In-flight muon spin resonance and muonium interferometry

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Abstract. The muon and muonium play a unique role in materials science as a tiny magnetometer and an emulator of hydrogen in matter. However, there are few examples of their application as matter waves. This is because the surface muon and its simple slowing-down in a degrader cannot keep sufficient coherence. Low-energy muons from laser ionization of muonium can be used to obtain slow muonium with small temporal and spatial spread. Like an ordinary atomic interferometer, a muonium interferometer has a variety of potential applications. For example, muonium spectroscopy using interference effects, studies of quantum interference effects such as a measurement of the Berry phase, and precise measurements of fundamental constants will be possible using muonium interferometry. In this contribution, we discuss the in-flight spectroscopy of muonium and the potential of muonium interferometry.

1. Introduction

Muonium, a hydrogen-like atom consisting of a positive muon and an electron, is a suitable two-body system to test the Standard Model of particle physics. Since muonium was found from the spin precession of muons stopped in a noble gas [1], its detailed properties have been investigated by muon spin rotation/relaxation/resonance (μSR) measurements and spectroscopic experiments using laser and microwave.

Over the long history of muonium spectroscopy, various techniques have been introduced to improve measurement precision. However, the short muon lifetime makes it challenging to apply cooling techniques widely used in atomic, molecular, and optical (AMO) physics. With recent improvements in beam intensity and advances in laser and micro techniques, the possibility of AMO physics experiments using cooled muons would be worth discussing.

2. Muonium spectroscopy

The energy levels of muonium are illustrated in Fig. 1. Muonium consists only of leptons and is free from the finite volume effect of nuclei. In addition, the uncertainty associated with recoil corrections is small due to the large muon-to-electron mass ratio m_μ/m_e. Because of these features, the energy levels of muonium can be calculated with high precision by adding radiative corrections of heavy particles to quantum electrodynamics (QED) of bound systems. In other words, the results obtained by muonium spectroscopy can be compared with the predictions of theory to test the Standard Model rigorously.

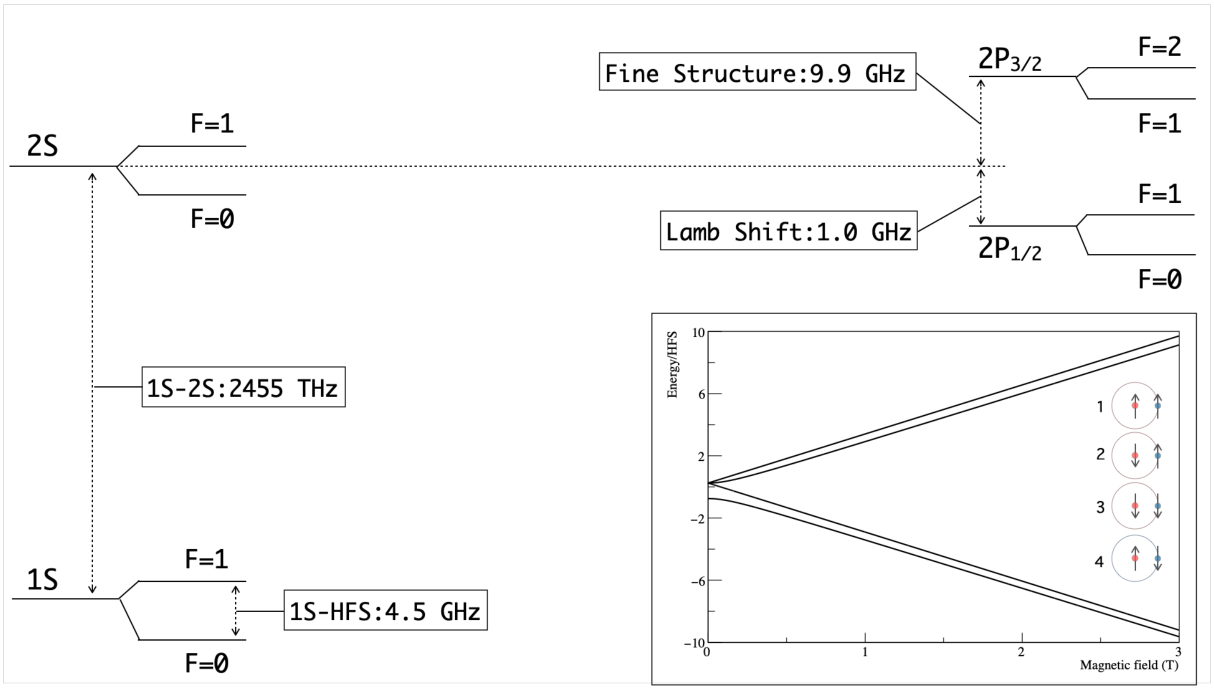

Figure 1. Muonium energy levels involved in spectroscopy experiments (not to scale). A small inset is a Breit-Rabi diagram of the Zeeman splitting in the ground state. Four Zeeman sub-levels are shown with muon and electron spin configurations.

