+0.15+0.06+0.68}_{-0.34-0.39-0.16-0.06-0.68} \times 10^{-9}$	$0.57^{+0.21+0.07+0.09+0.01}_{-0.17-0.09-0.07-0.01}$
$B^0 \to K^+[\rho(1700)^- \to]K^-K^0$	$9.95^{+1.87+1.83+1.31+0.44+4.56}_{-1.60-1.61-1.15-0.32-4.56} \times 10^{-8}$	$0.28^{+0.07+0.01+0.05+0.00}_{-0.09-0.01-0.04-0.00}$
$B^0 \to K^0[\rho(1700)^0 \to]K^+K^-$	$2.94^{+0.54+0.57+0.38+0.13+1.35}_{-0.53-0.56-0.36-0.09-1.35} \times 10^{-8}$	$-0.01^{+0.01+0.00+0.01+0.00}_{-0.01-0.00-0.01-0.00}$
$B^0 \to K^0[\omega(1650) \to]K^+K^-$	$1.89^{+0.43+0.39+0.22+0.08+0.87}_{-0.38-0.36-0.19-0.07-0.87} \times 10^{-8}$	$-0.01^{+0.04+0.00+0.01+0.00}_{-0.03-0.00-0.00-0.00}$
$B_s^0 \to \pi^-[\rho(1700)^+ \to]K^+\bar{K}^0$	$1.37^{+0.34+0.29+0.14+0.06+0.63}_{-0.31-0.27-0.14-0.05-0.63} \times 10^{-9}$	$-0.70^{+0.16+0.04+0.01+0.01}_{-0.15-0.07-0.04-0.01}$
$B_s^0 \to \pi^+[\rho(1700)^- \to]K^-K^0$	$3.57^{+0.94+0.30+0.54+0.16+1.63}_{-0.86-0.32-0.52-0.13-1.63} \times 10^{-9}$	$0.07^{+0.04+0.01+0.02+0.00}_{-0.05-0.02-0.02-0.00}$
$B_s^0 \to \pi^0[\rho(1700)^0 \to]K^+K^-$	$1.01^{+0.59+0.35+0.09+0.04+0.46}_{-0.51-0.30-0.11-0.03-0.46} \times 10^{-9}$	$-0.29^{+0.11+0.06+0.12+0.01}_{-0.18-0.08-0.15-0.01}$
$B_s^0 \to \pi^0[\omega(1650) \to]K^+K^-$	$3.14^{+1.35+1.10+0.53+0.19+1.44}_{-1.29-0.98-0.49-0.16-1.44} \times 10^{-11}$	$0.15^{+0.06+0.02+0.02+0.01}_{-0.05-0.01-0.03-0.01}$
$B_s^0 \to K^-[\rho(1700)^+ \to]K^+\bar{K}^0$	$1.14^{+0.07+0.25+0.12+0.06+0.52}_{-0.07-0.24-0.12-0.05-0.52} \times 10^{-7}$	$0.29^{+0.04+0.04+0.01+0.01}_{-0.04-0.03-0.01-0.01}$
$B_s^0 \to \bar{K}^0[\rho(1700)^0 \to]K^+K^-$	$4.21^{+1.90+0.47+0.55+0.29+1.93}_{-1.70-0.42-0.50-0.26-1.93} \times 10^{-10}$	$0.67^{+0.25+0.03+0.12+0.04}_{-0.26-0.02-0.16-0.03}$
$B_s^0 \to \bar{K}^0[\omega(1650) \to]K^+K^-$	$4.18^{+1.44+0.42+0.50+0.27+1.91}_{-1.17-0.38-0.43-0.23-1.91} \times 10^{-10}$	$-0.64^{+0.26+0.08+0.09+0.03}_{-0.19-0.08-0.12-0.05}$

$$\mathcal{B}(B^+ \to K^+\omega(782) \to K^+K^0\bar{K}^0) = 8.79^{+1.65+2.30+1.17+0.42+0.30}_{-1.44-2.15-1.03-0.33-0.30} \times 10^{-8}. \quad (31)$$

It is easy to check that these branching fractions are very close to the results in Table III for the corresponding decay modes with $\rho(770)^0$ and $\omega(782)$ decaying into K^+K^-. The impacts from the mass difference of $K^\pm$ and K^0 for the direct CP asymmetries for the relevant processes are even smaller, which could be inferred from the comparison of the results in Table III with

$$\mathcal{A}_{CP}(B^+ \to \pi^+\rho(770)^0 \to \pi^+K^0\bar{K}^0) = -0.22^{+0.04+0.01+0.01+0.01}_{-0.04-0.01-0.01-0.01}, \quad (32)$$

$$\mathcal{A}_{CP}(B^+ \to K^+\rho(770)^0 \to K^+K^0\bar{K}^0) = 0.39^{+0.03+0.04+0.04+0.01}_{-0.04-0.04-0.04-0.01}. \quad (33)$$

For the decay modes $B^+ \to \pi^+\rho(1450)^0$ and $B^+ \to K^+\rho(1450)^0$ with $\rho(1450)^0 \to K^0\bar{K}^0$, we have the central values 9.32×10^{-8} and -0.22, 3.30×10^{-8} and 0.42 as their branching fractions and direct CP asymmetries, respectively, which are also very close to the results in Table IV for the corresponding decay processes with $\rho(1450)^0 \to K^+K^-$. In view of the large errors for the predictions in Tables III, IV, and V, we set the concerned decays with the subprocess $\rho(770, 1450, 1700)^0 \to K^0\bar{K}^0$ or $\omega(782, 1420, 1650) \to K^0\bar{K}^0$ have the same results as their corresponding decay modes with the resonances decaying into K^+K^-. It should be stressed that the $K^0\bar{K}^0$ with the P-wave resonant origin in the final state of $B \to K\bar{K}h$ decays can not generate the $K_S^0K_S^0$ system because of the Bose-Einstein statistics.

From the branching fractions in Tables III and IV, one can find that the virtual contributions for $K\bar{K}$ from the BW tails of the intermediate states $\rho(770)$ and $\omega(782)$ in those quasi-two-body decays that have been ignored in experimental and theoretical studies are all larger than the corresponding results from $\rho(1450)$ and $\omega(1420)$. Specifically, the branching fractions in Table III with the resonances $\rho(770)^0$ and $\rho(770)^\pm$ are about 1.2–1.8 times of the corresponding results in Table IV for the decays with $\rho(1450)^0$ and $\rho(1450)^\pm$, while the six predictions for the branching fractions in Table III with $\omega(782)$ in the quasi-two-body decay processes are about 2.2–2.9 times of the corresponding values for the decays with the resonance $\omega(1420)$ in Table IV. The difference of the multiples between the results of the branching fractions with the resonances ρ and ω in Tables III and IV should mainly be attributed to the relatively small value for the $c^K_{\omega(1420)}$ adopted in this work comparing with $c^K_{\rho(1450)}$.