2.1. The 1S-2S interval

The gross structure of muonium between $1S$ and $2S$ states is 2455 THz, corresponding to a wavelength of 122-nm. When a slow muonium atom is obtained in a vacuum, its precise spectroscopy can be performed by the two-photon resonance of counter-propagating 244-nm laser lights. Laser spectroscopy of the $1S$-$2S$ interval can determine the muon-to-electron mass ratio m_μ/m_e with high precision.

Doppler-free two-photon laser spectroscopy was demonstrated at KEK Booster Meson Facility (BOOM) [2] and later improved to higher precision at the ISIS muon facility in Rutherford Appleton Laboratory (RAL) [3]. The most precise experiment to date determined the energy interval with a precision of 4 ppt [4]. The Mu-Mass collaboration is planning $1S$-$2S$ high precision spectroscopy at Paul Scherrer Institute (PSI) [5]. A group at Okayama University is at Japan Proton Accelerator Research Complex (J-PARC) [6].

2.2. The ground state hyperfine splitting

The hyperfine splitting (HFS) of muonium in the ground state is approximately 4.5 GHz. The transition between the spin singlet and triplet states can be induced by a microwave. Muonium atoms are obtained in a noble gas in a microwave cavity, and the HFS in a vacuum can be determined by extrapolating the results obtained at several gas densities to zero.

Early experiments were conducted at Nevis Synchrocyclotron at Columbia University [7, 8], then at NASA's Space Radiation Effects Laboratory (SREL) [9, 10], and eventually at the linear accelerator at Los Alamos Meson Physics Facility (LAMPF) [11, 12]. The most precise result to date is 12 ppb [13]. At J-PARC, the MuSEUM experiment is preparing to improve this precision by order of magnitude with a high-intensity pulsed muon beam[14].

2.3. The fine structure and the Lamb shift

The frequencies of the fine structure and the Lamb shift are 9.9 GHz and 1.0 GHz, respectively. They can be measured by applying microwaves to muonium atoms in the $2S$ state to induce a transition to the $2P$ states. Muonium with the $2S$ metastability is quenched by collisions with surrounding atoms, so in-flight spectroscopy in a vacuum is essential.

The fine structure was measured at LAMPF with a precision of 0.5% [15], and the Lamb shift in TRIUMF [16], and LAMPF [17]. Very recently, the Mu-Mass collaboration performed a new measurement at PSI and reported new results [18].

3. Matter wave interferometry

Like a coherent light, a matter wave can also constitute an interferometer [19]. Interferometers using electron and neutron were developed relatively early, but an atom interferometer was not realized until 1991 [20]. This was due to the short de Broglie wavelength of atoms, which made it difficult to develop mirrors and beam splitters.

Advances in atom cooling techniques and atomic state manipulation have led to the development of a wide variety of atom interferometers. Since atoms are much slower than light, they are extremely sensitive to slight phase changes in interferometry, making them excellent for detecting tiny effects.

Atom interferometers can be classified into those that use mechanical gratings to split atomic wave packets and those that use light or magnetic fields to manipulate atoms' internal degrees of freedom. Atom interferometry has been used for various applications, including precise measurement of fundamental constants such as atomic mass and gravitational constant and verification of quantum phenomena such as observation of the Aharonov-Bohm effect and detection of the Berry phase.

4. Low-energy muons

To date, no matter wave interferometer using muons has been realized. The main reasons are the short lifetime of muons and the large emittance of the muon beam. However, as the technology for obtaining low-energy, small-emittance muon beams has become more sophisticated, a few experiments have been proposed.

The low-energy muon (LEM) at PSI and the ultra-slow muon (USM) at J-PARC are known as low-energy muon beams that can be applied to matter wave interferometry. The former provides epithermal muon using a solid rare-gas moderator [21], and the latter delivers thermal muon generated by laser ionization of muonium [22].

In addition to LEM and USM, a new technique to compress the muon phase space by applying electric and magnetic fields in liquid helium with a temperature gradient is underway at PSI. Phase space compression has been demonstrated, and a method of beam extraction in a vacuum is being investigated [23].

At PSI, muonium interferometry to test the weak equivalent principle with a grating-based Mach-Zehnder interferometer has been proposed [24]. For this experiment, a new cryogenic muonium source using superfluid helium is being developed [25]. At J-PARC, a new project to develop a transmission muon microscope by accelerating USM is underway [26]. A compact cyclotron with a flat-top cavity will be developed to accelerate USMs from 30 keV to 5 MeV [27].

5. Muonium interferometry

Here we discuss some possible experiments using a muonium atom interferometer with USM. In the USM facility, a pulsed surface muon beam irradiates a muonium emitter, such as a current-heated tungsten foil or a disk of silica aerogel. Low-energy muons are obtained via laser ionization of thermal muonium in a vacuum by simultaneous injection of two coherent lights at wavelengths of 122 nm and 355 nm. For room temperature emitters, the initial energy of USM is 25 meV. USMs are extracted as a beam while focusing by an electrostatic lens. The extraction energy is tunable, up to 30 keV.

In the LEM facility, a beam of muonium atoms in the $2S$ metastable state was obtained by passing low-energy muons through a 10-nm thick carbon foil [28]. At an incident muon energy of 7.5 keV, the muonium fraction of 0.432(24), of which the $2S$ population is 0.11(4). This technique has been applied to the muonium Lamb shift measurement at PSI [18]. In future experiments, the use of a few-layer graphene foil is being considered based on a result of the proton neutralization [31]. The improvement in $2S$ muonium flux was estimated to be a factor of 15 to 25 due to suppression of angular spread. USM can also be converted to muonium using a similar technique.

Another method to obtain $2S$ muonium is Doppler-free two-photon resonance using a 244-nm wavelength laser light. In a photon spin-echo experiment using hydrogen atoms [32], the $2S$ state was prepared by overlapping a standing wave of 244-nm light on a hydrogen atom in the $1S$ state.

5.1. Longitudinal Stern-Gerlach experiment

A Stern-Gerlach interferometer uses an inhomogeneous magnetic field to induce phase changes depending on the magnetic quantum number. In the early 1990s, a series of experiments were performed using a hydrogen atomic beam, and longitudinal Stern-Gerlach interferometry was demonstrated [29, 30]. A collimated hydrogen atomic beam with $2S$ metastability passed through a sextupole magnetic field to select atoms according to their magnetic quantum numbers. The states were mixed in a zero-field region, and a transverse magnetic field induced a phase shift with a longitudinal gradient. The $2S$ atoms passing through a second zero-field region and sextupole field were then counted by a field-ionization detector.

It is believed that an experiment similar to those performed with hydrogen atoms can be conducted with muonium atoms. Here we examine the feasibility of such an experiment. Fig. 3 illustrates a possible experimental setup. The setup consists of USM generation in the first half and a longitudinal Stern-Gerlach interferometer in the second.

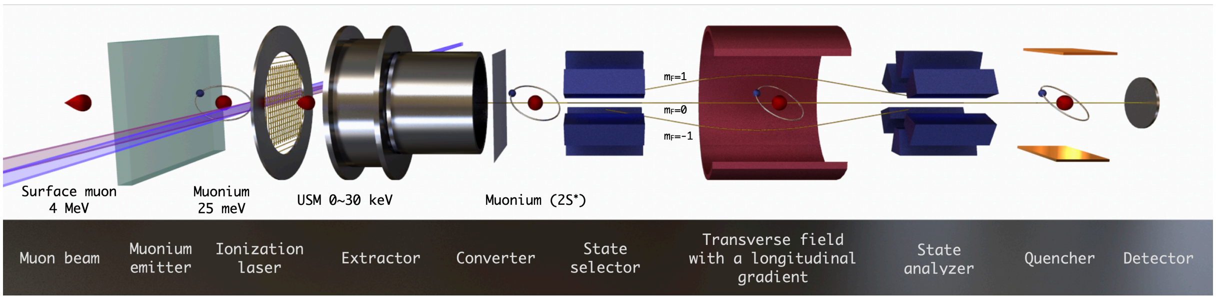

Figure 2. A conceptual diagram of the muonic longitudinal Stern-Gerlach experiment. USMs are generated from surface muons and neutralized by a charge conversion target while accelerating in an electrostatic lens. Muonium atoms in the $2S$ metastable are obtained and pass through an interferometer.