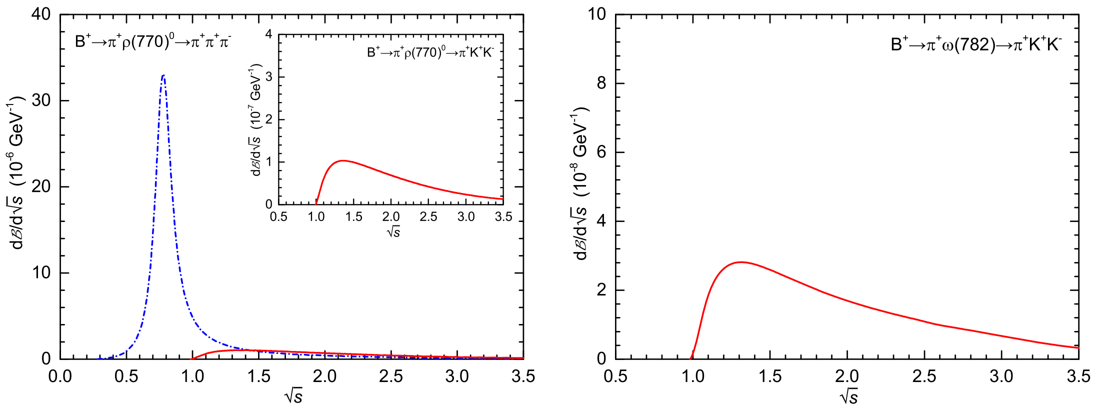

FIG. 2. The differential branching fractions for the decays $B^+ \to \pi^+[\rho(770)^0 \to]K^+K^-$ (left) and $B^+ \to \pi^+[\omega(782) \to]K^+K^-$ (right). The big diagram in the left is for the comparison for the differential branching fractions of $B^+ \to \pi^+[\rho(770)^0 \to]K^+K^-$ and $B^+ \to \pi^+[\rho(770)^0 \to]\pi^+\pi^-$, in which the solid line for $B^+ \to \pi^+[\rho(770)^0 \to]K^+K^-$ is magnified by a factor of 10.

It is remarkable for these virtual contributions in Table III that their differential branching fractions are nearly unaffected by the full widths of $\rho(770)$ and $\omega(782)$, which could be concluded from the Fig. 2. In this figure, the lines in the left diagram for $B^+ \to \pi^+[\rho(770)^0 \to]K^+K^-$ and in the right diagram for $B^+ \to \pi^+[\omega(782) \to]K^+K^-$ have very similar shape although there is a big difference between the values for the widths of $\rho(770)$ and $\omega(782)$ as listed in Table II. The best explanation for Fig. 2 is that the imaginary part of the denominator in the BW formula [Eq. (7)], which holds the energy dependent width for the resonances $\rho(770)$ or $\omega(782)$, becomes unimportant when the invariant mass square s is large enough even if one employs the effective mass defined by the *ad hoc* formula [26,131] to replace the m_R^2 in $|\vec{q}_0|$ in Eq. (8) or calculates the energy dependent width with the partial widths and the branching ratios for the intermediate state as in Refs. [39,41,43,50]. At this point, the BW expression for $\rho(770)$ or $\omega(782)$ is charged by the coefficient c_R^K in the timelike form factors for kaons and the gap between the invariant mass square s for kaon pair and the squared mass of the resonance. Although the threshold of kaon pair is not far from the pole masses of $\rho(770)$ and $\omega(782)$, thanks to the strong suppression from the factor $|\vec{q}|^3$ in Eq. (24), the differential branching fractions for those processes with $\rho(770)$ or $\omega(782)$ decaying into kaon pair will reach their peak at about 1.35 GeV as shown in Fig. 2.

As we have stated in Ref. [21], the bumps in Fig. 2 for $B^+ \to \pi^+[\rho(770)^0 \to]K^+K^-$ and $B^+ \to \pi^+[\omega(782)^0 \to]K^+K^-$ are generated by the tails of the BW formula for the resonances $\rho(770)$ and $\omega(782)$ along with the phase space factors in Eq. (24) and should not be taken as the evidence for a new resonant state at about 1.35 GeV. When we compare the curves for the differential branching fractions for $B^+ \to \pi^+[\rho(770)^0 \to]K^+K^-$ and $B^+ \to \pi^+[\rho(770)^0 \to]\pi^+\pi^-$, we can understand this point well. In order to make a better contrast, the differential branching fraction for $B^+ \to \pi^+[\rho(770)^0 \to]K^+K^-$ is magnified 10 times in the bigger one of Fig. 2 (left). The dash-dot line for $B^+ \to \pi^+[\rho(770)^0 \to]\pi^+\pi^-$ shall climb to its peak at about the pole mass of $\rho(770)^0$ and then descend as exhibited in Fig. 2. While this pattern is inapplicable for the decay process of $B^+ \to \pi^+[\rho(770)^0 \to]K^+K^-$, its curve can only show the existence from the threshold of kaon pair where the $\sqrt{s}$ has already crossed the peak of BW for $\rho(770)^0$. As $\sqrt{s}$ becoming larger, the effect of the full width for $\rho(770)$ fade from the stage, the ratio between the differential branching fractions for the quasi-two-body decays $B^+ \to \pi^+[\rho(770)^0 \to]K^+K^-$ and $B^+ \to \pi^+[\rho(770)^0 \to]\pi^+\pi^-$ will tend to be a constant that is proportional to the value of $|g_{\rho(770)K^+K^-}/g_{\rho(770)\pi^+\pi^-}|^2$ if the phase space for the decay process is large enough. This conclusion can also be demonstrated well from the curve of the ratio

$$R_{\rho(1450)}(\sqrt{s}) = \frac{d\mathcal{B}(B^+ \to \pi^+[\rho(1450)^0 \to]K^+K^-)/d\sqrt{s}}{d\mathcal{B}(B^+ \to \pi^+[\rho(1450)^0 \to]\pi^+\pi^-)/d\sqrt{s}} \tag{34}$$

for the decays $B^+ \to \pi^+[\rho(1450)^0 \to]K^+K^-$ and $B^+ \to \pi^+[\rho(1450)^0 \to]\pi^+\pi^-$ in Fig. 3. The solid line that stands for the $B^+ \to \pi^+[\rho(1450)^0 \to]K^+K^-$ decay and has been magnified 10 times will arise at the threshold of the kaon pair in Fig. 3 and contribute the zero for $R_{\rho(1450)}$ because of the factor $|\vec{q}|^3$ in Eq. (24). The following pattern of the curve for $R_{\rho(1450)}$ is a rapid rise to the value about 0.1 in the region where the main portion of the branching fractions for $B^+ \to \pi^+[\rho(1450)^0 \to]K^+K^-$ and $B^+ \to \pi^+[\rho(1450)^0 \to]\pi^+\pi^-$ concentrated, then $R_{\rho(1450)}$ is going to the value $|g_{\rho(1450)K^+K^-}/g_{\rho(1450)\pi^+\pi^-}|^2$ as s rises.

With the help of the factorization relation $\Gamma(B^+ \to \rho(1450)^0\pi^+ \to h^+h^-\pi^+) \approx \Gamma(B^+ \to \rho(1450)^0\pi^+) \times \mathcal{B}(\rho(1450)^0 \to h^+h^-)$ [132,133], the ratio $R_{\rho(1450)}$ can be

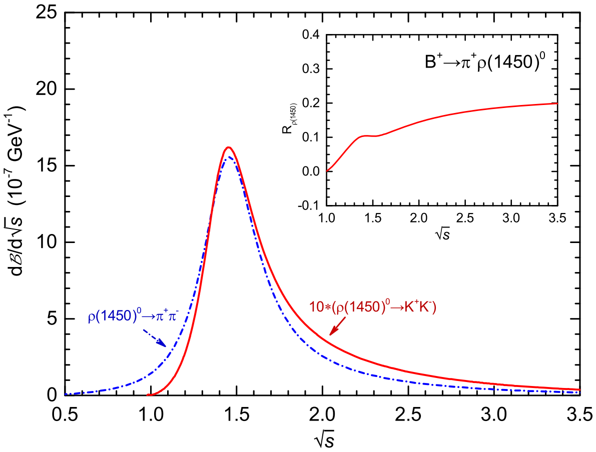

FIG. 3. The differential branching fractions for the decays $B^+ \to \pi^+[\rho(1450)^0 \to]K^+K^-$ (solid line), which is magnified by a factor of 10, and $B^+ \to \pi^+[\rho(1450)^0 \to]\pi^+\pi^-$ (dash-dot line) in the large diagram and curve for the $\sqrt{s}$ dependent ratio $R_{\rho(1450)}$ in the inset.

related to the coupling constants $g_{\rho(1450)^0\pi^+\pi^-}$ and $g_{\rho(1450)^0K^+K^-}$ with the expression

$$g_{\rho(1450)^0h^+h^-} = \sqrt{\frac{6\pi m^2_{\rho(1450)}\Gamma_{\rho(1450)}\mathcal{B}_{\rho(1450)^0\to h^+h^-}}{q^3}}, \quad (35)$$

here $q = \frac{1}{2}\sqrt{m^2_{\rho(1450)} - 4m_h^2}$ and h is pion or kaon. Utilizing the relation $g_{\rho(1450)^0K^+K^-} \approx \frac{1}{2} g_{\rho(1450)^0\pi^+\pi^-}$ [104] one has [21]

$$\begin{aligned} R_{\rho(1450)} &= \frac{\mathcal{B}(\rho(1450)^0 \to K^+K^-)}{\mathcal{B}(\rho(1450)^0 \to \pi^+\pi^-)} \\ &\approx \frac{g^2_{\rho(1450)^0K^+K^-}(m^2_{\rho(1450)} - 4m_K^2)^{3/2}}{g^2_{\rho(1450)^0\pi^+\pi^-}(m^2_{\rho(1450)} - 4m_\pi^2)^{3/2}} \\ &= 0.107. \end{aligned} \quad (36)$$

For the quasi-two-body decay $B^+ \to \pi^+[\rho(1450)^0 \to]\pi^+\pi^-$, we have its branching fraction as $8.73^{+2.73}_{-2.54} \times 10^{-7}$ with the BW formula for $\rho(1450)^0$ and the relation $|c^K_{\rho(1450)}| \approx |c^\pi_{\rho(1450)}|$ in Eq. (17), where the error has the same sources as the branching fractions in Table IV but have been added in quadrature. This result are consistent with the measurements $\mathcal{B} = 1.4^{+0.6}_{-0.9} \times 10^{-6}$ [15,134] from *BABAR* and $\mathcal{B} = (7.9 \pm 3.0) \times 10^{-7}$ [19,20] by LHCb and agree with the prediction $(9.97 \pm 2.25) \times 10^{-7}$ in [21] with the GS model for the resonance $\rho(1450)^0$. Then we have the ratio $R_{\rho(1450)} = 0.108^{+0.000}_{-0.001}$, which is very close to the 0.107 in Eq. (36) and the result in Fig. 3 for the ratio $R_{\rho(1450)}(\sqrt{s})$ in the region around the mass of $\rho(1450)$ where the main portion of the branching fractions for $B^+ \to \pi^+[\rho(1450)^0 \to]K^+K^-$ and $B^+ \to \pi^+[\rho(1450)^0 \to]\pi^+\pi^-$ is concentrated. The small error for $R_{\rho(1450)}$ from the PQCD predictions is caused by the cancellation, which means that the increase or decrease for the relevant numerical results from the uncertainties of those parameters will result in nearly identical change of the weight for these two decays. When the $\rho(1450)^0$ in Eq. (36) is replaced by $\rho(1700)^0$, one will have the ratio $R_{\rho(1700)} \approx 0.143$ [21]. With the results $\mathcal{B}(\rho(1450)^0 \to \pi^+\pi^-) = 15\%$ and $\mathcal{B}(\rho(1700)^0 \to \pi^+\pi^-) = 14\%$ in Ref. [135] from CMD-3 collaboration, one can estimate the branching fractions $\mathcal{B}(\rho(1450)^0 \to K^+K^-) \approx 1.6\%$ and $\mathcal{B}(\rho(1700)^0 \to K^+K^-) \approx 2.0\%$.

It is important to notice that the definition of the coupling constant the Eq. (35) for the resonant states $\rho(770)$ and $\omega(782)$ decaying to the final state $K\bar{K}$ are invalid, or rather, one could not define the partial decay width such as $\Gamma_{\rho(770)\to K^+K^-} = \Gamma_{\rho(770)}\mathcal{B}_{\rho(770)^0\to K^+K^-}$ or $\Gamma_{\omega(782)\to K^+K^-} = \Gamma_{\omega(782)}\mathcal{B}_{\omega(782)\to K^+K^-}$ for the virtual contribution. This conclusion can be extended to other strong decay processes with the virtual contributions that come from the tails of the resonances.