In the hydrogen experiments, the most-probable velocity of the hydrogen atoms was 10 km/s, and the time-of-flight through the interferometer was about $5\mu s$. A similar experiment with muonium atoms at thermal velocity will inevitably lose statistics due to decay-in-flight. As a first step in the feasibility study, an interference fringe considering decay loss and velocity distribution was calculated numerically.

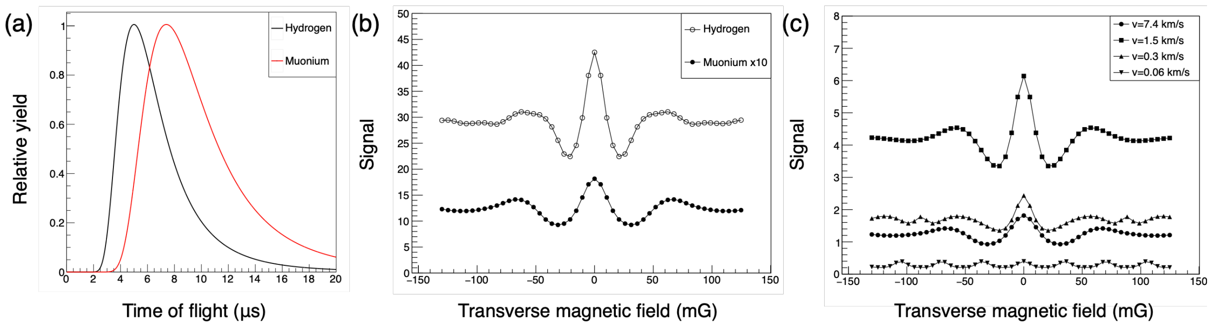

Figure 3. Simulation results on the muonium-based longitudinal Stern-Gerlach interferometry: (a) assumed TOF spectra for a case of room temperature, (b) comparison of interference fringes of hydrogen and muonium at room temperature, (c) muonium interference fringes at several velocities. Note that the muonium result shown in (b) has the signal multiplied by ten.

The $2S$ hydrogen beam flux was about 1 kHz; the USM flux is about 1/3 of this. When the conversion efficiency to obtain $2S$ muonium of 1/20 and the decay loss factor of 1/20 are considered, the statistics per unit time in the muonium interferometry is 1/1200 of the hydrogen. For the case of hydrogen, a measurement of 300 seconds for each magnetic field data point resulted in sufficient statistics. To achieve similar level of statistical significance, 100 hours of measurement per data point will be required. Assuming that eight data points are necessary to analyze an interference fringe, a month of beamtime is required. If the USM flux is improved several times in the near future, and the $2S$ muonium conversion with a graphene foil works, the time required for the measurement will be reduced by a factor of tens, and the experiment will be feasible. This should be a sufficient estimation to motivate further studies.

5.2. In-flight muon spin resonance with separated oscillatory fields

A method of spectroscopy applying two separated pulses to an atom and observing the fringes in the transition probability depending on the time interval is called separated oscillatory fields (SOF) or Ramsey resonance [33]. This method achieves a determination precision of the resonance frequency beyond the natural linewidth.

The SOF method has been applied to the spectroscopy of the muonium HFS [34]. In this experiment, a muonium atom formed by a muon stopped in a gas target was irradiated with two pulses having a time difference.

In addition to improving the statistical precision, the SOF method will be helpful for in-flight spectroscopy in a vacuum. Usually, the muonium HFS is measured in gas and extrapolated to zero density to correct the hyperfine pressure shift (HPS) [35]. This analysis introduces a systematic uncertainty that cannot be ignored in precise measurement. Spectroscopy in a vacuum is free from the HPS. Furthermore, one can search for muonium to anti-muonium conversion, which violates lepton number conservation. This process is difficult to be observed in a matter because a negative muon is easily transferred from anti-muonium to the surrounding atom.

In the long history of muonium HFS spectroscopy, only one experiment in a vacuum has been reported [36]. The result was statistically limited. In the experiment, the microwave cavity was enlarged with a higher-order mode, and a microwave was pulsed to suppress background events arising from muonium atoms reaching the cavity wall. The precision of the measurement was statistically limited because the signal induced by the pulsed microwave was counted in the experiment using a continuous muon beam. Spectroscopy in a vacuum combined with the pulsed beam at J-PARC and the SOF method is expected to enable significantly more precise measurements. A detailed simulation is under preparation to discuss the feasibility of the experiment.