In Ref. [14], the fit fraction of $\rho(1450)^0 \to K^+K^-$ for the three-body decays $B^\pm \to \pi^\pm K^+K^-$ was measured to be $(30.7 \pm 1.2 \pm 0.9)\%$ by LHCb Collaboration, implying $\mathcal{B} = (1.60 \pm 0.14) \times 10^{-6}$ for the quasi-two-body decay $B^+ \to \pi^+\rho(1450)^0 \to \pi^+K^+K^-$ [15]. This branching fraction is close to the measurement $\mathcal{B} = 1.4^{+0.6}_{-0.9} \times 10^{-6}$ in [15,134] and larger than the result $\mathcal{B} = (7.9 \pm 3.0) \times 10^{-7}$ from LHCb [19,20] for the $B^+ \to \pi^+[\rho(1450)^0 \to]\pi^+\pi^-$ process. In view of the mass difference between kaon and pion, the factor $|\vec{q}|^3$ in Eq. (24) will be about 4.76 times larger for the subprocess $\rho(1450)^0 \to \pi^+\pi^-$ when comparing with $\rho(1450)^0 \to K^+K^-$ for the decay $B^+ \to \pi^+\rho(1450)^0$ at $s = m^2_{\rho(1450)}$. It means that the coupling constant for $\rho(1450)^0 \to K^+K^-$ should roughly be $\sqrt{4.76}$ times larger than that for $\rho(1450)^0 \to \pi^+\pi^-$ in order to achieve the comparable branching fractions for the quasi-two-body decays $B^+ \to \pi^+[\rho(1450)^0 \to]K^+K^-$ and $B^+ \to \pi^+[\rho(1450)^0 \to]\pi^+\pi^-$. Clearly, a larger coupling constant for $\rho(1450)^0 \to K^+K^-$ is contrary to the naïve expectation [22] and the discussions in literature [43,104].

V. SUMMARY

In this work, we studied the contributions for kaon pair originating from the resonances $\rho(770)$, $\omega(782)$ and their excited states $\rho(1450, 1700)$ and $\omega(1420, 1650)$ in the three-body decays $B \to K\bar{K}h$ in the PQCD approach. The subprocesses $\rho(770, 1450, 1700) \to K\bar{K}$ and $\omega(782, 1420, 1650) \to K\bar{K}$, which can not be calculated in the

PQCD, were introduced into the distribution amplitudes for $K\bar{K}$ system via the kaon vector timelike form factors. With the coefficients $c^K_{\rho(770)} = 1.247 \pm 0.019$, $c^K_{\omega(782)} = 1.113 \pm 0.019$, $c^K_{\rho(1450)} = -0.156 \pm 0.015$, $c^K_{\omega(1420)} = -0.117 \pm 0.013$, and $c^K_{\omega(1650),\rho(1700)} = -0.083 \pm 0.019$ in the timelike form factors for kaons, we predicted the CP averaged branching fractions and the direct CP asymmetries for the quasi-two-body processes $B \to \rho(770, 1450, 1700)h \to K\bar{K}h$ and $B \to \omega(782, 1420, 1650)h \to K\bar{K}h$.

The branching fractions of the virtual contributions for $K\bar{K}$ in this work from the BW tails of the intermediate states $\rho(770)$ and $\omega(782)$ in the concerned decays that have been ignored in experimental and theoretical studies were found larger than the corresponding results from $\rho(1450, 1700)$ and $\omega(1420, 1650)$. A remarkable phenomenon for the virtual contributions discussed in this work is that the differential branching fractions for $B \to \rho(770)h \to K\bar{K}h$ and $B \to \omega(782)h \to K\bar{K}h$ are nearly unaffected by the quite different values of the full widths for $\rho(770)$ and $\omega(782)$. The definition of the partial decay width such as $\Gamma_{\rho(770)\to K^+K^-} = \Gamma_{\rho(770)}\mathcal{B}_{\rho(770)^0\to K^+K^-}$ for the virtual contribution are invalid. This conclusion can be extended to other strong decay processes with the virtual contributions come from the tails of the resonances. The bumps of the lines for the differential branching fractions for those virtual contributions, which are generated by the phase space factors and the tails of the BW formula of $\rho(770)$ or $\omega(782)$, should not be taken as the evidence for a new resonant state at about 1.35 GeV.

The PQCD predicted results for the branching fractions of the quasi-two-body decays $B^+ \to \pi^+\rho(1450)^0 \to \pi^+K^+K^-$ and $B^+ \to \pi^+\rho(1450)^0 \to \pi^+\pi^+\pi^-$ meet the requirement of the $SU(3)$ symmetry relation $g_{\rho(1450)^0K^+K^-} \approx \frac{1}{2} g_{\rho(1450)^0\pi^+\pi^-}$. The larger coupling constant for $\rho(1450)^0 \to K^+K^-$ deduced from the fit fraction $(30.7 \pm 1.2 \pm 0.9)\%$ for $\rho(1450)^0 \to K^+K^-$ in the $B^\pm \to \pi^\pm K^+K^-$ decays by LHCb collaboration is contrary to the discussions in literature. We estimated the branching fractions to be about 1.6% and 2.0% for the decays $\rho(1450)^0 \to K^+K^-$ and $\rho(1700)^0 \to K^+K^-$, respectively, according to the measurement results from CMD-3 collaboration for $\rho(1450, 1700)^0 \to \pi^+\pi^-$.

ACKNOWLEDGMENTS

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APPENDIX A: DISTRIBUTION AMPLITUDES

The B meson light cone matrix element can be decomposed as [130,136]

$$\Phi_B = \frac{i}{\sqrt{2N_c}}(\not{p}_B + m_B)\gamma_5\phi_B(k_B), \tag{A1}$$

where the distribution amplitude ϕ_B is of the form

$$\phi_B(x_B, b_B) = N_B x_B^2(1-x_B)^2 \exp\left[-\frac{(x_B m_B)^2}{2\omega_B^2} - \frac{1}{2}(\omega_B b_B)^2\right], \tag{A2}$$

with N_B the normalization factor. The shape parameters $\omega_B = 0.40 \pm 0.04$ GeV for $B^\pm$ and B^0 and $\omega_{B_s} = 0.50 \pm 0.05$ for B_s^0, respectively.

The light cone wave functions for pion and kaon are written as [137–140]

$$\Phi_h = \frac{i}{\sqrt{2N_c}}\gamma_5[\not{p}_3\phi^A(x_3) + m_0^h\phi^P(x_3) + m_0^h(\not{n}\not{v} - 1)\phi^T(x_3)]. \tag{A3}$$

The distribution amplitudes of $\phi^A(x_3)$, $\phi^P(x_3)$, and $\phi^T(x_3)$ are

$$\phi^A(x_3) = \frac{f_h}{2\sqrt{2N_c}}6x_3(1-x_3)\left[1 + a_1^h C_1^{3/2}(t) + a_2^h C_2^{3/2}(t) + a_4^h C_4^{3/2}(t)\right], \tag{A4}$$

$$\phi^P(x_3) = \frac{f_h}{2\sqrt{2N_c}}\left[1 + \left(30\eta_3 - \frac{5}{2}\rho_h^2\right)C_2^{1/2}(t) - 3\left[\eta_3\omega_3 + \frac{9}{20}\rho_h^2(1+6a_2^h)\right]C_4^{1/2}(t)\right], \tag{A5}$$

$$\phi^T(x_3) = \frac{f_h}{2\sqrt{2N_c}}(-t)\left[1 + 6\left(5\eta_3 - \frac{1}{2}\eta_3\omega_3 - \frac{7}{20}\rho_h^2 - \frac{3}{5}\rho_h^2 a_2^h\right)(1 - 10x_3 + 10x_3^2)\right], \tag{A6}$$

with $t = 2x_3 - 1$, $C_{2,4}^{1/2}(t)$, and $C_{1,2,4}^{3/2}(t)$ are Gegenbauer polynomials. The chiral scale parameters $m_0^h = \frac{m_h^2}{m_q + m_{q'}}$ for pion and kaon are $m_0^\pi = (1.4 \pm 0.1)$ GeV and $m_0^K = (1.9 \pm 0.1)$ GeV as they are in [130]. The decay constants $f_\pi = 130.2(1.2)$ MeV and $f_K = 155.7(3)$ MeV can be found in Ref. [15]. The Gegenbauer moments $a_1^\pi = 0$, $a_1^K = 0.06$, $a_2^h = 0.25$, $a_4^h = -0.015$ and the parameters $\rho_h = m_h/m_0^h$, $\eta_3 = 0.015$, $\omega_3 = -3$ are adopted in the numerical calculation.

APPENDIX B: DECAY AMPLITUDES

With the subprocesses $\rho^+ \to K^+\bar{K}^0$, $\rho^- \to K^-K^0$, $\rho^0 \to K^+K^-$, $\rho^0 \to K^0\bar{K}^0$, $\omega \to K^+K^-$, and $\omega \to K^0\bar{K}^0$, and ρ is $\rho(770), \rho(1450)$, or $\rho(1700)$ and ω is

$\omega(782)$, $\omega(1420)$, or $\omega(1650)$, the Lorentz invariant decay amplitudes for the quasi-two-body decays $B \to \rho h \to K\bar{K}h$ and $B \to \omega h \to K\bar{K}h$ are given as follows:

$$\mathcal{A}(B^+ \to \rho^+\pi^0) = \frac{G_F}{2} V^*_{ub}V_{ud}\{a_1[F^{LL}_{Th} + F^{LL}_{Ah} - F^{LL}_{a\rho}] + a_2 F^{LL}_{T\rho} + C_1[M^{LL}_{Th} + M^{LL}_{Ah} - M^{LL}_{A\rho}] + C_2 M^{LL}_{T\rho}\} - \frac{G_F}{2} V^*_{tb}V_{td}\left\{\left[-a_4 + \frac{5C_9}{3} + C_{10} - \frac{3a_7}{2}\right]F^{LL}_{T\rho} - \left[a_6 - \frac{a_8}{2}\right]F^{SP}_{T\rho} + \left[\frac{C_9 + 3C_{10}}{2} - C_3\right]M^{LL}_{T\rho} - \left[C_5 - \frac{C_7}{2}\right]M^{LR}_{T\rho} + \frac{3C_8}{2}M^{SP}_{T\rho} + [a_4 + a_{10}][F^{LL}_{Th} + F^{LL}_{Ah} - F^{LL}_{A\rho}] + [a_6 + a_8][F^{SP}_{Ah} - F^{SP}_{A\rho}] + [C_3 + C_9][M^{LL}_{Th} + M^{LL}_{Ah} - M^{LL}_{A\rho}] + [C_5 + C_7][M^{LR}_{Th} + M^{LR}_{Ah} - M^{LR}_{A\rho}]\right\}, \tag{B1}$$

$$\mathcal{A}(B^+ \to \rho^0\pi^+) = \frac{G_F}{2} V^*_{ub}V_{ud}\{a_1[F^{LL}_{T\rho} + F^{LL}_{A\rho} - F^{LL}_{Ah}] + a_2 F^{LL}_{Th} + C_1[M^{LL}_{T\rho} + M^{LL}_{A\rho} - M^{LL}_{Ah}] + C_2 M^{LL}_{Th}\} - \frac{G_F}{2} V^*_{tb}V_{td}\left\{[a_4 + a_{10}][F^{LL}_{T\rho} + F^{LL}_{A\rho} - F^{LL}_{Ah}] + [a_6 + a_8][F^{SP}_{T\rho} + F^{SP}_{A\rho} - F^{SP}_{Ah}] + [C_3 + C_9][M^{LL}_{T\rho} + M^{LL}_{A\rho} - M^{LL}_{Ah}] + [C_5 + C_7][M^{LR}_{T\rho} + M^{LR}_{A\rho} - M^{LR}_{Th}] + \left[\frac{5}{3}C_9 + C_{10} + \frac{3a_7}{2} - a_4\right]F^{LL}_{Th} + \left[\frac{C_9 + 3C_{10}}{2} - C_3\right]M^{LL}_{Th} - \left[C_5 - \frac{C_7}{2}\right]M^{LR}_{Th} + \frac{3C_8}{2}M^{SP}_{Th}\right\}, \tag{B2}$$

$$\mathcal{A}(B^+ \to \omega\pi^+) = \frac{G_F}{2} V^*_{ub}V_{ud}\{a_1[F^{LL}_{T\omega} + F^{LL}_{A\omega} + F^{LL}_{Ah}] + a_2 F^{LL}_{Th} + C_1[M^{LL}_{T\omega} + M^{LL}_{A\omega} + M^{LL}_{Ah}] + C_2 M^{LL}_{Th}\} - \frac{G_F}{2} V^*_{tb}V_{td}\left\{[a_4 + a_{10}][F^{LL}_{T\omega} + F^{LL}_{A\omega} + F^{LL}_{Ah}] + [a_6 + a_8][F^{SP}_{T\omega} + F^{SP}_{A\omega} + F^{SP}_{Ah}] + [C_3 + C_9][M^{LL}_{T\omega} + M^{LL}_{A\omega} + M^{LL}_{Ah}] + [C_5 + C_7][M^{LR}_{T\omega} + M^{LR}_{A\omega} + M^{LR}_{Ah}] + \left[(7C_3 + 5C_4 + C_9 - C_{10})/3 + 2a_5 + \frac{a_7}{2}\right]F^{LL}_{Th} + \left[C_3 + 2C_4 - \frac{C_9 - C_{10}}{2}\right]M^{LL}_{Th} + \left[C_5 - \frac{C_7}{2}\right]M^{LR}_{Th} + \left[2C_6 + \frac{C_8}{2}\right]M^{SP}_{Th}\right\}, \tag{B3}$$

$$\mathcal{A}(B^+ \to \rho^+K^0) = \frac{G_F}{\sqrt{2}} V^*_{ub}V_{us}\{a_1 F^{LL}_{A\rho} + C_1 M^{LL}_{A\rho}\} - \frac{G_F}{\sqrt{2}} V^*_{tb}V_{ts}\left\{\left[a_4 - \frac{a_{10}}{2}\right]F^{LL}_{T\rho} + \left[a_6 - \frac{a_8}{2}\right]F^{SP}_{T\rho} + \left[C_3 - \frac{C_9}{2}\right]M^{LL}_{T\rho} + \left[C_5 - \frac{C_7}{2}\right]M^{LR}_{T\rho} + [a_4 + a_{10}]F^{LL}_{A\rho} + [C_3 + C_9]M^{LL}_{A\rho} + [a_6 + a_8]F^{SP}_{A\rho} + [C_5 + C_7]M^{LR}_{A\rho}\right\}, \tag{B4}$$