5.3. Ramsey-Bordé interferometry

Two pairs of counter-propagating laser beams can be used to induce Ramsey resonance while canceling the phase shift associated with the angular spread of the atomic beam. This setup can be regarded as an atom interferometer so-called Ramsey-Bordé interferometer [37].

Figure 4 illustrates a conceptual diagram of a Ramsey-Bordé interferometer using $2S$ muonium. The scheme is similar to that of a photon spin echo experiment using hydrogen atoms [32]. A counter-propagating laser beam with a wavelength of 244 nm excites muonium atoms from $1S$ to $2S$. A pulsed 371-nm laser irradiates muonium to induce a transition from $2S$ to $15P$, A trajectory of the excited muonium shifts due to the momentum transfer from the laser light to the atom.

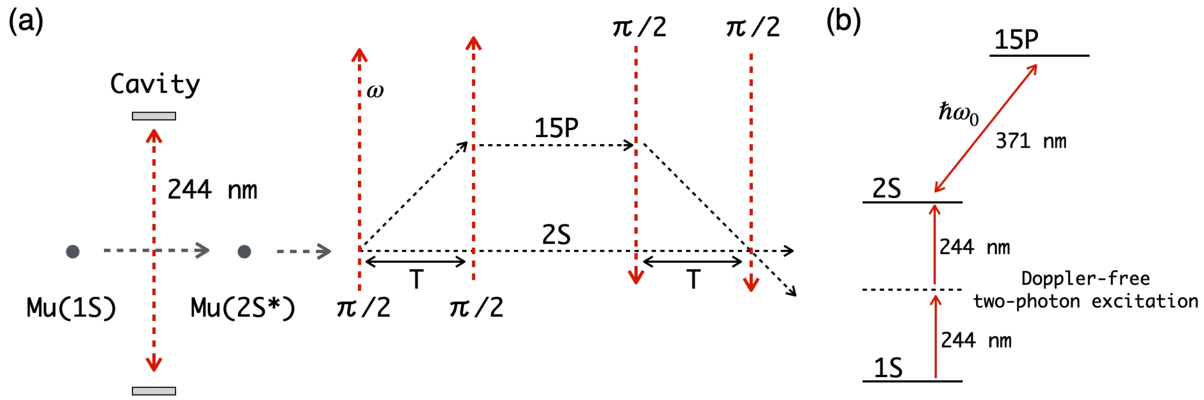

Figure 4. A Ramsey-Bordé interferometer for muonium mass measurement: (a) interferometer configuration, (b) state transitions involved in the interferometer.

The phase difference $\delta\phi$ between trajectories in the interferometer is described by the following equation,

$$\delta\phi = 2(\omega - \omega_0)T - \frac{\hbar k^2}{2m}T \tag{1}$$

where ω is laser light frequency, ω_0 is the transition frequency, T is the time interval between light pulses, $\hbar k$ is the photon momentum, and m is the mass of muonium. Since ω, ω_0, and $\hbar k$ can be determined precisely, the mass of muonium can be obtained from the equation above. This method has been applied to rubidium atoms to obtain the fine structure constant with high precision [38].

The phase difference can be enhanced by irradiating additional π-pulses to improve the measurement precision. The phase difference is multiplied by a factor of $2N+1$, where N is the number of π-pulses. According to the reference [32], a precision of 1 ppb can be expected from a proposed experiment using hydrogen atoms with ten additional π-pulses. In naive estimation, one can expect precision of 21 ppb even without additional π-pulses.

6. Summary

Atom Interferometers based on low-energy muons could lead to a breakthrough in muonium spectroscopy. Three muonium atom interferometers were examined for possible configurations regarding previous studies using hydrogen atoms and relevant muonium experiments. A muonium-based longitudinal Stern-Gerlach interferometer seems feasible with USM after some improvements. Spectroscopy of the muonium HFS in a vacuum using the Ramsey resonance technique is a promising experiment using intense pulsed muon beams at J-PARC MUSE. A measurement of muon mass using Ramsey-Bordé interferometry, although challenging, is worth considering as a new method. We will continue to evaluate the feasibility and requirements for the beam and instruments.

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