$$\mathcal{A}(B^+ \to \rho^0K^+) = \frac{G_F}{2} V^*_{ub}V_{us}\{a_1[F^{LL}_{T\rho} + F^{LL}_{A\rho}] + a_2 F^{LL}_{Th} + C_1[M^{LL}_{T\rho} + M^{LL}_{A\rho}] + C_2 M^{LL}_{Th}\} - \frac{G_F}{2} V^*_{tb}V_{ts}\left\{[a_4 + a_{10}][F^{LL}_{T\rho} + F^{LL}_{A\rho}] + [a_6 + a_8][F^{SP}_{T\rho} + F^{SP}_{A\rho}] + [C_3 + C_9][M^{LL}_{T\rho} + M^{LL}_{A\rho}] + [C_5 + C_7][M^{LR}_{T\rho} + M^{LR}_{A\rho}] + \frac{3}{2}[a_7 + a_9]F^{LL}_{Th} + \frac{3C_{10}}{2}M^{LL}_{Th} + \frac{3C_8}{2}M^{SP}_{Th}\right\}, \tag{B5}$$

$$\mathcal{A}(B^+ \to \omega K^+) = \frac{G_F}{2} V_{ub}^* V_{us} \{a_1 [F_{T\omega}^{LL} + F_{A\omega}^{LL}] + a_2 F_{Th}^{LL} + C_1 [M_{T\omega}^{LL} + M_{A\omega}^{LL}] + C_2 M_{Th}^{LL}\} - \frac{G_F}{2} V_{tb}^* V_{ts} \Big\{ [a_4 + a_{10}][F_{T\omega}^{LL} + F_{A\omega}^{LL}] + [a_6 + a_8][F_{T\omega}^{SP} + F_{A\omega}^{SP}] + [C_3 + C_9][M_{T\omega}^{LL} + M_{A\omega}^{LL}] + [C_5 + C_7][M_{T\omega}^{LR} + M_{A\omega}^{LR}] + [2a_3 + 2a_5 + a_7/2 + a_9/2] F_{Th}^{LL} + \left[2C_4 + \frac{C_{10}}{2}\right] M_{Th}^{LL} + \left[2C_6 + \frac{C_8}{2}\right] M_{Th}^{SP} \Big\}, \tag{B6}$$

$$\mathcal{A}(B^0 \to \rho^+ \pi^-) = \frac{G_F}{\sqrt{2}} V_{ub}^* V_{ud} \{a_2 F_{A\rho}^{LL} + C_2 M_{A\rho}^{LL} + a_1 F_{Th}^{LL} + C_1 M_{Th}^{LL}\} - \frac{G_F}{\sqrt{2}} V_{tb}^* V_{td} \Big\{ [a_3 + a_9 - a_5 - a_7] F_{A\rho}^{LL} + [C_4 + C_{10}] M_{A\rho}^{LL} + [C_6 + C_8] M_{A\rho}^{SP} + [a_4 + a_{10}] F_{Th}^{LL} + [C_3 + C_9] M_{Th}^{LL} + [C_5 + C_7] M_{Th}^{LR} + \left[\frac{4}{3}\left[C_3 + C_4 - \frac{C_9}{2} - \frac{C_{10}}{2}\right] - a_5 + \frac{a_7}{2}\right] F_{Ah}^{LL} + \left[a_6 - \frac{a_8}{2}\right] F_{Ah}^{SP} + \left[C_3 + C_4 - \frac{C_9}{2} - \frac{C_{10}}{2}\right] M_{Ah}^{LL} + \left[C_5 - \frac{C_7}{2}\right] M_{Ah}^{LR} + \left[C_6 - \frac{C_8}{2}\right] M_{Ah}^{SP} \Big\}, \tag{B7}$$

$$\mathcal{A}(B^0 \to \rho^- \pi^+) = \frac{G_F}{\sqrt{2}} V_{ub}^* V_{ud} \{a_1 F_{T\rho}^{LL} + a_2 F_{Ah}^{LL} + C_1 M_{T\rho}^{LL} + C_2 M_{Ah}^{LL}\} - \frac{G_F}{\sqrt{2}} V_{tb}^* V_{td} \Big\{ [a_4 + a_{10}] F_{T\rho}^{LL} + [a_6 + a_8] F_{T\rho}^{SP} + [C_3 + C_9] M_{T\rho}^{LL} + [C_5 + C_7] M_{T\rho}^{LR} + \left[\frac{4}{3}\left[C_3 + C_4 - \frac{C_9 + C_{10}}{2}\right] - a_5 + \frac{a_7}{2}\right] F_{A\rho}^{LL} + \left[a_6 - \frac{a_8}{2}\right] F_{A\rho}^{SP} + \left[C_3 + C_4 - \frac{C_9 + C_{10}}{2}\right] M_{A\rho}^{LL} + \left[C_5 - \frac{C_7}{2}\right] M_{A\rho}^{LR} + \left[C_6 - \frac{C_8}{2}\right] M_{A\rho}^{SP} + [a_3 + a_9 - a_5 - a_7] F_{Ah}^{LL} + [C_4 + C_{10}] M_{Ah}^{LL} + [C_6 + C_8] M_{aP}^{SP} \Big\}, \tag{B8}$$

$$\mathcal{A}(B^0 \to \rho^0 \pi^0) = \frac{G_F}{2\sqrt{2}} V_{ub}^* V_{ud} \{a_2 [F_{A\rho}^{LL} + F_{Ah}^{LL} - F_{T\rho}^{LL} - F_{Th}^{LL}] + C_2 [M_{A\rho}^{LL} + M_{Ah}^{LL} - M_{T\rho}^{LL} - M_{Th}^{LL}]\} - \frac{G_F}{2\sqrt{2}} V_{tb}^* V_{td} \Big\{ \left[a_4 - \frac{5C_9}{3} - C_{10} + \frac{3a_7}{2}\right] F_{T\rho}^{LL} + \left[a_6 - \frac{a_8}{2}\right] [F_{T\rho}^{SP} + F_{A\rho}^{SP} + F_{Ah}^{SP}] + \left[C_3 - \frac{C_9 + 3C_{10}}{2}\right] [M_{T\rho}^{LL} + M_{Th}^{LL}] + \left[C_5 - \frac{C_7}{2}\right] [M_{T\rho}^{LR} + M_{A\rho}^{LR} + M_{Th}^{LR} + M_{Ah}^{LR}] - \frac{3C_8}{2} [M_{T\rho}^{SP} + M_{Th}^{SP}] + \left[(7C_3 + 5C_4 + C_9 - C_{10})/3 - 2a_5 - \frac{a_7}{2}\right] [F_{A\rho}^{LL} + F_{Ah}^{LL}] + \left[C_3 + 2C_4 - \frac{C_9 - C_{10}}{2}\right] [M_{A\rho}^{LL} + M_{Ah}^{LL}] + \left[2C_6 + \frac{C_8}{2}\right] [M_{A\rho}^{SP} + M_{Ah}^{SP}] + \left[a_4 - \frac{5C_9}{3} - C_{10} - \frac{3a_7}{2}\right] F_{Th}^{LL} \Big\}, \tag{B9}$$

$$\mathcal{A}(B^0 \to \omega \pi^0) = \frac{G_F}{2\sqrt{2}} V_{ub}^* V_{ud} \{a_2 [F_{A\omega}^{LL} + F_{Ah}^{LL} + F_{T\omega}^{LL} - F_{Th}^{LL}] + C_2 [M_{A\omega}^{LL} + M_{Ah}^{LL} + M_{T\omega}^{LL} - M_{Th}^{LL}]\} - \frac{G_F}{2\sqrt{2}} V_{tb}^* V_{td} \Big\{ \left[-a_4 + \frac{5C_9}{3} + C_{10} - \frac{3a_7}{2}\right] [F_{T\omega}^{LL} + F_{A\omega}^{LL} + F_{Ah}^{LL}] - \left[a_6 - \frac{a_8}{2}\right] [F_{T\omega}^{SP} + F_{A\omega}^{SP} + F_{Ah}^{SP}] - \left[(7C_3 + 5C_4 + C_9 - C_{10})/3 + 2a_5 + \frac{a_7}{2}\right] F_{Th}^{LL} - \left[C_3 - \frac{C_9 + 3C_{10}}{2}\right] [M_{T\omega}^{LL} + M_{A\omega}^{LL} + M_{Ah}^{LL}] - \left[C_5 - \frac{C_7}{2}\right] [M_{T\omega}^{LR} + M_{A\omega}^{LR} + M_{Th}^{LR} + M_{Ah}^{LR}] + \frac{3C_8}{2} [M_{T\omega}^{SP} + M_{A\omega}^{SP} + M_{Ah}^{SP}] - \left[C_3 + 2C_4 - \frac{C_9 - C_{10}}{2}\right] M_{Th}^{LL} - \left[2C_6 + \frac{C_8}{2}\right] M_{Th}^{SP} \Big\}, \tag{B10}$$

$$\mathcal{A}(B^0\to\rho^-K^+)=\frac{G_F}{\sqrt{2}}V_{ub}^*V_{us}\{a_1F_{T\rho}^{LL}+C_1M_{T\rho}^{LL}\}-\frac{G_F}{\sqrt{2}}V_{tb}^*V_{ts}\bigg\{[a_4+a_{10}]F_{T\rho}^{LL}+[a_6+a_8]F_{T\rho}^{SP}+[C_3+C_9]M_{T\rho}^{LL}+[C_5+C_7]M_{T\rho}^{LR}+\left[a_4-\frac{a_{10}}{2}\right]F_{A\rho}^{LL}+\left[a_6-\frac{a_8}{2}\right]F_{A\rho}^{SP}+\left[C_3-\frac{C_9}{2}\right]M_{A\rho}^{LL}+\left[C_5-\frac{C_7}{2}\right]M_{A\rho}^{LR}\bigg\}, \tag{B11}$$

$$\mathcal{A}(B^0\to\rho^0K^0)=\frac{G_F}{2}V_{ub}^*V_{us}\{a_2F_{Th}^{LL}+C_2M_{Th}^{LL}]\}-\frac{G_F}{2}V_{tb}^*V_{ts}\bigg\{-\left[a_4-\frac{a_{10}}{2}\right][F_{T\rho}^{LL}+F_{A\rho}^{LL}]-\left[a_6-\frac{a_8}{2}\right][F_{T\rho}^{SP}+F_{A\rho}^{SP}]-\left[C_3-\frac{C_9}{2}\right][M_{T\rho}^{LL}+M_{A\rho}^{LL}]-\left[C_5-\frac{C_7}{2}\right][M_{T\rho}^{LR}+M_{A\rho}^{LR}]+\frac{3}{2}[a_7+a_9]F_{Th}^{LL}+\frac{3C_{10}}{2}M_{Th}^{LL}+\frac{3C_8}{2}M_{Th}^{SP}\bigg\}, \tag{B12}$$

$$\mathcal{A}(B^0\to\omega K^0)=\frac{G_F}{2}V_{ub}^*V_{us}\{a_2F_{Th}^{LL}+C_2M_{Th}^{LL}\}-\frac{G_F}{2}V_{tb}^*V_{ts}\bigg\{\left[a_4-\frac{a_{10}}{2}\right][F_{T\omega}^{LL}+F_{A\omega}^{LL}]+\left[a_6-\frac{a_8}{2}\right][F_{T\omega}^{SP}+F_{A\omega}^{SP}]+\left[C_3-\frac{C_9}{2}\right][M_{T\omega}^{LL}+M_{A\omega}^{LL}]+\left[C_5-\frac{C_7}{2}\right][M_{T\omega}^{LR}+M_{A\omega}^{LR}]+\left[2a_3+2a_5+\frac{a_7+a_9}{2}\right]F_{Th}^{LL}+\left[2C_4+\frac{C_{10}}{2}\right]M_{Th}^{LL}+\left[2C_6+\frac{C_8}{2}\right]M_{Th}^{SP}\bigg\}, \tag{B13}$$

$$\mathcal{A}(B_s^0\to\rho^+\pi^-)=\frac{G_F}{\sqrt{2}}V_{ub}^*V_{us}\{a_2F_{A\rho}^{LL}+C_2M_{A\rho}^{LL}\}-\frac{G_F}{\sqrt{2}}V_{tb}^*V_{ts}\bigg\{[a_3+a_9-a_5-a_7]F_{A\rho}^{LL}+[C_4+C_{10}]M_{A\rho}^{LL}+[C_6+C_8]M_{A\rho}^{SP}+\left[a_3-\frac{a_9}{2}-a_5+\frac{a_7}{2}\right]F_{Ah}^{LL}+\left[C_4-\frac{C_{10}}{2}\right]M_{Ah}^{LL}+\left[C_6-\frac{C_8}{2}\right]M_{Ah}^{SP}\bigg\}, \tag{B14}$$

$$\mathcal{A}(B_s^0\to\rho^-\pi^+)=\frac{G_F}{\sqrt{2}}V_{ub}^*V_{us}\{a_2F_{Ah}^{LL}+C_2M_{Ah}^{LL}\}-\frac{G_F}{\sqrt{2}}V_{tb}^*V_{ts}\bigg\{\left[a_3-\frac{a_9}{2}-a_5+\frac{a_7}{2}\right]F_{A\rho}^{LL}+\left[C_4-\frac{C_{10}}{2}\right]M_{A\rho}^{LL}+\left[C_6-\frac{C_8}{2}\right]M_{A\rho}^{SP}+[a_3+a_9-a_5-a_7]F_{Ah}^{LL}+[C_4+C_{10}]M_{Ah}^{LL}+[C_6+C_8]M_{Ah}^{SP}\bigg\}, \tag{B15}$$

$$\mathcal{A}(B_s^0\to\rho^0\pi^0)=\frac{G_F}{2\sqrt{2}}V_{ub}^*V_{us}\{a_2[F_{A\rho}^{LL}+F_{Ah}^{LL}]+C_2[M_{A\rho}^{LL}+M_{Ah}^{LL}]\}-\frac{G_F}{2\sqrt{2}}V_{tb}^*V_{ts}\bigg\{\left[2a_3+\frac{a_9}{2}-2a_5-\frac{a_7}{2}\right][F_{A\rho}^{LL}+F_{Ah}^{LL}]+\left[2C_4+\frac{C_{10}}{2}\right][M_{A\rho}^{LL}+M_{Ah}^{LL}]+\left[2C_6+\frac{C_8}{2}\right][M_{A\rho}^{SP}+M_{Ah}^{SP}]\bigg\}, \tag{B16}$$

$$\mathcal{A}(B_s^0\to\omega\pi^0)=\frac{G_F}{2\sqrt{2}}V_{ub}^*V_{us}\{a_2[F_{A\omega}^{LL}+F_{Ah}^{LL}]+C_2[M_{A\omega}^{LL}+M_{Ah}^{LL}]\}-\frac{G_F}{2\sqrt{2}}V_{tb}^*V_{ts}\bigg\{\frac{3}{2}[a_9-a_7][F_{A\omega}^{LL}+F_{Ah}^{LL}]+\frac{3C_{10}}{2}[M_{A\omega}^{LL}+M_{Ah}^{LL}]+\frac{3C_8}{2}[M_{A\omega}^{SP}+M_{Ah}^{SP}]\bigg\}, \tag{B17}$$

$$\mathcal{A}(B_s^0\to\rho^+K^-)=\frac{G_F}{\sqrt{2}}V_{ub}^*V_{ud}\{a_1F_{Th}^{LL}+C_1M_{Th}^{LL}\}-\frac{G_F}{\sqrt{2}}V_{tb}^*V_{td}\bigg\{[a_4+a_{10}]F_{Th}^{LL}+[C_3+C_9]M_{Th}^{LL}+[C_5+C_7]M_{Th}^{LR}+\left[a_4-\frac{a_{10}}{2}\right]F_{Ah}^{LL}+\left[a_6-\frac{a_8}{2}\right]F_{Ah}^{SP}+\left[C_3-\frac{C_9}{2}\right]M_{Ah}^{LL}+\left[C_5-\frac{C_7}{2}\right]M_{Ah}^{LR}\bigg\}, \tag{B18}$$

$$\mathcal{A}(B_s^0 \to \rho^0 \bar{K}^0) = \frac{G_F}{2} V_{ub}^* V_{ud} \{a_2 F_{Th}^{LL} + C_2 M_{Th}^{LL}\} - \frac{G_F}{2} V_{tb}^* V_{td} \bigg\{ \left[\frac{5C_9}{3} + C_{10} + \frac{3a_7}{2} - a_4\right] F_{Th}^{LL} + \left[\frac{C_9}{2} + \frac{3C_{10}}{2} - C_3\right] M_{Th}^{LL} - \left[C_5 - \frac{C_7}{2}\right][M_{Th}^{LR} + M_{Ah}^{LR}] + \frac{3C_8}{2} M_{Th}^{SP} - \left[a_4 - \frac{a_{10}}{2}\right] F_{Ah}^{LL} - \left[a_6 - \frac{a_8}{2}\right] F_{Ah}^{SP} - \left[C_3 - \frac{C_9}{2}\right] M_{Ah}^{LL} \bigg\}, \quad \text{(B19)}$$

$$\mathcal{A}(B_s^0 \to \omega \bar{K}^0) = \frac{G_F}{2} V_{ub}^* V_{ud} \{a_2 F_{Th}^{LL} + C_2 M_{Th}^{LL}\} - \frac{G_F}{2} V_{tb}^* V_{td} \bigg\{ \left[(7C_3 + 5C_4 + C_9 - C_{10})/3 + 2a_5 + \frac{a_7}{2}\right] F_{Th}^{LL} + \left[C_3 + 2C_4 - \frac{C_9 - C_{10}}{2}\right] M_{Th}^{LL} + \left[C_5 - \frac{C_7}{2}\right][M_{Th}^{LR} + M_{Ah}^{LR}] + \left[2C_6 + \frac{C_8}{2}\right] M_{Th}^{SP} + \left[a_4 - \frac{a_{10}}{2}\right] F_{Ah}^{LL} + \left[a_6 - \frac{a_8}{2}\right] F_{Ah}^{SP} + \left[C_3 - \frac{C_9}{2}\right] M_{Ah}^{LL} \bigg\}, \quad \text{(B20)}$$

where G_F is the Fermi coupling constant and Vs are the CKM matrix elements. The combinations a_i with $i = 1$–10 are defined as

$$a_1 = C_2 + C_1/3, \quad a_2 = C_1 + C_2/3, \quad a_3 = C_3 + C_4/3, \quad a_4 = C_4 + C_3/3,$$
$$a_5 = C_5 + C_6/3, \quad a_6 = C_6 + C_5/3, \quad a_7 = C_7 + C_8/3, \quad a_8 = C_8 + C_7/3,$$
$$a_9 = C_9 + C_{10}/3, \quad a_{10} = C_{10} + C_9/3, \quad \text{(B21)}$$

for the Wilson coefficients.

The general amplitudes for the quasi-two-body decays $B \to \rho h \to K\bar{K}h$ and $B \to \omega h \to K\bar{K}h$ in the decay amplitudes Eqs. (B1)–(B20) are given according to Fig. 1, the typical Feynman diagrams for the PQCD approach. The symbols LL, LR, and SP are employed to denote the amplitudes from the $(V-A)(V-A)$, $(V-A)(V+A)$, and $(S-P)(S+P)$ operators, respectively. The emission diagrams are depicted in Figs. 1(a) and 1(c), while the annihilation diagrams are shown by Figs. 1(b) and 1(d). For the factorizable diagrams in Fig. 1, we name their expressions with F, while the others are nonfactorizable diagrams: we name their expressions with M. The specific expressions for these general amplitudes are the same as in the appendix of [71] but with the replacements $\phi \to \rho$ and $\phi \to \omega$ for their subscripts for the subprocesses $\rho \to K\bar{K}$ and $\omega \to K\bar{K}$, respectively, in this work. It should be understood that the Wilson coefficients C and the amplitudes F and M for the factorizable and nonfactorizable contributions, respectively, appear in convolutions in momentum fractions and impact parameters b.

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